Title: Generalizing Bell non locality without global causal constraints

Speakers: Ravi Kunjwal

Series: Quantum Foundations, Quantum Information

Date: September 17, 2024 - 1:45 PM

URL: https://pirsa.org/24090123

Generalizing Bell nonlocality without global causal assumptions

Ravi Kunjwal
Aix-Marseille University

Causalworlds
Perimeter Institute

September 17, 2024

(Based on arXiv:2307.02565 with Ognyan Oreshkov)

amidex Aix Marseille Université   FRANCE 2030   amU Aix Marseille Université

Quantum $\Longrightarrow$ Nonclassical

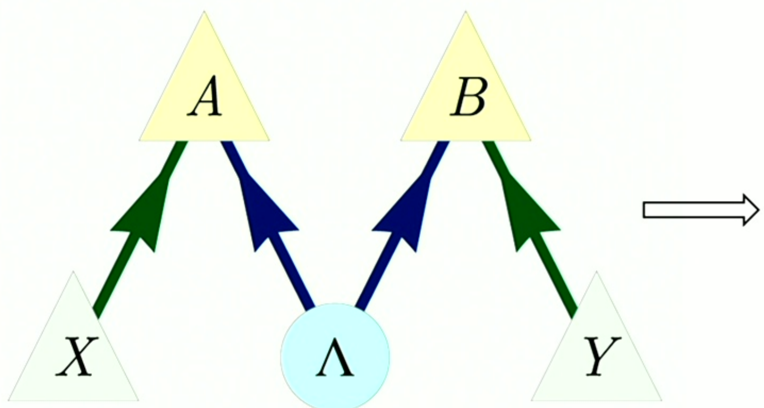
(probabilistic notions of nonclassicality)

Three observations:

- quantum theory is *intrinsically* probabilistic
- these probabilities are *not* classical, *e.g.*, Bell nonlocality
- probabilities can be nonclassical in more ways than those allowed by quantum theory

What do nonclassical probabilities look like?

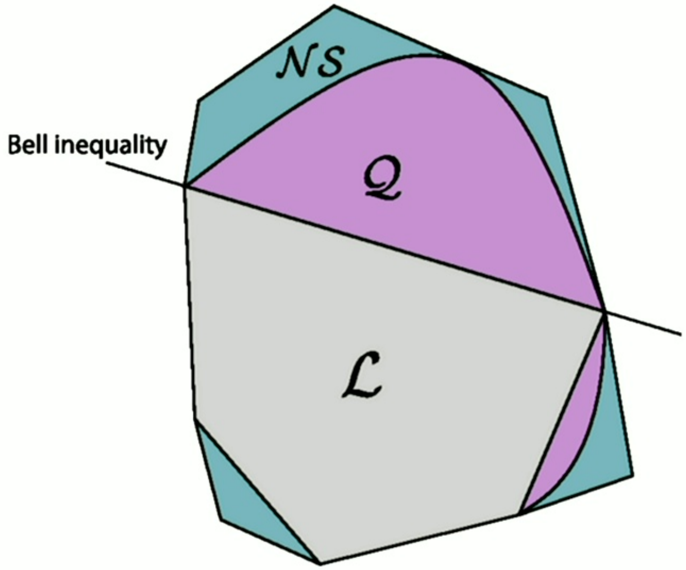
Bell scenario



Bell inequalities

(violation certifies nonclassicality)

What do nonclassical probabilities look like?



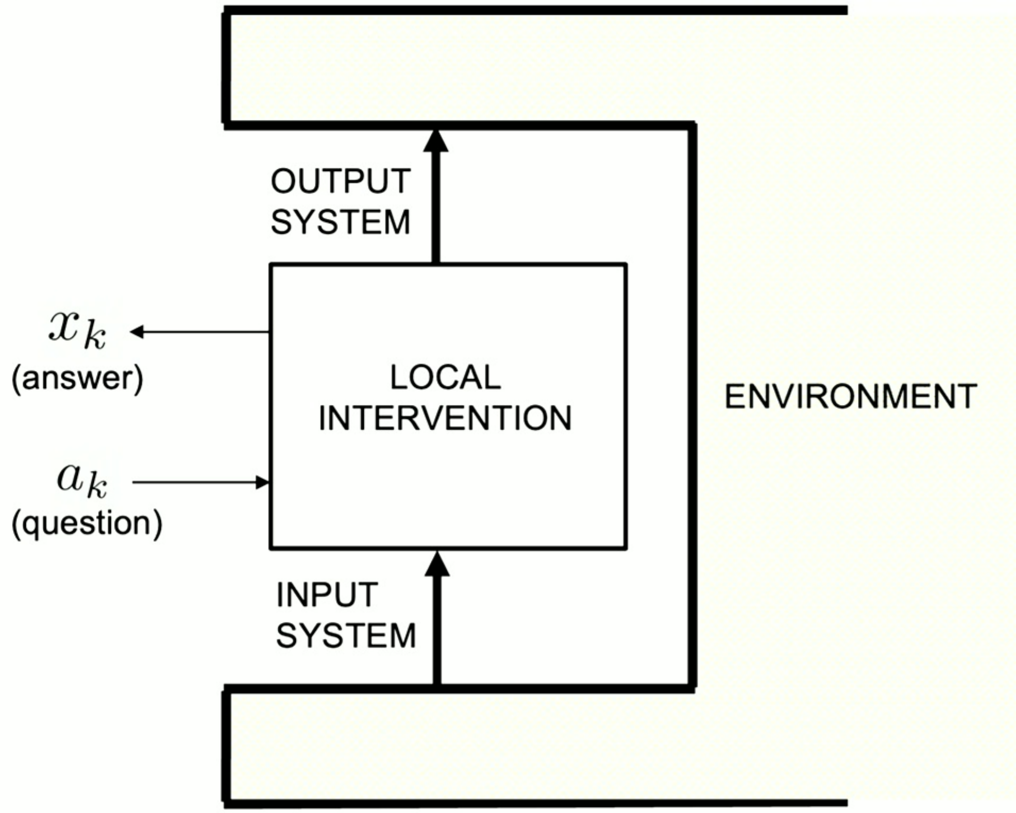
Bell nonlocality, Brunner *et al.*, Rev. Mod. Phys. 86, 419 (2014)

Outline

The set-up ⟶ **Causal ineq. viol.** ⟶ **Not always nonclassical**

Antinomicity

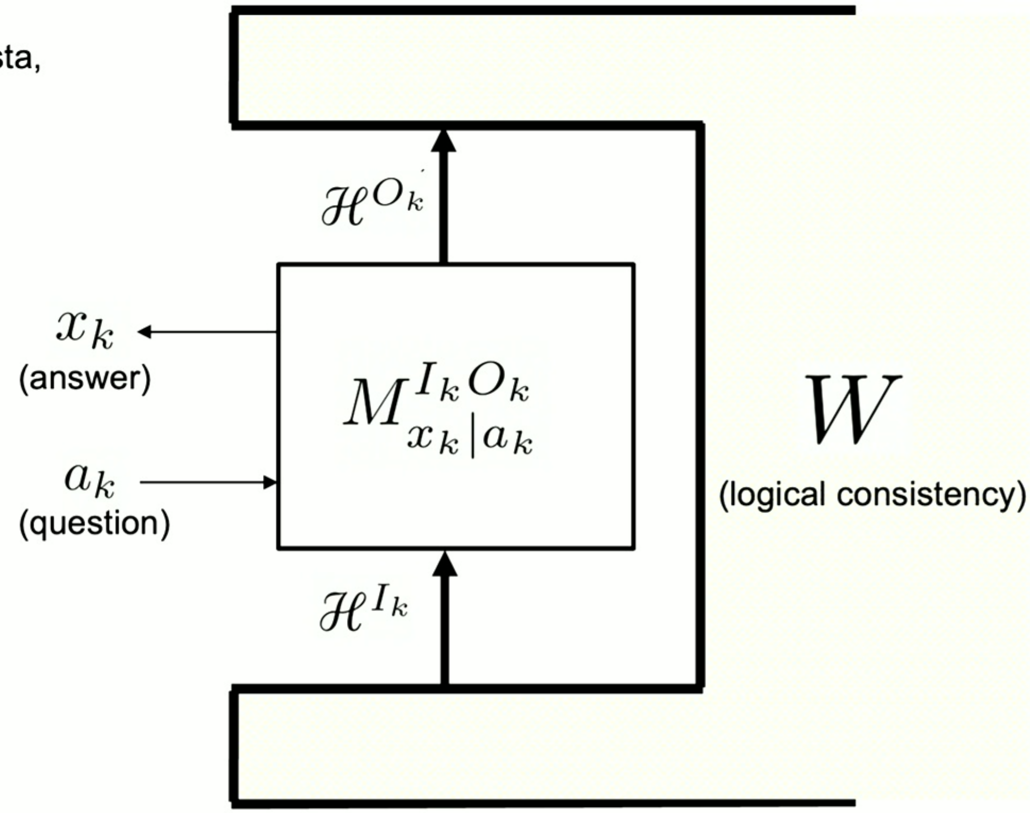
General operational paradigm



$$p(x_1, x_2, \ldots, x_N | a_1, a_2, \ldots, a_N)$$

Process-matrix framework

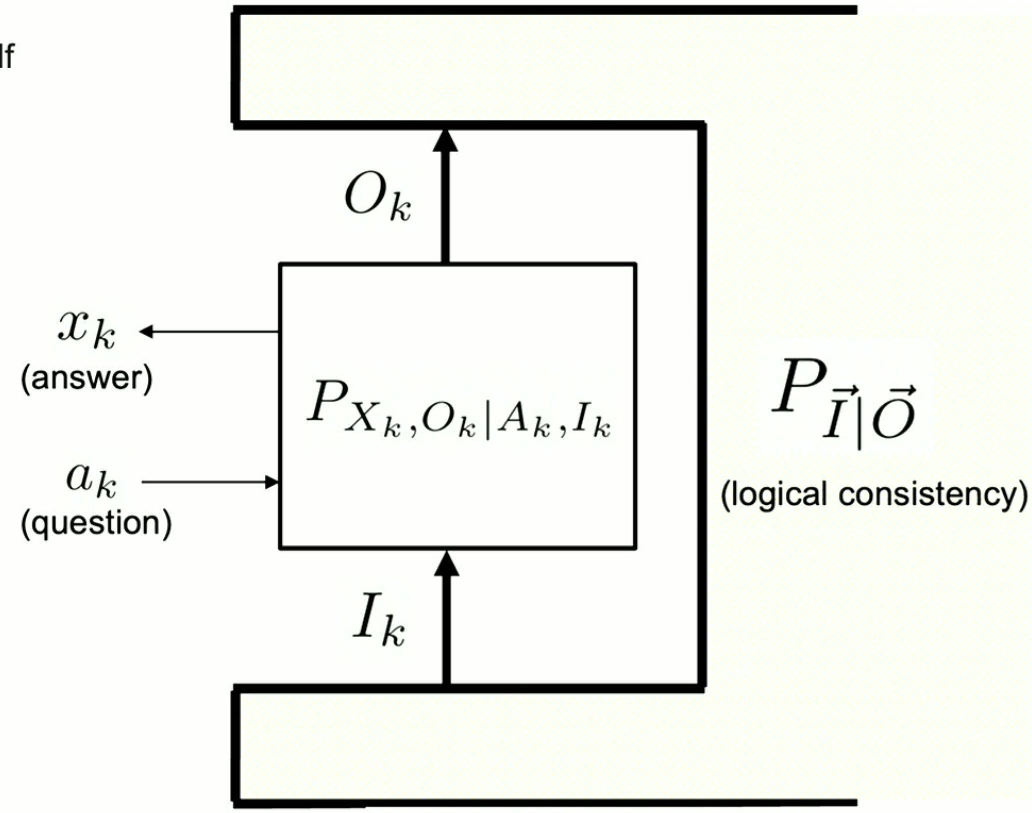
O. Oreshkov, F. Costa,
Č. Brukner (OCB)
arXiv:1105.4464



$$p(\vec{x}|\vec{a}) = \mathrm{Tr}(W M^{I_1 O_1}_{x_1|a_1} \otimes M^{I_2 O_2}_{x_2|a_2} \otimes \cdots \otimes M^{I_N O_N}_{x_N|a_N})$$

Classical process framework

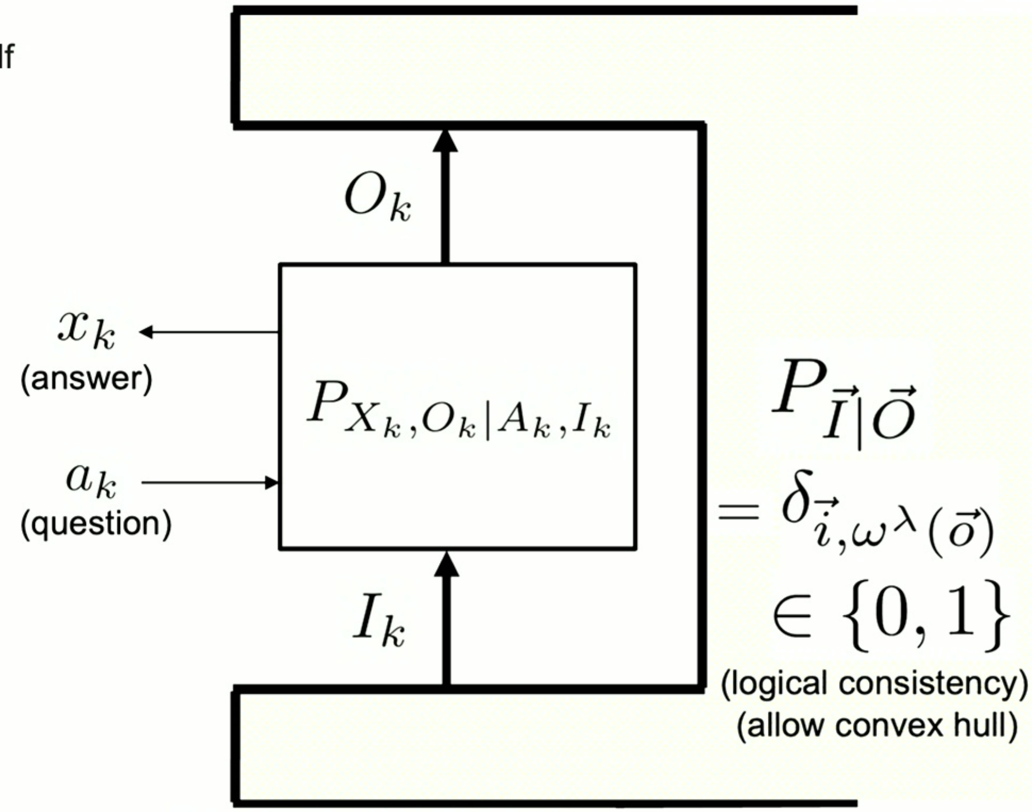
Ä. Baumeler, S. Wolf
arXiv:1511.05444



$$p(\vec{x}|\vec{a}) = \sum_{\vec{i},\vec{o}} \left(\prod_{k=1}^{N} p(x_k, o_k | a_k, i_k) \right) p(\vec{i}|\vec{o})$$

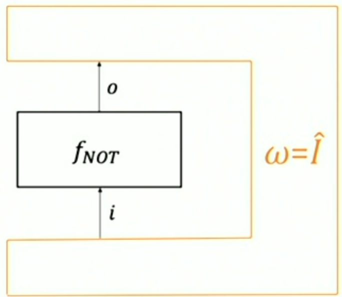
Process function framework

Ä. Baumeler, S. Wolf
arXiv:1511.05444

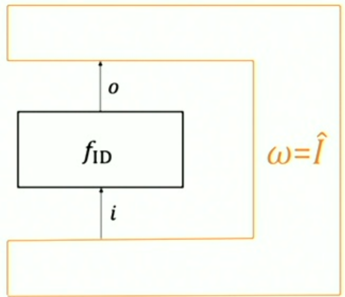


$$p(\vec{x}|\vec{a}) = \sum_{\vec{i},\vec{o}} \left(\prod_{k=1}^{N} p(x_k, o_k | a_k, i_k) \right) p(\vec{i}|\vec{o}) \quad \text{where} \quad p(\vec{i}|\vec{o}) = \sum_{\lambda} p(\lambda) \delta_{\vec{i},\omega^\lambda(\vec{o})}$$

Examples of logical inconsistency



(a) If the function ω is the identity and the operation of the party is to flip the inputs, then there is no fixed point (grandfather antinomy).



(b) If both f and ω are the identity channel, then every possible input is a fixed point. If the input variable i is binary then there are two fixed points (information antinomy).

[Baumeler and Tselentis, arXiv:2004.12921]

Causal inequalities

Operational constraints from a definite causal order

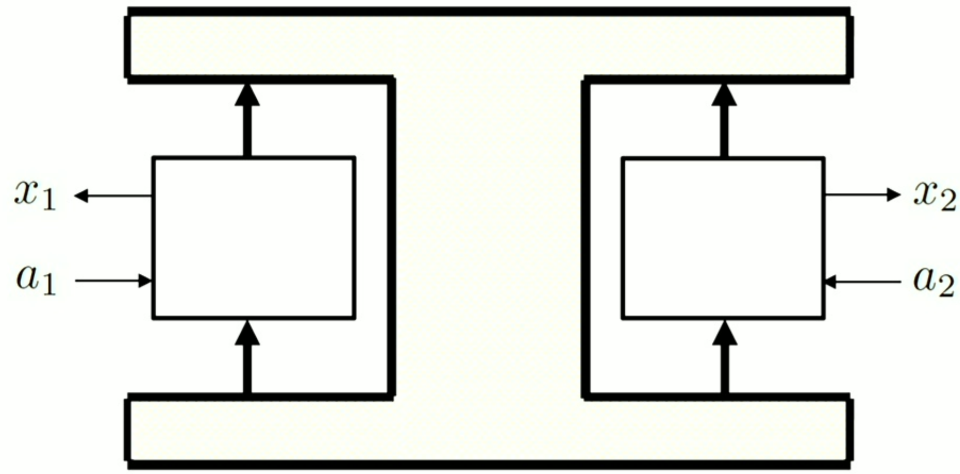
Example: Guess Your Neighbour's Input (GYNI) inequality

$$x_1 = a_2 \text{ and } x_2 = a_1$$

Optimal causal strategy:

Alice sends a_1 to Bob who reports $x_2 = a_1$

Alice makes a uniformly random guess x_1 for a_2



$$\frac{1}{4}\sum_{a_1,a_2,x_1,x_2} \delta_{x_1,a_2}\delta_{x_2,a_1} P(x_1,x_2|a_1,a_2) \leq \frac{1}{2}$$

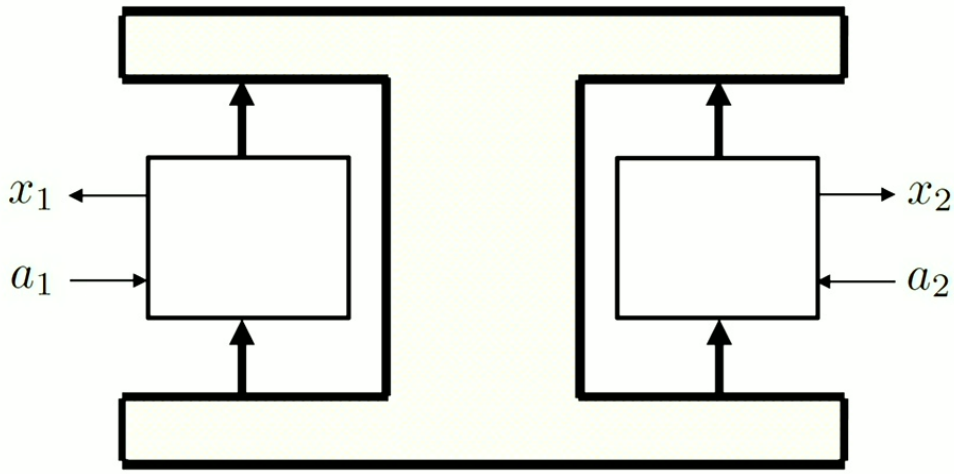
Example: Guess Your Neighbour's Input (GYNI) inequality

$$x_1 = a_2 \text{ and } x_2 = a_1$$

Optimal causal strategy:

Alice sends a_1 to Bob who reports $x_2=a_1$

Alice makes a uniformly random guess x_1 for a_2



$$\frac{1}{4} \sum_{a_1,a_2,x_1,x_2} \delta_{x_1,a_2} \delta_{x_2,a_1} P(x_1, x_2 | a_1, a_2) \leq \frac{1}{2}$$

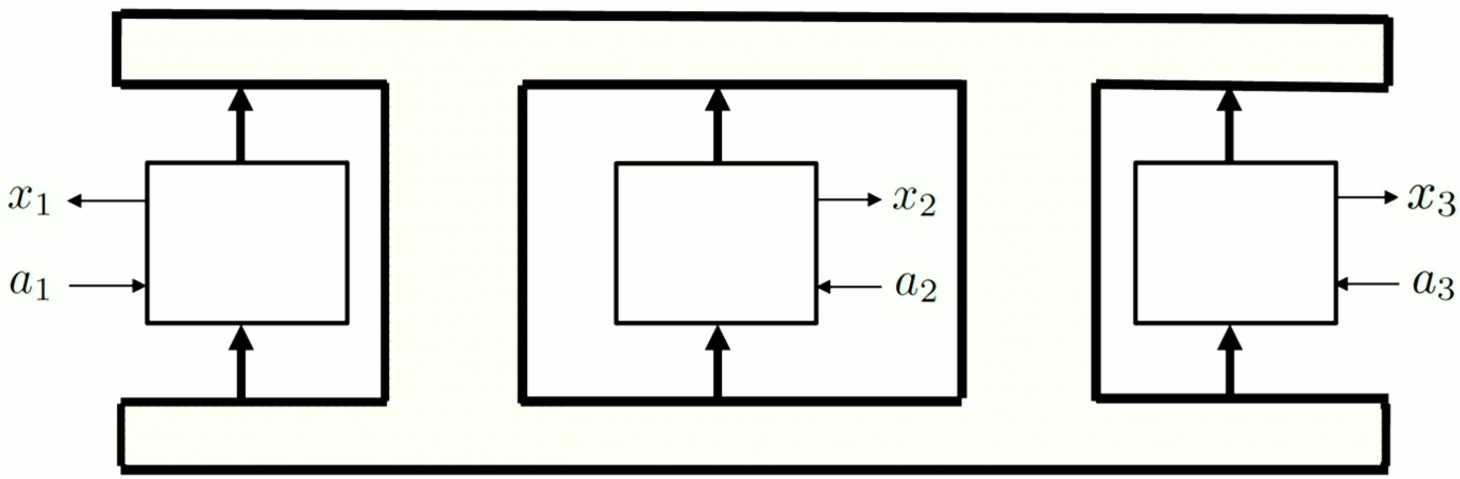
Violated by process-matrix correlations! arXiv:1508.01704

Does the diagonal limit of the process-matrix framework imply causality?

Bipartite: Yes! (OCB)
In general: No! (BFW, AF/BW)

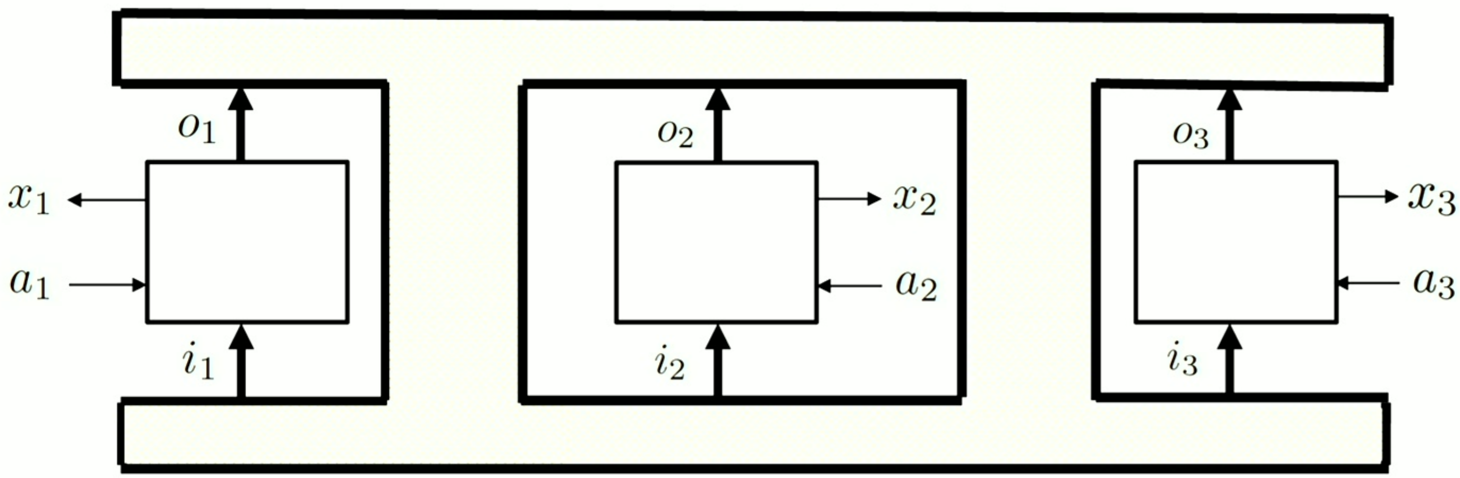
OCB: arXiv:1105.4464
BFW: arXiv:1403.7333
AF/BW: arXiv:1507.01714

Example: a tripartite causal inequality



$$\frac{1}{2}\sum_{\vec{x},\vec{a}} p(x_1,x_2,x_3|a_1,a_2,a_3)\Big(\delta_{x_1,a_3}\delta_{x_2,a_1}\delta_{x_3,a_2}\delta_{\mathrm{maj}(a_1,a_2,a_3),0} + \delta_{x_1,\bar{a}_2}\delta_{x_2,\bar{a}_3}\delta_{x_3,\bar{a}_1}\delta_{\mathrm{maj}(a_1,a_2,a_3),1}\Big) \leq \frac{3}{4}.$$

AF/BW or “Lugano” process function



$$i_1 = \bar{o}_2 o_3, i_2 = \bar{o}_3 o_1, i_3 = \bar{o}_1 o_2$$

Causal inequality violations do not require nonclassical resources

A notion of classicality: Deterministic Consistency (or "nomicity")

A multipartite correlation satisfies deterministic consistency if and only if it can be achieved in the process function framework, *i.e.*,

$$p(\vec{x}|\vec{a}) = \sum_{\vec{i},\vec{o}} \prod_{k=1}^{N} p(x_k, o_k|a_k, i_k)p(\vec{i}|\vec{o})$$

where $p(\vec{i}|\vec{o}) = \sum_{\lambda} p(\lambda)\delta_{\vec{i},\omega^{\lambda}(\vec{o})}$

For a non-signalling environment, this describes a Bell-local model for the correlation!

Baumeler-Wolf: arXiv:1507.01714

Antinomicity is the failure of deterministic consistency for a correlation

intuitively, it's the property that a classical environment must admit "hidden logical contradictions" to reproduce the correlation

Correlational scenario (N, M, D)

Settings: $\vec{a} := (a_1, a_2, \ldots, a_N)$

Outcomes: $\vec{x} := (x_1, x_2, \ldots, x_N)$

$$p(\vec{x}|\vec{a}) \geq 0 \quad \forall \vec{x}, \vec{a},$$

$$\sum_{\vec{x}} p(\vec{x}|\vec{a}) = 1 \quad \forall \vec{a}.$$

Four sets of correlations

- $\mathscr{DC}$: deterministically consistent correlations
  (achievable via process functions)

- $\mathscr{PC}$: probabilistically consistent correlations
  (achievable via diagonal process matrices)

- $\mathscr{QP}$: quantum process correlations
  (achievable via process matrices)

- $q\mathscr{C}$: quasi-consistent correlations
  (the full set of correlations, achievable via arbitrary classical channels)

$$\mathscr{DC} \subsetneq \mathscr{PC} \subsetneq \mathscr{QP} \subsetneq q\mathscr{C}$$

Key theorem

A deterministic correlation can be realized by a process matrix if and only if it can also be realized by a process function

Theorem 4 in arXiv:2307.02565

Key theorem

A deterministic correlation can be realized by a process matrix if and only if it can also be realized by a process function

(Generalizes the non-signalling Bell case)

Theorem 4 in arXiv:2307.02565

Key theorem

Hence: any deterministic correlation unachievable by a process function is also unachievable by a process matrix!

Theorem 4 in arXiv:2307.02565

Logic of the strict inclusions

$$\mathcal{QP} \subsetneq q\mathcal{C}$$

- Every deterministic correlation achievable by a process matrix is achievable by a process function
- Bipartite case: perfect GYNI correlation unachievable by any process function (bipartite diagonal limit => no causal inequality violation)
- Hence, perfect GYNI correlation unachievable by any process matrix

Logic of the strict inclusions

$$\mathscr{DC} \subsetneq \mathscr{PC}$$

Guess Your Neighbour's Input or NOT (GYNIN) game

$$(x_1, x_2, x_3) = (a_3, a_1, a_2) \text{ OR } (x_1, x_2, x_3) = (\bar{a}_3, \bar{a}_1, \bar{a}_2)$$

$$p_{\text{gynin}}$$

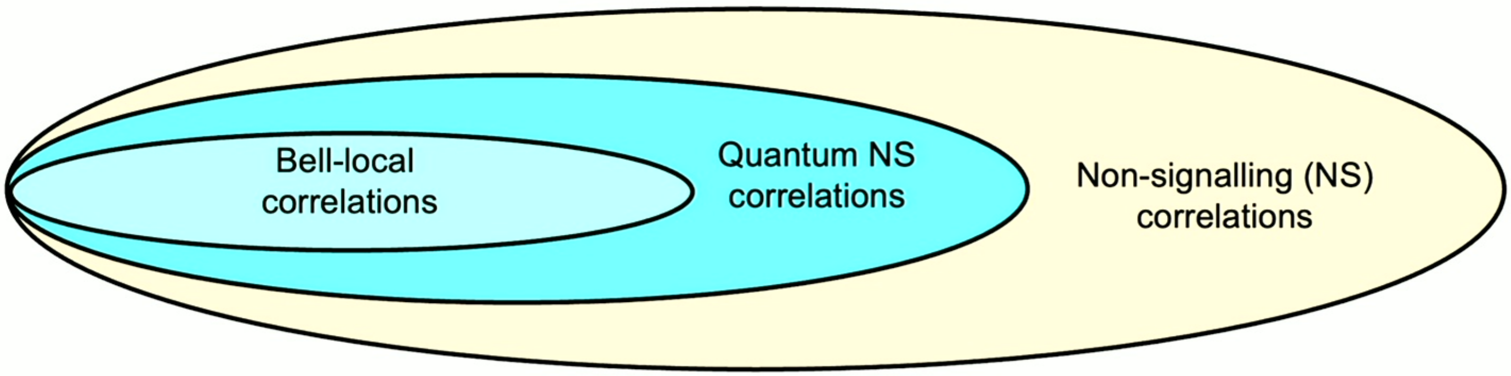
$$:= \frac{1}{8} \sum_{\vec{x},\vec{a}} p(\vec{x}|\vec{a}) \left(\delta_{x_1,a_3} \delta_{x_2,a_1} \delta_{x_3,a_2} + \delta_{x_1,\bar{a}_3} \delta_{x_2,\bar{a}_1} \delta_{x_3,\bar{a}_2} \right)$$

Logic of the strict inclusions

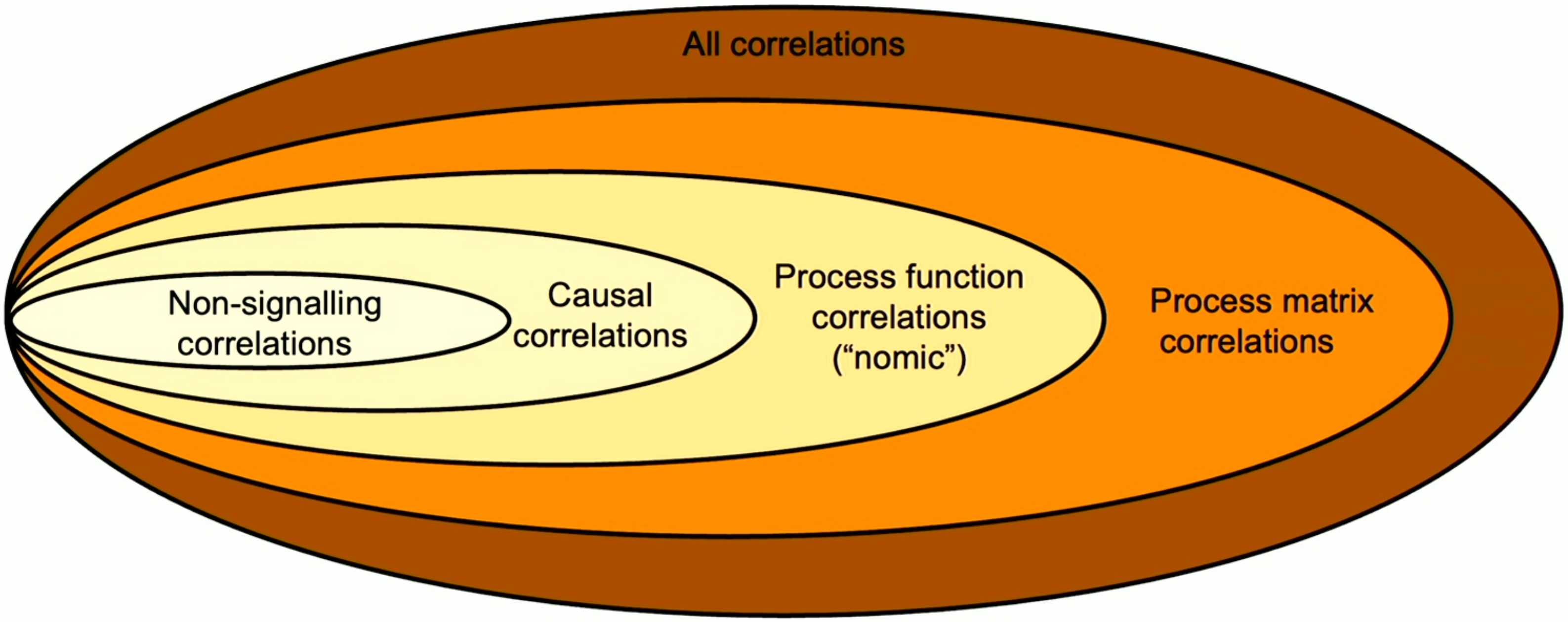
$$p_{\text{gynin}} \overset{\text{causal}}{\leq} \frac{1}{2} \overset{\text{classical}}{\leq} \frac{5}{8} \overset{\text{antinomic}}{\leq} 1$$

AF/BW: arXiv:1507.01714 $\mathscr{DC} \subsetneq \mathscr{PC}$ BFW: arXiv:1403.7333

Takeaway



Bell inequalities: separate Bell-local correlations from the rest of the non-signalling correlations

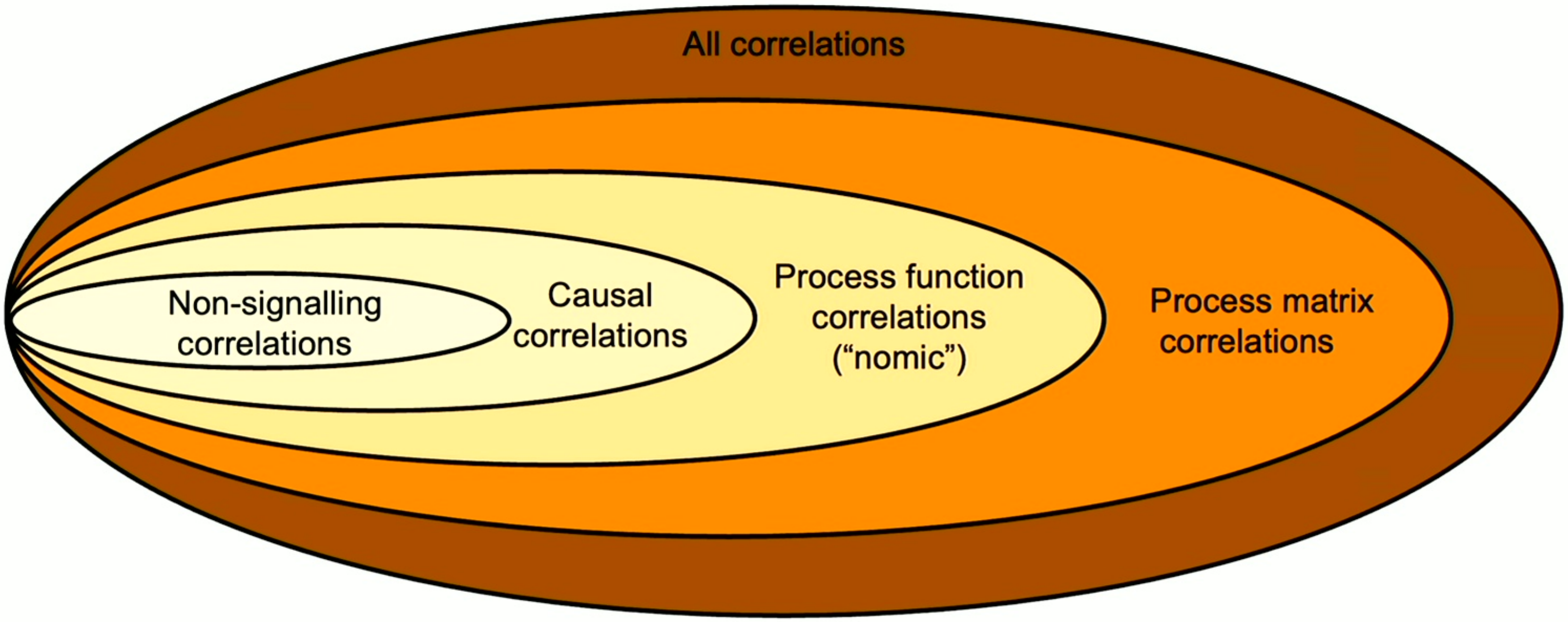


Causal inequalities: separate causal correlations from the rest of the correlations

Antinomicity inequalities: separate process function correlations from the rest

Open questions

- Fully characterize the classical polytope in the simplest non-trivial scenario, i.e., (3,2,2)
- Can one witness antinomicity with unitary processes?
- Tsirelson-type bounds on process-matrix correlations? [See arXiv:2403.02749]
- Infinite-dimensional surprises?



Causal inequalities: separate causal correlations from the rest of the correlations

Antinomicity inequalities: separate process function correlations from the rest