Title: Cyclic causal modelling with a graph separation property in classical and quantum theories

Speakers: Carla Ferradini

Series: Quantum Foundations, Quantum Information

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Cyclic causal modelling with a graph separation property in classical and quantum theories

(on arXiv soon)

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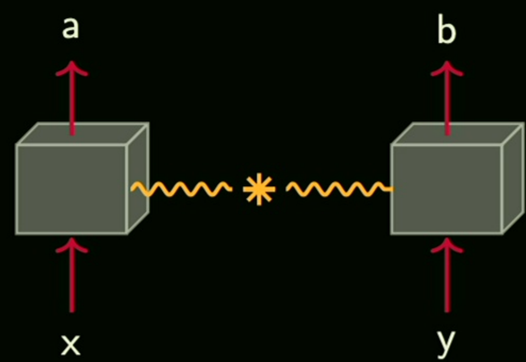
ETH zürich   Causalworlds 2024 — September 18, 2024

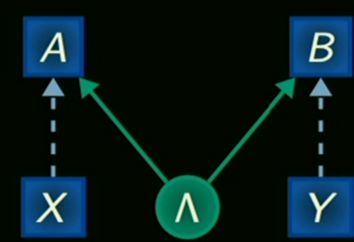
Cyclic causal modelling

with a graph separation property

in classical and quantum theories



Quantum circuit

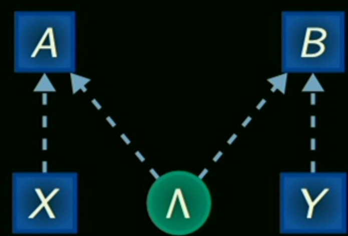


Causal model

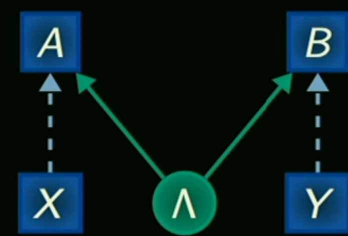
Cyclic causal modelling

with a graph separation property

in classical and quantum theories



Hidden variable model



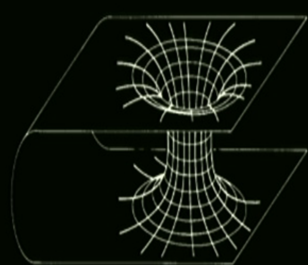
Quantum scenario

(Robert Spekkens tutorial)

Cyclic causal modelling with a graph separation property in classical and quantum theories

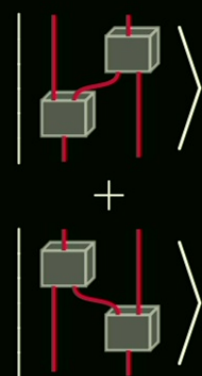
CTCs

Solutions of GR



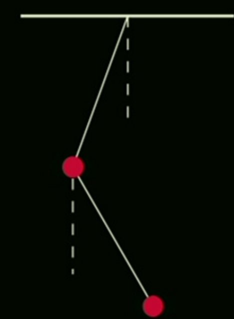
Indefinite causality

(Cyril Branciard tutorial)

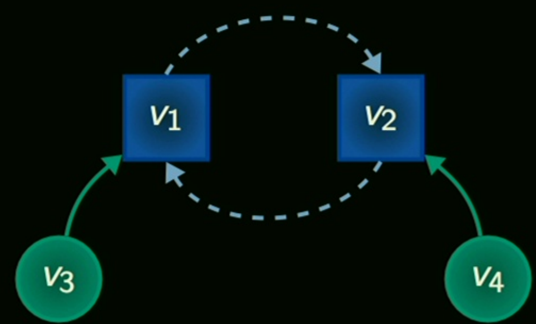


Feedback processes

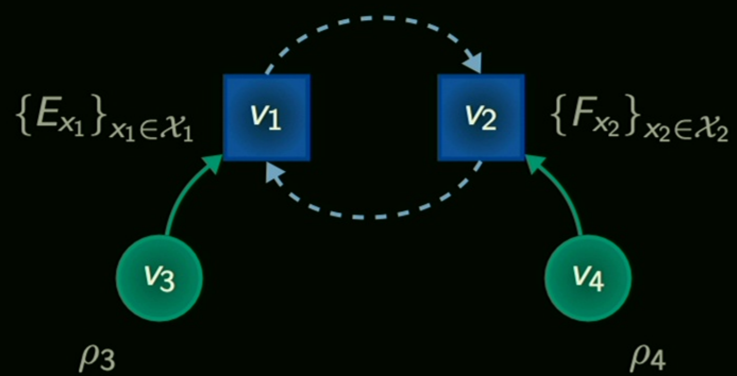
(Joris M. Mooij talk)



Evaluating probabilities



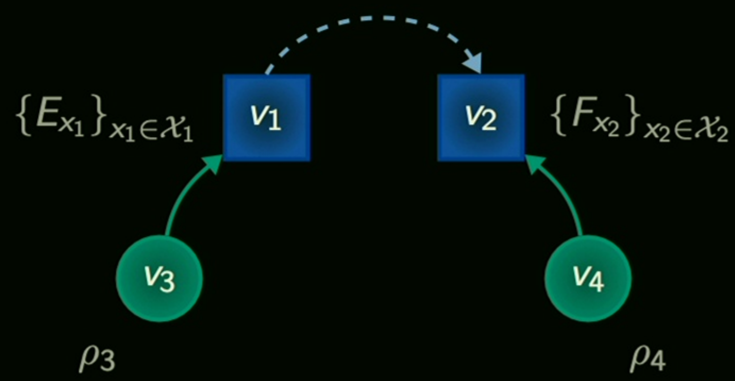
Evaluating probabilities



$P(x_1, x_2)$?

Evaluating probabilities

For any causal model on acyclic G

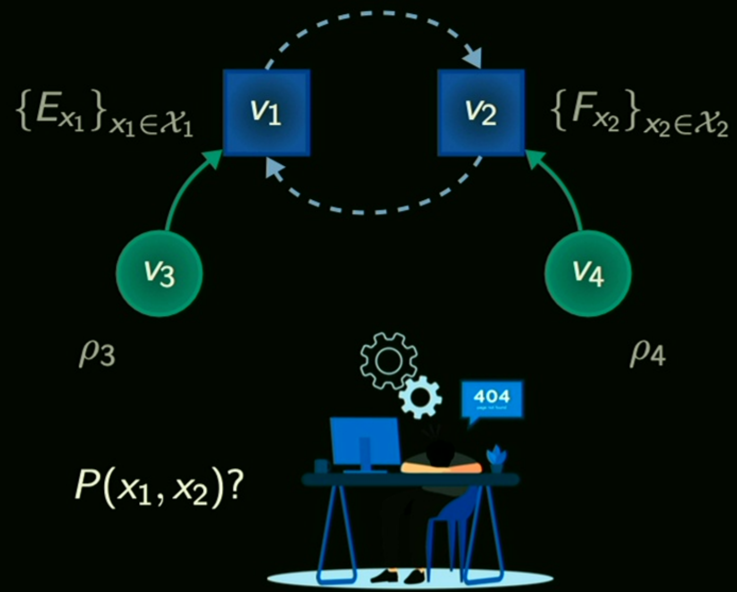


$$P(x_1, x_2) = \mathrm{Tr}\,[(E_{x_1} \otimes F_{x_2})(\rho_3 \otimes |x_1\rangle\langle x_1| \otimes \rho_4)]$$

[Henson et al. *New J. Phys.* 16 113043 (2014)]
[Barrett et al. arXiv:1906.10726 (2019)]

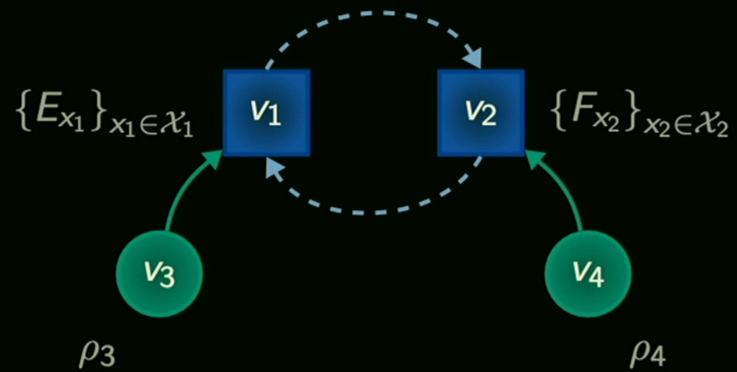
Evaluating probabilities

For any causal model on arbitrary G



Evaluating probabilities

For a subset of cyclic causal models on G

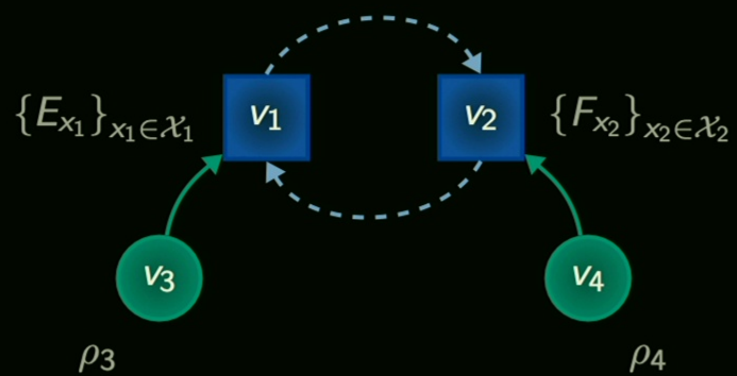


$$P(x_1, x_2)$$

[Forré et al. arXiv:1710.08775 (2017)]
[Bongers et al. *Ann. Statist.* 49(5): 2885-2915 (2021)]
[Barrett et al. *Nat. Commun.* 12, 885 (2021)]

Evaluating probabilities

For any causal model* on arbitray G

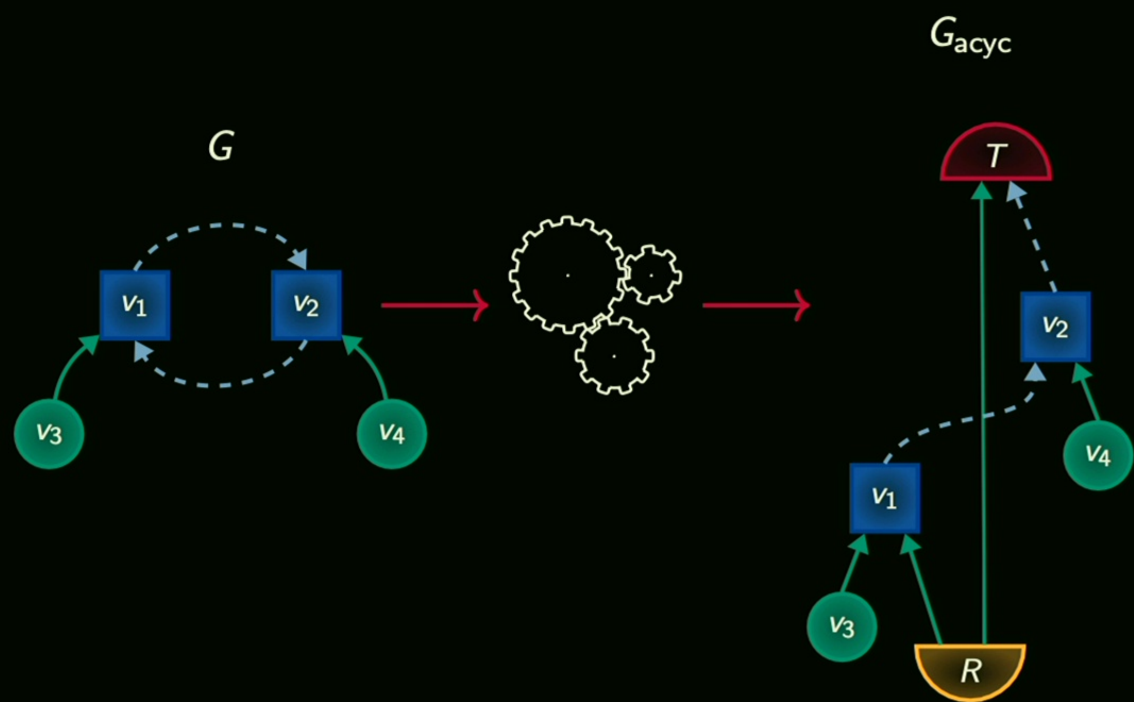


$$P(x_1, x_2)$$

*with finite dimensional $\mathcal{H}$ and discrete random variables

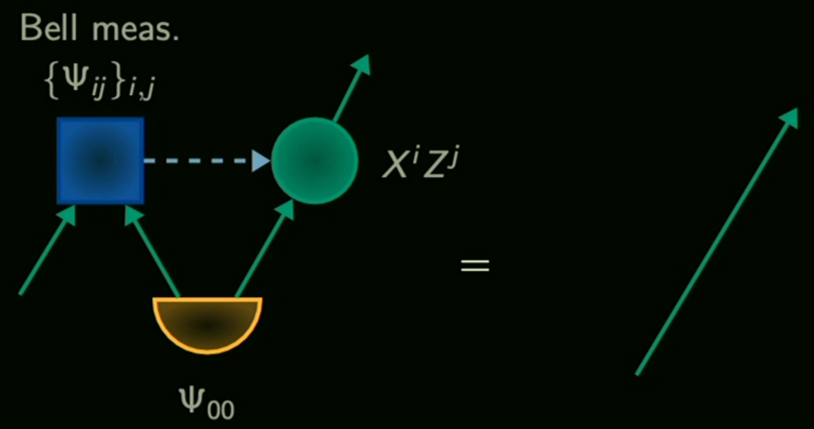
Breaking the cycles

Cyclic $\mapsto$ acyclic with postselection



Ingredient

Goal: simulate an identity channel

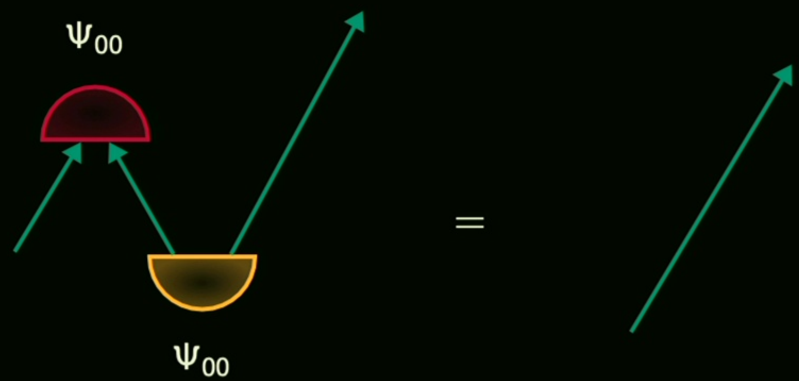


Teleportation protocol

Ingredient

Goal: simulate an identity channel

Postselected teleportation protocol ✓

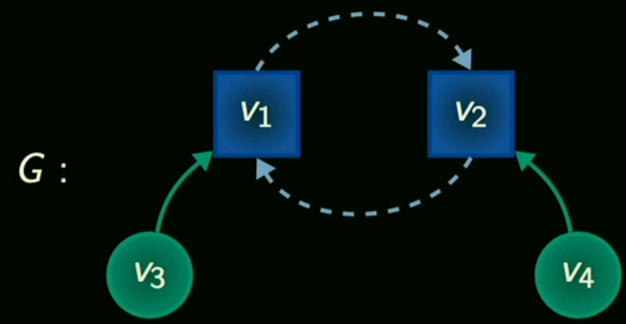


$$\mathrm{Tr}_{AB}[|\Psi_{00}\rangle\langle\Psi_{00}|_{AB}\rho_A|\Psi_{00}\rangle\langle\Psi_{00}|_{BC}] = p_{\checkmark}\rho_C$$

[Lloyd et al. *Phys. Rev. Lett.* 106, 040403 (2011)]
[Lloyd et al. *Phys. Rev. D* 84, 025007 (2011)]

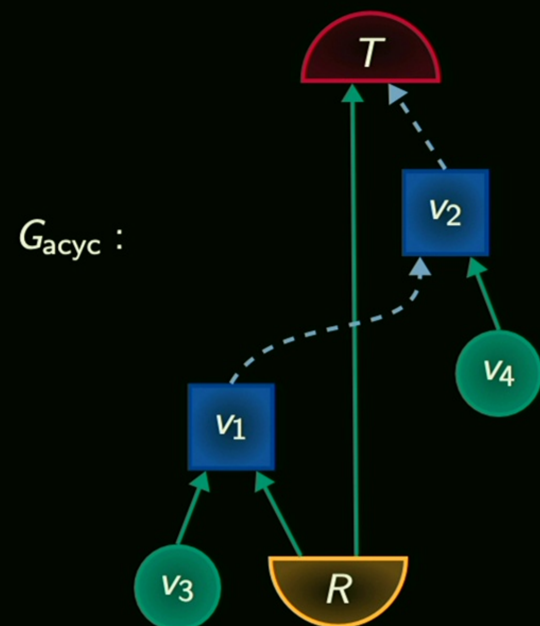
Recipe

Given a causal model on $G = (V, E)$



Recipe

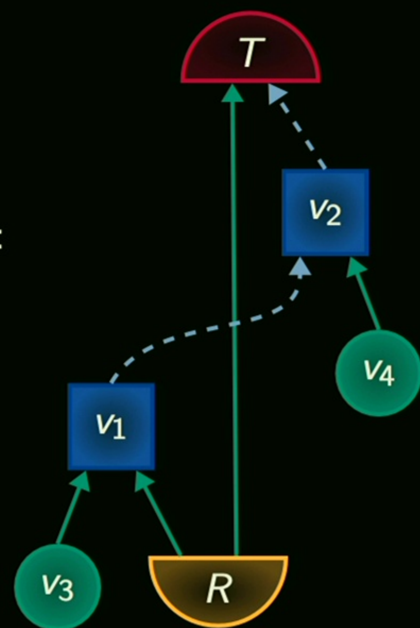
Step 2: add postselected teleportations to missing edges



Recipe

Step 2: add postselected teleportations to missing edges

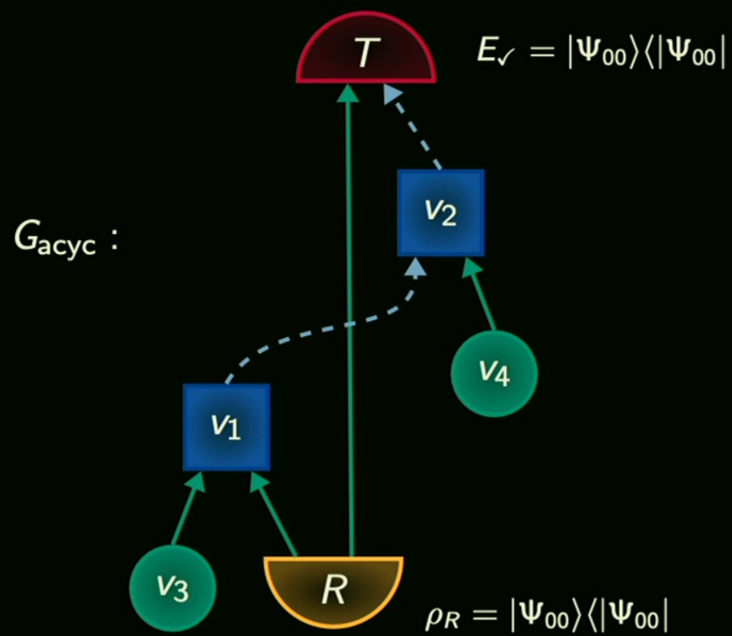
G_{acyc} :



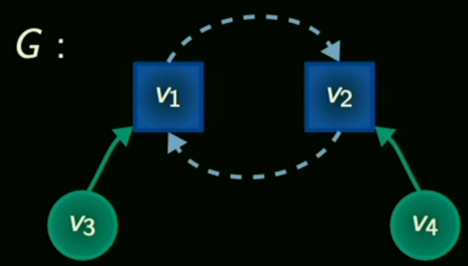
Defines a family of acyclic graphs $G_{acyc} \in \mathbb{G}^{acyc}(G)$

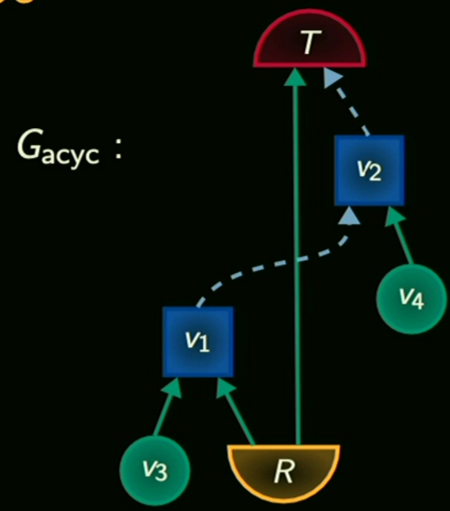
Recipe

Step 3: define a cm with postselected teleportation protocols on the added nodes of G_{acyc}



Recipe

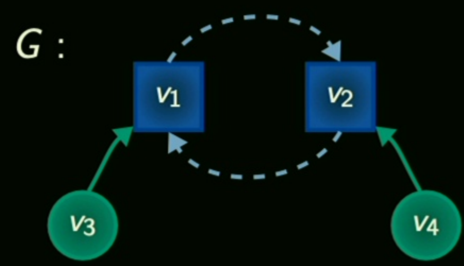


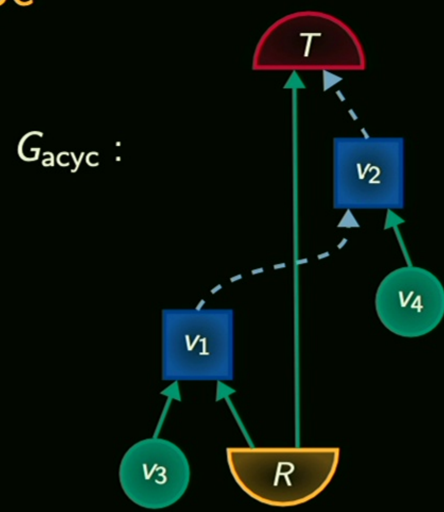


$\Pr_G(x_1, x_2)$

$\Pr_{G_{acyc}}(x_1, x_2 | t = \checkmark)$

Recipe



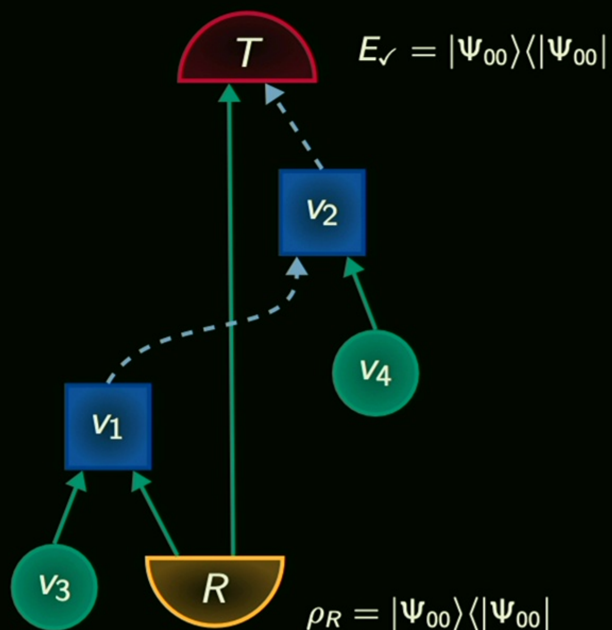


Define $\Pr_G(x_1, x_2) \quad := \quad \Pr_{G_{\text{acyc}}}(x_1, x_2 | t = \checkmark)$

Thm: the probability rule is independent on the choice of $G_{\text{acyc}} \in \mathbb{G}^{\text{acyc}}(G)$

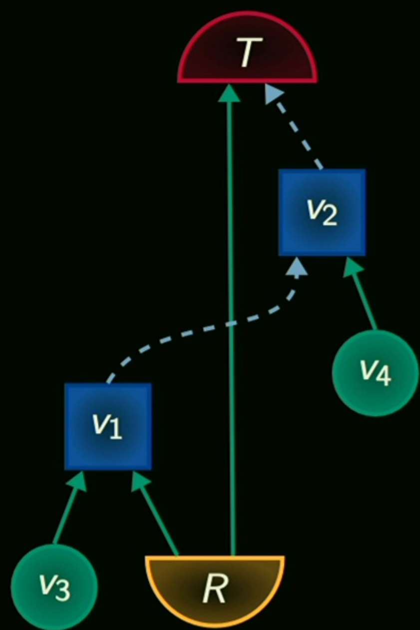
Fine tuning?

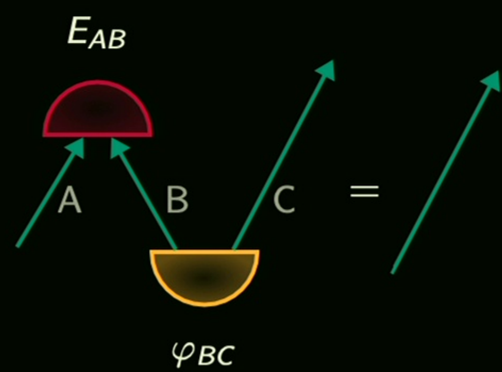
Step 3: define a cm with postselected teleportation protocols on the added nodes of G_{acyc}



Fine tuning?

Step 3: define a cm with any postselected teleportation protocols on the added nodes of G_{acyc}





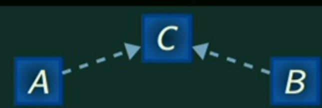
$$\mathrm{Tr}_{AB}[E_{AB}\rho_A\varphi_{BC}] = p_{\checkmark}\rho_C$$

Graph separation property: p-separation

Graph separation property: *d*-separation

Given a directed graph G and a compatible probability Pr

Graph structure	Causal mechanisms
↓	↓
d-separation in G	*independencies* in Pr
$\perp^d$	$\perp\!\!\!\perp$



$A \perp^d B$ and $A \not\perp^d B|C$

$A \perp\!\!\!\perp B|C$ if
$\Pr(a,b|c) = \Pr(a|c)\Pr(b|c)$

[Pearl. *Cambridge University Press* (2009)]

d-separation theorem

G is acyclic

Soundness: If $A \perp^d B|C$ in G, then for all causal models $A \perp\!\!\!\perp B|C$ in Pr

$\implies \Pr(a,b) = \Pr(a)\Pr(b)$

$A \perp^d B$

[Pearl. *Cambridge University Press* (2009)]

d-separation theorem

G is acyclic

Completeness: If $A \not\perp^d B|C$ in G, then there exists a causal model where $A \not\perp\!\!\!\perp B|C$ in Pr

$A \not\perp^d B|C$

[Pearl. *Cambridge University Press* (2009)]

d-separation theorem

G is acyclic

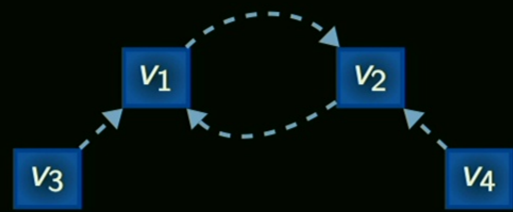
Completeness: If $A \not\perp^d B|C$ in G, then there exists a causal model where $A \not\perp\!\!\!\perp B|C$ in Pr

$A \not\perp^d B|C$

$$c = \delta_{a,b}$$
$$\Pr(a, b|c = 1) \propto \delta_{a,b}$$

[Pearl. *Cambridge University Press* (2009)]

Failure of *d*-separation in cyclic graphs



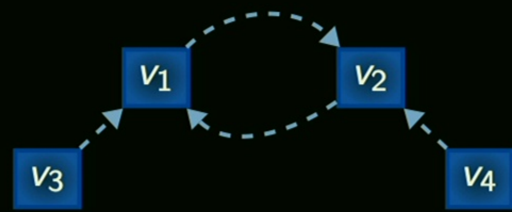
$$v_3 \perp^d v_4 \overset{d\text{-sep.}}{\Longrightarrow} x_3 \perp\!\!\!\perp x_4 \text{ for all causal models}$$

Causal model:

$$x_1 = x_3 \oplus x_2 \qquad \Longrightarrow \qquad x_3 = x_4$$
$$x_2 = x_1 \oplus x_4$$

[Neal. *JAIR*, 12:87–91, (2000)]
[Pearl. *Cambridge University Press* (2009)]

Failure of d-separation in cyclic graphs



$$v_3 \perp^d v_4 \overset{d\text{-sep.}}{\Longrightarrow} x_3 \perp\!\!\!\perp x_4 \text{ for all causal models}$$

Causal model:

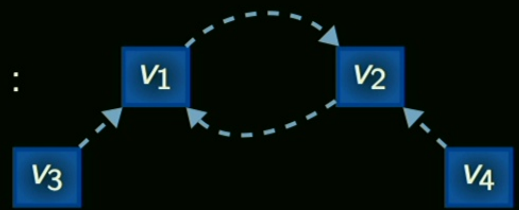
$$x_1 = x_3 \oplus x_2 \qquad \Longrightarrow \qquad x_3 = x_4 \qquad \Longrightarrow \qquad \Pr(x_3, x_4) \propto \delta_{x_3, x_4}$$

$$x_2 = x_1 \oplus x_4 \qquad\qquad\qquad\qquad\qquad x_3 \not\perp\!\!\!\perp x_4$$

[Neal. *JAIR*, 12:87–91, (2000)]
[Pearl. *Cambridge University Press* (2009)]

Defining p-separation

G :



$(v_3 \perp^p v_4 | V)_G$

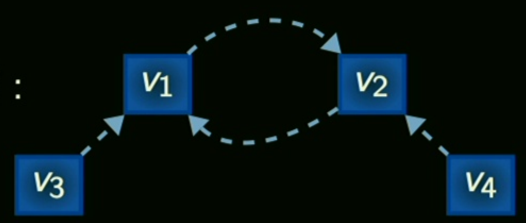
if and only if

$\exists\, G_{\text{acyc}} \in \mathbb{G}^{\text{acyc}}(G)$ such that $(v_3 \perp^d v_4 | V, T)$

"p" for *postselection* — name suggested by Yìlè Yīng

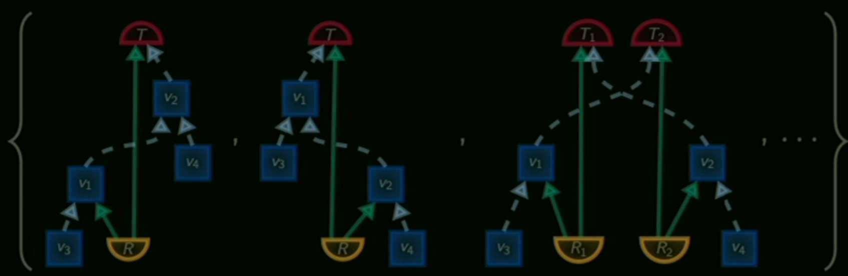
Defining p-separation

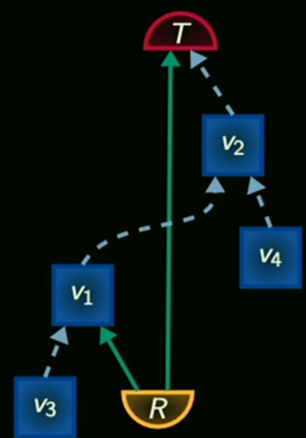
G :



$(v_3 \perp^p v_4)_G$?

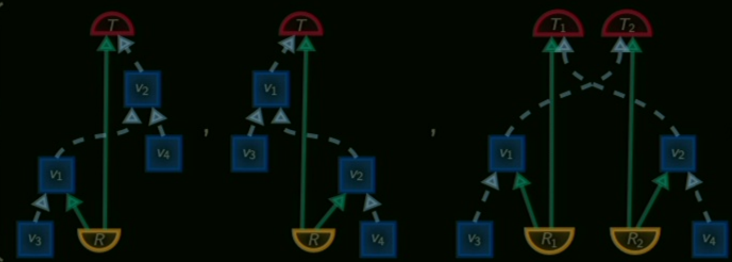
Defining p-separation

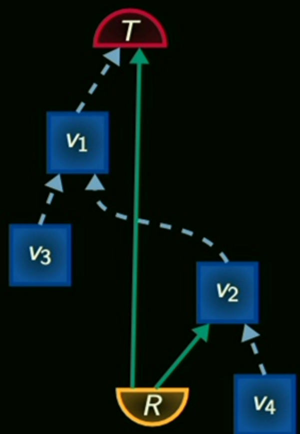
$\mathbb{G}^{\text{acyc}}(G) =$ 



$(v_3 \perp^d v_4 | T)_{G_{\text{acyc}}}$?

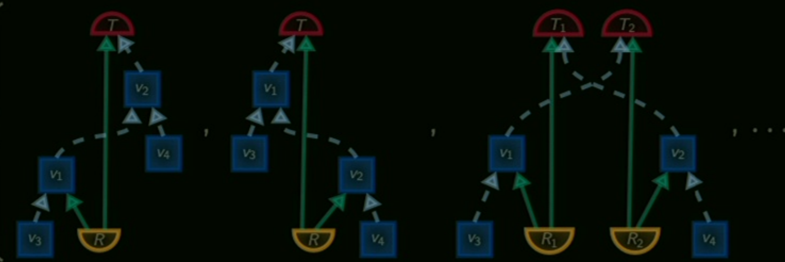
Defining p-separation

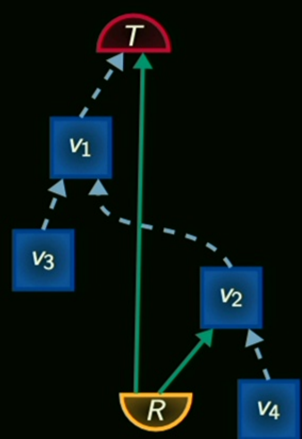
$\mathbb{G}^{\mathrm{acyc}}(G) = \{$  $, \dots \}$



$(v_3 \perp^d v_4 | T)_{G_{\mathrm{acyc}}}$?

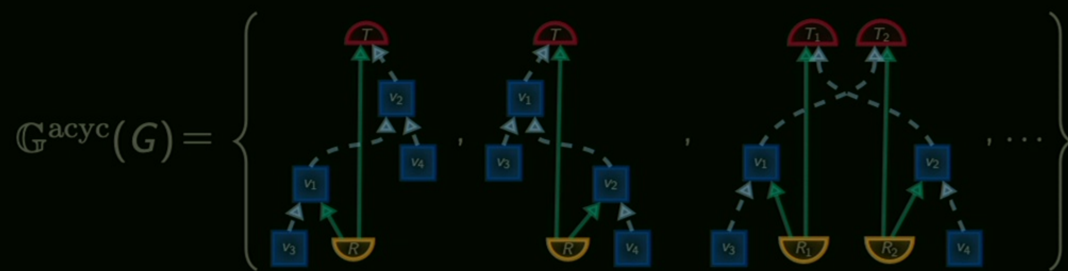
Defining p-separation

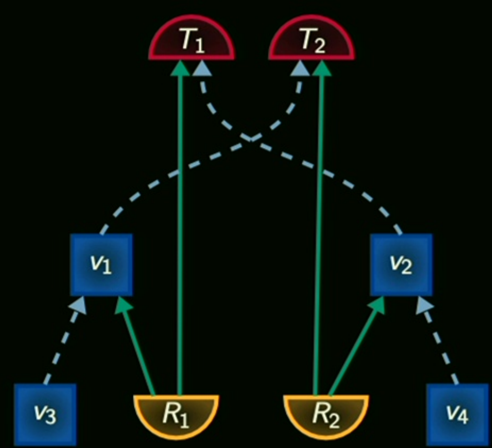
$\mathbb{G}^{\text{acyc}}(G) = \{$  $\}$



$(v_3 \perp^d v_4 | T)_{G_{\text{acyc}}}$? no

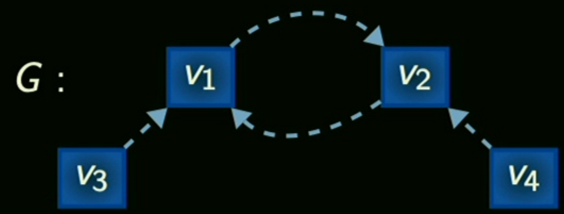
Defining p-separation





$(v_3 \perp^d v_4 | T_1, T_2)_{G_{\text{acyc}}}$? no

Defining p-separation



$(v_3 \perp^p v_4)_G$

if and only if

$\exists\, G_{\text{acyc}} \in \mathbb{G}^{\text{acyc}}(G)$ such that $(v_3 \perp^d v_4 | T)$

$\implies (v_3 \not\perp^p v_4)_G$

p-separation theorem

for any G

Soundness: If $A \perp^p B|C$ in G, then for all causal models $A \perp\!\!\!\perp B|C$ in Pr

Completeness: If $A \not\perp^p B|C$ in G, then there exists a causal model where $A \not\perp\!\!\!\perp B|C$ in Pr

For causal models* on arbitrary G

*with finite dimensional $\mathcal{H}$ and discrete random variables

Further results and Outlook

Further results

- Simplified methodology for functional causal models

→ number and existence of solutions

- Describe quantum Bayesian networks, process matrices, ...

→ characterising subsets of cyclic models

[Henson et al. *New J. Phys.* 16 113043 (2014)]
[Barrett et al. arXiv:1906.10726 (2019)]
[Costa et al. *New J. Phys.* 18 063032 (2016)]

Further results

- Simplified methodology for functional causal models

→ number and existence of solutions

- Describe quantum Bayesian networks, process matrices, . . .

→ characterising subsets of cyclic models

- Definition of interventions (in preparation)

→ effect of cyclicity on signalling

- Mapping tensor networks to cyclic causal models (in preparation)

→ emergence of space-time and notion of causality

[Cotler et al. *J. High Energ. Phys.* 42 (2019)]

Outlook

Causal discovery algorithms use d-separation for acyclic G

$\rightarrow$ use p-separation in the cyclic case

Indefinite causal structures are described with cyclic models

$\rightarrow$ characterise special subsets, e.g., violating causal inequalities or causally non-separable

Causal compatibility problems compatibility of Pr with G

$\rightarrow$ extend known tecniques for acyclic G, e.g., inflation, to cyclic mapping them to acyclic with postselection

Studying spacetime emergence using tensor networks

$\rightarrow$ emergence of space time geometry from operational properties of causal models

[Wolfe et al. *J. Causal Inference* (2019)]

Thank you