

Title: Partitions in quantum theory, and causal decompositions of 1D Quantum Cellular Automata

Speakers: Augustin Vanrietvelde

Series: Quantum Foundations, Quantum Information

Date: September 17, 2024 - 4:10 PM

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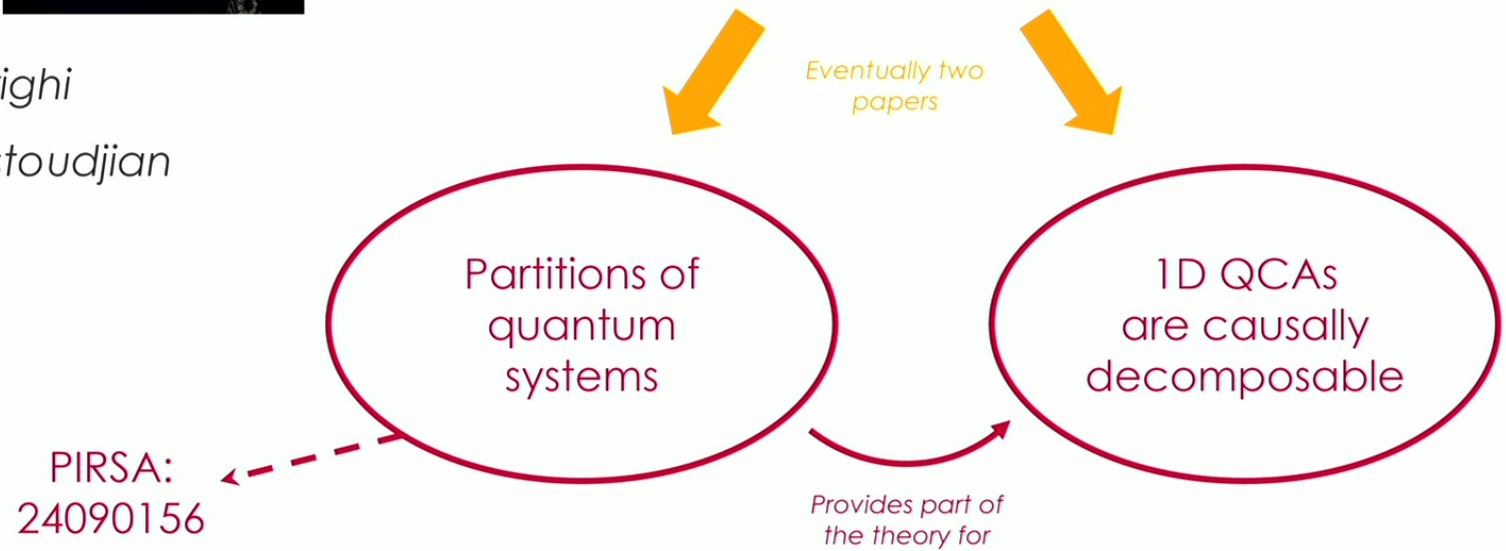
# Causal decompositions of 1D QCAs

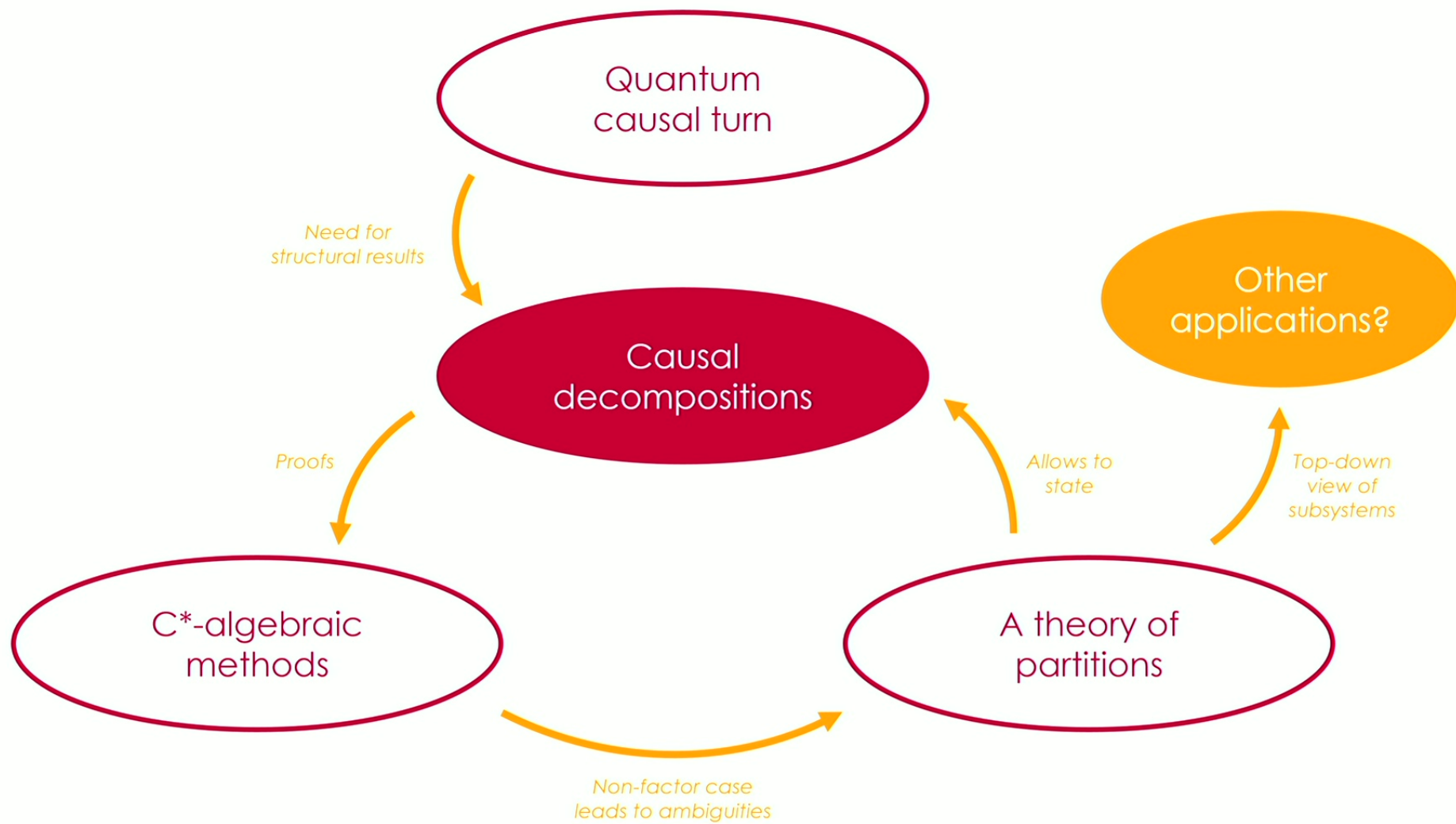
Augustin Vanrietvelde  
Causalworlds 2024  
September 17<sup>th</sup>, 2024



Pablo Arrighi  
+ Octave Mestoudjian

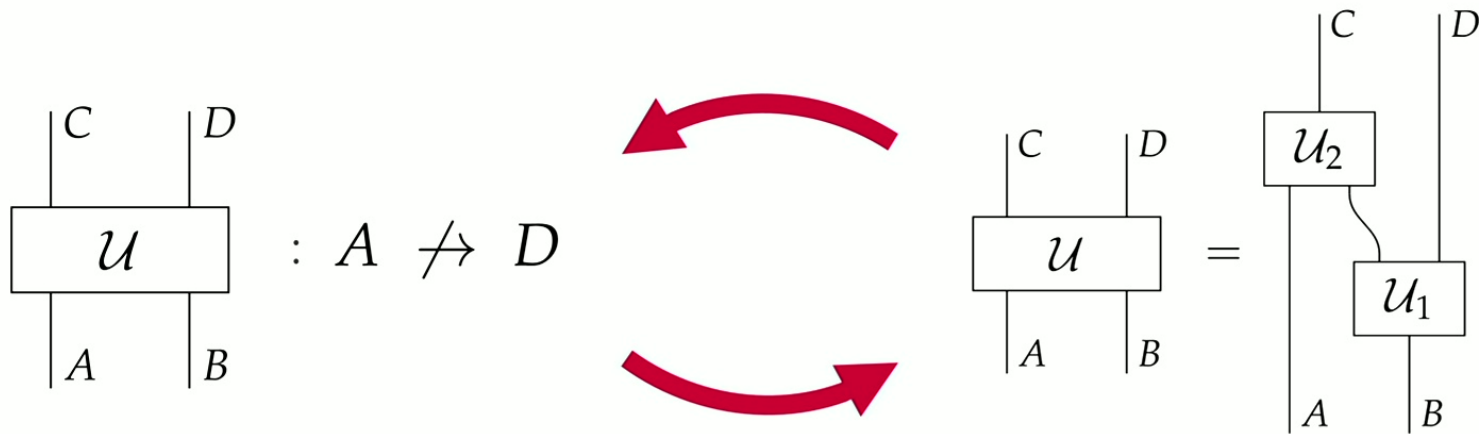
## Ongoing work





# Causal decompositions

# Causal decompositions: one-way causation

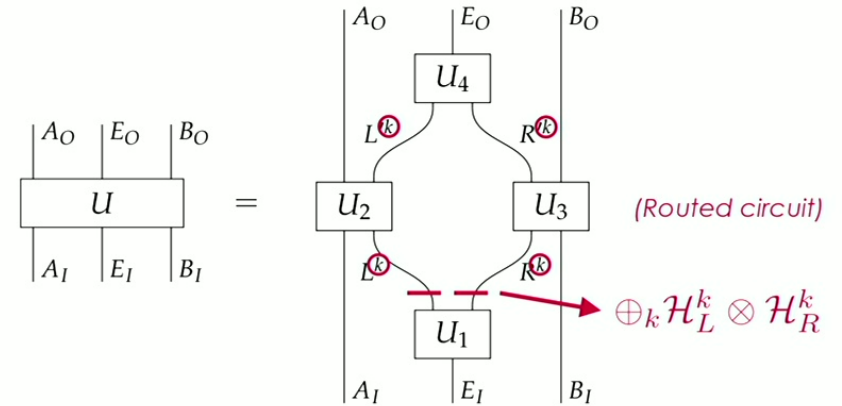
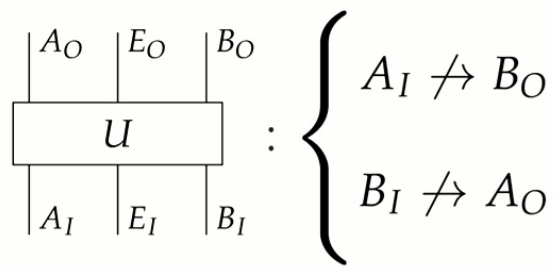


Causal structure  
(operational /  
phenomenological)

Compositional  
structure  
(mathematical / structural)

Eggeling, Schlingemann and Werner, "Semicausal operations are semilocalizable", 2001 (quant-ph/0104027)

# Causal decompositions: the diamond

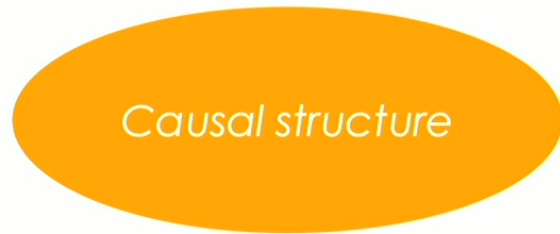


Causal structure  
(operational /  
phenomenological)

Compositional  
structure  
(mathematical / structural)

Lorenz and Barrett, "Causal and compositional structure of unitary transformations", 2020 (2001.07774)

# Causal decompositions: the general conjecture

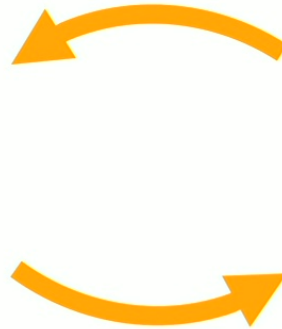


“What I can probe at the phenomenological level”

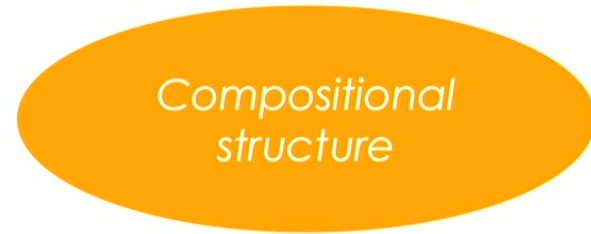
*Fuzzy*

*No handles*

*No clear view of what is feasible*



*(still to be proven in general)*



“What might be going on deep down in the Universe”

*Sharp*

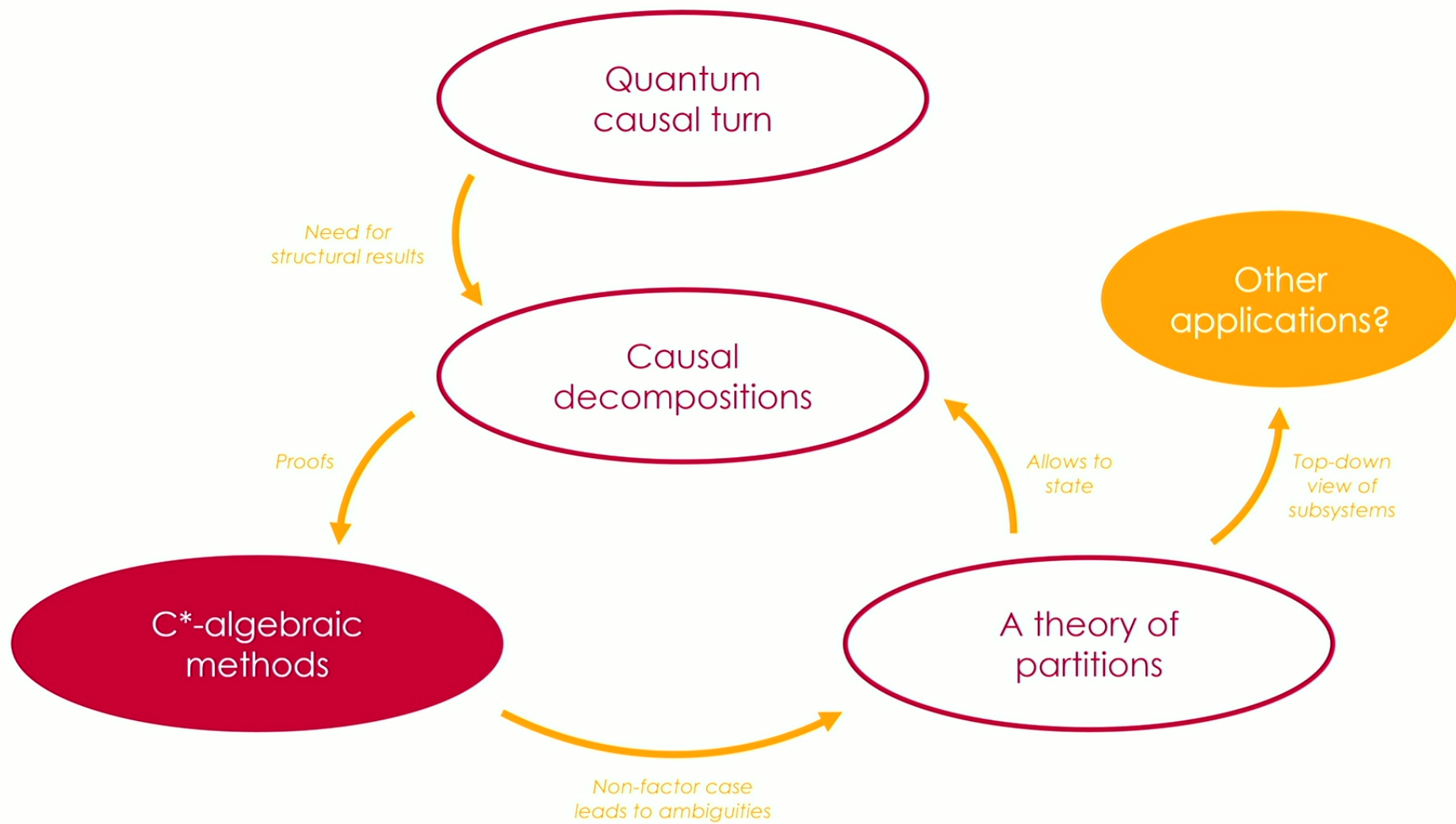
*Easy to manipulate*

*Constructive*

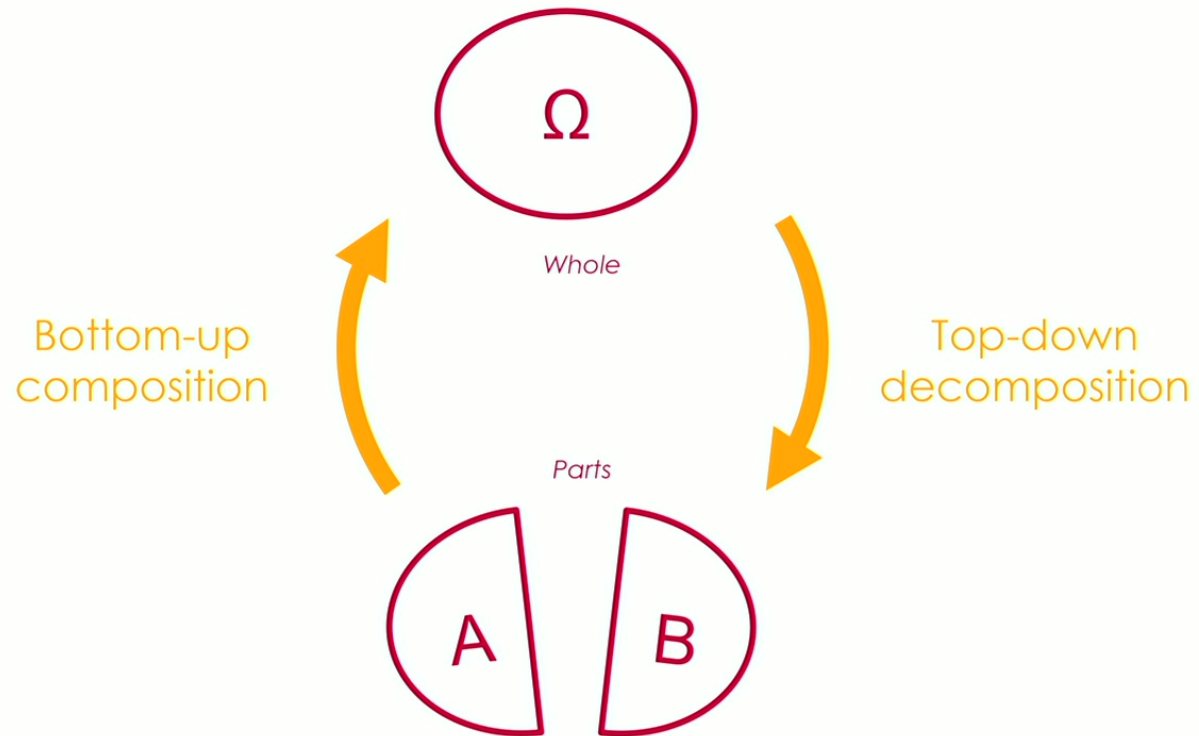
*(see e.g. the classification of bipartite supermaps)*

Barrett, Lorenz and Oreshkov, “Cyclic quantum causal models”, 2020 (2002.12157)

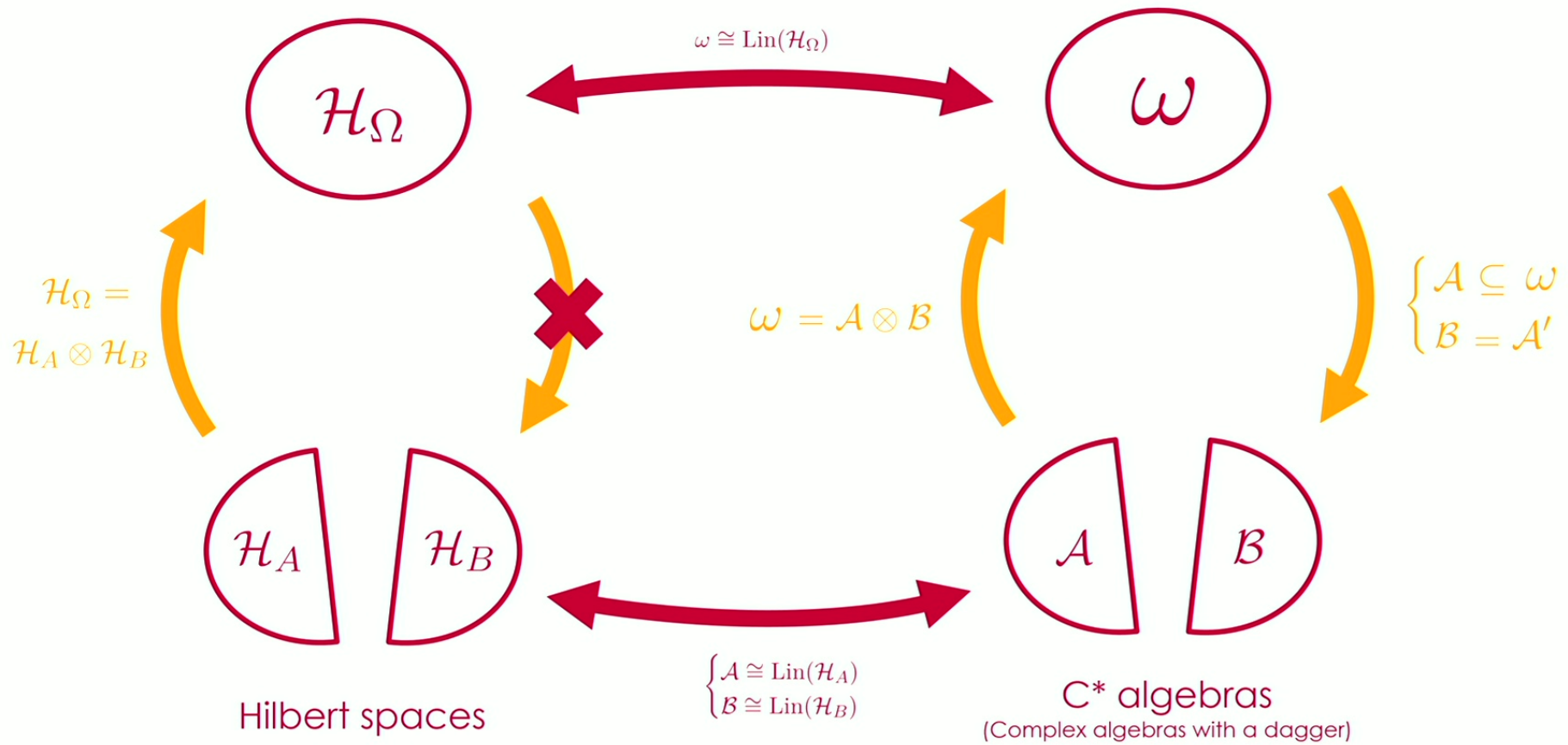




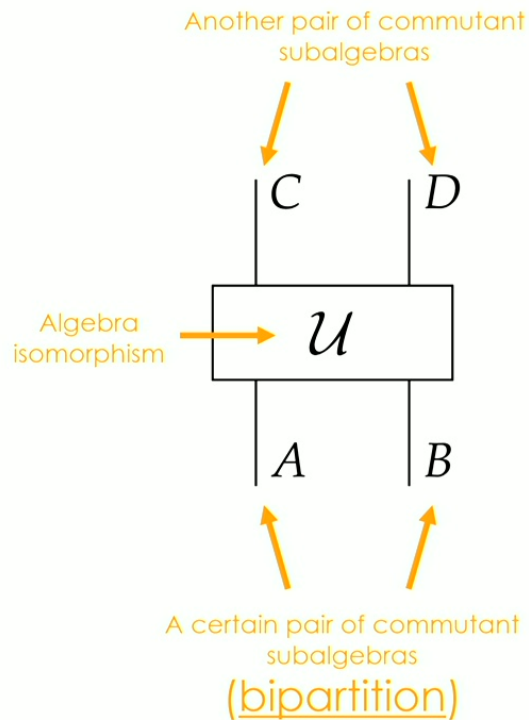
# A bit of mereology



# A bit of quantum mereology



# A better view of quantum causal structure

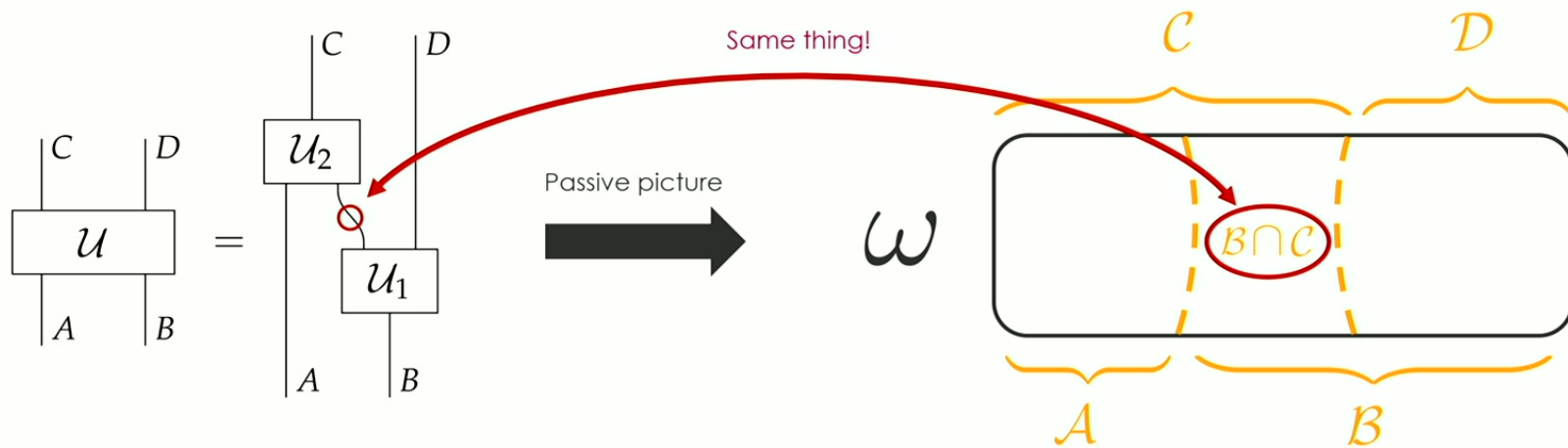


“Passive picture”:  $\mathcal{U}$  specifies the relation between two pairs of subalgebras of the same global algebra

Re-expressing causality

$$A \not\leftrightarrow D \iff [\mathcal{A}, \mathcal{D}] = 0$$

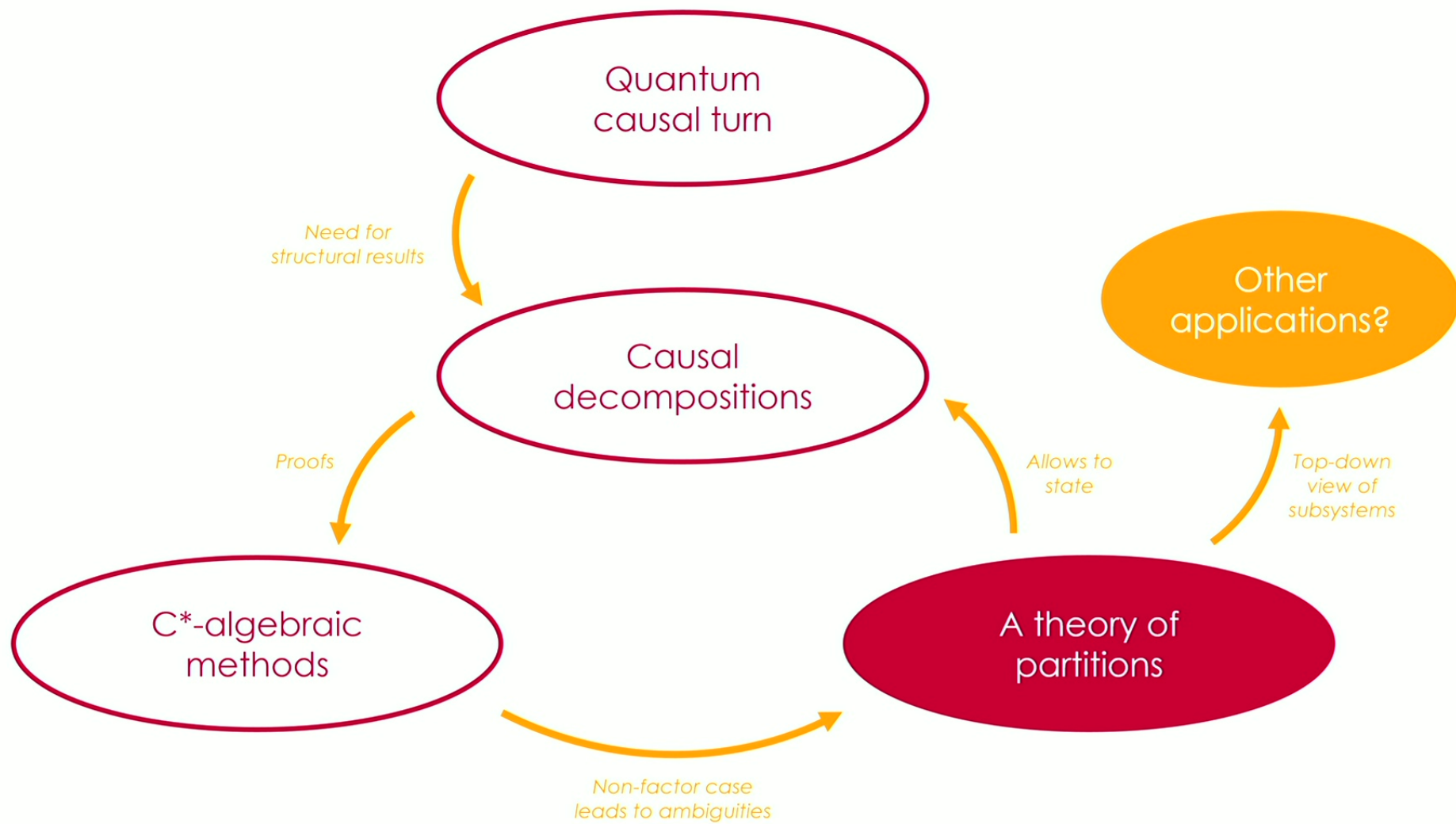
# Causal decompositions as repartitioning



Causal decompositions are a problem of **fine-graining partitions**

# How it feels to reason with $C^*$ algebras





# A theory of partitions of quantum systems

*– a surprisingly subtle problem*

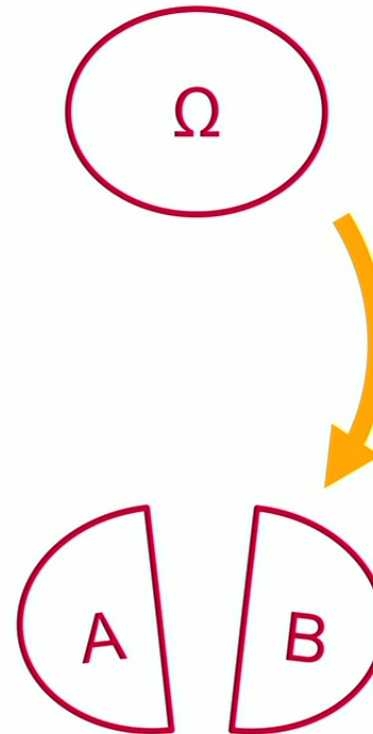


# Subsystems as sub- $C^*$ algebras

What counts as a good physical subsystem?

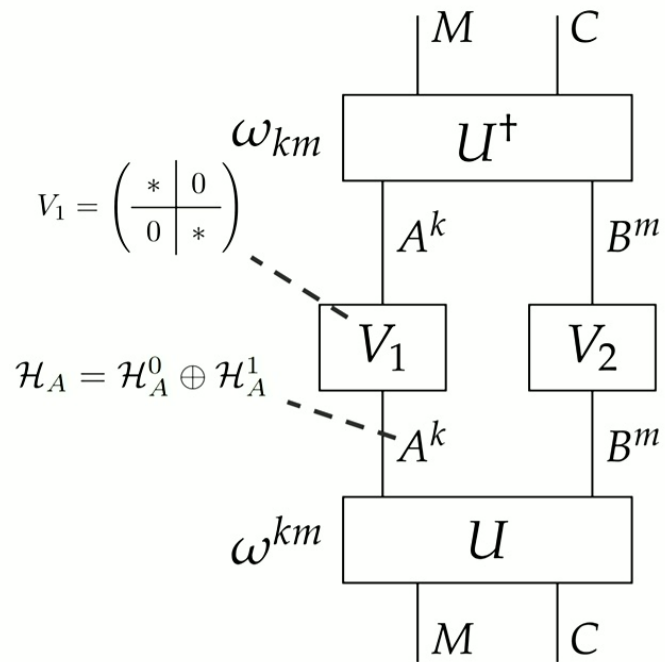
→ a sub- $C^*$  algebra.

(see e.g. algebraic QFT,  
quantum error-correction,  
operational approaches...)



Chiribella, "Agents, subsystems, and the conservation of information", 2018 (1804.01943)

# An example of non-factor subsystems



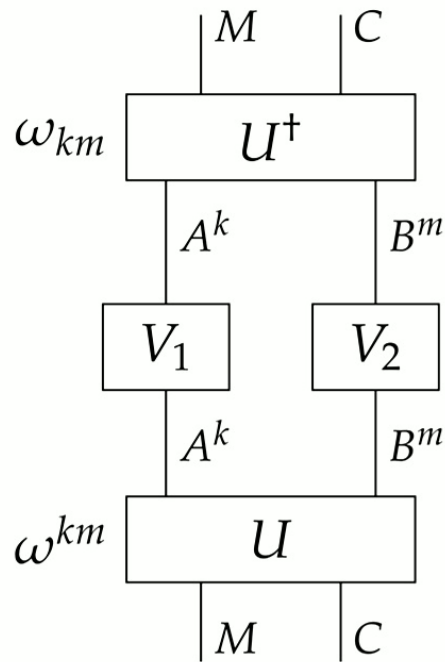
Superposition of trajectories

If we believe that A&B are acting on subsystems, then these have to correspond to non-factor C\*-subalgebras.

*(Can be linked with partial classicality / superselection rules, but with subtleties!!)*

AV, "Routed quantum circuits: an extended framework for coherent control and indefinite causal order", 2022 (PhD thesis)

# A disturbing feature: Failure Of Local Tomography (FOLT)



Mathematical level

The algebraic span of  $\mathcal{A}$  and  $\mathcal{B}$  is not equal to the whole algebra.



Operational level

The relative phase between paths is inaccessible to  $A$ ,  $B$ , and even to their correlations, while it is accessible to a global agent.

# A theory of partitions (summed up)

**Goal:** defining  
partitions into  $\geq 3$  parts



Non-factoriness and FOLT lead to  
a surprising wealth of ambiguities

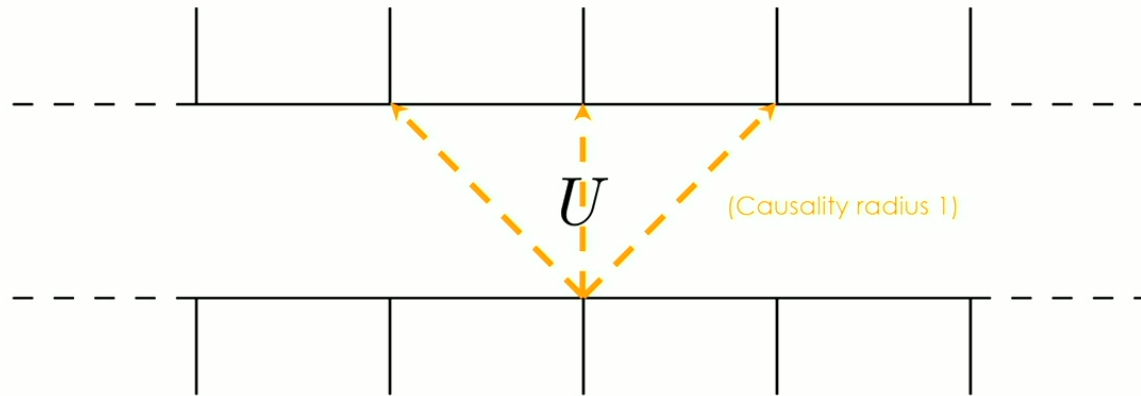


It has a lot of deep  
structure to explore!!



We overcome that, and provide a  
**well-defined and consistent theory**

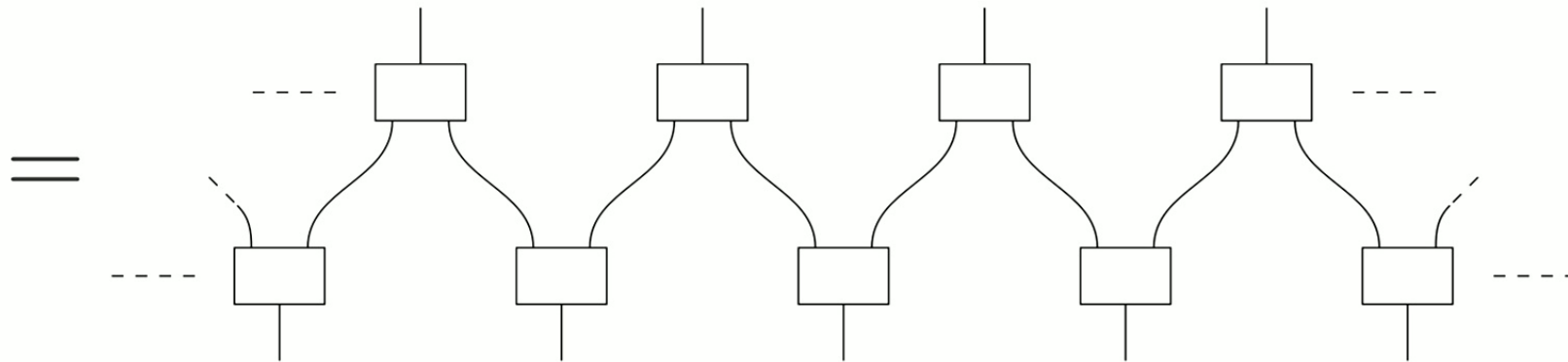
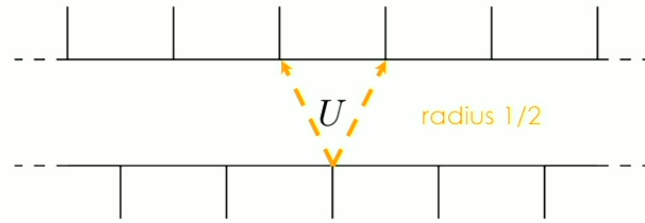
# 1D Quantum Cellular Automata



Usually translation-inv (but unnecessary here)  
Can be looped around in a circle  
Restricting to a finite number of sites here

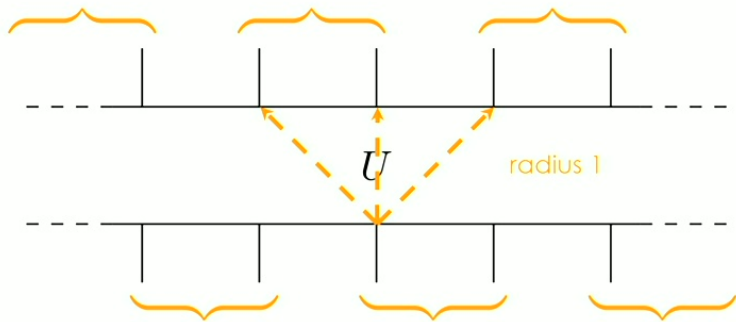
➔ Models of  $\left\{ \begin{array}{l} \text{discretised spacetime} \\ \text{quantum computation} \end{array} \right.$

# Decompositions of 1D QCAs for $r = 1/2$



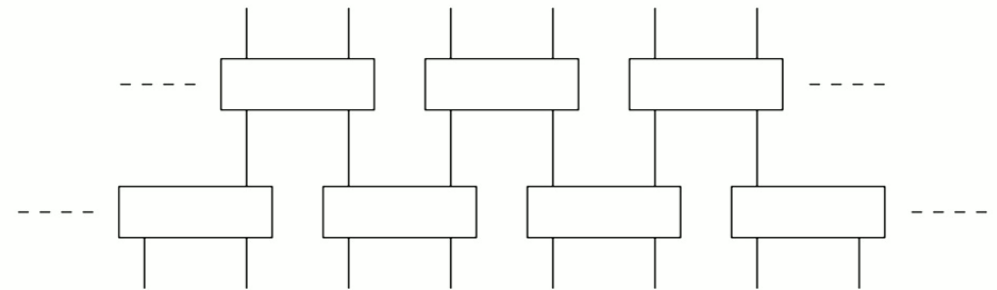
(Also yields index theory)

# Higher radii?



Usual story:  
"just regroup" ...

=

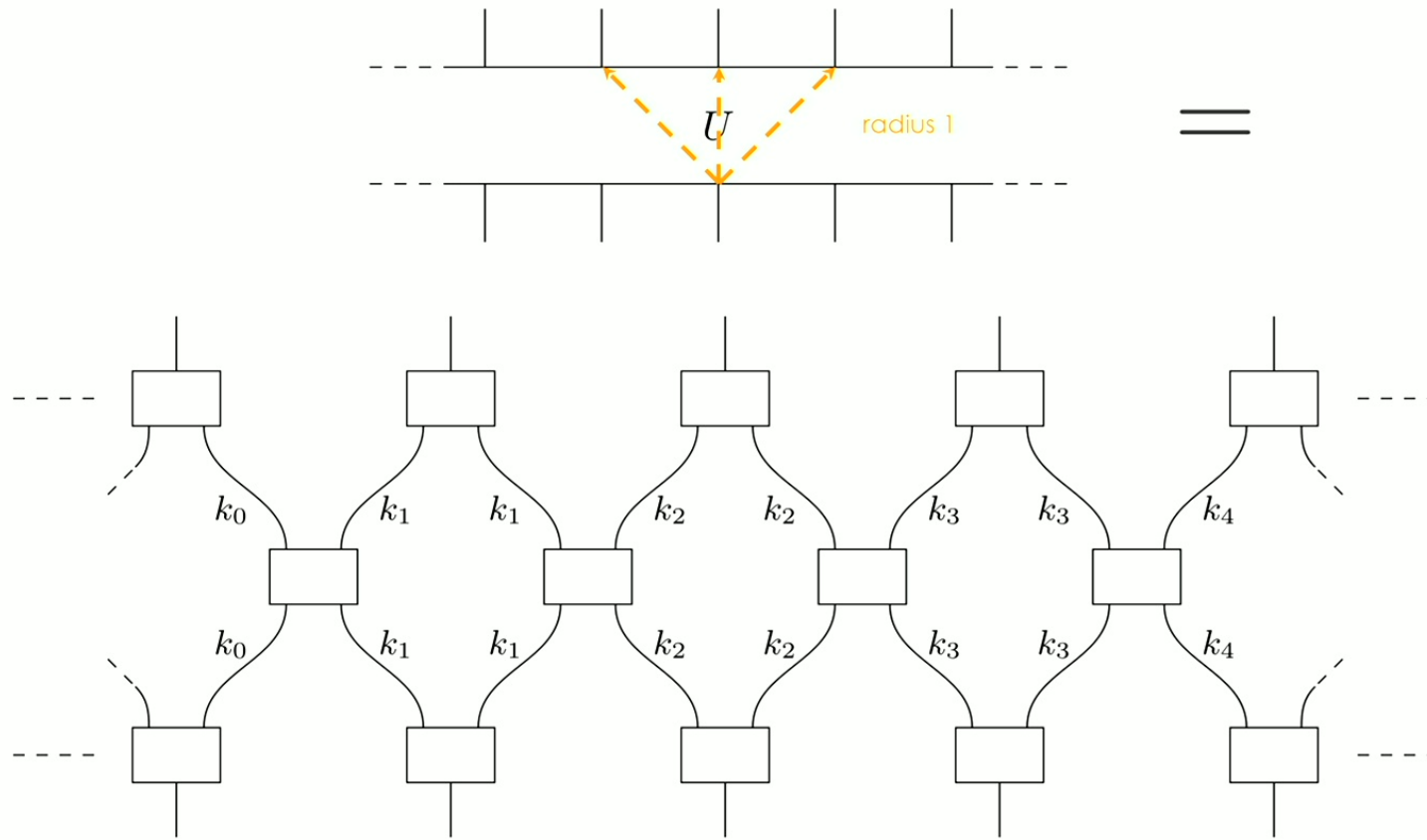


...but that gives a  
decomposition of radius  $3/2$



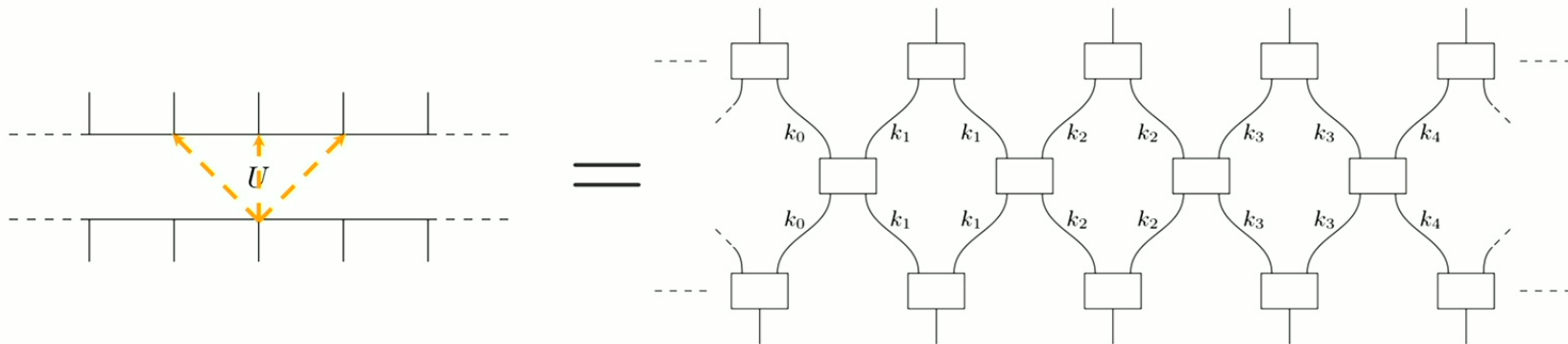
**At radius  $> 1/2$ , we don't have a constructive form of 1D QCAs yet**

# Our result: all 1D QCAs are causally decomposable





# 1D QCAs are causally decomposable



## Implications

- All causal dynamics in (discretised) 1+1 spacetime are (unitarily) localisable
- **Constructive form** of all 1D QCAs  $\rightarrow$  Extremely tight handle on them

## Some lessons from this talk

Causal decompositions are cool

Causal decompositions are a problem of **fine-graining partitions**

We would be wise to pay more attention to non-factor  $C^*$  algebras

In 1+1D discretised spacetime, **the phenomenological and ontological formulations of relativistic causality are equivalent**

# Prospects

Spatiotemporal  
relevance of the  
decomposability?

*Link with special properties  
of 1+1 spacetimes*

Other uses of  
C\*-algebraic  
methods?

*Classifying dual unitaries,  
perfect tensors...*

Other  
applications of  
partition theory?

*A new take on superselection rules  
Aharonov-Bohm effect  
Quantum Reference Frames...*

Pushing causal  
decs further?

# Prospects

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**E a counter-example!**

