Title: Limitations of causal models of the triangle network, in the symmetric subspace and beyond

Speakers: Tamás Kriváchy

Collection/Series: Causalworlds

Subject: Quantum Foundations, Quantum Information

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Limitations of causal models of the triangle network, in the symmetric subspace and beyond

2024.09.20, Waterloo (online)
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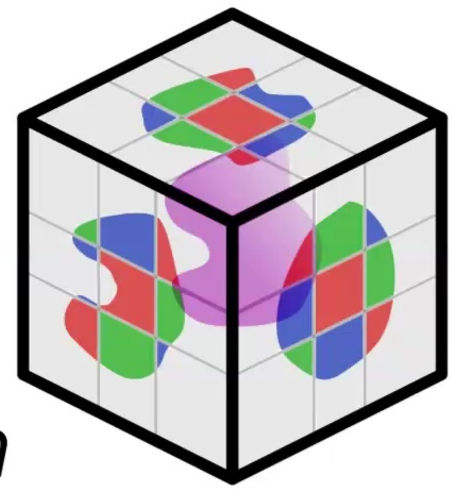

Exploring the Local Landscape in the Triangle Network,
E. Bäumer*, V. Gitton*, T. Kriváchy*, N. Gisin, R. Renner,
arXiv:2405.08939 (2024)

Ongoing work
T. Kriváchy, S. Boreiri, A. Girardin, P. Sekatski, ...

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2024 Anniversary

~~60 years of Bell's theorem~~

50 years of Rubik's cube!!!

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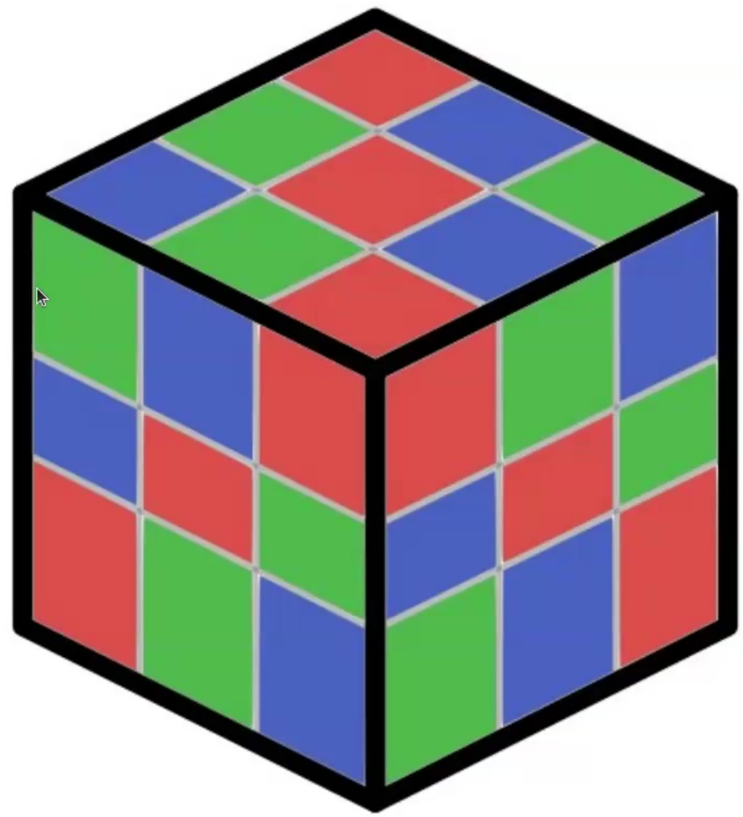
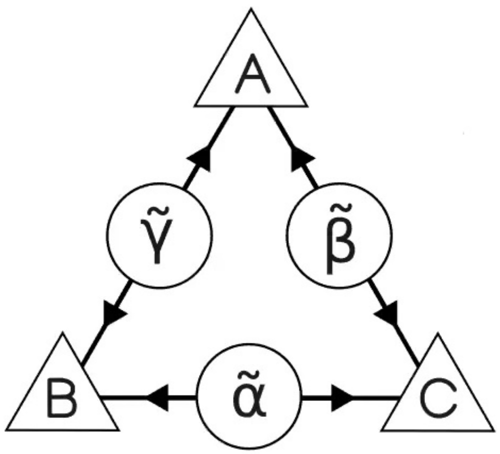


Context



$$p(abc) = \mathrm{Tr}\left(\rho_\gamma \otimes \rho_\alpha \otimes \rho_\beta M^a \otimes M^b \otimes M^c\right) \neq \int d\alpha\, d\beta\, d\gamma\, p_A(a|\beta\gamma) p_B(b|\gamma\alpha) p_C(c|\alpha\beta) p(\alpha) p(\beta) p(\gamma)$$

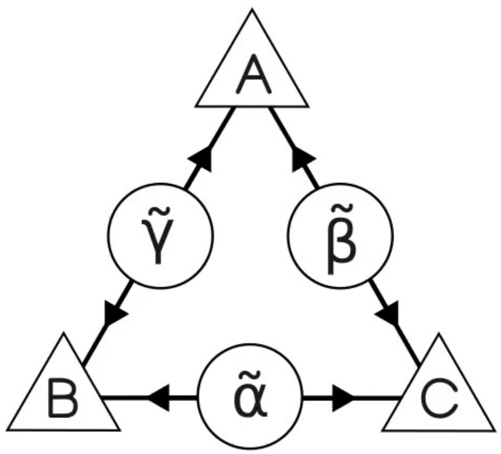
quantum

(non–)local

(non–)classical

[Renou et al., PRL 123, 140401 (2019)]
[Gisin, Entropy 21, 325 (2019)]

Context



Why triangle?

Simple network, yet many questions.

Genuine qu. nonlocality exists. (RGB4)

Why symmetric distributions?

Not RGB4.

Genuine nonlocality exists? (Elegant)

$$p(abc) = \mathrm{Tr}\left(\rho_\gamma \otimes \rho_\alpha \otimes \rho_\beta M^a \otimes M^b \otimes M^c\right) \neq \int d\alpha\, d\beta\, d\gamma\, p_A(a|\beta\gamma) p_B(b|\gamma\alpha) p_C(c|\alpha\beta) p(\alpha) p(\beta) p(\gamma)$$

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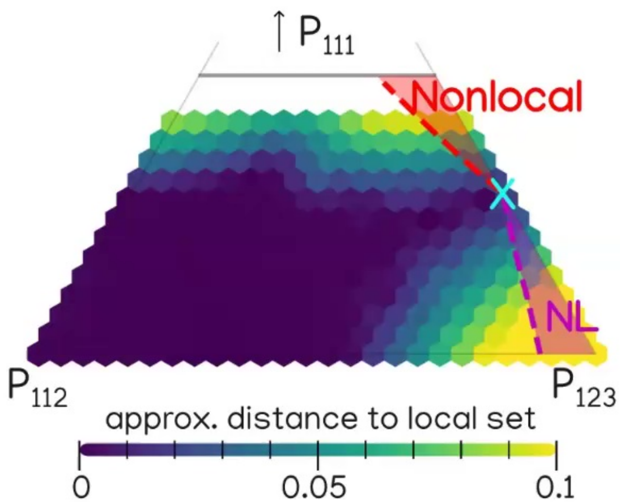
Goal



What techniques?

→ Neural-network techniques for conjecturing Bell inequalities.

→ Rigidity-based techniques for nonlocality.

What did we learn?

→ It is hard for classical models to exhibit GHZ-type correlations AND be symmetric.

→ A feel for why EJM distr. is nonlocal

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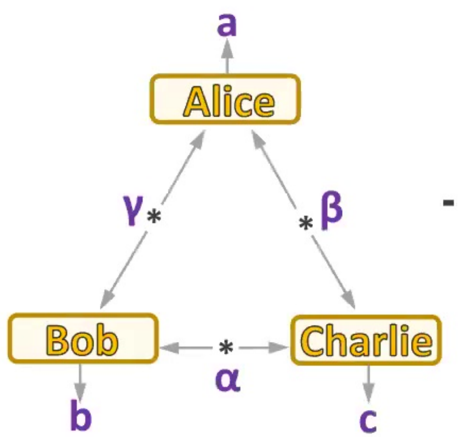
LHV-Net

$a \in \{1, 2, 3, 4\}$



Quantum: $p(abc) = \mathrm{Tr}\left(\rho_{A_1 B_2} \otimes \rho_{B_1 C_2} \otimes \rho_{C_1 A_2} M^a_{A_1 A_2} \otimes M^b_{B_1 B_2} \otimes M^c_{C_1 C_2}\right)$

Local: $p(abc) = \int d\alpha\, d\beta\, d\gamma\, p_A(a|\beta\gamma) p_B(b|\gamma\alpha) p_C(c|\alpha\beta)$

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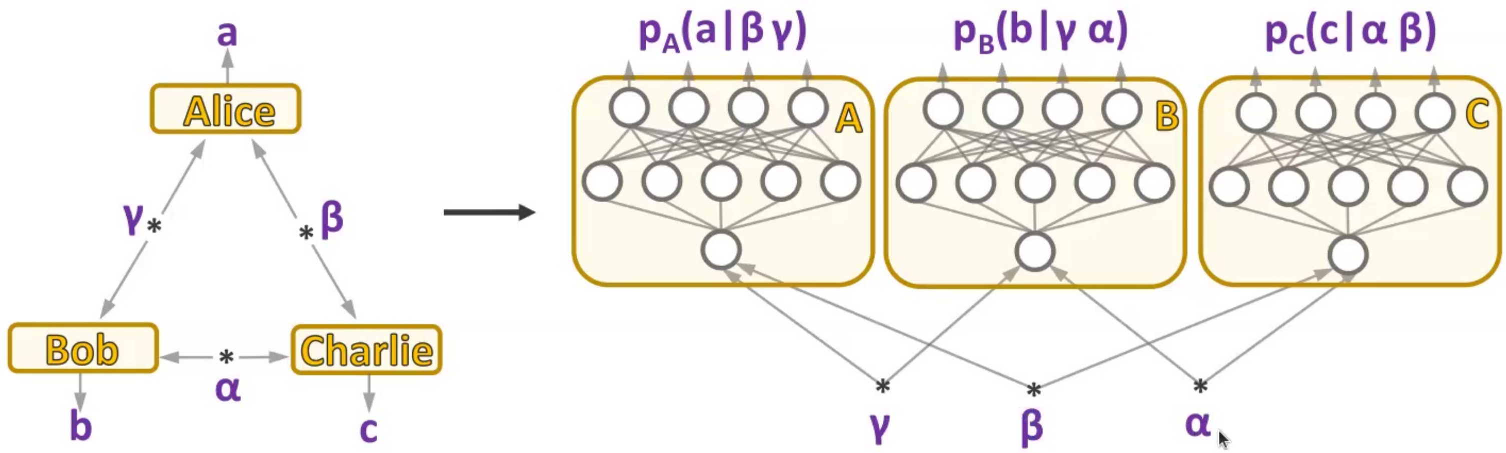
LHV–Net

$a \in \{1, 2, 3, 4\}$



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[TK et al., npj Quantum Inf. 6, 70 (2020)]

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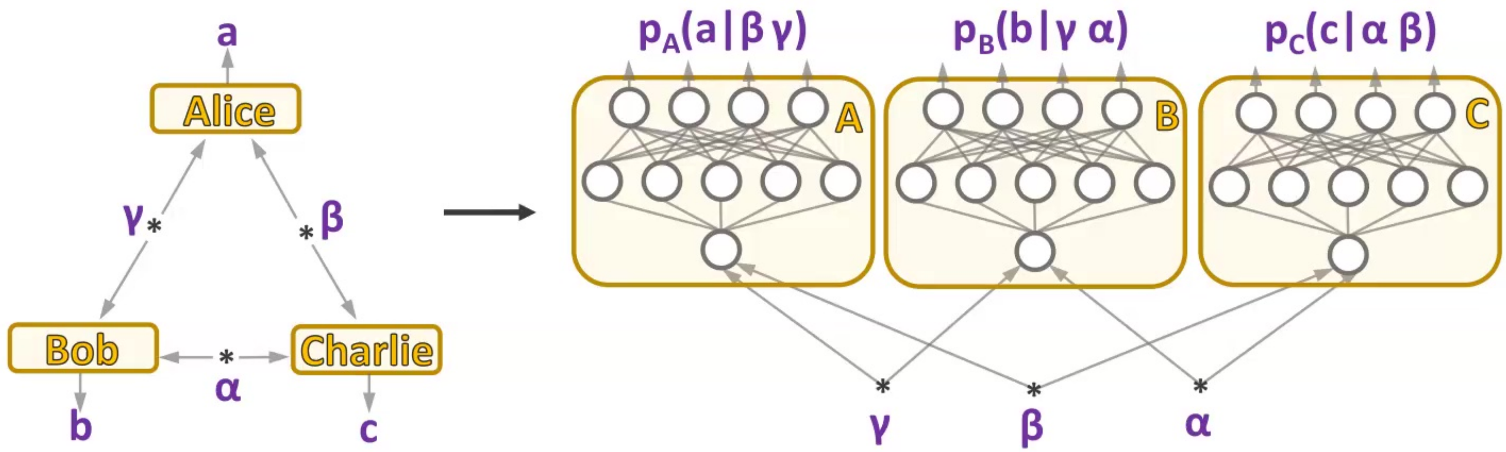

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Local: $p(abc) = \int d\alpha \, d\beta \, d\gamma \, p_A(a|\beta\gamma) p_B(b|\gamma\alpha) p_C(c|\alpha\beta)$

Goal: min dist. → Heuristic to find closest **local** distribution to a target

[TK et al., npj Quantum Inf. 6, 70 (2020)]

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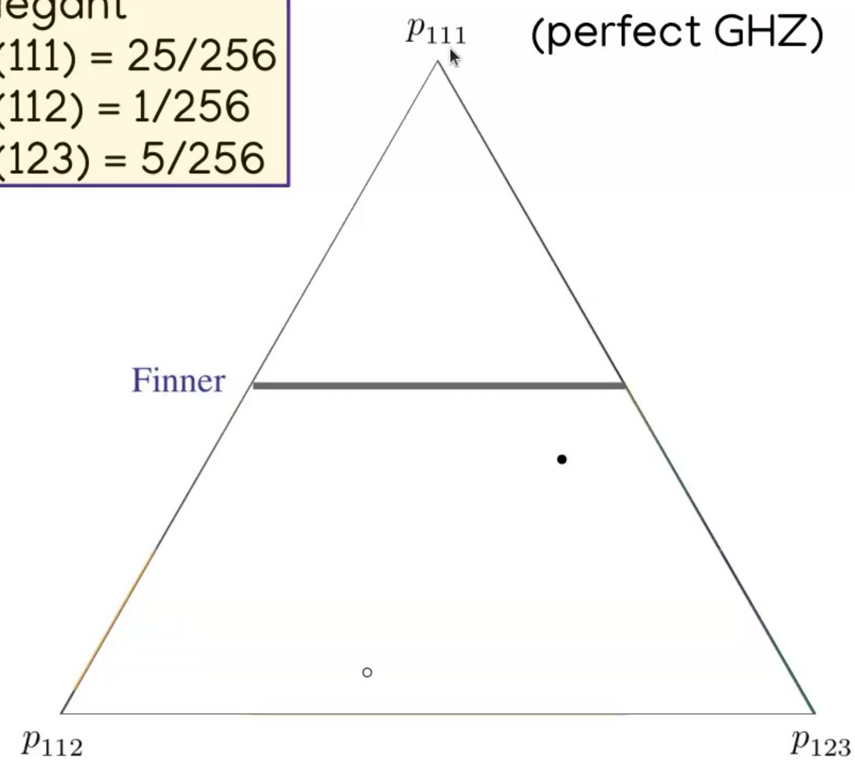
Symmetric subspace

Symmetric (N=4)
p(111) = p(222) = … = p(a=b=c)
p(112) = p(313) = … = p(a=b≠c) = cycl.
p(123) = p(243) = … = p(a≠b≠c≠a)

Elegant
p(111) = 25/256
p(112) = 1/256
p(123) = 5/256

Symmetric subspace
$s_{111} = \sum p(a=b=c)$
$s_{112} = \sum p(a=b\neq c)$ = cycl.
$s_{123} = \sum p(a\neq b\neq c\neq a)$
$s_{111} + s_{112} + s_{123} = 1$

$$|\Phi_i\rangle = \frac{\sqrt{3}+1}{2\sqrt{2}}\,|\vec{m}_i, -\vec{m}_i\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}}\,|-\vec{m}_i, \vec{m}_i\rangle$$



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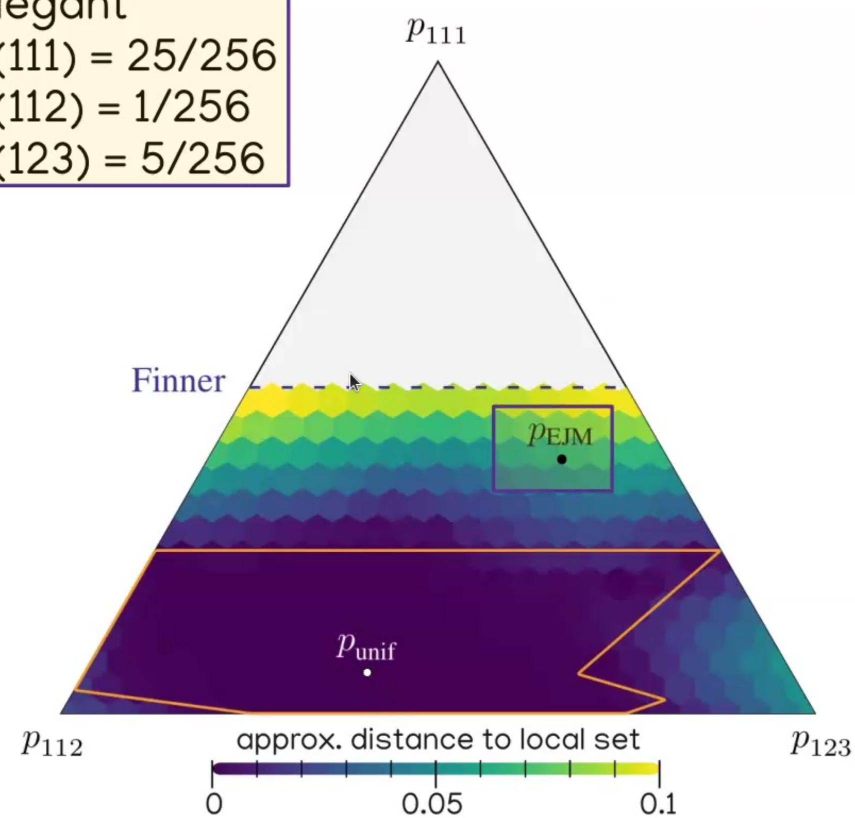

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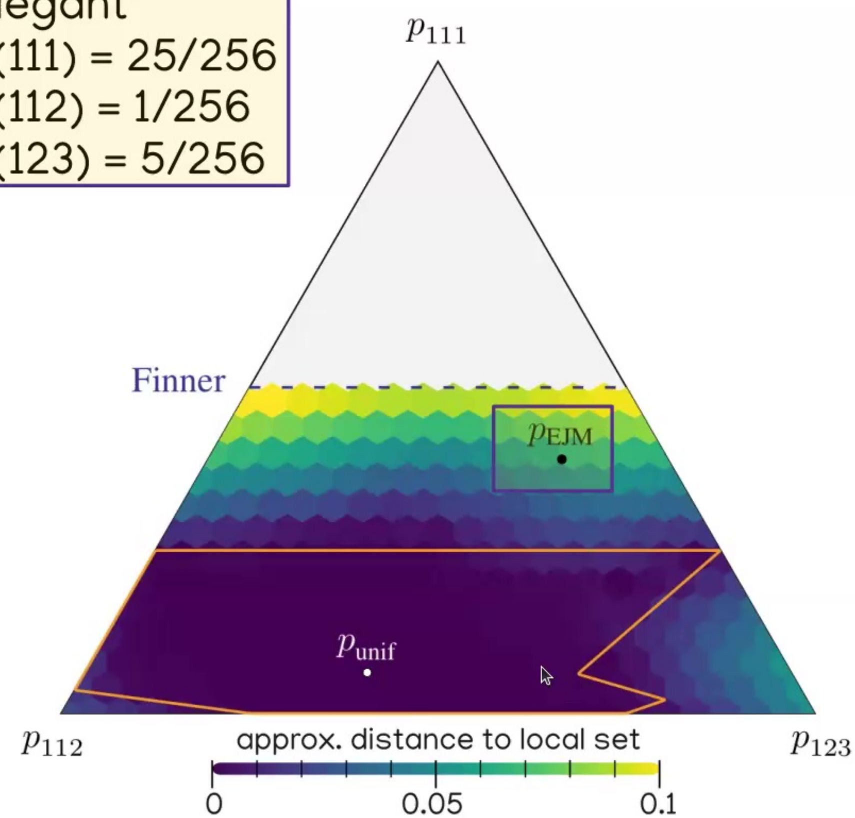
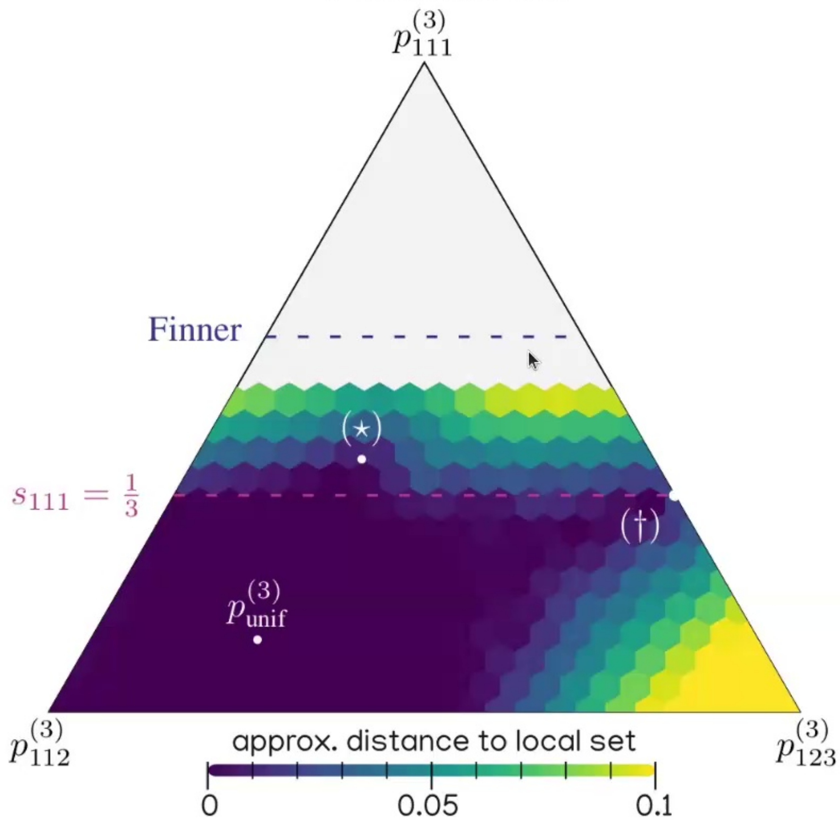
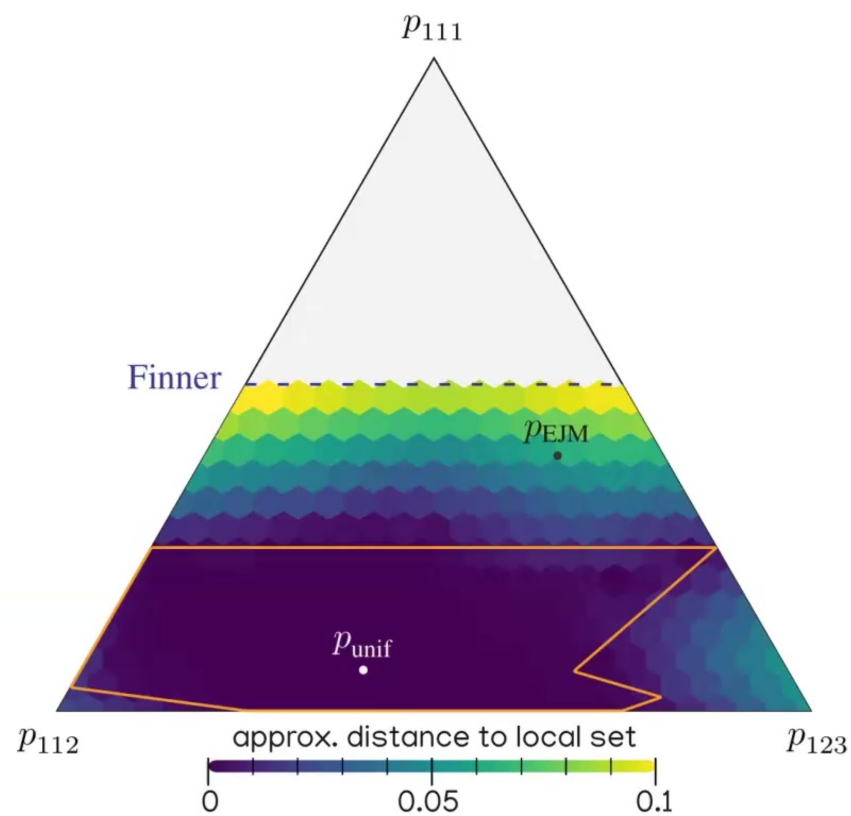


Symmetric subspace

3 outcomes

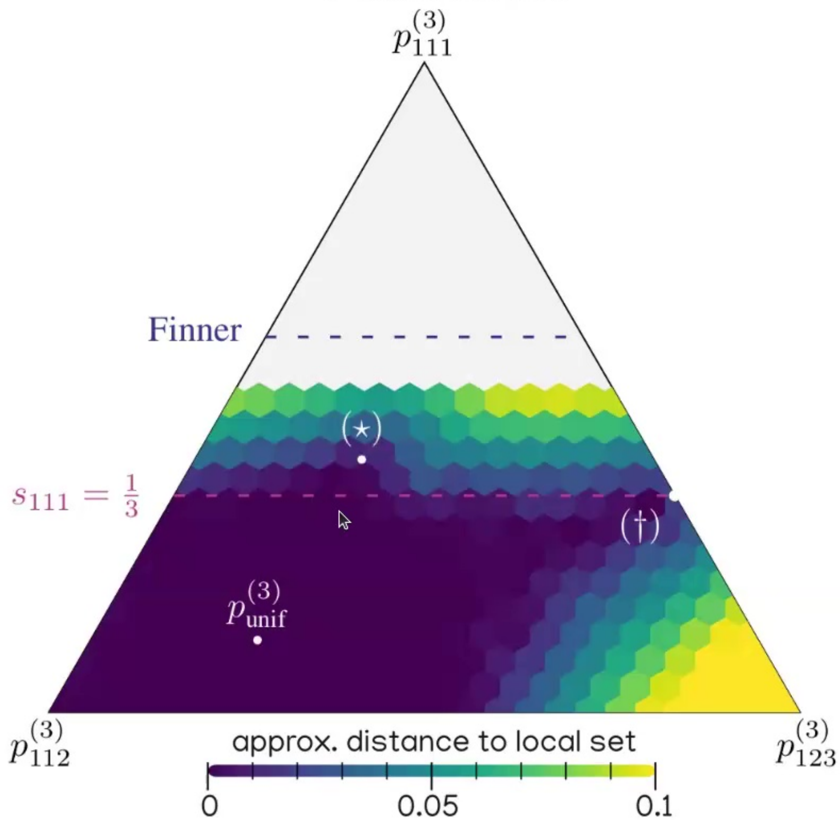


4 outcomes

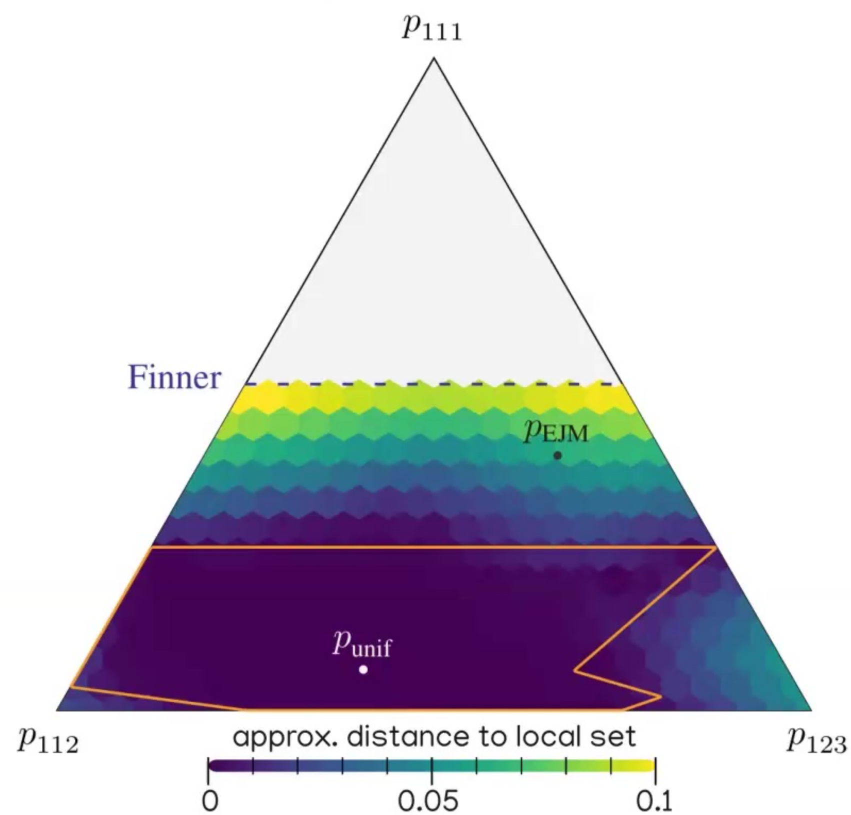


Symmetric subspace

3 outcomes



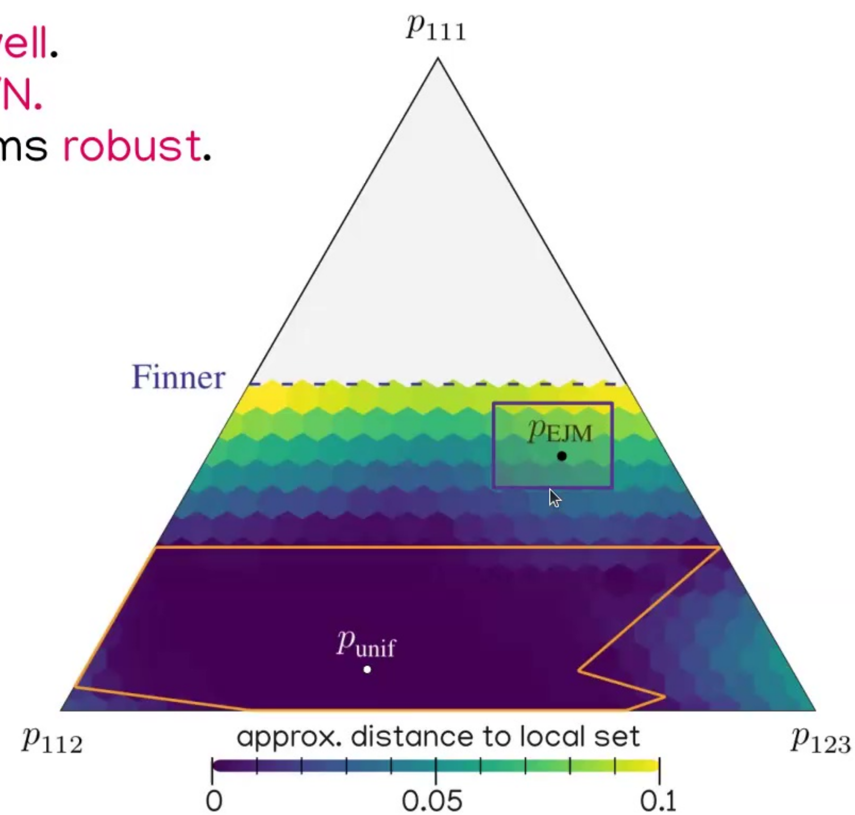
4 outcomes



Symmetric subspace

Take-away:

- Analytic modeling and numerics agree well.
- Local models can reach s_{111} of approx. 1/N.
- Nonlocality in symmetric subspace seems robust.



A Bell inequality

Symmetric subspace lesson: it is hard to be symmetric and have high 111-type correlations simultaneously.

$$\text{max.} \quad w \cdot s_{111} - (1-w)\Delta_l < w\frac{100}{256} \approx w \cdot 0.39$$

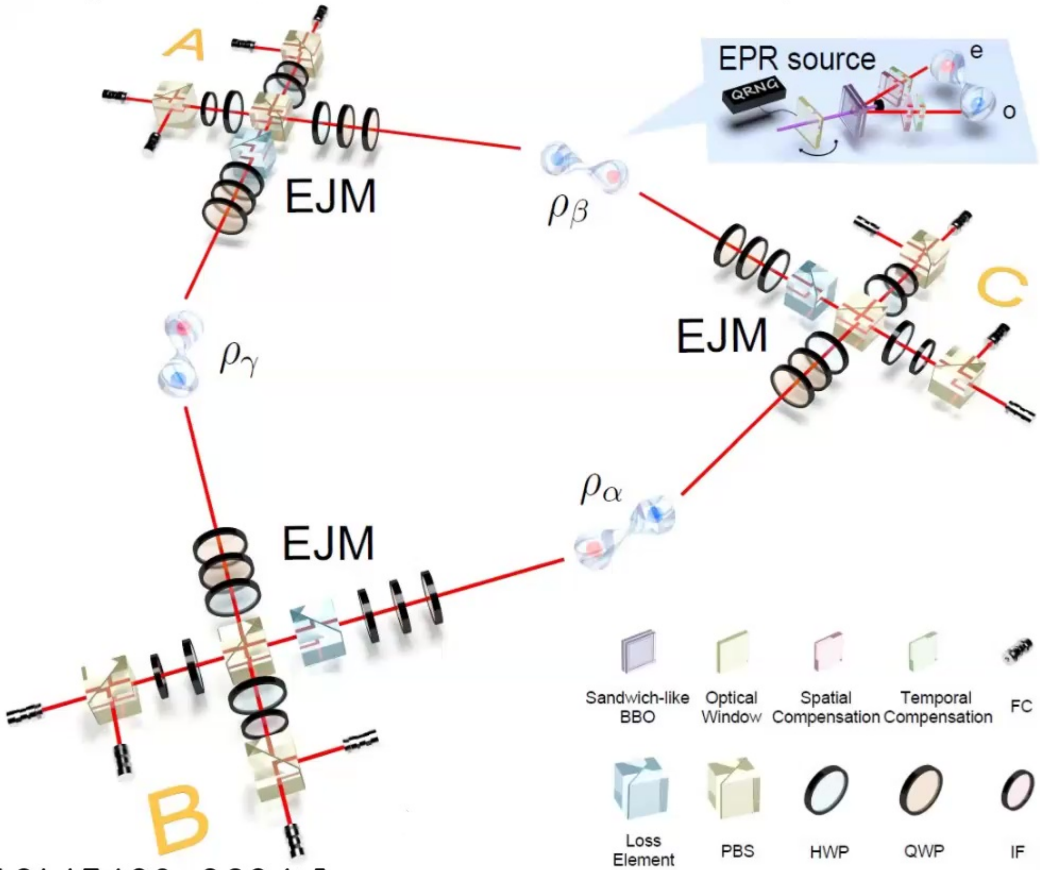
Penalty for being asymmetric

$$\Delta_l = \sum_{X \in \{111,112,123\}} \Delta_{l,X},$$

$$\Delta_{l,X} = \sum_{\{a,b,c\} \in \mathcal{I}_X} |M_X - p(a,b,c)|^l,$$

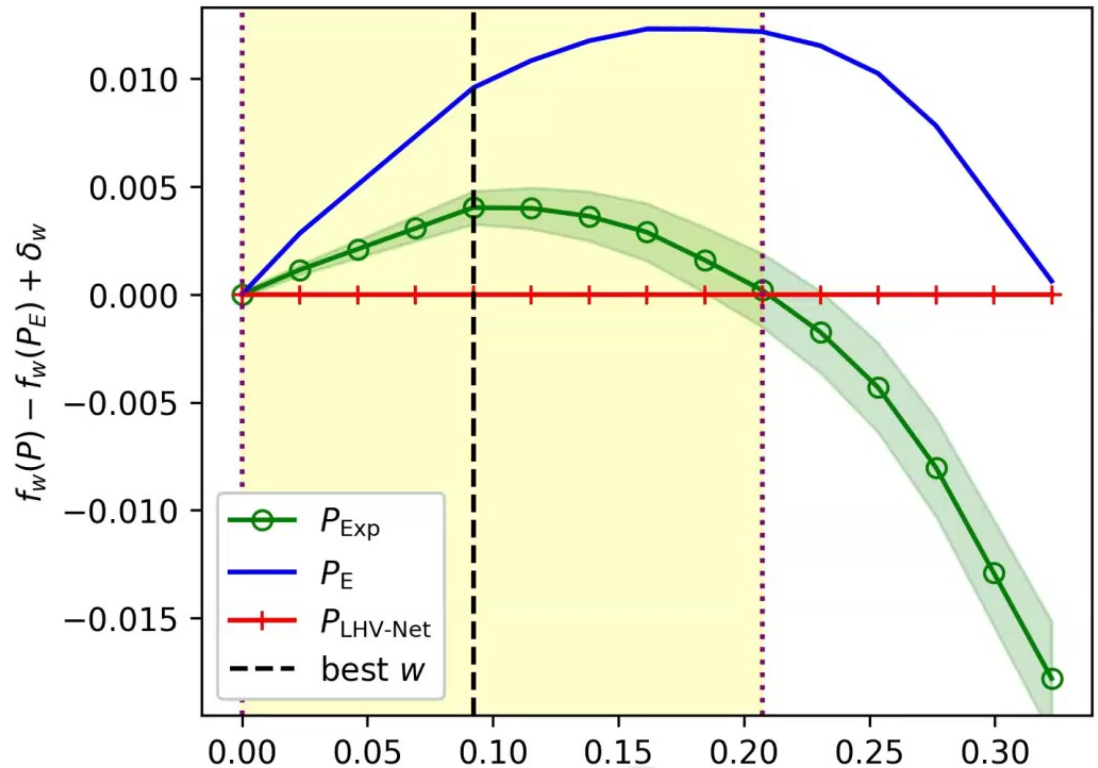
$$M_X = \frac{1}{|\mathcal{I}_X|} \sum_{\{a,b,c\} \in \mathcal{I}_X} p(a,b,c),$$

Elegant distribution: Experiment



[Wang et al., arXiv:2401.15428 (2024)]

Elegant distribution: Experiment



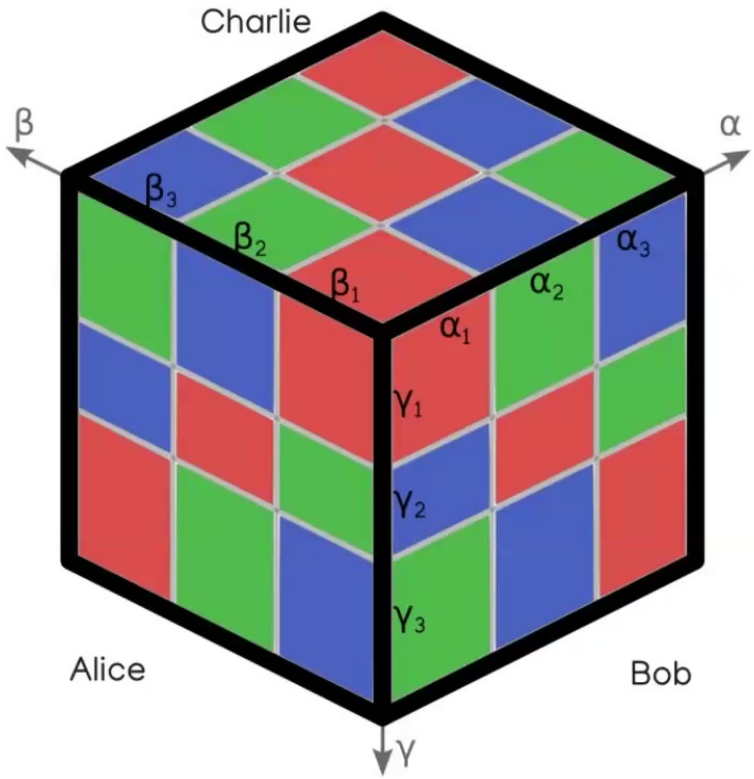
[Wang et al., arXiv:2401.15428 (2024)]

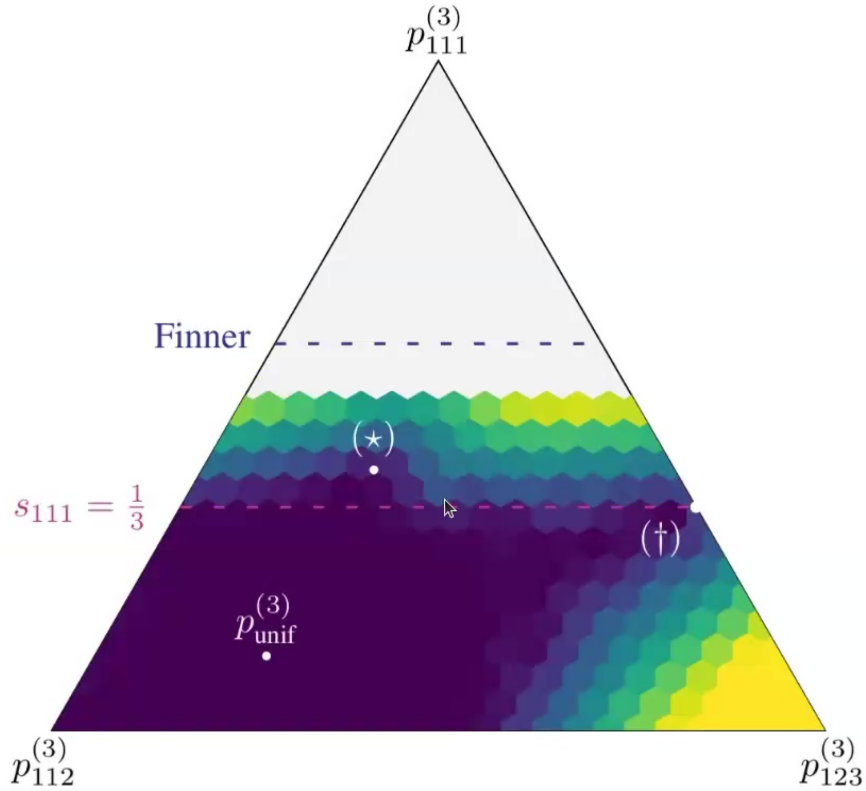
Ok but why?

Why are classical models limited like this?

Cube picture (3 outcomes)

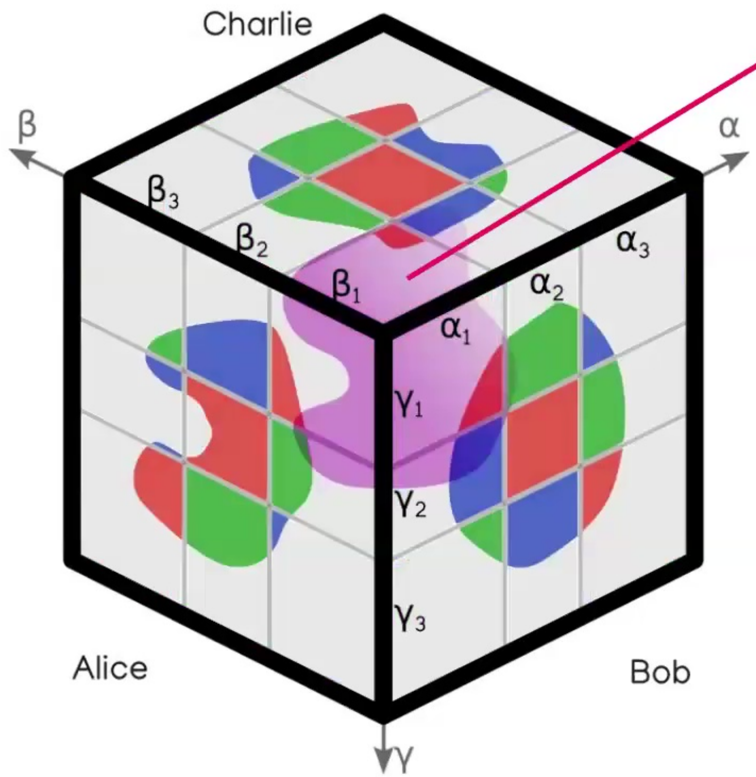
ONLY classical strategy such that no (1,1,2)-type outcomes and nonzero probability of all other outcomes. Rigidity.





Cube picture (3 outcomes)

How about a little bit of (1,1,2)-type outcomes?



All (1,1,1) and (1,2,3)-type outcomes.

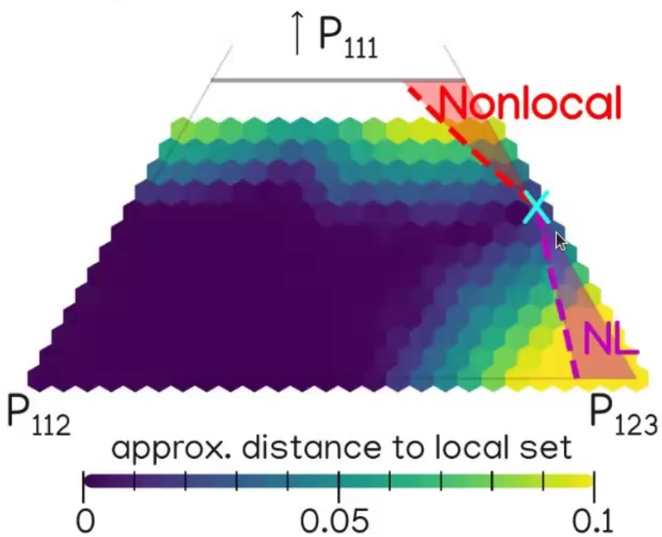
The response function parts fixed by this volume already define almost the whole distribution.

Caveat: ordering of rows/columns could be different in different regions...

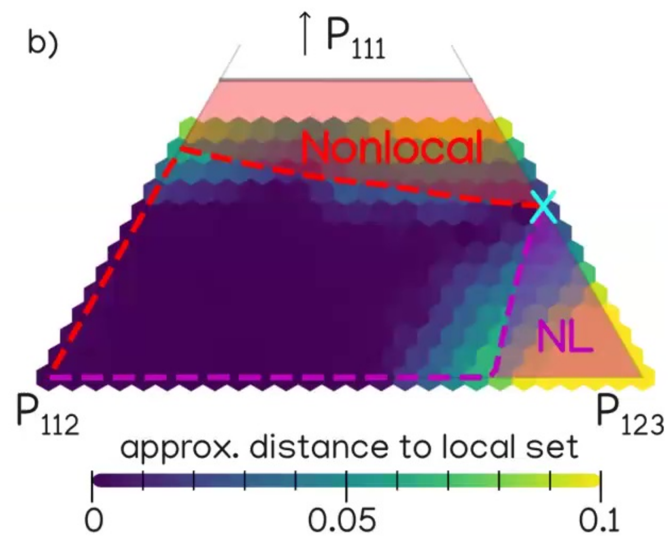
Outlook

Recall: x amount of (1,1,2)–volume allowed.

Proof for $\approx(1-3x)$ volume having rigid structure.



What if $(1-x)$ volume had rigid structure?



Summary

Take-home message

→ Numerical and analytical heuristics give excellent map of sym. subspace.

→ Insights lead to a starting point for rigidity-based proofs of nonlocality, *beyond* RGB4's rigidity-based proof.

→ Rigidity-based proofs are insightful and easy to learn from.

→ Hopefully strong enough to work for EJM-distribution. (???)

→ +1: Have a Bell inequality idea? Use LHV-Net!

Thank you for the attention!