

Title: Observational-Interventional Bell Inequalities

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Series: Quantum Foundations, Quantum Information

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Observational-Interventional inequalities

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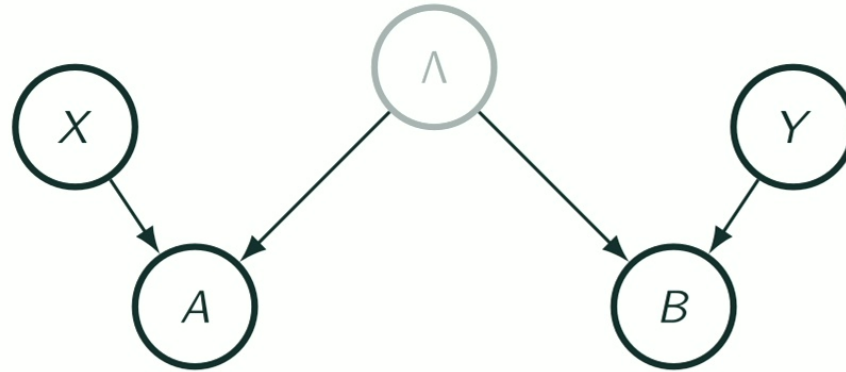


CAUSALWORLDS

at



Bell Causal Scenario



Bell theorem assumptions are encoded in the DAG.

Decomposition of the distribution

$$p(a|x\lambda), p(b|y\lambda), p(\lambda)$$

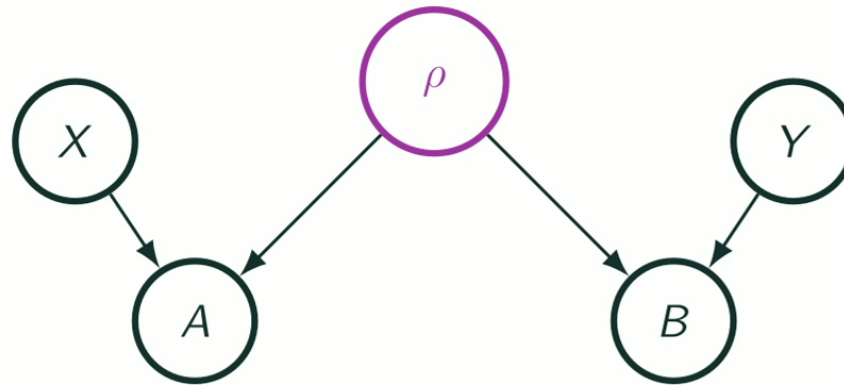
$$p(a, b|x, y) = \sum_{\lambda} p(a|x, \lambda)p(b|y, \lambda)p(\lambda)$$

↓

Bell inequalities.

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Witnessing Nonclassicality



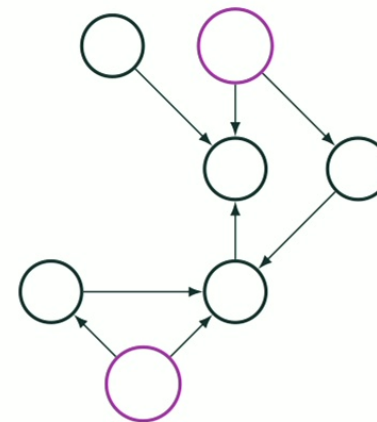
Violation of inequalities means

- Wrong causal assumptions.
- Nonclassicality of the source.

Beyond Bell Scenario

We can do it with arbitrary scenarios:

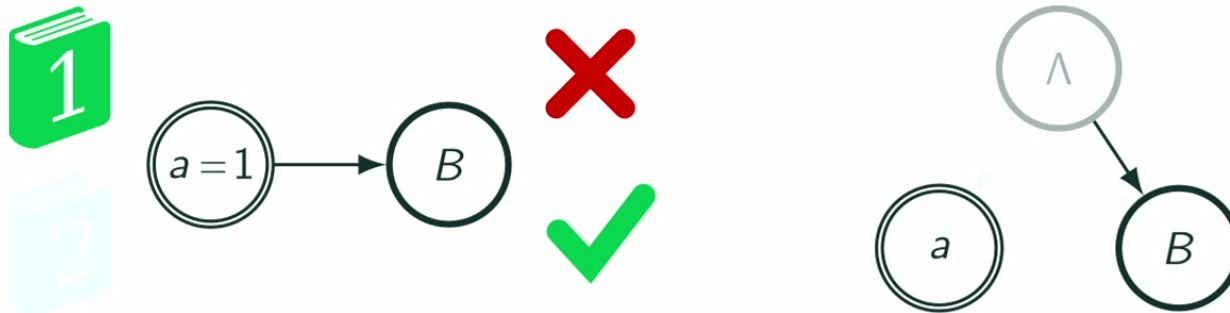
1. Model the scenario with a DAG.
 2. Derive the constraints.
 3. Use them as a witness of nonclassicality (if possible).
- Explore different forms of nonclassicality.
 - Model real situations with complicated networks topologies.
 - Use tools from Causal Modeling.



Using Interventions

Quantifying Causal Influence

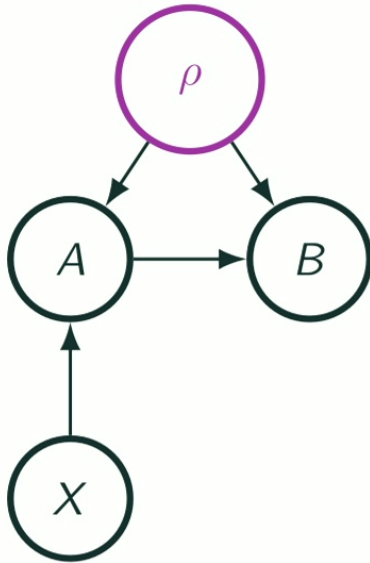
Possible causal explanations for correlated variables A and B :



- Intervention: $\text{do}(a)$, Forces $A = a$ erasing incoming edges.
- Quantifier of causal influence (Average Causal Effect):

$$ACE_{A \rightarrow B} = \max_{a, a', b} |p(b | \text{do}(a)) - p(b | \text{do}(a'))|$$

Interventions and observations (Instrumental scenario)



$$F(\{p(a, b|x)\}_{a,b,x}) \geq \underset{A \rightarrow B}{ACE} \geq f(\{p(a, b|x)\}_{a,b,x}).$$

Interventions are **constrained** by observations

Suppose I can measure both

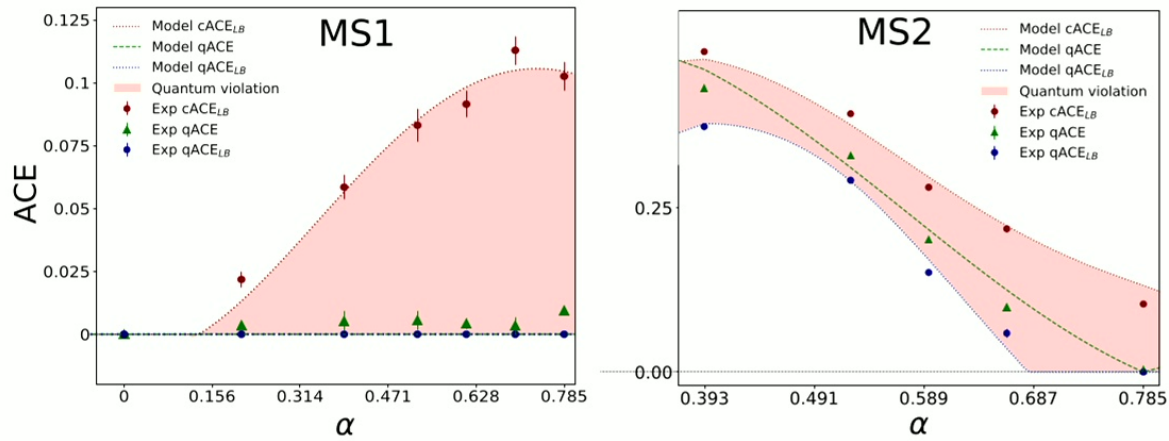
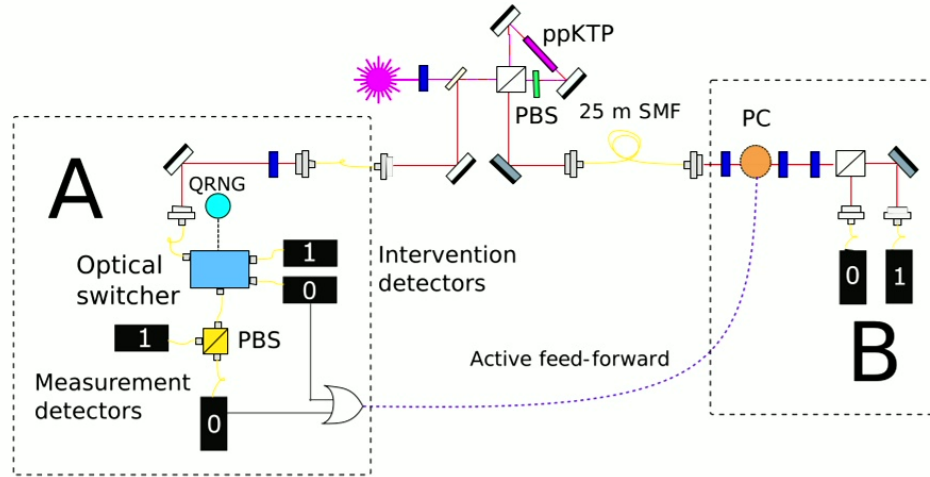
- Observations: $P_{\text{obs}} = \{p(ab|x)\}_{a,b,x}$.
- Interventions: $P_{\text{do}(a)} = \{p(b|\text{do}(a))\}_{b,a}$.

I have a compatibility inequality.



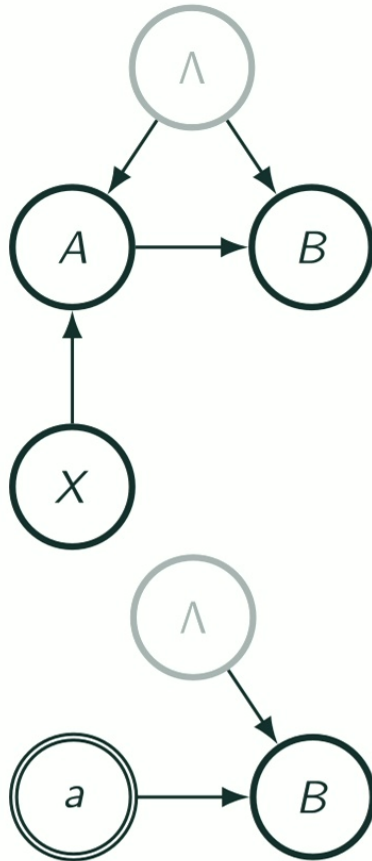
Nonclassicality witness

Demonstrated in a photonic platform:



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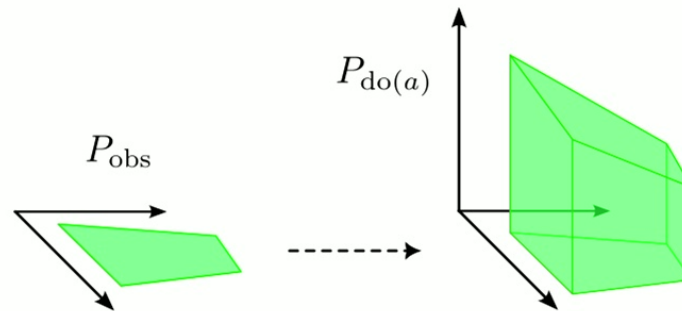
The complete description



Observational-Interventional probability vector

- Observational data: $P_{\text{obs}} = \{p(ab|x)\}_{a,b,x}$.
- Interventional: $P_{\text{do}(a)} = \{p(b|\text{do}(a))\}_{b,a}$.
- $P = (P_{\text{obs}}, P_{\text{do}(a)})$.

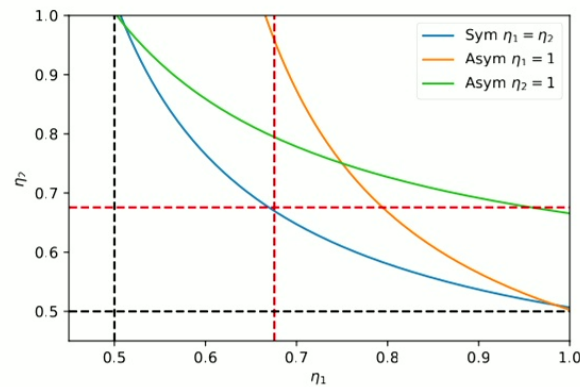
Set of compatible P s



Why is it useful?

- Observational only \rightarrow classical **but** Observation+Interventions \rightarrow nonclassical (example: 222 Instrumental scenario).
- Detects nonclassicality for observationally classical states (better noise robustness).

Example: Detection efficiencies: $\eta_{\text{obs}} \approx 0.92$ while $\eta_{\text{int}} \approx 2/3$



Similar to Bell?

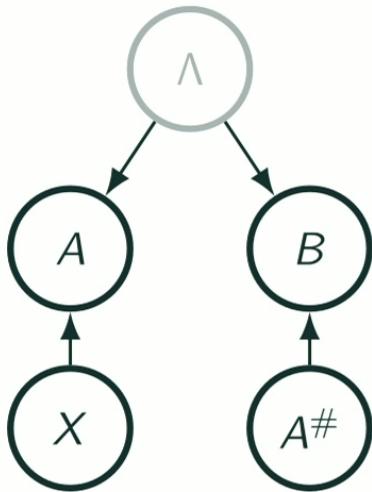
Dichotomic case

Simple description

For the case $|X| = m$ $|A| = |B| = 2$.

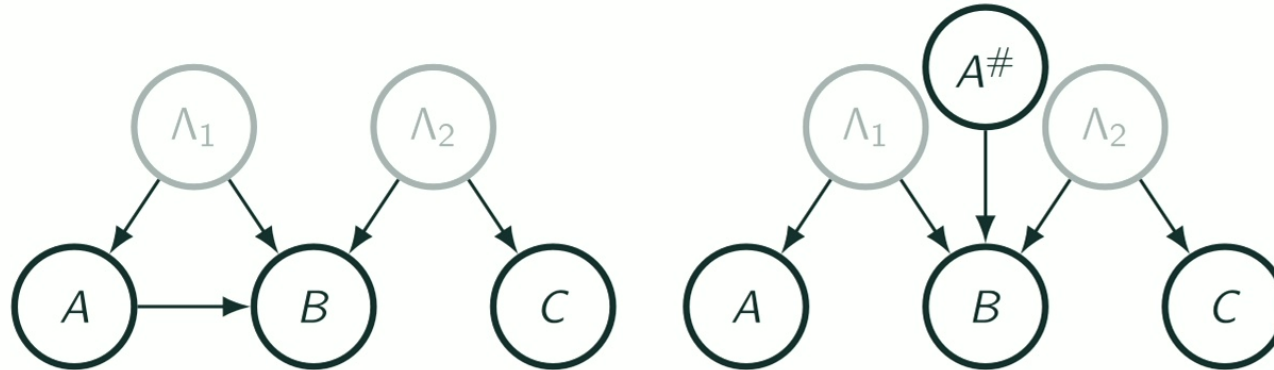
$$p(b|do\ a) - p(a, b|x) \geq 0$$

$$p(b|do\ a) - p(a, b|x') + p(a, \bar{b}|x) + p(\bar{a}, b|x) - p(\bar{a}, b|x') \geq 0$$



- Mapping to **observational** distribution:
 $p(a, b|x) = p(a, b|x, a^\# = a)$.
- Mapping to **interventional** distribution:
 $p(b|do(a = a^\#)) = \sum_a p(a, b|a^\#, x)$.
- Mapping back
 $p(\bar{a}, b|x, a^\# = a) = p(b|do(a)) - p(a, b|x)$.
- **Hardy inequality.**
- Not bijective in general.

General scenarios

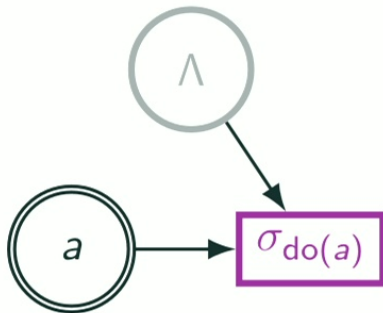
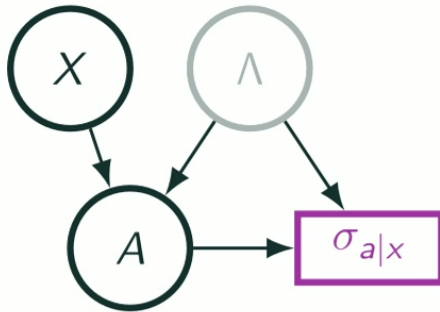


- **Interruption:** Split the node: $p(a, b, c) \rightarrow p(a, a^\#, b, c)$.
- Formulate the problem in the interrupted graph.
- Mapping back to **observational** and **interventional** distributions.

Application \rightarrow **Pedro Lauand's** talk (tomorrow)



Semi-DI (Steering)



Assemblage $\{\sigma_{a|x}\}_{a,x}$

- Classical: $\sigma_{a|x} = \sum_{\lambda} p(\lambda) p(a|x, \lambda) \rho_{a,\lambda}^B$
- Quantum: $\sigma_{a|x} = \text{Tr}_A [M_x^{(a)} \otimes \mathcal{E}_a(\rho_{\Lambda})]$
- Classical: $\sigma_{\text{do}(a)} = \sum_{\lambda} p(\lambda) \rho_{a,\lambda}^B$
- Quantum: $\sigma_{\text{do}(a)} = \text{Tr}_A [I \otimes \mathcal{E}_a(\rho_{\Lambda})]$

Extended assemblage $\{\sigma_{a|x}\}_{a,x}, \{\sigma_{\text{do}(a)}\}_a$

- Still describes a convex set
- Opt/Witness can be obtained by SDP
- Nonclassicality for $|X| = 2$

Take home

Interventions give precious data for nonclassicality witnesses.
(Simple scenarios, Networks, semi-DI)

Directions

- Protocols using interventions?
- More general kinds interventions (see Rafael Chaves' talk this morning).

Thank You!

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Pedro Lauand, Davide Poderini, Rafael Rabelo, et al. (2024). **“Quantum non-classicality in the simplest causal network”**. In: *arXiv preprint arXiv:2404.12790*



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