Title: Zero Inflation as a Missing Data Problem: a Proxy-based Approach

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Series: Quantum Foundations, Quantum Information

Date: September 16, 2024 - 2:10 PM

URL: https://pirsa.org/24090108

Zero Inflation as a Missing Data Problem: a Proxy-based Approach

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Content

1. Zero Inflation Data
2. Review of missing data
3. Zero Inflation graphical models
4. Non-identifiability
5. Partial Identification
6. Application

What is Zero Inflation?

Age	Therapy	Device	Recorded infection
23	1	3	0
42	2	2	0
67	3	1	1
34	2	3	1

Covariates/Features Outcome

What is Zero Inflation?

			Current test results	If all tests were done
Age	Therapy	Device	Recorded infection	True infection
23	1	3	0	1
42	2	2	0	0
67	3	1	1	1
34	2	3	1	1

Covariates/Features | Outcome | Hidden

Goals in a Zero Inflation problem

Age	Therapy	Device	Recorded infection	True infection
23	1	3	0	1
42	2	2	0	0
67	3	1	1	1
34	2	3	1	1

Goals in a Zero Inflation problem

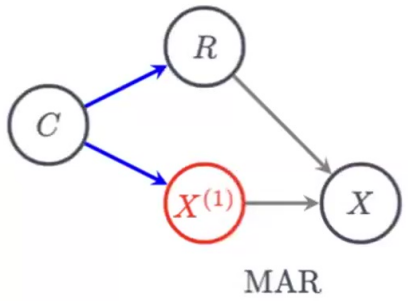
Age	Therapy	Device	Recorded infection	True infection
23	1	3	0	1
42	2	2	0	0
67	3	1	1	1
34	2	3	1	1

Which statistical model fit the data the best?
Pr(Age, Therapy, Device, Recorded Infection)

- Hurdle model
- ⇒ Zero-inflated Poisson ⇐
- …

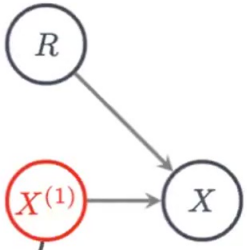
Missing data: the cousin

Age (C1)	Therapy (C2)	Device (C3)	Recorded infection (X)	True infection (X(1))	Indicator (R)
23	1	3	?	1	0
42	2	2	0	0	1
67	3	1	1	1	1
34	2	3	?	0	0



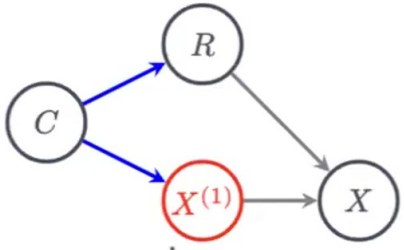
Consistency: when R=1, X = X(1), when R=0, X = ?

Missing data identification for the target law p(X(1))



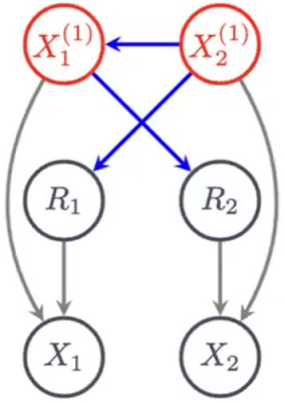
MCAR

$$p(X^{(1)}) = p(X \mid R = 1)$$



MAR

$$p(X^{(1)}) = \sum_c p(X \mid R = 1, c)p(c)$$



MNAR

If $\mathcal{G}$ has no self-censoring edge and no colluder, then $p(X^{(1)})$ is identified given $p(X, R)$

Nabi, Razieh, Rohit Bhattacharya, and Ilya Shpitser. 2020. "Full Law Identification in Graphical Models of Missing Data: Completeness Results." ICML.

Missing Data vs Zero Inflation

Missing data Consistency: when R=1, X = X(1), when R=0, X = ?

1) **Know** which values are incorrect (“?”)
2) Don’t know the true values for the incorrects

C1	C2	C3	X	R
23	1	3	**?**	0
42	2	2	0	1
67	3	1	1	1

ZI Consistency: when R=1, X = X(1), when R=0, X = 0

1) **Don’t know** which 0s are incorrect
2) Don’t know the true values for the incorrects

C1	C2	C3	X
23	1	3	**0**
42	2	2	**0**
67	3	1	1

Missing Data vs Zero Inflation

Missing data Consistency: when R=1, X = X(1), when R=0, X = ?

1) **Know** which values are incorrect (“?”)
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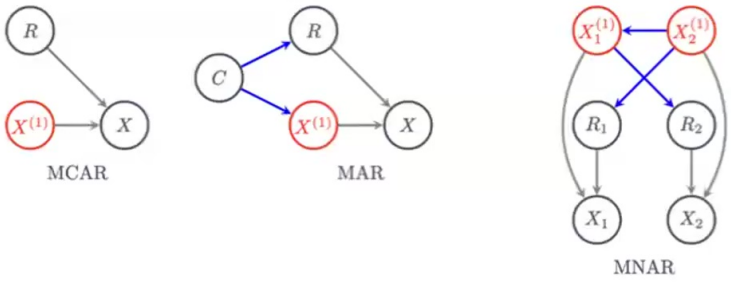

ZI Consistency: when R=1, X = X(1), when R=0, X = 0

1) **Don’t know** which 0s are incorrect
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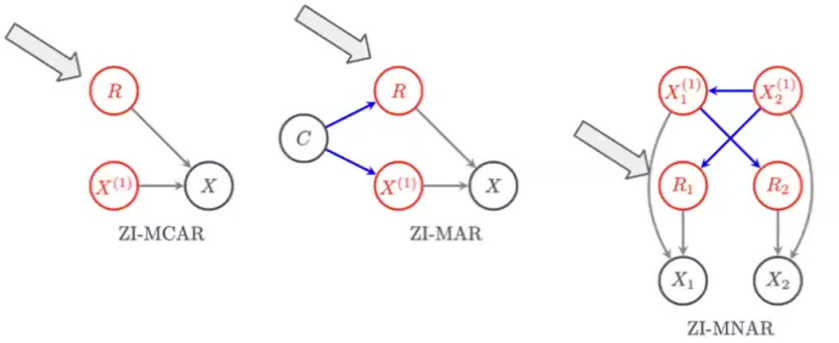
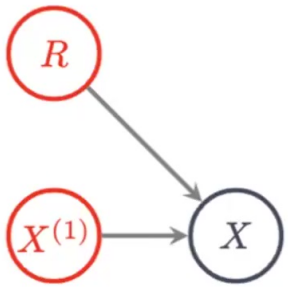


Zero Inflation non-id

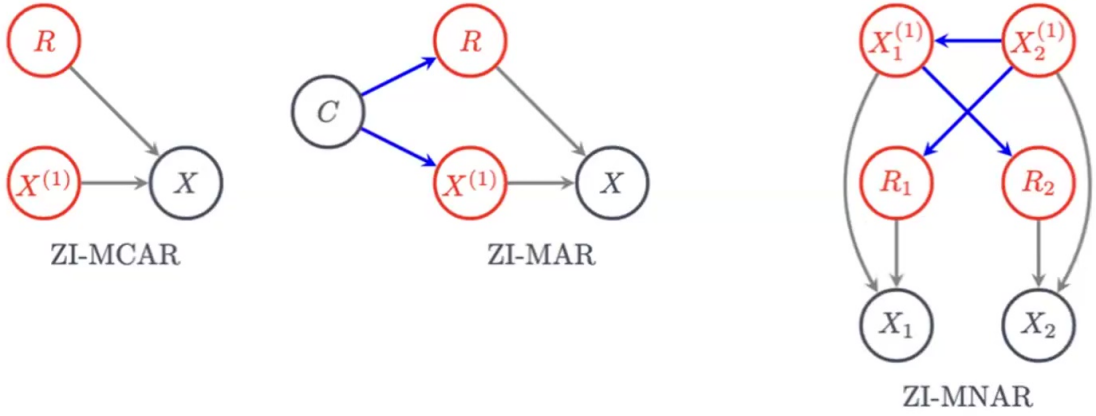
Now R is unobserved, is p(X(1)) identified from p(X)?



ZI-MCAR

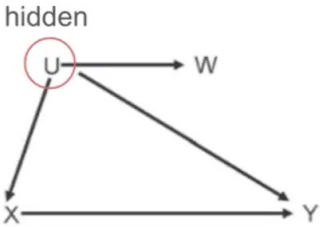
No. p(X(1)) in any ZI m-DAG is non-id non-parametrically.

Identification: what's next?



Recall: if we know p(X, R), we get identification ⇒ Find p(X,R)

Kuroki Pearl method



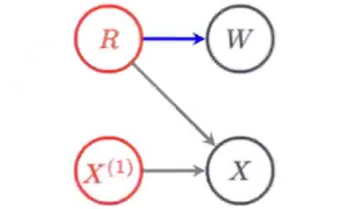
Question: Identify p(y|x,u) from p(x,y,w) and p(w|u)?

Observed proxy W of hidden U satisfying

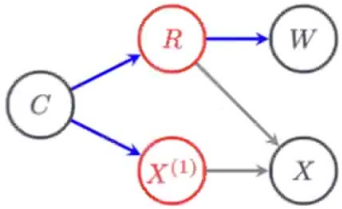
1. W is independent of everything given U
2. p(W|U) forms an invertible matrix
3. p(W|U) is given

Kuroki, Manabu, and Judea Pearl. 2014. “Measurement Bias and Effect Restoration in Causal Inference.” *Biometrika* 101 (2): 423–37.

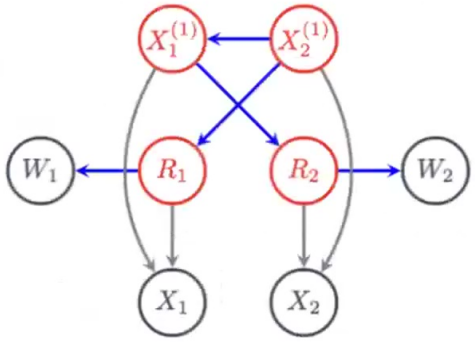
Kuroki Pearl method in ZI



Proxy-augmented ZI MCAR



Proxy-augmented ZI MAR



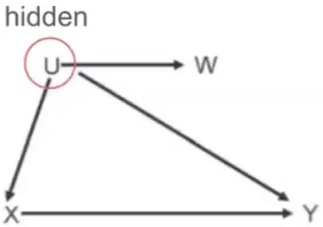
Proxy-augmented ZI MNAR

Question: Identify p(X, R) from p(X,W) and p(W|R)?

Conditions:

1) W indep others given R.
2) p(W|R) form an invertible matrix
3) p(W|R) is known.

Kuroki Pearl method



Question: Identify p(y|x,u) from p(x,y,w) and p(w|u)?

$$\mathrm{pr}(y, w \mid x) = \sum_{i=1}^{k} \mathrm{pr}(y, u_i \mid x)\mathrm{pr}(w \mid u_i).$$

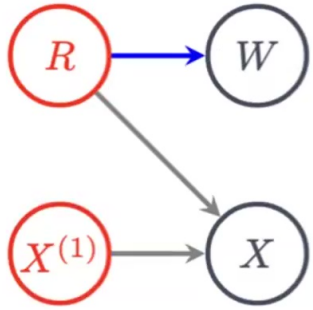
$$\underbrace{\begin{pmatrix} \mathrm{pr}(y, w_1 \mid x) \\ \vdots \\ \mathrm{pr}(y, w_k \mid x) \end{pmatrix}}_{V_{xy}(w)} = \underbrace{\begin{pmatrix} \mathrm{pr}(w_1 \mid u_1) & \cdots & \mathrm{pr}(w_1 \mid u_k) \\ \vdots & \ddots & \vdots \\ \mathrm{pr}(w_k \mid u_1) & \cdots & \mathrm{pr}(w_k \mid u_k) \end{pmatrix}}_{M(w,u)} \underbrace{\begin{pmatrix} \mathrm{pr}(y, u_1 \mid x) \\ \vdots \\ \mathrm{pr}(y, u_k \mid x) \end{pmatrix}}_{V_{xy}(u)}$$

$V_{xy}(w)$ (known) — $M(w,u)$ (known) — $V_{xy}(u)$ (unknown)

$$V_{xy}(u) = M(w, u)^{-1} V_{xy}(w).$$

Kuroki, Manabu, and Judea Pearl. 2014. “Measurement Bias and Effect Restoration in Causal Inference.” *Biometrika* 101 (2): 423–37.

Kuroki Pearl method in ZI



Proxy-augmented ZI MCAR

$$p(W, X) = \sum_r p(W \mid r) p(r, X)$$

$$\underbrace{\begin{pmatrix} p_{w_0,x_0} & p_{w_0,x_1} \\ p_{w_1,x_0} & p_{w_1,x_1} \end{pmatrix}}_{\mathbf{p}_{W,X}} = \underbrace{\begin{pmatrix} p_{w_0|r_0} & p_{w_0|r_1} \\ p_{w_1|r_0} & p_{w_1|r_1} \end{pmatrix}}_{\mathbf{p}_{W|R}} \underbrace{\begin{pmatrix} p_{r_0,x_0} & p_{r_0,x_1} \\ p_{r_1,x_0} & p_{r_1,x_1} \end{pmatrix}}_{\mathbf{p}_{R,X}}$$

$$\mathbf{p}_{R,X} = \mathbf{p}_{W|R}^{-1} \mathbf{p}_{W,X}$$

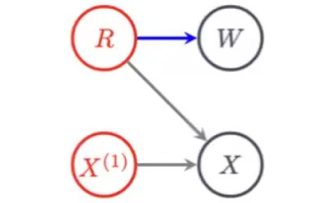
Kuroki-Pearl Step

Finally, identify p(X(1)) from p(x,w) and p(w|r)?

Know p(R, X) ⇒ know p(X(1)) = p(X|R=1)

missing data ID step

Kuroki Pearl method in ZI: Compatibility Problem



Proxy-augmented ZI MCAR

Conditions:

1) W indep others given R.
2) p(W|R) form an invertible matrix
3) p(W|R) is known.

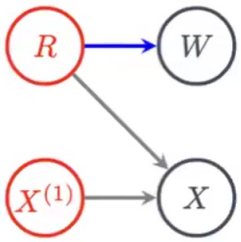
In ZI: p(W|R) is **not given** ⇒ guess

⇒ Not all choices of p(W|R) are compatible to the given p(W, X)
⇒ **Task:** Find compatible p(W|R)

Compatibility: There is some p(X(1), R, X, W) in the ZI model yielding both observed p(W,X) and the guess p(W|R).

Find compatibility set

Full model for p(X(1), X, R, W)



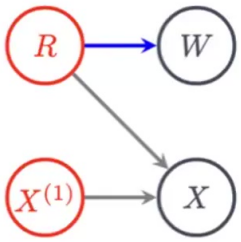
Proxy-augmented ZI MCAR

Model for marginal p(X, R, W)

$$\mathscr{P} = \left\{ (\mathbf{q}_{W|R}, \mathbf{q}_{RX}) \;\middle|\; \begin{array}{l} \mathbf{q}_{W|R} \geq 0, \forall r \left(\sum_w q_{w|r} = 1 \right), q_{w_0|r_0} \neq q_{w_0|r_1}, \\ \mathbf{q}_{RX} \geq 0, \sum_{rx} q_{rx} = 1, \forall x \neq 0 (q_{r_0 x} = 0) \end{array} \right\}.$$

⇒ Compatibility: There is some p(R, X, W) in the ZI model yielding both p(W,X) and p(W|R).

Compatibility bound



Proxy-augmented ZI MCAR

$$\underbrace{\begin{pmatrix} p_{w_0|r_0} & p_{w_0|r_1} \\ p_{w_1|r_0} & p_{w_1|r_1} \end{pmatrix}}_{\mathbf{p}_{W|R}}$$

identified

$$p_{w_0|r_1} = p_{w_0|x_1}$$

Find the compatible set by solving

$$\begin{aligned} \max_{q_{w_0|r_0}} \quad & \pm q_{w_0|r_0} \\ \text{s.t.} \quad & \mathbf{q}_{W|R}\mathbf{q}_{RX} = \mathbf{p}_{WX}, \\ & \mathbf{q}_{W|R} \geq 0, \forall r(\textstyle\sum_w q_{w|r} = 1), q_{w_0|r_0} \neq q_{w_0|r_1}, \\ & \mathbf{q}_{RX} \geq 0, \textstyle\sum_{rx} q_{rx} = 1, q_{w_0|r_1} = p_{w_0|x_1}. \end{aligned}$$

p(w=0|r=0) and p(RX) are free variables

⇒ quadratic program

⇒ Can be solve numerically

Compatibility bound: linearize

$$\begin{aligned}\max_{q_{w_0|r_0}} \quad & \pm q_{w_0|r_0} \\ \text{s.t.} \quad & \mathbf{q}_{W|R}\mathbf{q}_{RX} = \mathbf{p}_{WX}, \\ & \mathbf{q}_{W|R} \geq 0,\ \forall r\left(\textstyle\sum_w q_{w|r} = 1\right),\ q_{w_0|r_0} \neq q_{w_0|r_1}, \\ & \mathbf{q}_{RX} \geq 0,\ \textstyle\sum_{rx} q_{rx} = 1,\ q_{w_0|r_1} = p_{w_0|x_1}.\end{aligned}$$

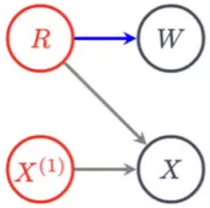
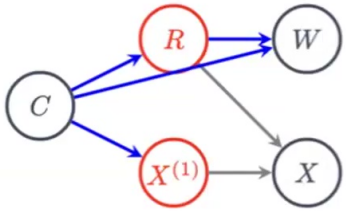
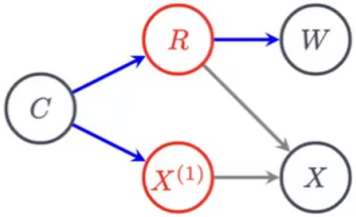
Just need to check it is a probability

$$\mathbf{q}_{RX} = [\mathbf{q}_{W|R}]^{-1}\mathbf{p}_{WX}.$$

$$\frac{1}{q_{w_0|r_0} - q_{w_0|r_1}} \begin{pmatrix} 1 - q_{w_0|r_1} & -q_{w_0|r_1} \\ q_{w_0|r_0} - 1 & q_{w_0|r_0} \end{pmatrix}$$

Linear program with only p(w=0|R=0) is the variable

$$\begin{aligned}\max_{q_{w_0|r_0}} \quad & \pm q_{w_0|r_0} \\ \text{s.t.} \quad & s \cdot \begin{pmatrix} 1 - q_{w_0|r_1} & -q_{w_0|r_1} \\ q_{w_0|r_0} - 1 & q_{w_0|r_0} \end{pmatrix} \mathbf{p}_{WX} \geq \mathbf{0}, \\ & s \cdot q_{w_0|r_0} > s \cdot q_{w_0|r_1}, 0 \leq q_{w_0|r_0} \leq 1, \\ & q_{w_0|r_1} = p_{w_0|x_1}.\end{aligned}$$

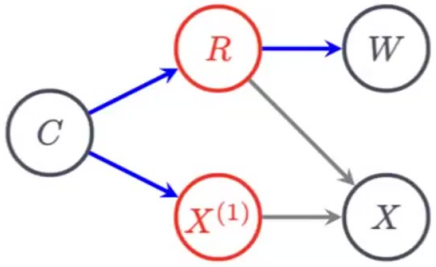
 Proxy-augmented ZI MCAR	 Proxy-augmented ZI MAR *	 Proxy-augmented ZI MAR
(A1) $W \perp\!\!\!\perp X^{(1)} \mid R$, **(A2)** The matrix $\mathbf{p}_{W\mid R}$ is invertible.	**(A1†)** $\forall i, W_i \perp\!\!\!\perp X^{(1)}, R_{-i} \mid C, R_i$. **(A2†)** The matrix $\mathbf{p}_{W\mid R,c}$ is invertible for every value c.	**(A1*)** $\forall i, W_i \perp\!\!\!\perp X^{(1)}, C, R_{-i} \mid R_i$. **(A2*)** The matrix $\mathbf{p}_{W\mid R}$ is invertible.
$\forall x \neq 0, p_{w_0\mid x} = p_{w_0\mid x_1}$	$\forall c, \forall x \neq 0, p_{w_0\mid x,c} = p_{w_0\mid x_1,c}$.	$\forall c, \forall x \neq 0, p_{w_0\mid x,c} = p_{w_0\mid x_1}$, *either* $\forall c \left(p_{w_0\mid x_0,c} \leq p_{w_0\mid x_1}\right)$ *or* $\forall c \left(p_{w_0\mid x_0,c} \geq p_{w_0\mid x_1}\right)$
$q_{w_0\mid r_1} = p_{w_0\mid x_1}$ $q_{w_0\mid r_0} \in \begin{cases} [p_{w_0\mid x_0}, 1] \text{ if } p_{w_0\mid x_0} > p_{w_0\mid x_1} \\ [0, p_{w_0\mid x_0}] \text{ if } p_{w_0\mid x_0} < p_{w_0\mid x_1} \\ (0,1) \setminus \{p_{w_0\mid x_0}\} \text{ if } p_{w_0\mid x_0} = p_{w_0\mid x_1} \end{cases}$	$q_{w_0\mid r_1,c} = p_{w_0\mid x_1,c}$ $q_{w_0\mid r_0,c} \in \begin{cases} [p_{w_0\mid x_0,c}, 1] \text{ if } p_{w_0\mid x_0,c} > p_{w_0\mid x_1,c} \\ [0, p_{w_0\mid x_0,c}] \text{ if } p_{w_0\mid x_0,c} < p_{w_0\mid x_1,c} \\ (0,1) \setminus \{p_{w_0,\mid x_0,c}\} \text{ if } p_{w_0\mid x_0,c} = p_{w_0\mid x_1,c} \end{cases}$	$q_{w_0\mid r_1} = p_{w_0\mid x_1}$ $q_{w_0\mid r_0} \in \begin{cases} [\max_c p_{w_0\mid x_0,c}, 1] \text{ if } \exists \tilde{c}, p_{w_0\mid x_0,\tilde{c}} > p_{w_0\mid x_1}, \\ [0, \min_c p_{w_0\mid x_0,c}] \text{ if } \exists \tilde{c}, p_{w_0\mid x_0,\tilde{c}} < p_{w_0\mid x_1}, \\ (0,1) \setminus \{p_{w_0\mid x_1}\} \text{ if } \forall c, p_{w_0\mid x_0,c} = p_{w_0\mid x_1}. \end{cases}$

CLABSI

Covariates C | Outcome (Y) | Proxy (W)

Peds	Chemotherapy	OPAT	TPN	OtherTherapy	PICC	Port	TunneledCVC	NHSN_CLABSI	EPIC
0	0	1	0	0	1	0	0	0	0
1	0	0	1	0	0	0	1	1	0
0	1	0	0	0	0	1	0	1	0
0	1	0	0	0	0	1	0	0	0
0	0	0	1	0	1	0	0	1	0
0	0	0	1	0	0	1	0	1	0

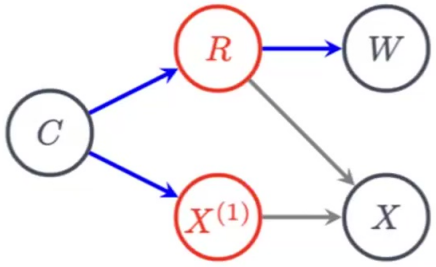
CLABSI



Proxy-augmented ZI MAR

Why R → W not W → R?

CLABSI

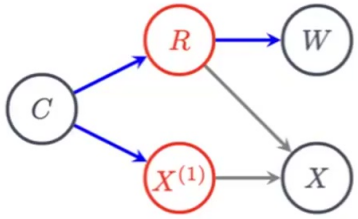


Proxy-augmented ZI MAR

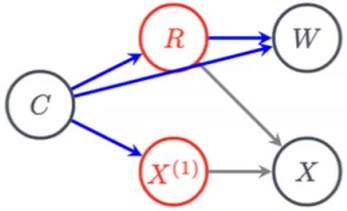
1. Full law model $p(X^{(1)}, X, R, W, C)$ Markov to the graph.
2. Fit $\hat{p}(X, W, C)$ using EM.
3. Compatible bound $p(W \mid R)$.
4. Grid search the bound
    1. for each $p(W \mid R)$ get a $p(R, X, C)$.
    2. True rate $p(X^{(1)}) = \sum_c p(X \mid R = 1, c)p(c)$.

$$q_{w_0|r_1} = p_{w_0|x_1}$$

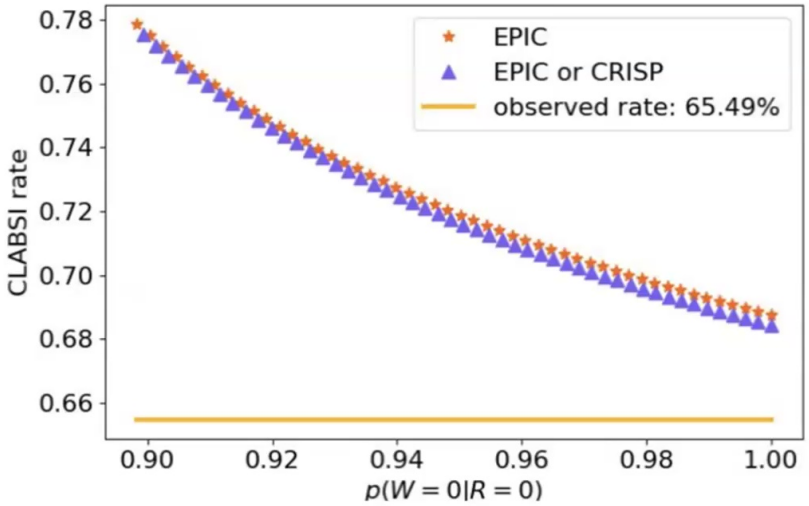
$$q_{w_0|r_0} \in \begin{cases} [\max_c p_{w_0|x_0,c}, 1] \text{ if } \exists \tilde{c}, p_{w_0|x_0,\tilde{c}} > p_{w_0|x_1} \\ [0, \min_c p_{w_0|x_0,c}] \text{ if } \exists \tilde{c}, p_{w_0|x_0,\tilde{c}} < p_{w_0|x_1} \\ (0,1) \setminus \{p_{w_0|x_1}\} \text{ if } \forall c, p_{w_0|x_0,c} = p_{w_0|x_1} \end{cases}$$



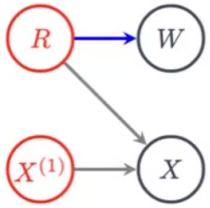
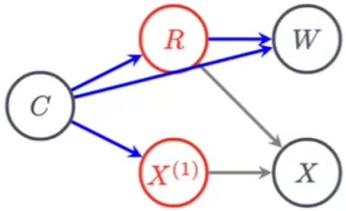
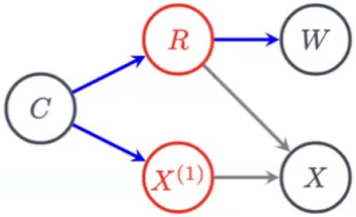
Proxy-augmented ZI MAR



Proxy-augmented ZI MAR *



CLABSI rate in [0.68, 1.0]

Proxy-augmented ZI MCAR	Proxy-augmented ZI MAR *	Proxy-augmented ZI MAR
		
(A1) $W \perp\!\!\!\perp X^{(1)} \mid R$, **(A2)** The matrix $\mathbf{p}_{W\mid R}$ is invertible.	**(A1†)** $\forall i, W_i \perp\!\!\!\perp X^{(1)}, R_{-i} \mid C, R_i$. **(A2†)** The matrix $\mathbf{p}_{W\mid R,c}$ is invertible for every value c.	**(A1*)** $\forall i, W_i \perp\!\!\!\perp X^{(1)}, C, R_{-i} \mid R_i$. **(A2*)** The matrix $\mathbf{p}_{W\mid R}$ is invertible.
$\forall x \neq 0, p_{w_0\mid x} = p_{w_0\mid x_1}$	$\forall c, \forall x \neq 0, p_{w_0\mid x,c} = p_{w_0\mid x_1,c}$.	$\forall c, \forall x \neq 0, p_{w_0\mid x,c} = p_{w_0\mid x_1}$, *either* $\forall c \left(p_{w_0\mid x_0,c} \leq p_{w_0\mid x_1}\right)$ *or* $\forall c \left(p_{w_0\mid x_0,c} \geq p_{w_0\mid x_1}\right)$
$q_{w_0\mid r_1} = p_{w_0\mid x_1}$ $q_{w_0\mid r_0} \in \begin{cases} [p_{w_0\mid x_0}, 1] \text{ if } p_{w_0\mid x_0} > p_{w_0\mid x_1} \\ [0, p_{w_0\mid x_0}] \text{ if } p_{w_0\mid x_0} < p_{w_0\mid x_1} \\ (0,1) \setminus \{p_{w_0\mid x_0}\} \text{ if } p_{w_0\mid x_0} = p_{w_0\mid x_1} \end{cases}$	$q_{w_0\mid r_1,c} = p_{w_0\mid x_1,c}$ $q_{w_0\mid r_0,c} \in \begin{cases} [p_{w_0\mid x_0,c}, 1] \text{ if } p_{w_0\mid x_0,c} > p_{w_0\mid x_1,c} \\ [0, p_{w_0\mid x_0,c}] \text{ if } p_{w_0\mid x_0,c} < p_{w_0\mid x_1,c} \\ (0,1) \setminus \{p_{w_0,\mid x_0,c}\} \text{ if } p_{w_0\mid x_0,c} = p_{w_0\mid x_1,c} \end{cases}$	$q_{w_0\mid r_1} = p_{w_0\mid x_1}$ $q_{w_0\mid r_0} \in \begin{cases} [\max_c p_{w_0\mid x_0,c}, 1] \text{ if } \exists \tilde{c}, p_{w_0\mid x_0,\tilde{c}} > p_{w_0\mid x_1}, \\ [0, \min_c p_{w_0\mid x_0,c}] \text{ if } \exists \tilde{c}, p_{w_0\mid x_0,\tilde{c}} < p_{w_0\mid x_1}, \\ (0,1) \setminus \{p_{w_0\mid x_1}\} \text{ if } \forall c, p_{w_0\mid x_0,c} = p_{w_0\mid x_1}. \end{cases}$