Title: Indefinite causal order and quantum reference frames

Speakers: Anne-Catherine de la Hamette

Series: Quantum Foundations, Quantum Information

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Abstract: Recent research on quantum reference frames (QRFs) has shown that whether a system is in a superposed state of locations, momenta, and other properties can depend on the quantum reference frame relative to which it is being described. Whether an event is localized in spacetime or not can change under QRF transformations, in that case so-called quantum-controlled diffeomorphisms. This raises a critical question: can quantum reference frame transformations render indefinite causal order definite? In this talk, I propose a relativistic definition of causal order based on worldline coincidences and proper time differences, establishing it as an operationally meaningful observable in both general relativity and quantum mechanics. Using this definition, we can analyse the indefiniteness of causal order in the optical and gravitational quantum switch on equal footing. This analysis suggests an operational rather than a spacetime-based understanding of events. I will compare these findings to other recent results and conclude with broader implications for events in non-classical contexts

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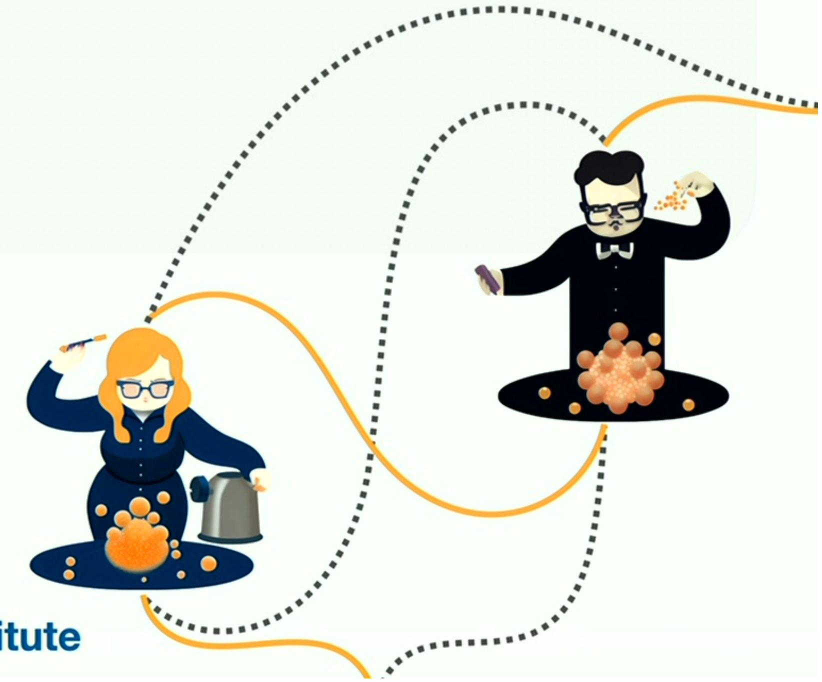
Indefinite causal order and quantum reference frames

Anne-Catherine de la Hamette
University of Vienna, IQOQI Vienna



Causalworlds 2024 - 20th September 2024 - Perimeter Institute

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IQI Vienna

based on joint works with

V. Kabel

Č. Brukner

M. Christodoulou

L. Apadula

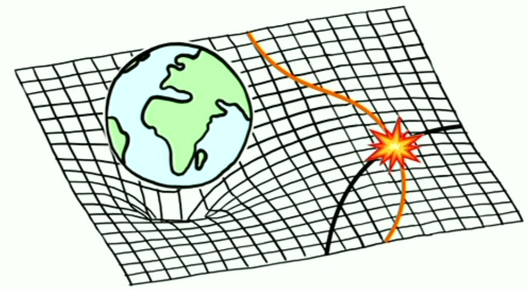
C. Cepollaro

H. Gomes

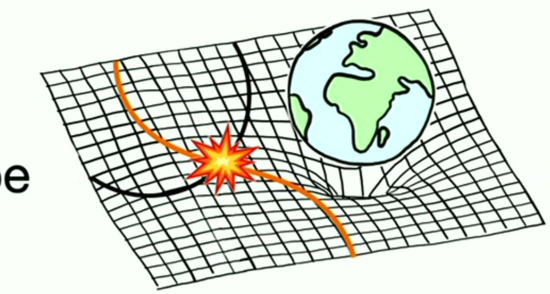
J. Butterfield

arXiv:2211.15685 *arXiv:2402.10267* *arXiv:2404.00159*

Questions…



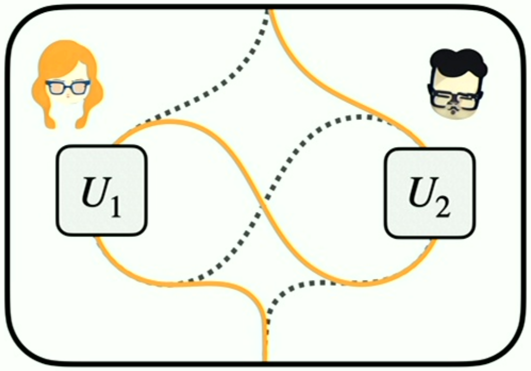
What defines an event in a non-classical spacetime?



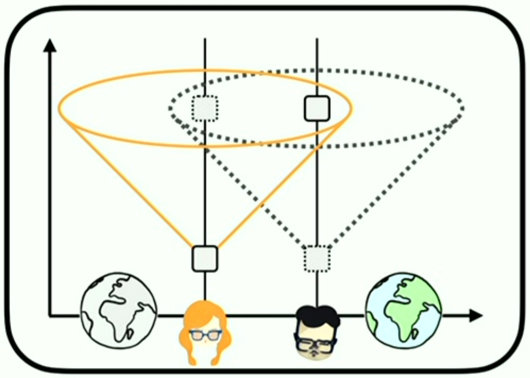
What does it mean for such an event to be spacetime localised?

What are inherent properties of an event, in particular in quantum theory?

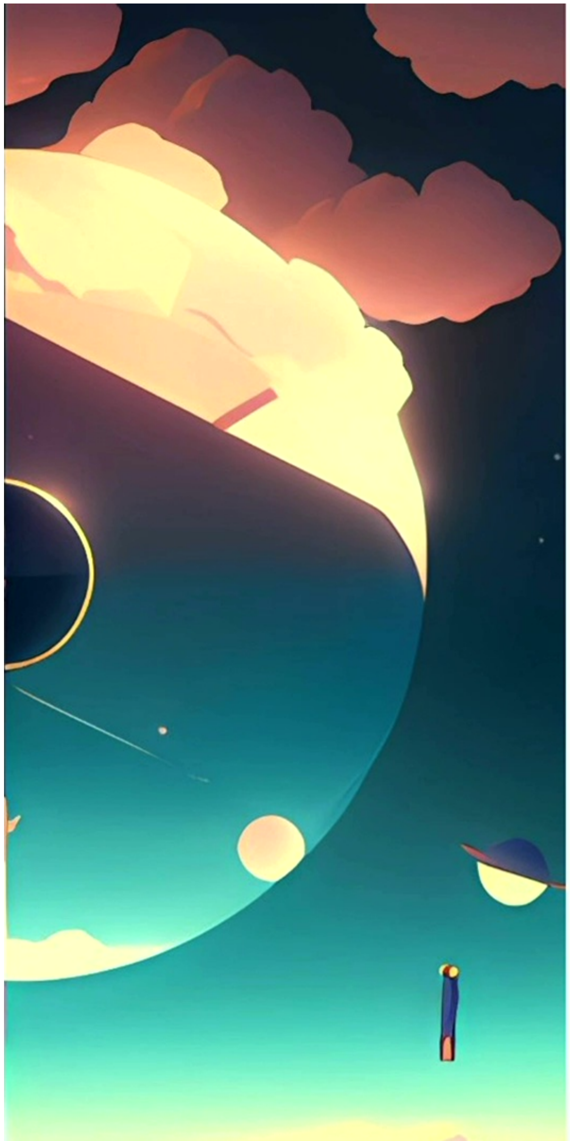
Questions…



To what extent is indefinite causal order a (quantum) coordinate artefact?



What does the indefiniteness tell us about the quantumness of the underlying spacetime?

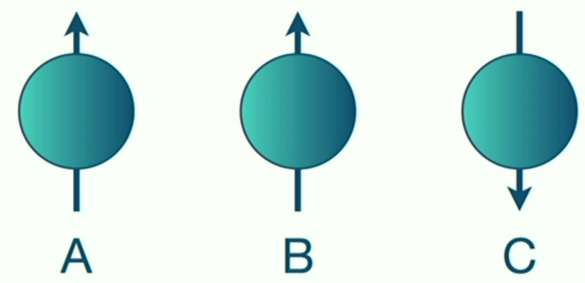


Outline

- Introduction to quantum reference frames
- Quantum reference frames for GR
- Localisation of events
- Implications for the quantum switch
- A GR notion of indefinite causal order
- Operational vs. spacetime notions of events
- Take home messages

Classical Reference Frames

- **Classical reference frame transformations.**
  - Example: system with two possible configurations, ↑ or ↓.



ACdlH, Galley, Quantum 4, 367 (2020).

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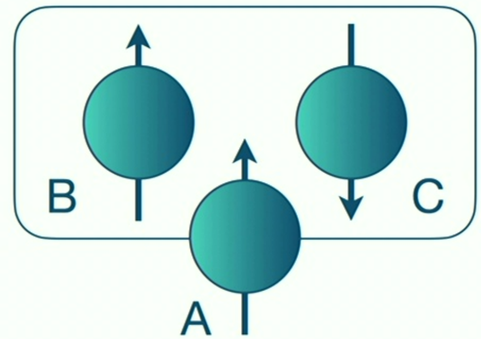

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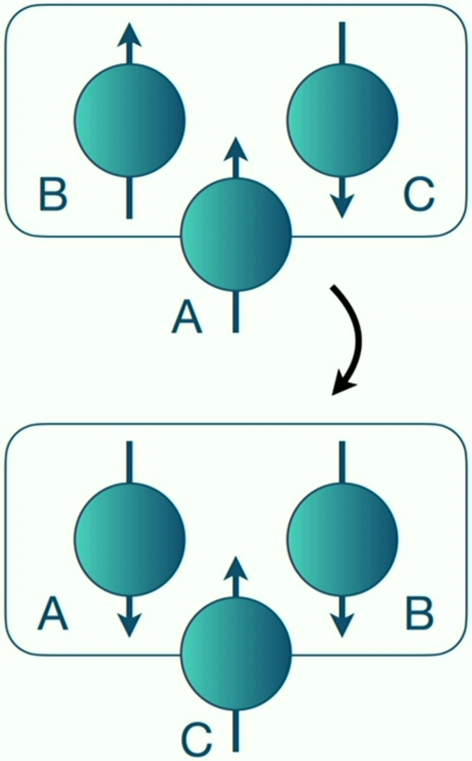

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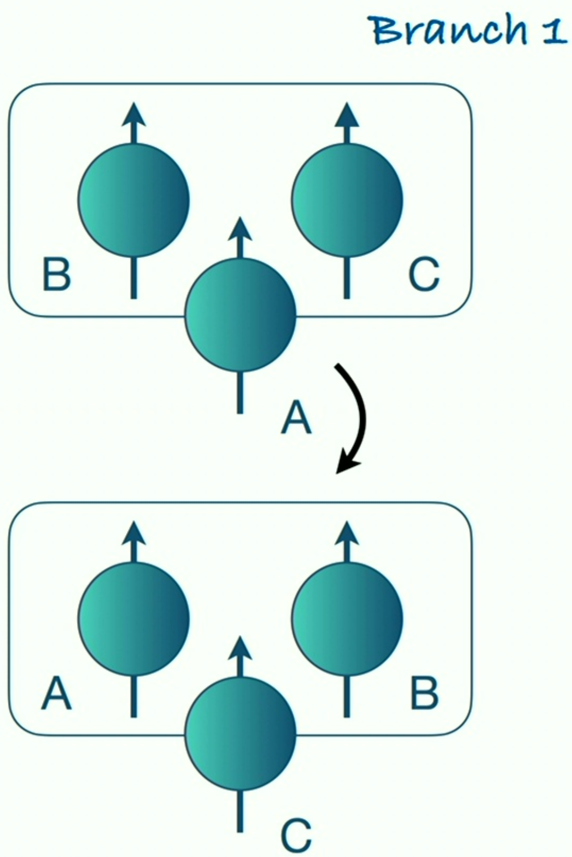
Quantum Reference Frames

The Idea

- **Quantum reference frame transformations.**
    - Example: quantum system $\mathbb{C}^2$.
    - Implemented by quantum-controlled symmetry transformation.

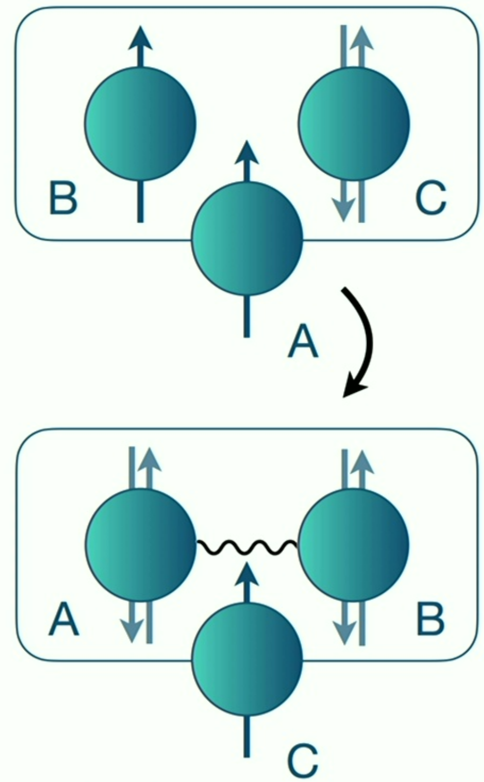


ACdlH, Galley, Quantum 4, 367 (2020).

Quantum Reference Frames

The Idea

- **Quantum reference frame transformations.**
  - Example: quantum system $\mathbb{C}^2$.
  - Implemented by quantum-controlled symmetry transformation.
  - Superposition and entanglement are frame-dependent.



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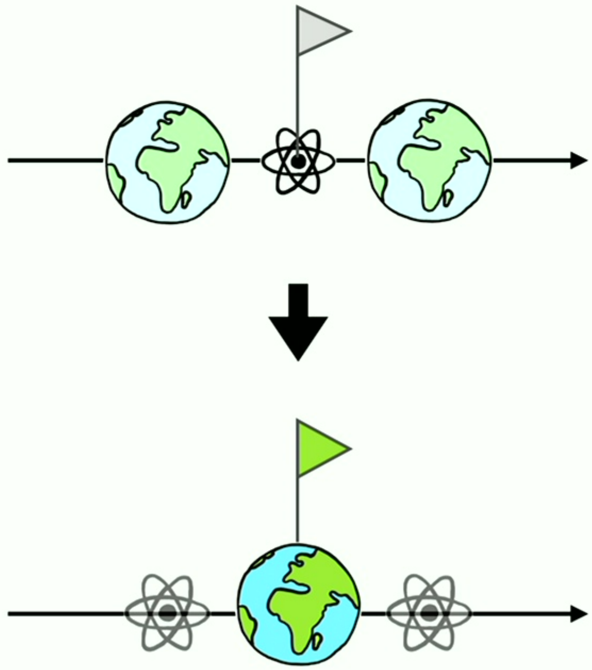
Quantum Reference Frames

The Idea

- **Quantum reference frame transformations.**
    - Example: location of a quantum particle.



ACdlH, V. Kabel, E. Castro-Ruiz, Č. Brukner, Commun Phys 6, 231 (2023).

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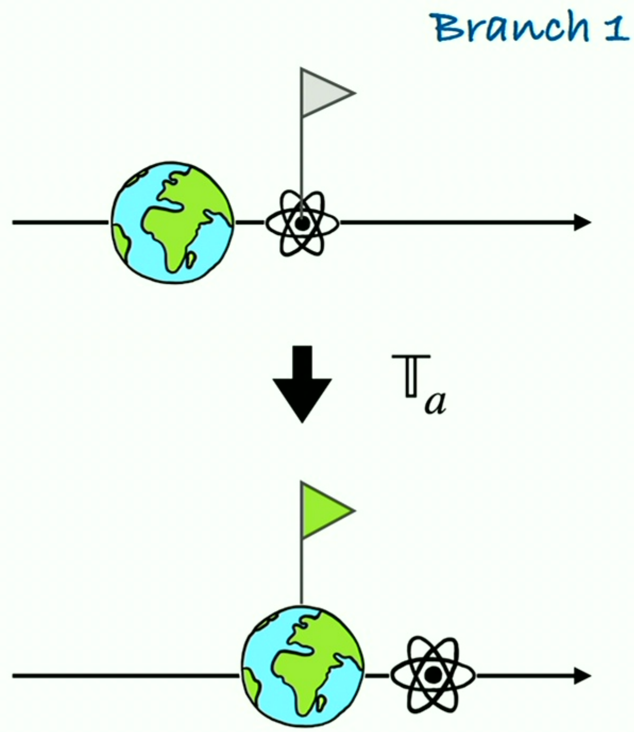

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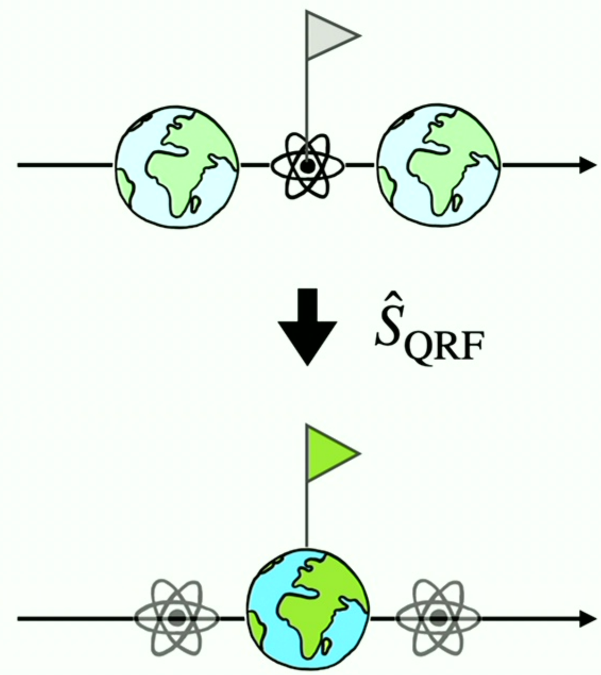
Quantum Reference Frames

The Idea

- **Quantum reference frame transformations.**
  - Example: location of a quantum particle.
  - Implemented by quantum-controlled symmetry transformation.
  - Superposition as a frame-dependent feature.

Superposition can be "shifted" from one subsystem to another.



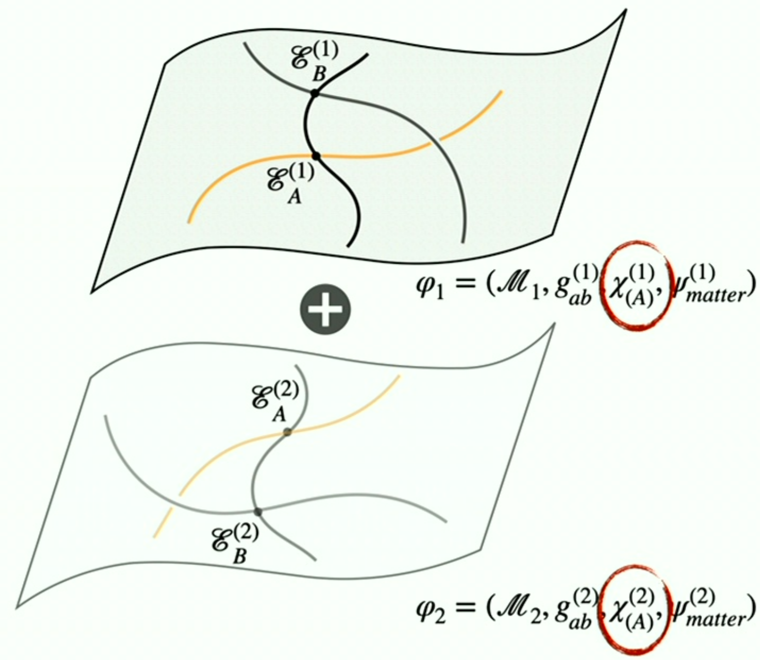
$$\hat{S}_{\text{QRF}} = |0\rangle\langle 0| \otimes \mathbb{T}_a + |1\rangle\langle 1| \otimes \mathbb{T}_{-a}$$

Quantum Reference Frames for GR

Construction

- **Quantum coordinates:** independent choice of coordinate system (or **reference field**) for each branch of the superposition.
- Pick **4 scalar fields** $\{\chi_{(A)}\}_{A=0,..,3}$ as quantum reference fields
- Change implemented by a **quantum-controlled coordinate transformation** (~ quantum diffeomorphism):

$$\hat{S} = |0\rangle\langle 0| \otimes \mathscr{U}_1 + |1\rangle\langle 1| \otimes \mathscr{U}_2$$

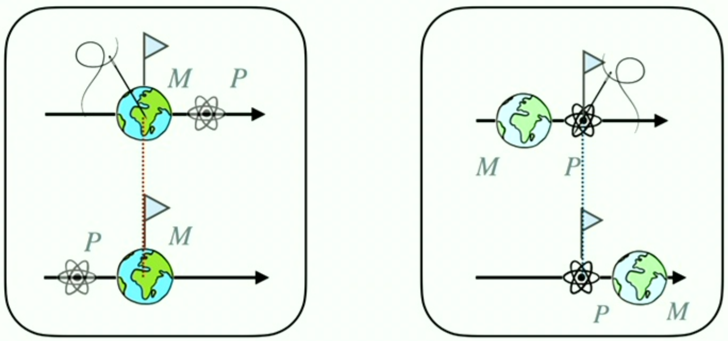


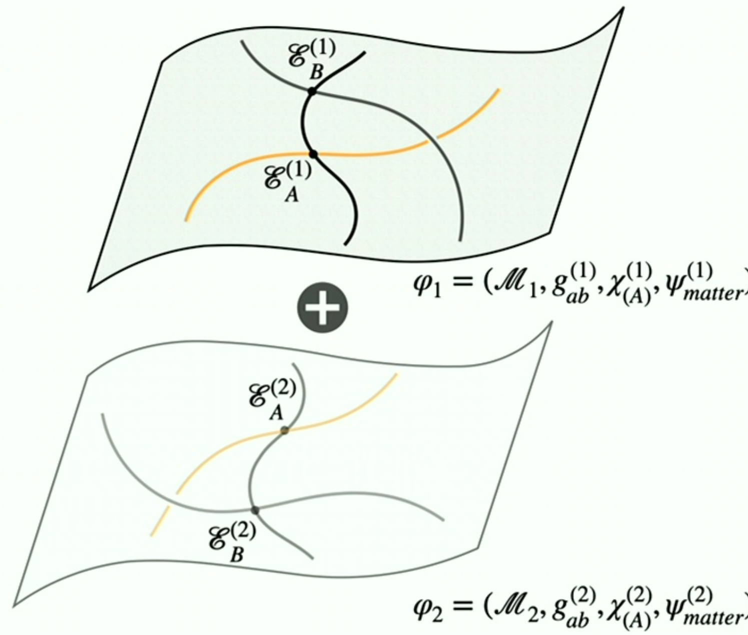
$$|\Psi\rangle = \alpha|0\rangle|g^{(1)}\rangle|\chi_{(A)}^{(1)}, \psi_{matter}^{(1)}\rangle + \beta|1\rangle|g^{(2)}\rangle|\chi_{(A)}^{(2)}, \psi_{matter}^{(2)}\rangle$$

Quantum Reference Frames for GR

Construction

- **Previously:** identify the points at which the Earth or the particle are located.





$$|\Psi\rangle = \alpha|0\rangle|g^{(1)}\rangle|\chi_{(A)}^{(1)}, \psi_{matter}^{(1)}\rangle + \beta|1\rangle|g^{(2)}\rangle|\chi_{(A)}^{(2)}, \psi_{matter}^{(2)}\rangle$$

Kabel, ACdlH, Apadula, Cepollaro, Gomes, Butterfield, Brukner, 2402.10267

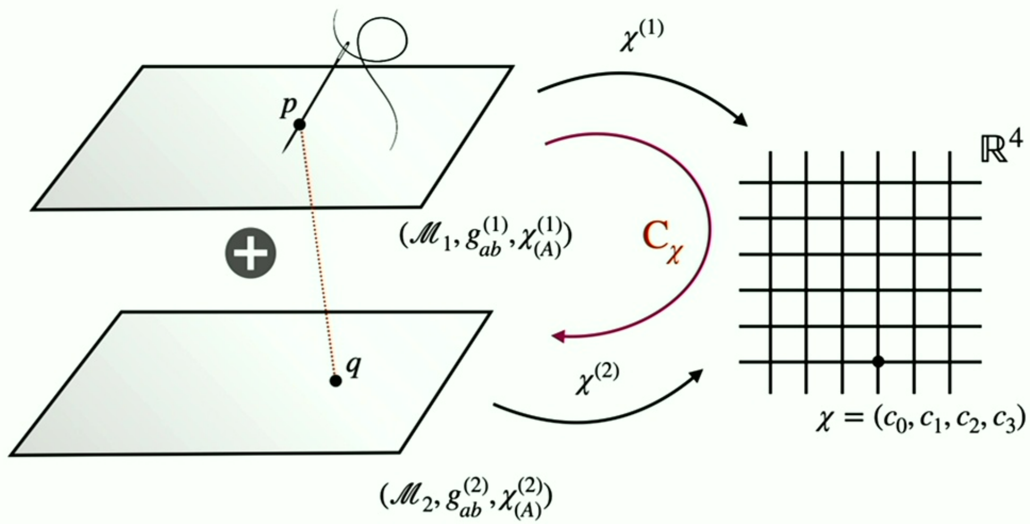
Quantum Reference Frames for GR

- **Now:** Use the values of the four scalar fields $\{\chi_{(A)}\}_{A=0,1,2,3}$ to **label** the points and **identify** them across the branches in superposition.

- **Identify** a point $p \in \mathscr{M}_1$ with a point $q \in \mathscr{M}_2$ iff $\chi^{(1)}(p) = \chi^{(2)}(q)$

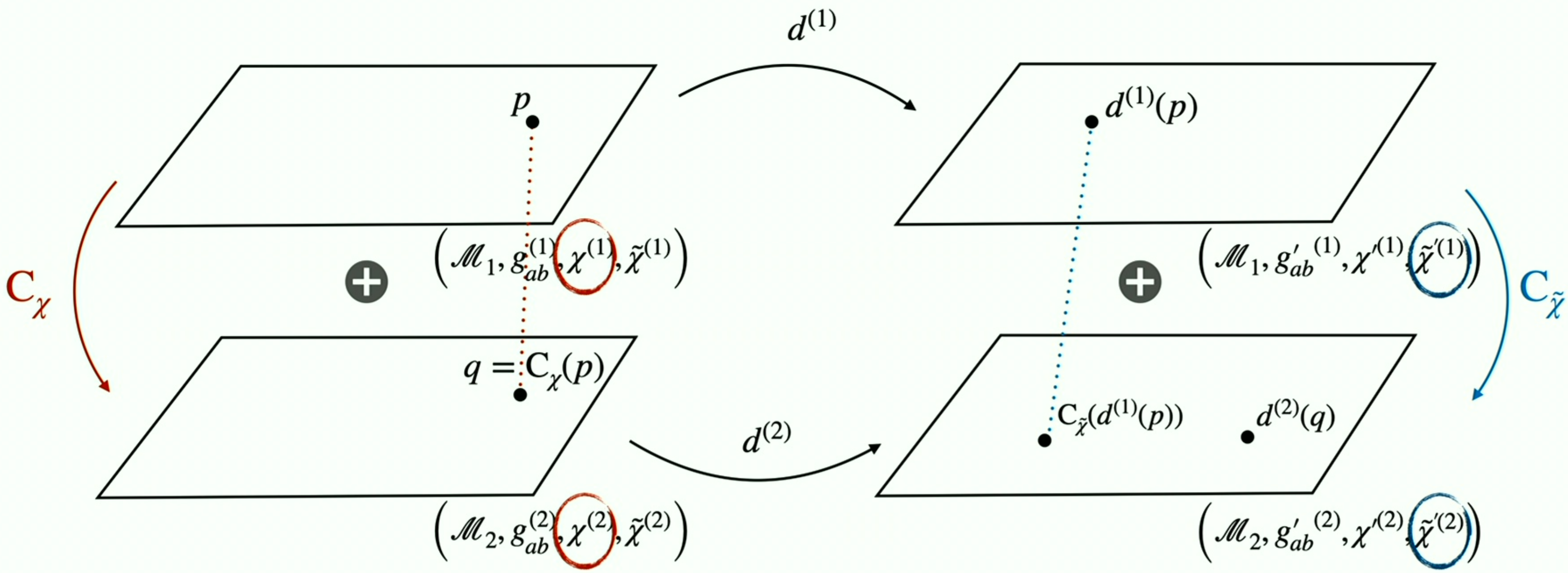
- **Comparison map** relative to χ-fields:

$$\mathrm{C}_\chi \equiv (\chi^{(2)})^{-1} \circ \chi^{(1)} : \mathscr{M}_1 \to \mathscr{M}_2$$



Kabel, ACdlH, Apadula, Cepollaro, Gomes, Butterfield, Brukner, 2402.10267

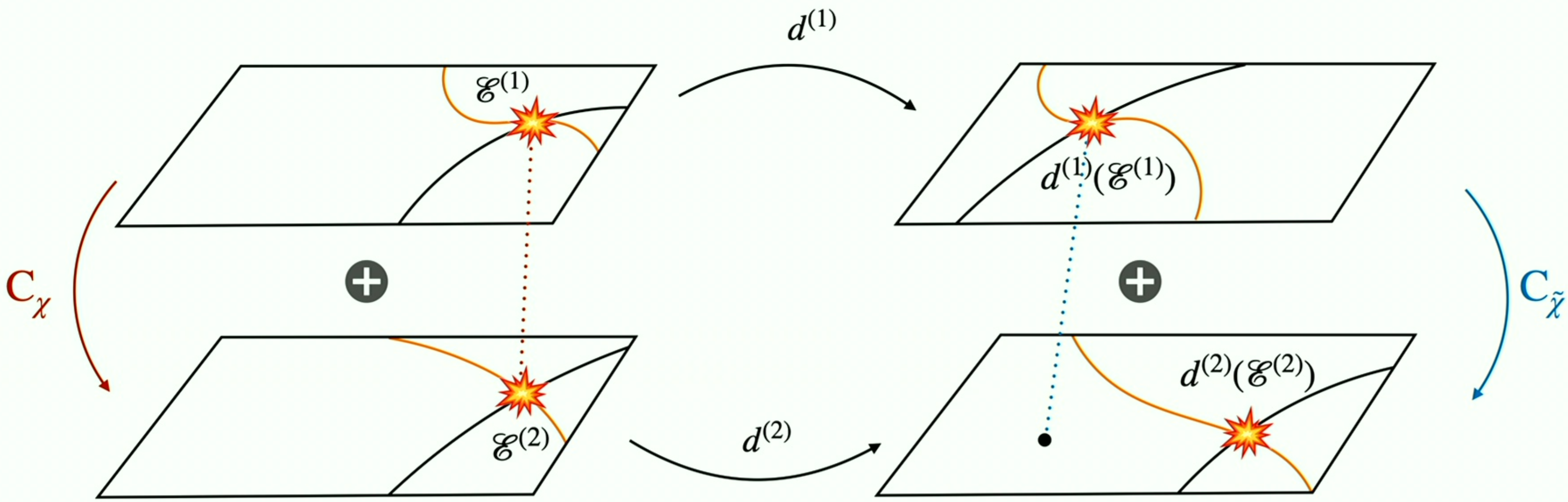
Identification of points



The pair (p, q) where $p \in \mathscr{M}_1$ and $q \in \mathscr{M}_2$ is localised iff $q = C_\chi(p)$.

The pair $(d^{(1)}(p), d^{(2)}(q))$ will in general not be localised: $d^{(2)}(q) \neq C_{\tilde{\chi}}(d^{(1)}(p))$.

Spacetime Localisation of Events

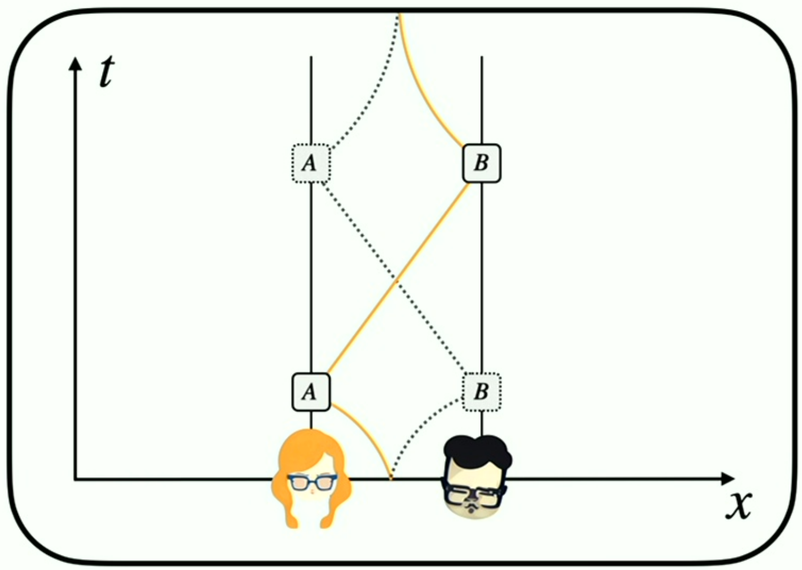


Identification of spacetime points and **localisation of events** are **frame-dependent** and have no absolute physical meaning.

Intermezzo: Debate on the Quantum Switch

The optical quantum switch

- Optical QS: Implementation of ICO through superposition of paths
- Issue: Is this a proper **realisation** of indefinite causal order? There is still a fixed Minkowski background…
- Ongoing debate: realisation vs. simulation
- "Spatiotemporalists":
  - *"There are four points in spacetime, thus **four events** involved in this process."*
- "Dynamicists" (or "operationalists"):
  - *"There are **two events** involved in this process, each of them in a superposition of two spacetime locations."*



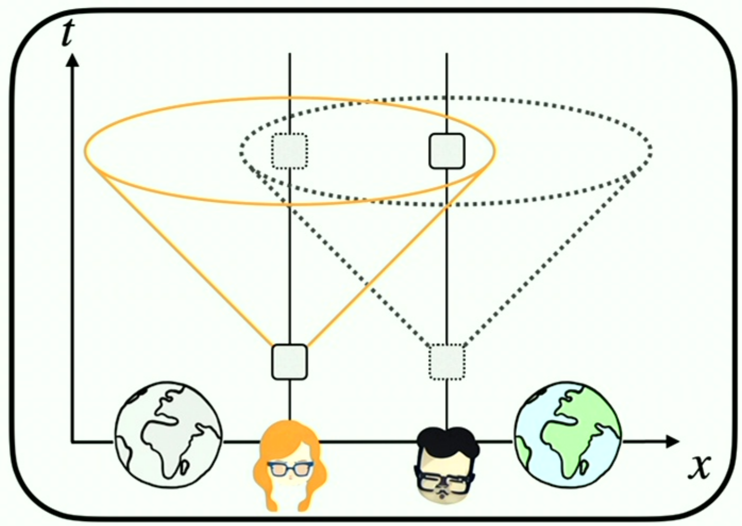
Oreshkov (2019)
Paunković, Vojinović (2020)
Ormrod, Vanrietvelde, Barrett (2023)

Intermezzo: Debate on the Quantum Switch

The gravitational quantum switch

- Gravitational QS: Implementation of ICO through superposition of gravitational fields
- Now a superposition of gravitational fields, thus of causal structures
- Would this be a proper implementation of indefinite causal order?

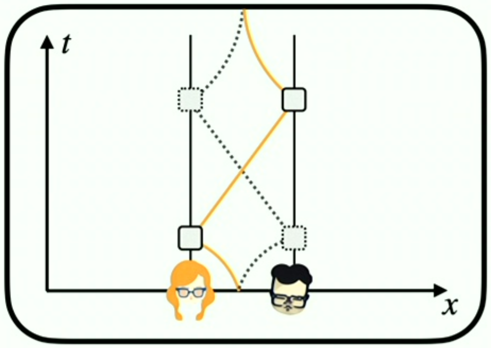
"**Spatiotemporalists**": Yes! Only two events!



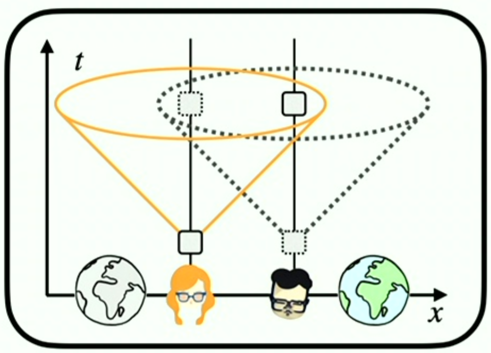
"**Dynamicists**": But again four spacetime points relative to far away observer?!

Oreshkov (2019)
Paunković, Vojinović (2020)
Ormrod, Vanrietvelde, Barrett (2023)

Intermezzo: Debate on the Quantum Switch



Optical Quantum Switch



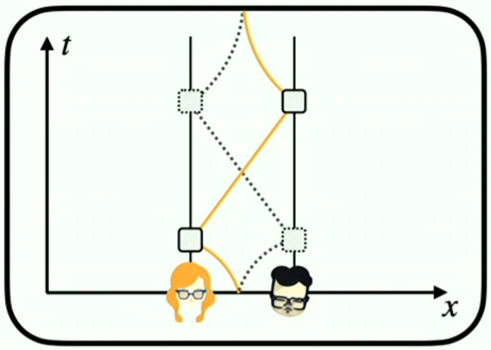
Gravitational Quantum Switch

- Ongoing debate regarding the implementation of the "quantum switch" exhibiting **indefinite causal order**.
- Core of the debate: **how many events** are there?
  - **"Spatiotemporalists":** 4 events (spacetime points)
  - **"Dynamicists":** 2 events (application of operation)

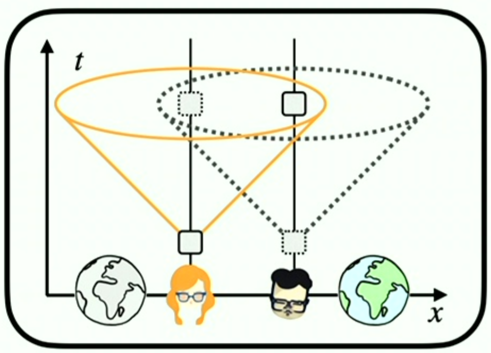
> demonstrated experimentally. However, the interpretation of such experiments as realizations of a process with indefinite causal structure as opposed to some form of simulation of such a process has remained controversial. Where exactly are the local

Oreshkov (2019)

Intermezzo: Debate on the Quantum Switch



Optical Quantum Switch



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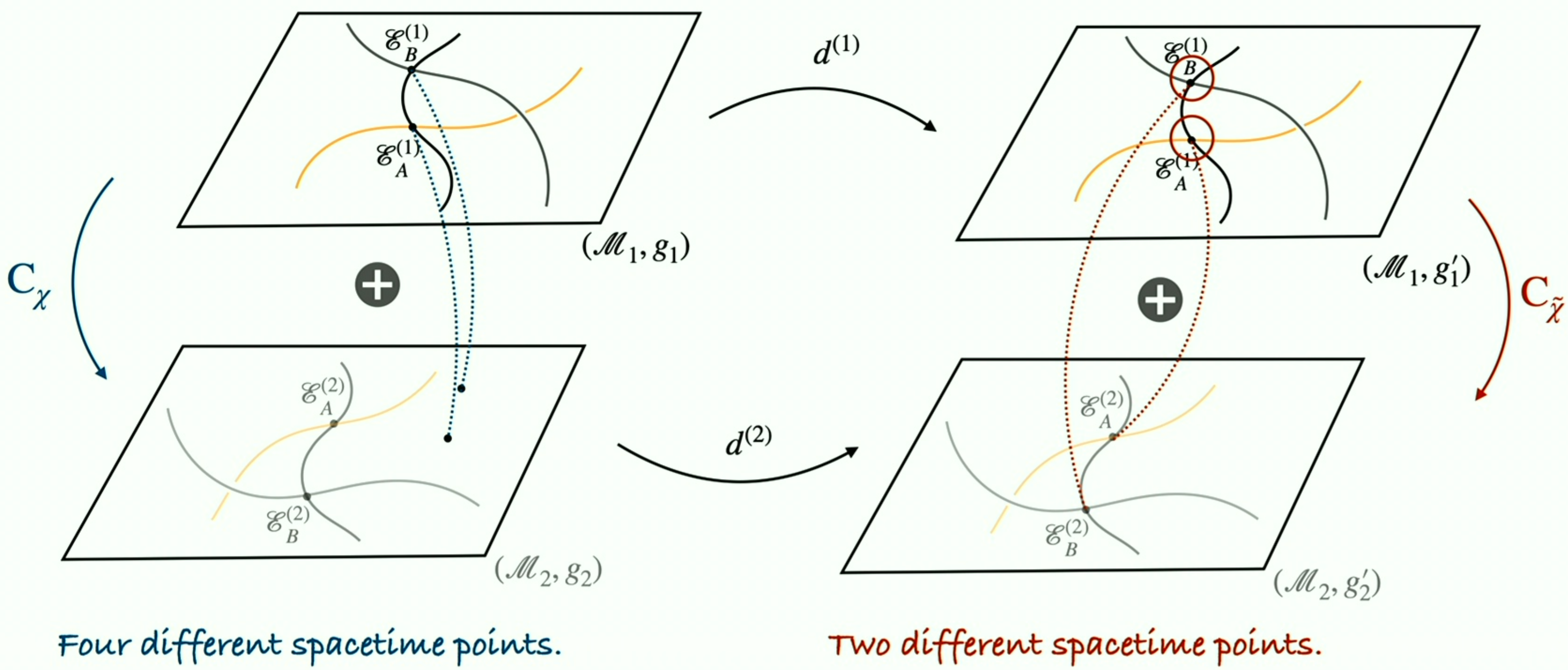
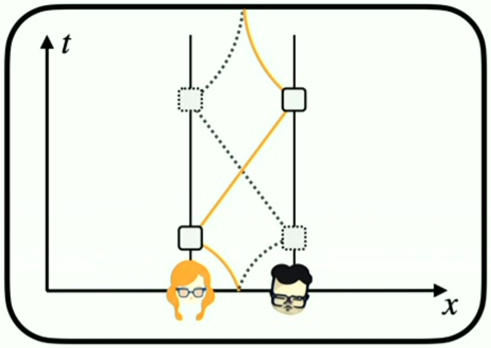
Oreshkov (2019)

- Can quantum reference frames - in particular **quantum diffeomorphisms** - bring any insight here?

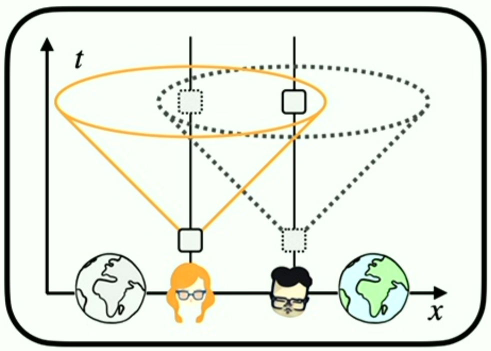
Implications for Indefinite Causal Order



Implications for Indefinite Causal Order



Optical Quantum Switch



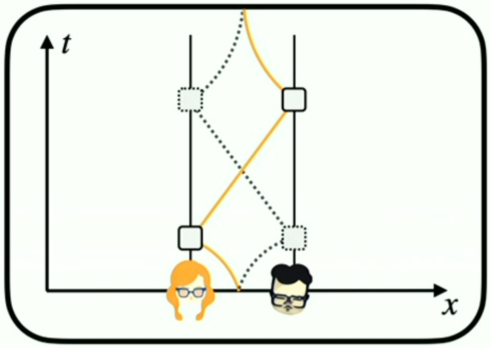
Gravitational Quantum Switch

- **Insight:** A change of QRF can change the number of spacetime points involved in a process.

Should not take the number of spacetime points nor the spacetime location of event as relevant property.

ACdlH, Kabel, Christodoulou, Brukner, 2211.15685; ACdlH, Kabel, Brukner, 2404.00159

Implications for Indefinite Causal Order



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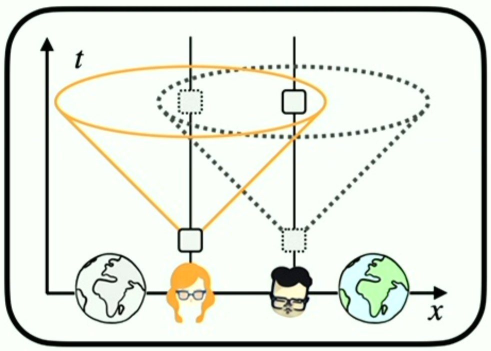


Gravitational Quantum Switch

- **Insight:** A change of QRF can change the number of spacetime points involved in a process.

  → Should not take the number of spacetime points nor the spacetime location of event as relevant property.

- How does causal order transform under changes of QRFs?

- Can QRF changes make indefinite causal order definite?

ACdlH, Kabel, Christodoulou, Brukner, 2211.15685; ACdlH, Kabel, Brukner, 2404.00159

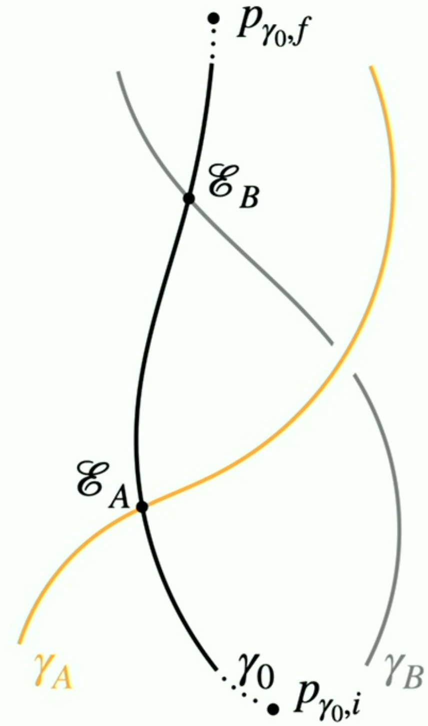
A GR Notion of Indefinite Causal Order

Classical Definitions

- Define causal order between **events**.

But what is an "event"?

- Define an **event** as the coincidence of two specific worldlines (coordinate independent notion).
  - Coincidence of γ_0 with γ_A defines Event $\mathscr{E}_A$.
  - Coincidence of γ_0 with γ_B defines Event $\mathscr{E}_B$.



A GR Notion of Indefinite Causal Order

Classical Definitions

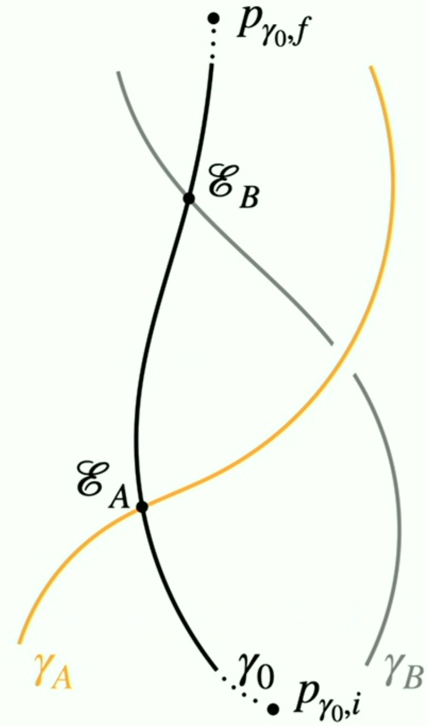
- Use **proper time** of γ_0....

$$\tau \equiv \tau_{\gamma_0} = \frac{1}{c} \int_{\gamma_0} \sqrt{-g_{\mu\nu} dx^\mu dx^\nu}$$

- ... to define the **causal order between the events** $\mathscr{E}_A$ and $\mathscr{E}_B$.

$$\Delta\tau = \tau_B - \tau_A \equiv s\,|\tau_B - \tau_A|$$

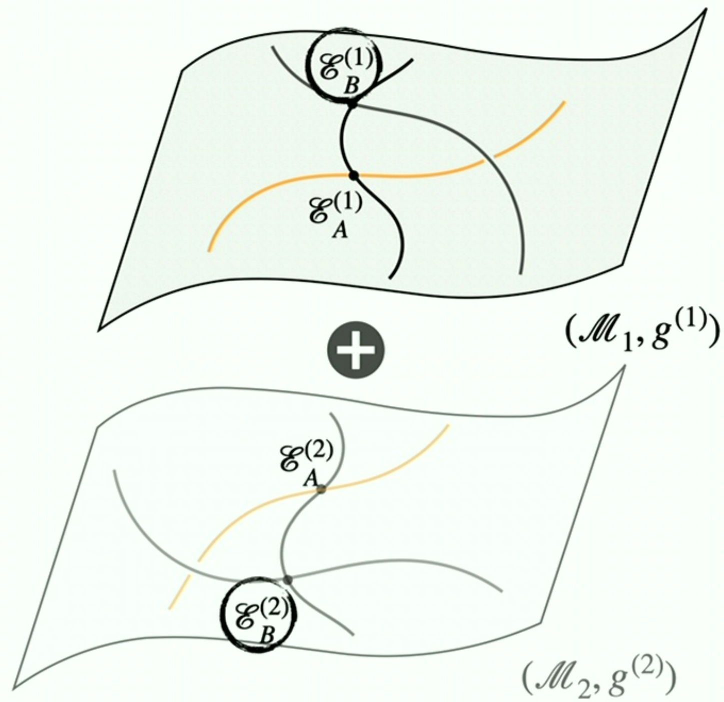
Sign of proper time difference defines causal order.



A GR Notion of Indefinite Causal Order

Quantum Definitions

- Define a **quantum event** as the coincidence of two specific worldlines.
  - Coincidence of γ_0 with γ_A defines Event $\mathscr{E}_A^{(1)} = \mathscr{E}_A^{(2)} \equiv \mathscr{E}_A$.
  - Coincidence of γ_0 with γ_B defines Event $\mathscr{E}_B^{(1)} = \mathscr{E}_B^{(2)} \equiv \mathscr{E}_B$.



A GR Notion of Indefinite Causal Order

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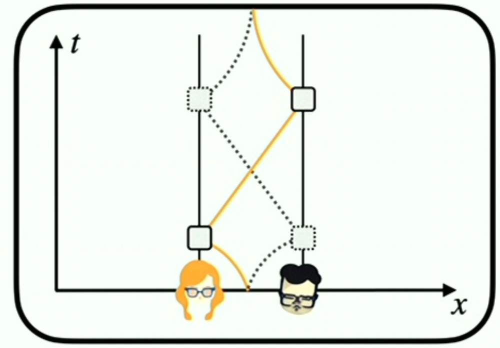
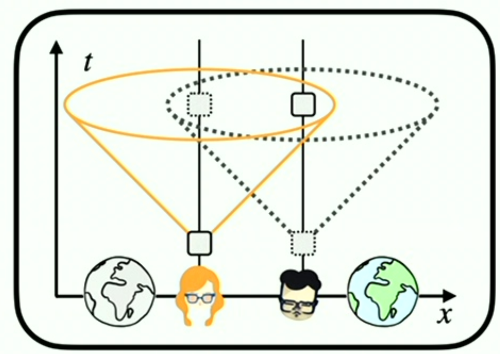
  - Coincidence of γ_0 with γ_B defines Event $\mathscr{E}_B^{(1)} = \mathscr{E}_B^{(2)} \equiv \mathscr{E}_B$.

There are <u>two</u> quantum events in both the optical and gravitational quantum switch.



Optical Quantum Switch



Gravitational Quantum Switch

A GR Notion of Indefinite Causal Order

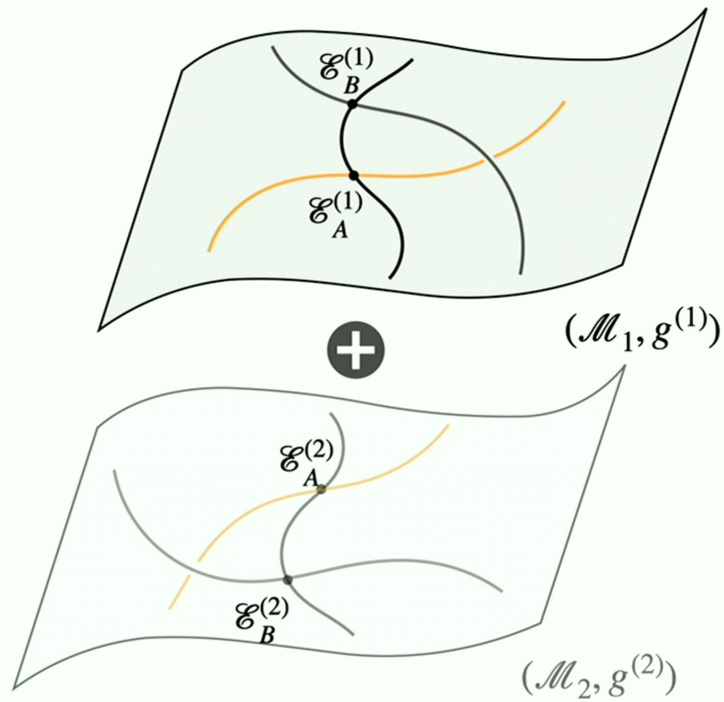
Quantum Definitions

- In each branch (i), define the causal order between the events $\mathscr{E}_A^{(i)}$ and $\mathscr{E}_B^{(i)}$ via the sign $s^{(i)}$ of the proper time difference, where $i = 1,2$.
- **Definite** causal order iff

$$s^{(1)} \cdot s^{(2)} = 1$$

- **Indefinite** causal order iff

$$s^{(1)} \cdot s^{(2)} = -1$$



A GR Notion of Indefinite Causal Order

Implications

- The causal order is defined through the sign of the difference between proper times.
- It is a **global**, **diffeomorphism-invariant** quantity.

↳ *Observable in the sense of general relativity.*

Using this definition of causal order, the causal order between two events is **invariant** under quantum diffeomorphisms.

- To see that it is an observable in the quantum mechanical sense, see proposal for implementation in arXiv:2211.15685.

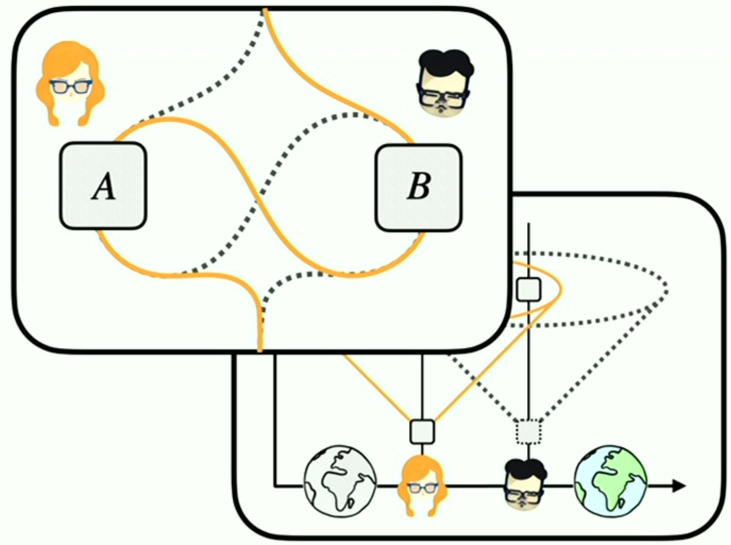
A GR Notion of Indefinite Causal Order

Implications

- The causal order is defined through the sign of the difference between proper times.
- The proper time depends on **both** the **path** γ_0 and the **metric** $g_{\mu\nu}$.

$$\tau \equiv \tau_{\gamma_0} = \frac{1}{c} \int_{\gamma_0} \sqrt{-g_{\mu\nu} dx^\mu dx^\nu}$$

- If **any** of the two is indefinite, so is the causal order.

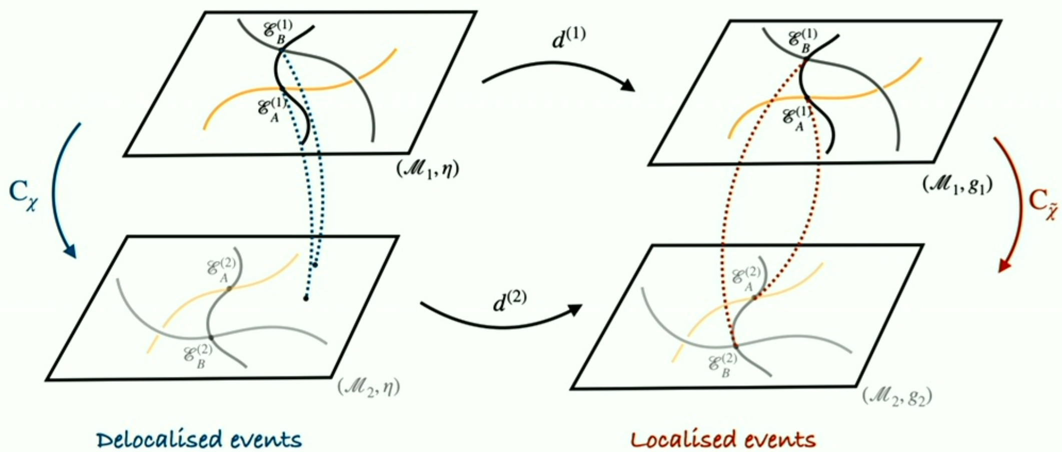


A GR Notion of Indefinite Causal Order

Implications

- Whether a process displays ICO 'due' to delocalised events or a superposition of spacetime metrics can change under quantum diffeomorphisms.

1. **ICO 'due' to delocalised events in fixed spacetime.**



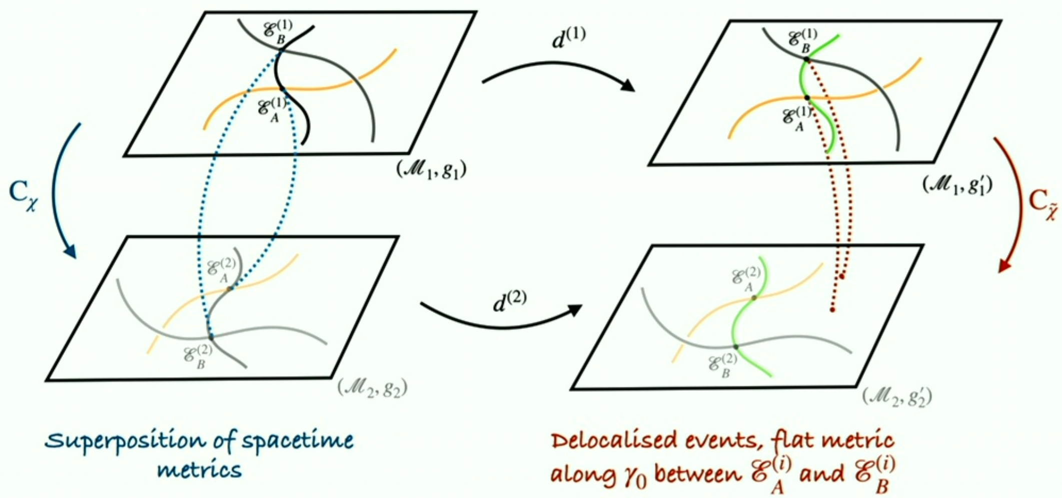
ICO due to delocalised events in fixed spacetime
↓
ICO due to superposition of spacetime metrics at localised events

A GR Notion of Indefinite Causal Order

Implications

- Whether a process displays ICO 'due' to delocalised events or a superposition of spacetime metrics can change under quantum diffeomorphisms.

2. ICO 'due' to a superposition of spacetime metrics.



ICO due to superposition of spacetime metrics

↓

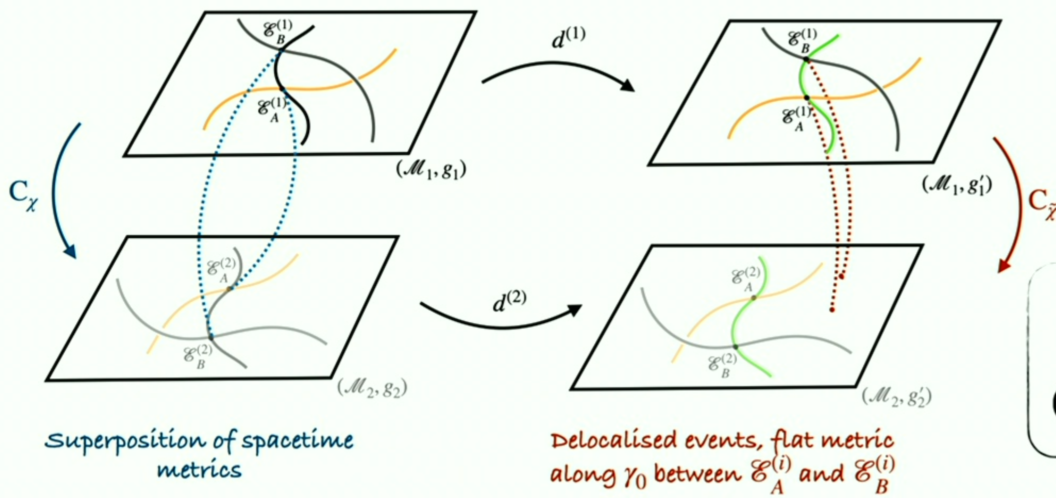
ICO due to delocalised events

A GR Notion of Indefinite Causal Order

Implications

- Whether a process displays ICO ‘due’ to delocalised events or a superposition of spacetime metrics can change under quantum diffeomorphisms.

2. ICO ‘due’ to a superposition of spacetime metrics.



ICO due to superposition of spacetime metrics

↓

ICO due to delocalised events

Fermi normal coordinates along γ_0 :

$$(d^{(1)})_*^{-1}(g^{(1)})\,|_{\gamma_0} = (d^{(2)})_*^{-1}(g^{(2)})\,|_{\gamma_0} = \eta\,|_{\gamma_0}$$

A GR Notion of Indefinite Causal Order

A no-go theorem

Whether ICO is due to delocalised events or a superposition of metrics depends on the choice of quantum coordinates.

There is no quantum coordinate system in which the following three statements hold:

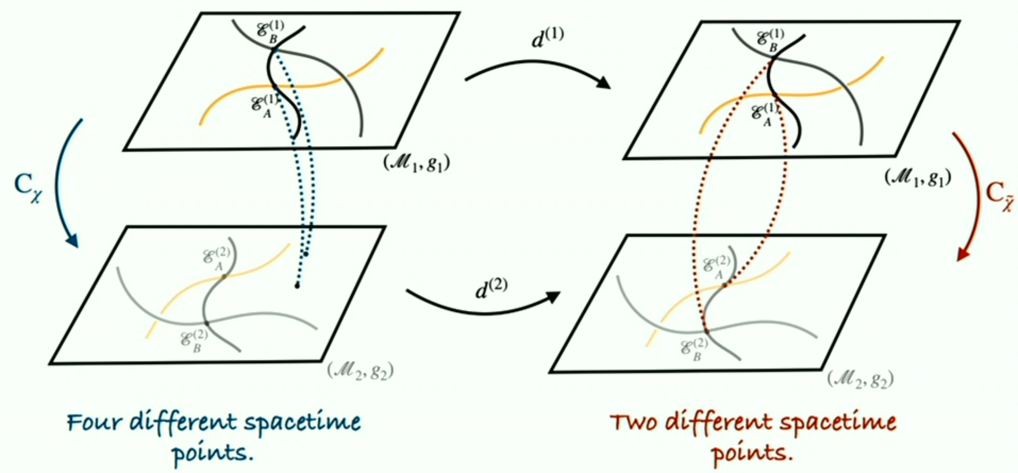
1. Both events $\mathscr{E}_A$ and $\mathscr{E}_B$ are localised.
2. The spacetime metric is definite.
3. The causal order between $\mathscr{E}_A$ and $\mathscr{E}_B$ is indefinite.

*consistent with Corollary 8.2 in Vilasini Renner PRA 110, 022227 (2024).

A GR Notion of Indefinite Causal Order

Disclaimers

- We can map a 4-point switch to a 2-point switch, and vice versa.
- This does **not** mean that we can map the **optical** switch to the **gravitational** one, and vice versa.
- Diffeomorphism-invariant quantities, such as the curvature scalar, remain invariant under quantum diffeomorphisms.



A GR Notion of Indefinite Causal Order

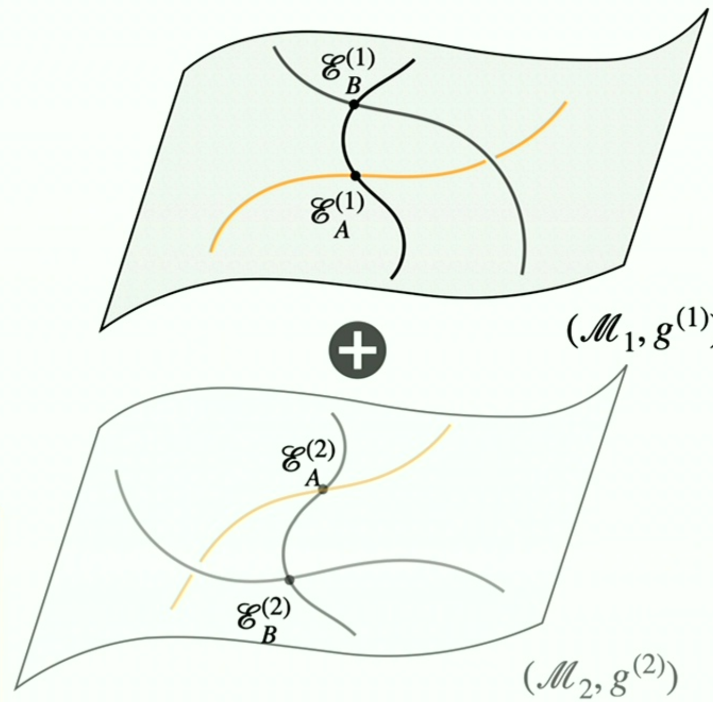
Disclaimers

- This definition of causal order is restricted to
  - particles on worldlines.

More generally we can use the coincidence of any physical systems* (e.g. quantum fields).

  - two events and two spacetimes in superposition.
- This notion only captures *whether* there is ICO and not "how much".

See arXiv:2409.11448 and talk by Samuel Fedida on Wednesday for
- more ICO quantifiers
- extension to N events and M spacetimes
- QM quantifiers of ICO in this setting



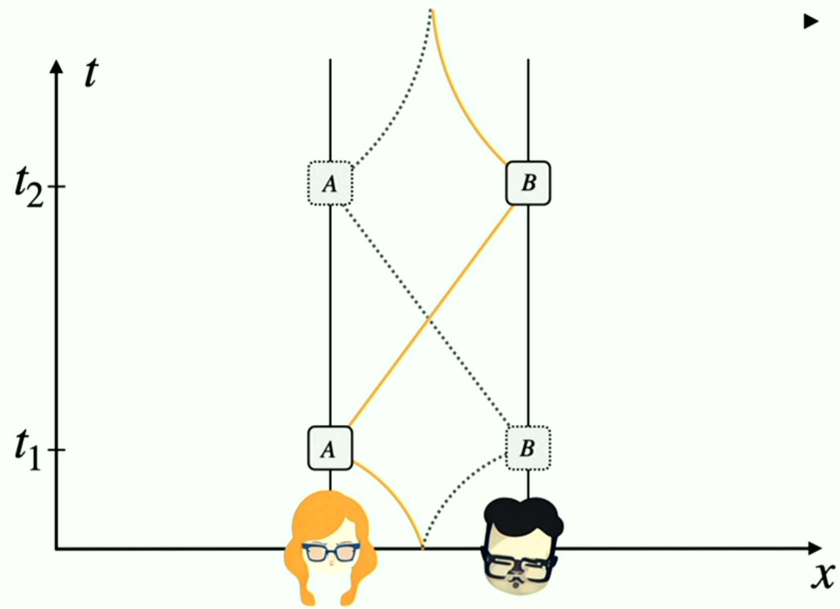
*arXiv:2402.10267

Outline

- Introduction to quantum reference frames
- Quantum reference frames for GR
- Localisation of events
- Implications for the quantum switch
- A GR notion of indefinite causal order
- Operational vs. spacetime notions of events
- Take home messages

Operational vs. spacetime notions of events

Coarse-grained vs. fine-grained



- **Fine-grained notion of event**
  - Takes the spacetime location of an event* as a defining property.
  - Differentiates between (A, t_1) and (A, t_2).
  - Counts 4 different events in total.
  - There exists an acyclic classical causal structure of this process.

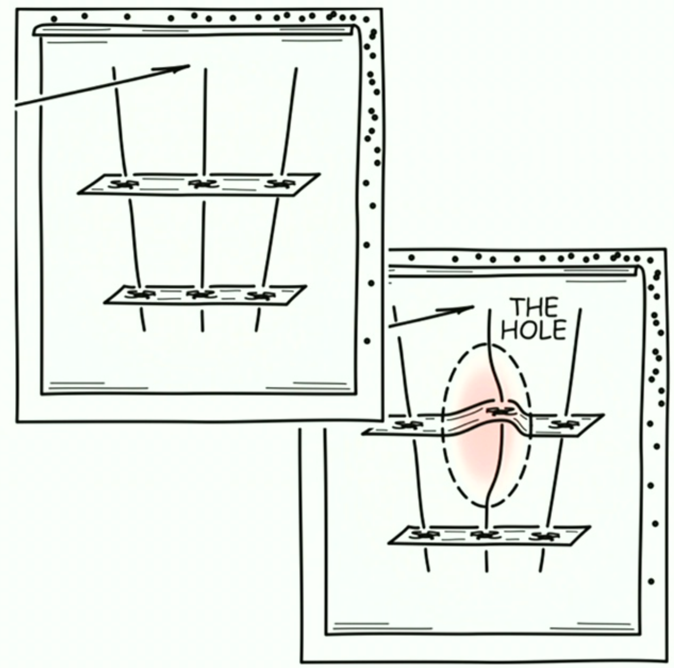
Vilasini, Renner, PRA 110, 022227 (2024)
Vilasini, Renner, PRL 133, 080201 (2024)

**or locus of intervention*

Operational vs. spacetime notions of events

Implications of the fine-grained notion

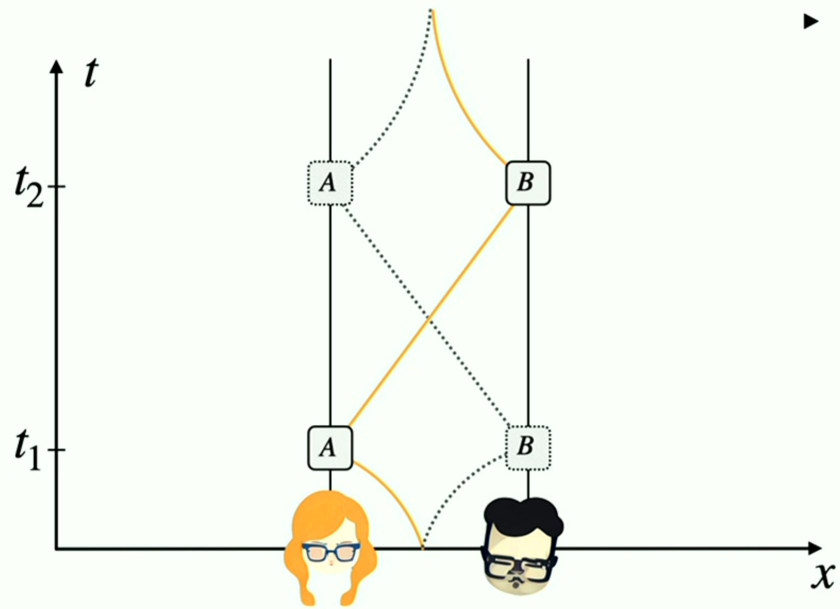
- Einstein's **hole argument**: assigning physical meaning to spacetime points leads to issues due to the diffeomorphism-invariance of GR.
- An event defined as a spacetime point (or region) is problematic.
- The **spacetime location** of an event should **not** be taken as an **inherent** property.



Norton, SEP (2022)

Operational vs. spacetime notions of events

Coarse-grained vs. fine-grained

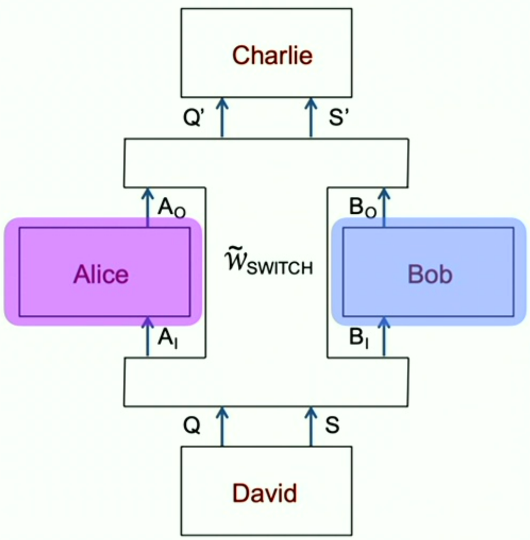


- **Coarse-grained notion of event**
  - Restricts to operationally accessible properties of an event.
  - This excludes the (space)time location of the event.
  - Does not differentiate between (A, t_1) and (A, t_2).

Oreshkov, Quantum 3, 206 (2019); ACdlH, Kabel, Brukner, 2404.00159

Operational vs. spacetime notions of events

Coarse-grained vs. fine-grained

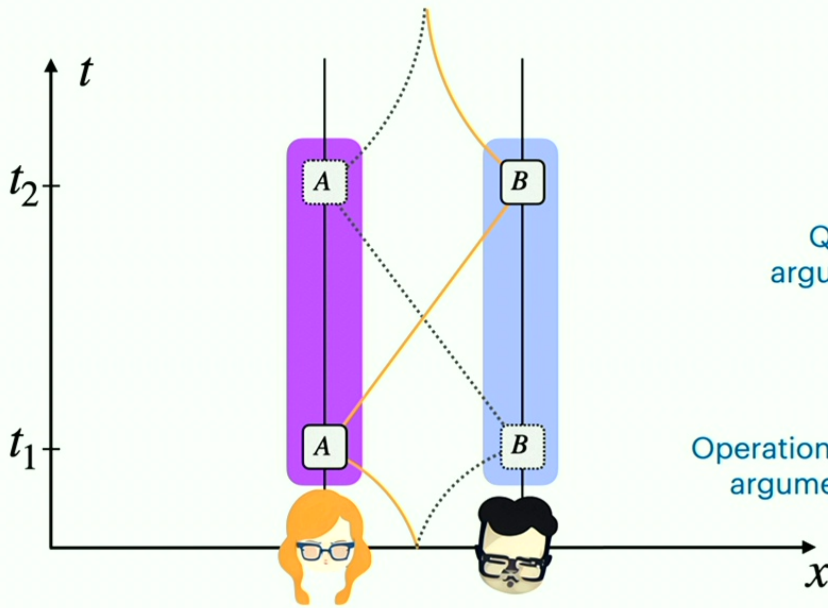


- **Coarse-grained notion of event**
  - Restricts to operationally accessible properties of an event.
  - This excludes the (space)time location of the event.
  - Does not differentiate between (A, t_1) and (A, t_2).
  - 2 **time-delocalised** operational events in total.

Oreshkov, Quantum 3, 206 (2019); ACdlH, Kabel, Brukner, 2404.00159

Operational vs. spacetime notions of events

Coarse-grained vs. fine-grained



- **Coarse-grained notion of event**
  - Only assign **individual identity** to events which can be **distinguished** based on inherent properties.

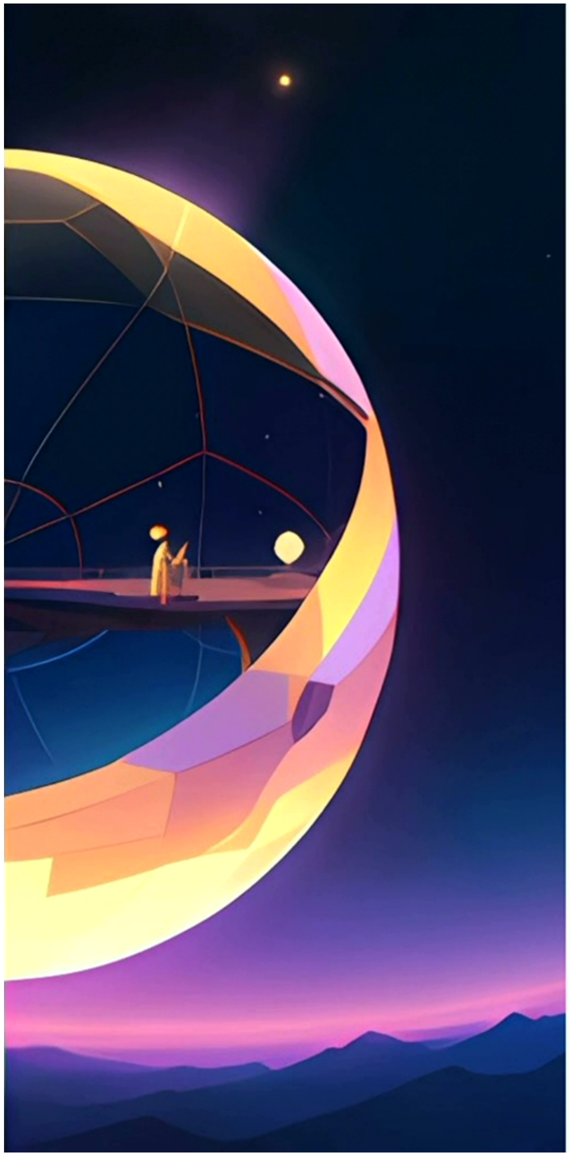
QRF argument

- Importantly, inherent properties should not change under a frame transformation.

Operationalism argument

- Inherent properties should be operationally accessible within the experimental (or measurement) context.

ACdlH, Kabel, Brukner, 2404.00159

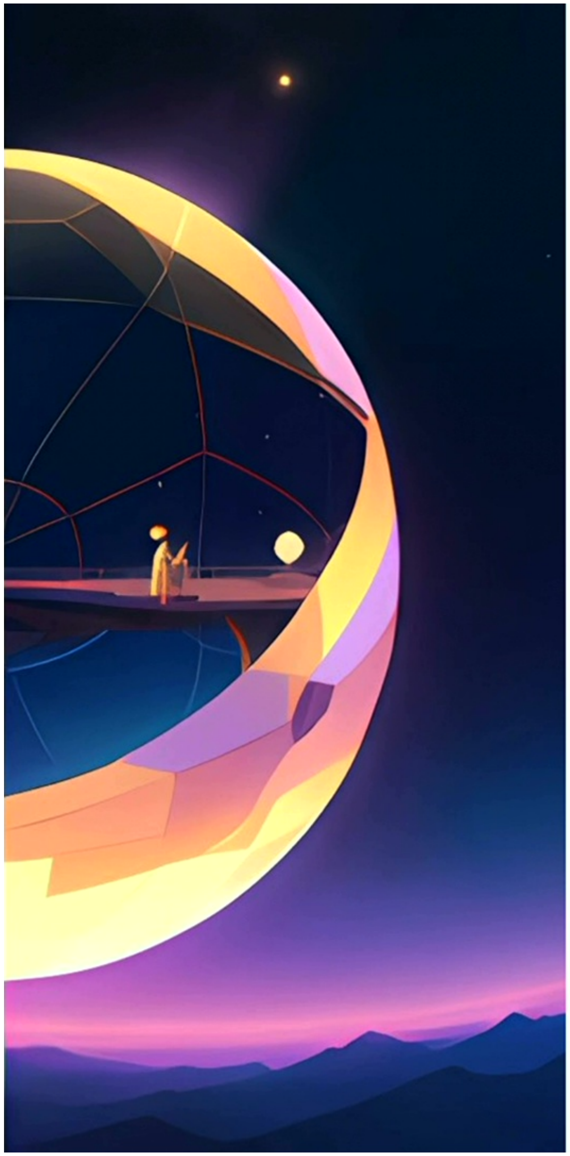


Take home message 1

The **localisation** of **events** in spacetime is **frame-dependent** and has no absolute physical meaning.

→ identification is pointless :)

Kabel, ACdlH, Apadula, Cepollaro, Gomes, Butterfield, Brukner, 2402.10267

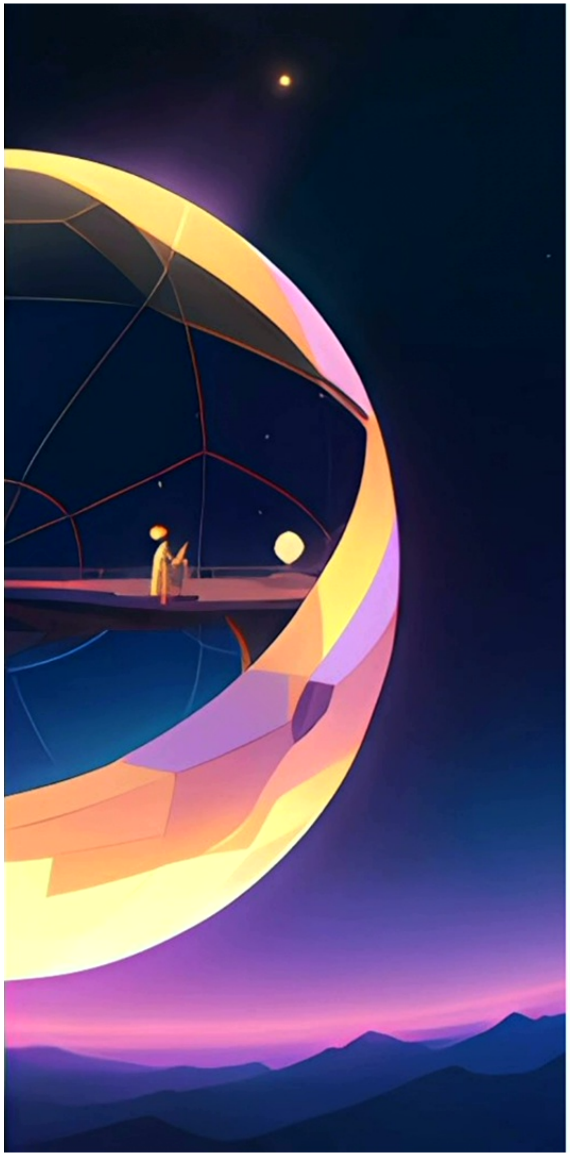


Take home message 2

Whether ICO is due to delocalised **events** or a superposition of **metrics** depends on the choice of quantum coordinates.

Using this notion of causal order, there is **no difference** in the ICO between the **optical** and the **gravitational** quantum switch.

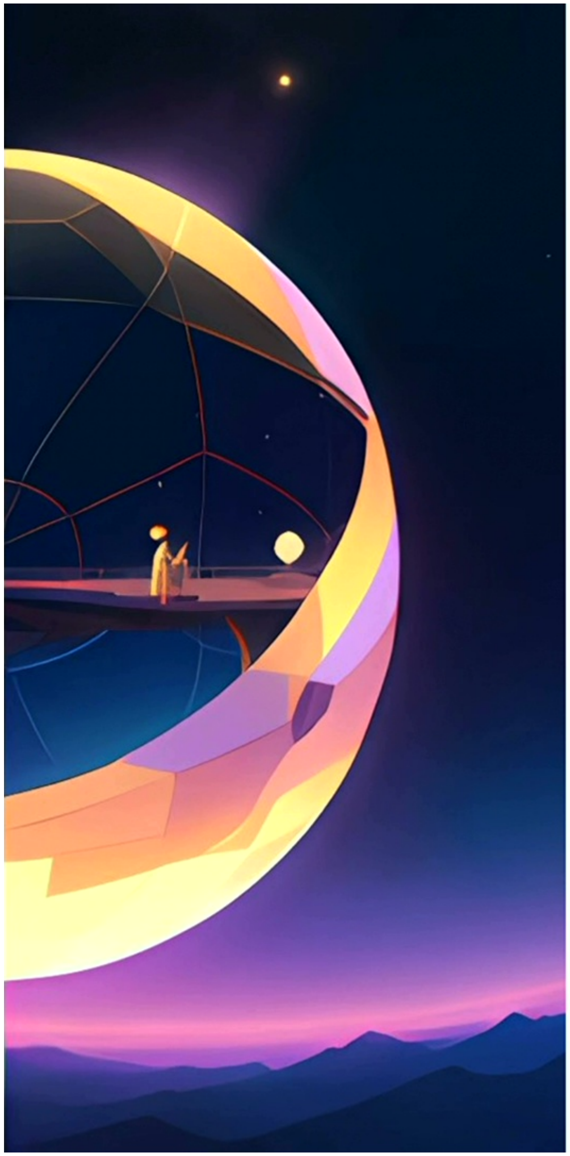
ACdlH, Kabel, Christodoulou, Brukner, 2211.15685



Take home message 3

The **causal order** *between* **events** remains **invariant** but the number of “spacetime events” can vary under changes of quantum frame.

ACdlH, Kabel, Christodoulou, Brukner, 2211.15685



Take home message 3

The **causal order** *between* **events** remains **invariant** but the number of “spacetime events” can vary under changes of quantum frame.

The **spacetime location** of an event is **not** an **inherent** property of the event.

Thank you for your attention!

ACdlH, Kabel, Brukner, 2404.00159

ACdlH, Kabel, Christodoulou, Brukner, 2211.15685