Title: Causally faithful circuits for relativistic realisability, or: What can you do in a spacetime?

Speakers: Tein van der Lugt

Collection/Series: Causalworlds

Subject: Quantum Foundations, Quantum Information

Date: September 20, 2024 - 9:40 AM

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Abstract:

Multipartite quantum channels realisable in a spacetime obey the no-superluminal-signalling constraints imposed by relativistic causality. But what about the converse: Can every channel that exhibits no superluminal signalling also be realised through relativistically valid dynamics? To our knowledge, only special cases of this question have been studied. For bipartite channels, the answer has been found to be negative in general (Beckman et al., 2001), though we will argue that counterexamples must necessarily involve a form of fine-tuning. Another special case of the question has been extensively explored under the name of nonlocal quantum computation in the context of position-based cryptography. We will pose and motivate the question in generality, conjecture a positive answer for all but the fine-tuned channels, and present results towards proving it, drawing on insights from nonlocal quantum computation and the new field of causally faithful circuit decompositions of unitary transformations (see also Tuesday). Beyond their relevance to spacetime realisability, the circuit decompositions involved in addressing the question also find applications in quantum causal modelling.

Causal decompositions for relativistic realisability

Tein van der Lugt [1]
Robin Lorenz [2]
Augustin Vanrietvelde [3]
Rob Spekkens [4]
Jon Barrett [1]

CAUSALWORLDS @ PI

[1] Oxford [2] Quantinuum [3] Télécom Paris [4] Perimeter Institute

20 Sep 2024

What can you do in a spacetime?

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20 Sep 2024

probably pretty much anything

probably pretty much anything

as long as it's not signalling faster than light

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need causal decompositions

Three examples

General conjecture

Progress towards proof

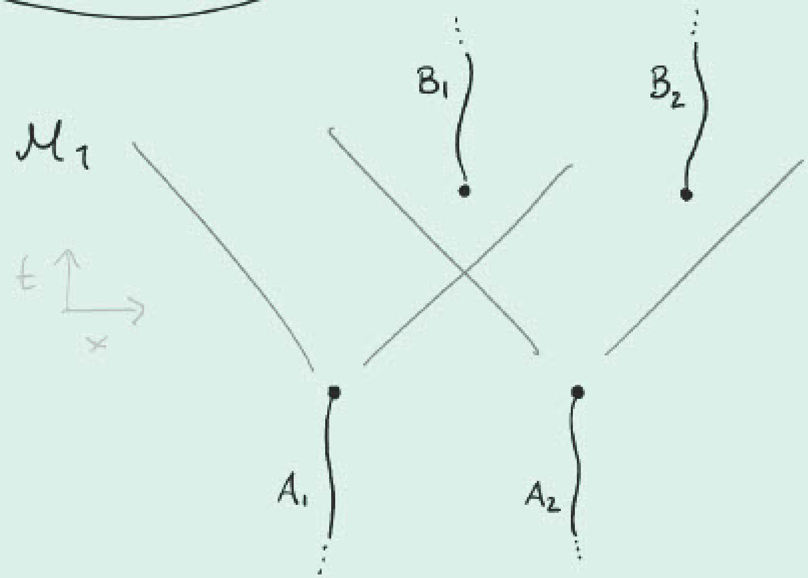
→ Three examples

General conjecture

Progress towards proof

Example 1

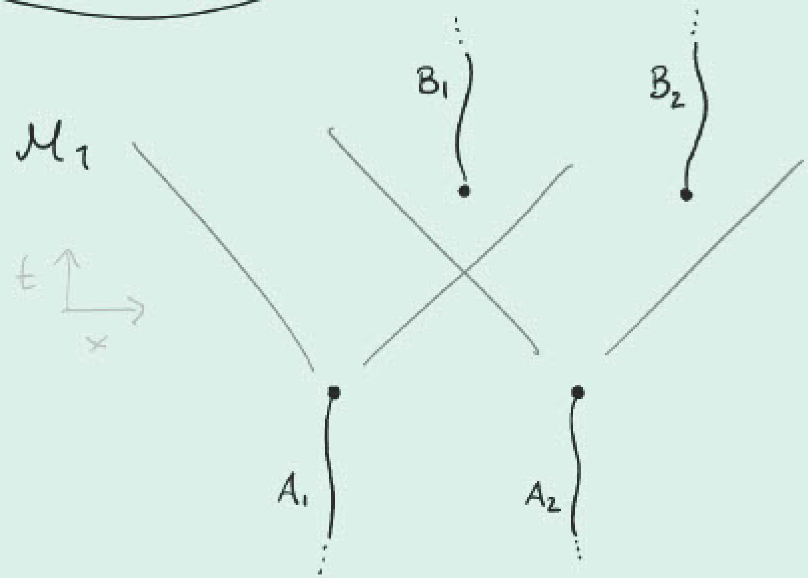
Beckman, Gottesman, Nielsen, Preskill 2001



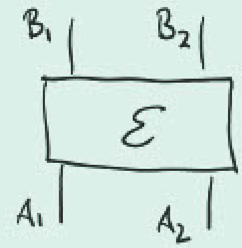
Example 1

Beckman, Gottesman, Nielsen, Preskill 2001

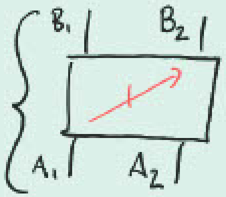
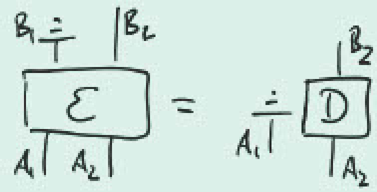
$\mathcal{M}_1$

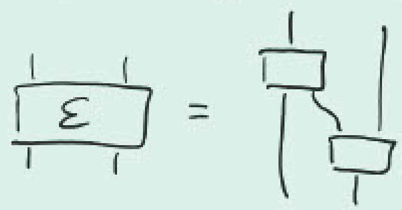


$\xrightarrow[\text{spacetime}]{\text{forget}}$



(CPTP)

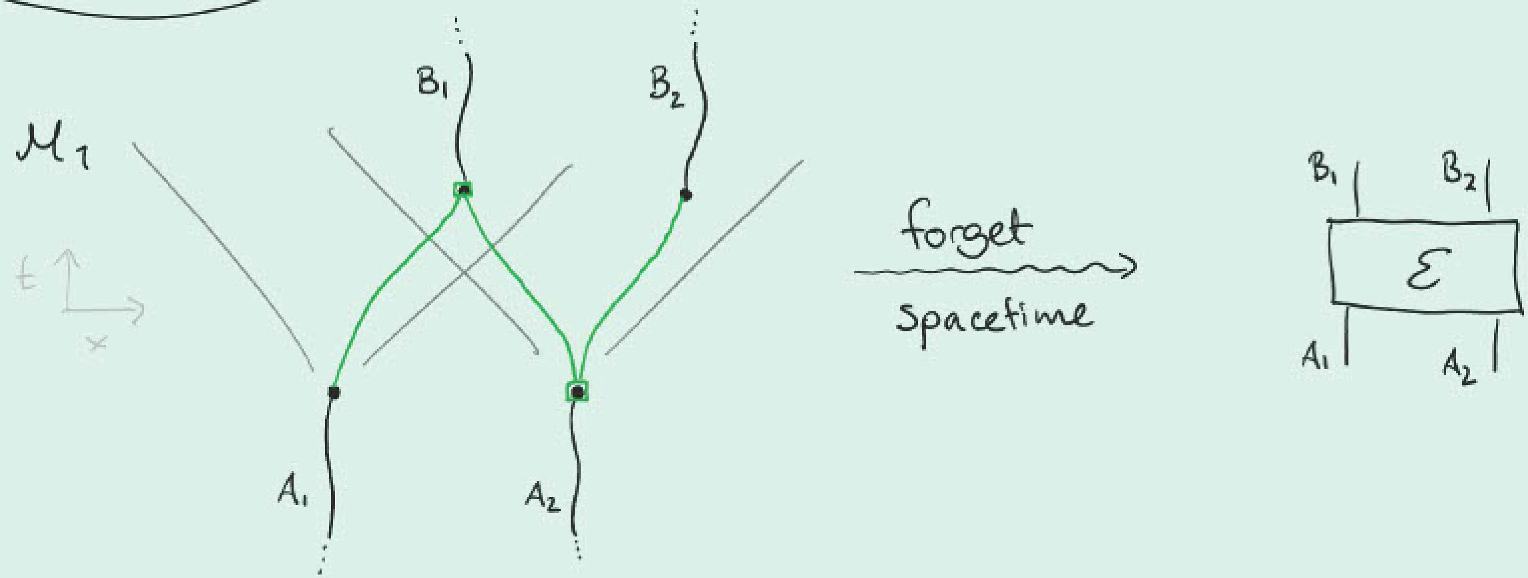
"top-down": $Rel_{\mathcal{M}_1} = \Big\{$  $\Big\} = \Big\{ \mathcal{E} :$  $\Big\}$ ("semicausal")

"bottom-up": $Real_{\mathcal{M}_1} = \Big\{ \mathcal{E} :$  $\Big\}$ ("semilocalisable")

$Real_{\mathcal{M}_1} \subseteq Rel_{\mathcal{M}_1}$

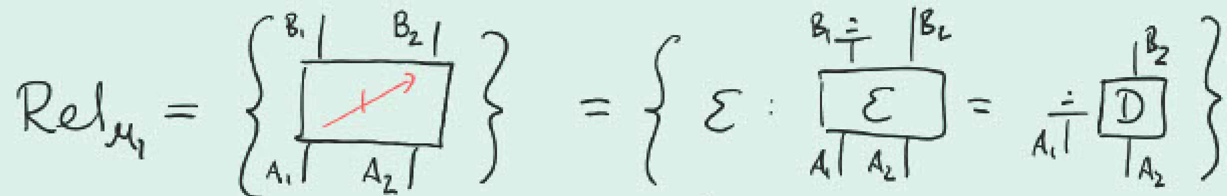
Example 1

Beckman, Gottesman, Nielsen, Preskill 2001



(CPTP)

"top-down":



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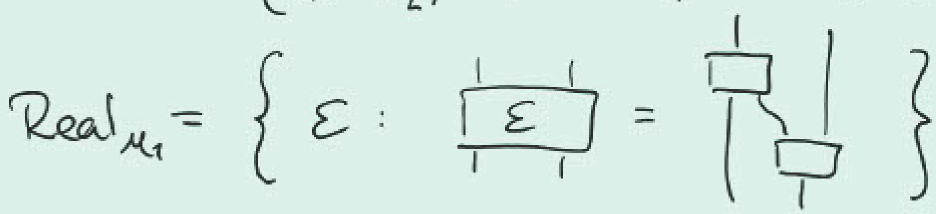
"bottom-up":

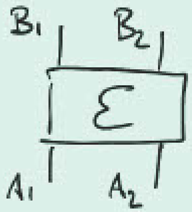


("semilocalisable")

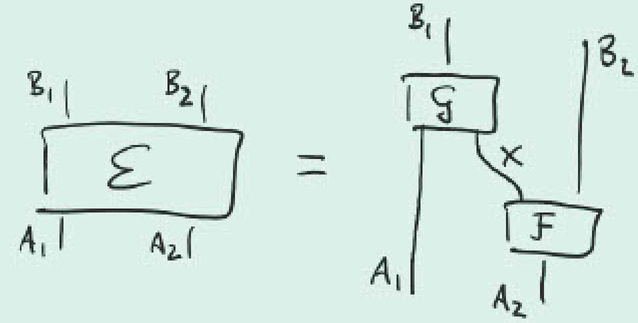
$$\mathrm{Real}_{\mathcal{M}_1} \subseteq \mathrm{Rel}_{\mathcal{M}_1}$$

(Information-theoretic)

$\boxed{\text{Thm}}$ (Eggeling, Schlingemann, Werner 2002)



has no signalling $A_1 \to B_2$ $\Longrightarrow$



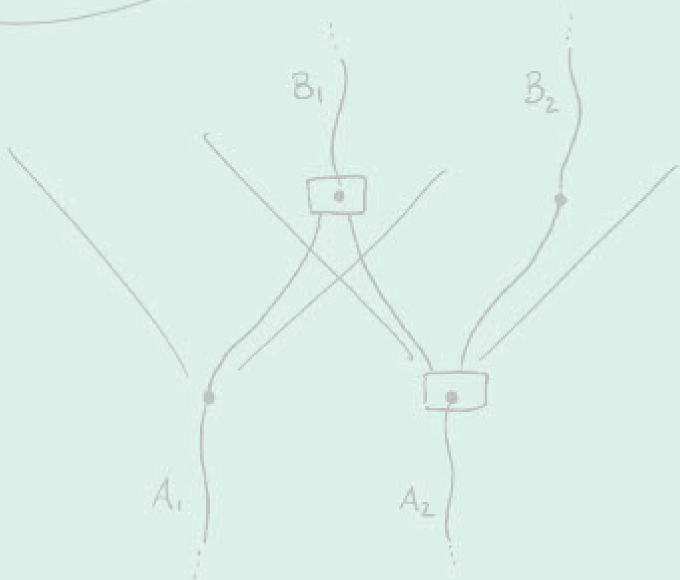
for some $X, \mathcal{F}, \mathcal{G}$.

"Every semicausal channel is semilocalisable"

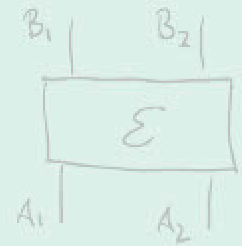
Example 1

Beckman, Gottesman, Nielsen, Preskill 2001

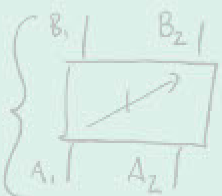
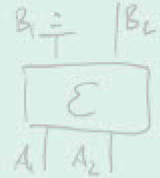
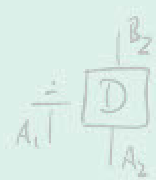
$\mathcal{M}_1$

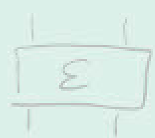
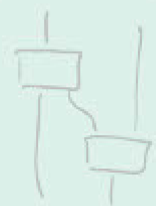


$\xrightarrow[\text{spacetime}]{\text{forget}}$



(CPTP)

"top-down": $\mathrm{Rel}_{\mathcal{M}_1} = \Bigg\{$  $\Bigg\} = \Bigg\{ \mathcal{E} :$  $=$  $\Bigg\}$ ("semicausal")

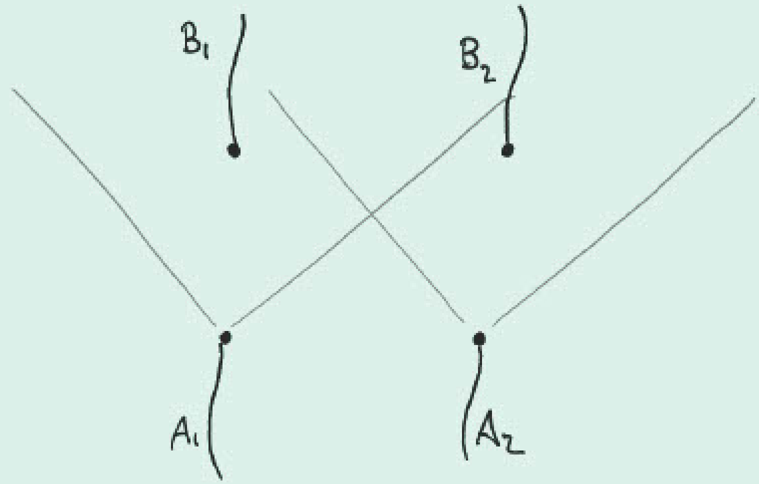
"bottom-up": $\mathrm{Real}_{\mathcal{M}_1} = \Bigg\{ \mathcal{E} :$  $=$  $\Bigg\}$ ("semilocalisable")

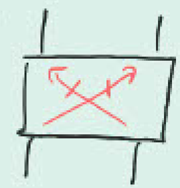
$$\boxed{\mathrm{Real}_{\mathcal{M}_1} = \mathrm{Rel}_{\mathcal{M}_1}}$$

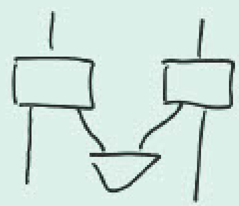
Example 2

Beckman, Gottesman, Nielsen, Preskill 2001

$\mathcal{M}_2$



$Rel_{\mathcal{M}_2} = \{$  $\}$ ("causal")

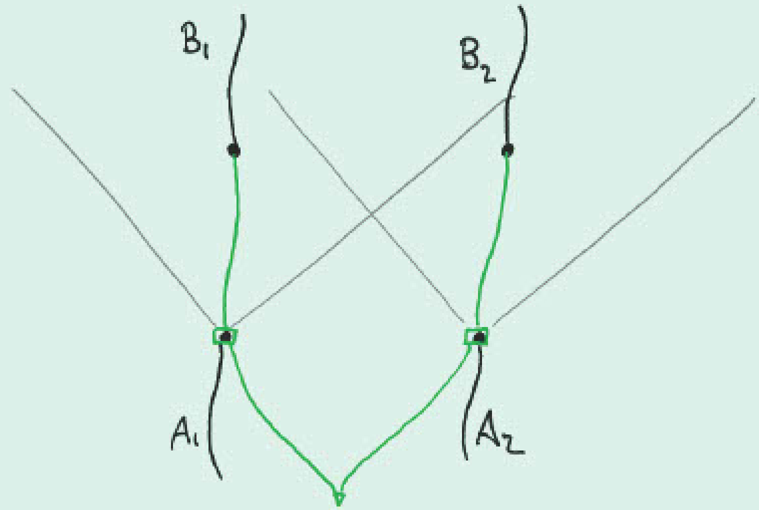
$Real_{\mathcal{M}_2} = \{$  $\}$ ("localisable")

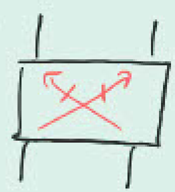
$$Real_{\mathcal{M}_2} \subseteq Rel_{\mathcal{M}_2}$$

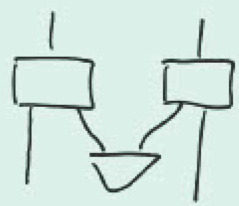
Example 2

Beckman, Gottesman, Nielsen, Preskill 2001

$\mathcal{M}_2$



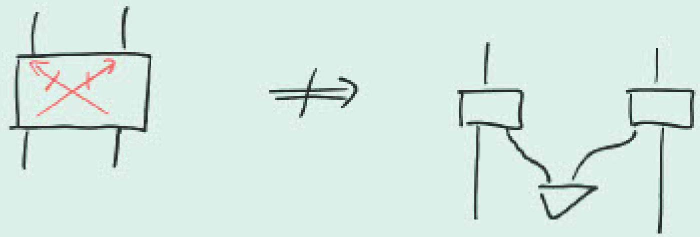
$\mathrm{Rel}_{\mathcal{M}_2} = \{$  $\}$ ("causal")

$\mathrm{Real}_{\mathcal{M}_2} = \{$  $\}$ ("localisable")

$$\mathrm{Real}_{\mathcal{M}_2} \subseteq \mathrm{Rel}_{\mathcal{M}_2}$$

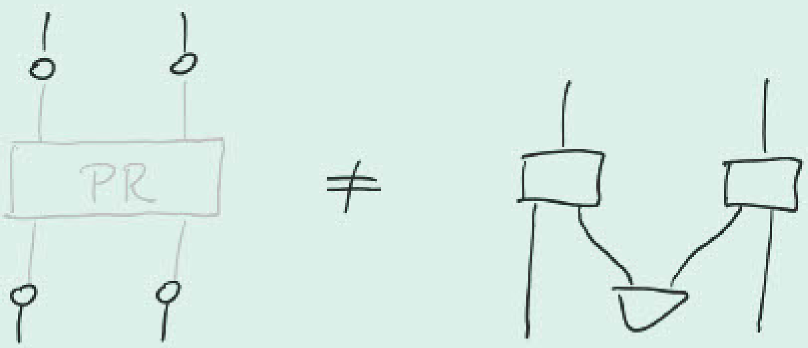
(Information-theoretic)

$\boxed{\text{Thm}}$ (Beckman, Gottesman, Nielsen, Preskill, 2001)



"Not all causal channels are localisable"

Counterexample:



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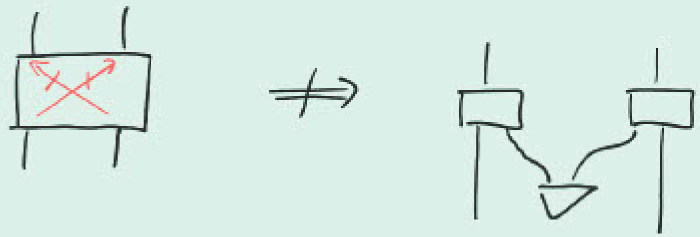

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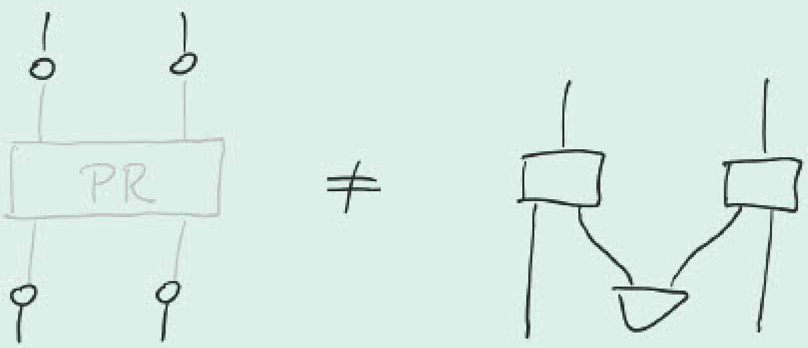
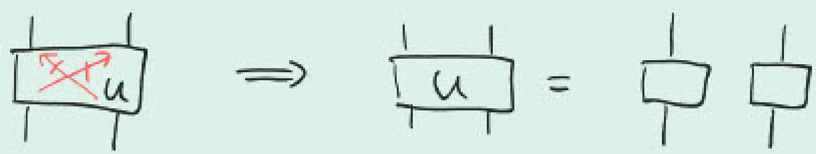
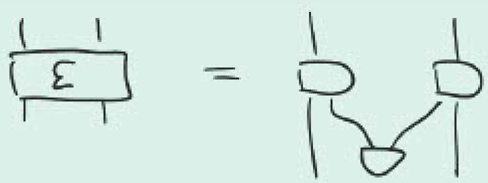
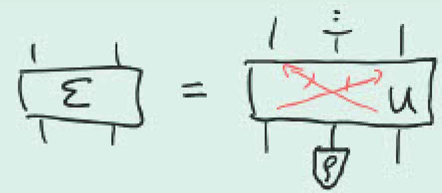
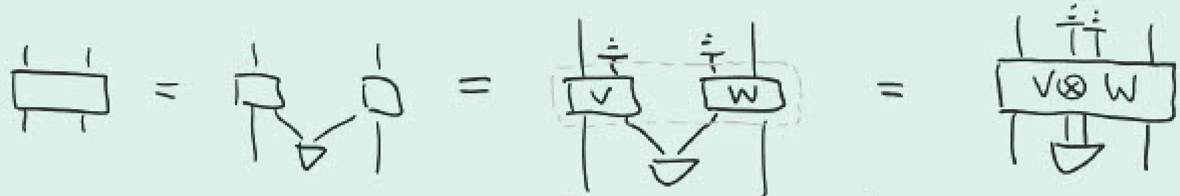
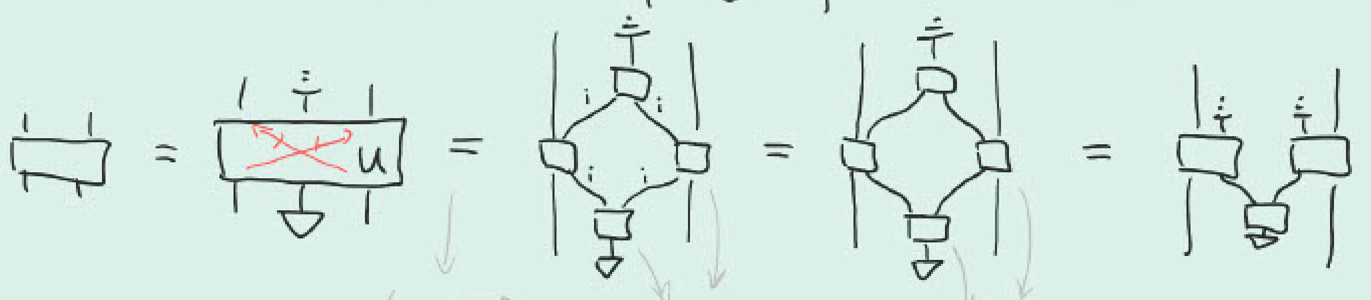


$\boxed{\text{Thm}}$ (BGNP'01) U unitary and



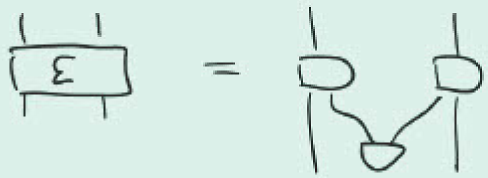
Thm  $\Longleftrightarrow$ $\exists$ unitary dilation 

Prf $\Longrightarrow$: 

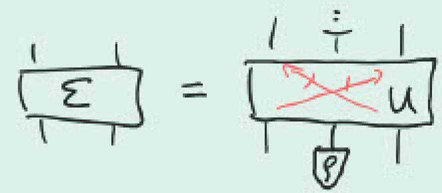
$\Longleftarrow$: 

Lorenz, Barrett '21 routed unitaries general channels

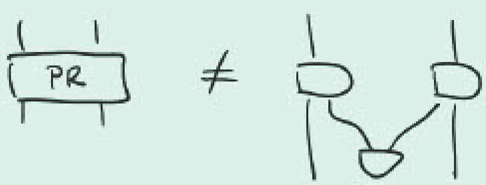
Thm



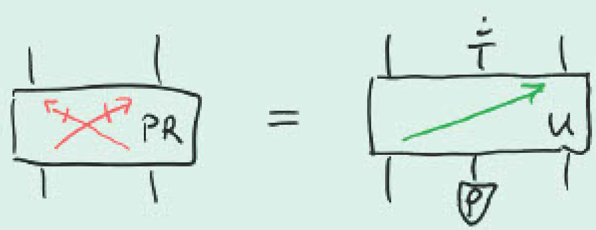
$\iff$ $\exists$ unitary dilation

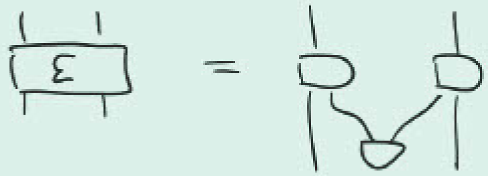


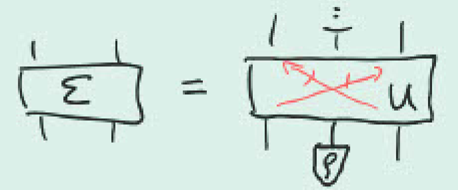
What about the PR box?



so any unitary dilation has



Thm 

$\iff$ $\exists$ unitary dilation 

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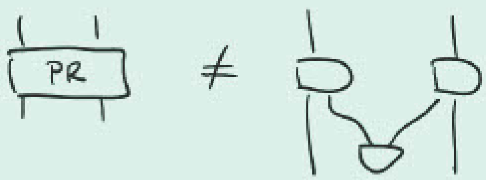

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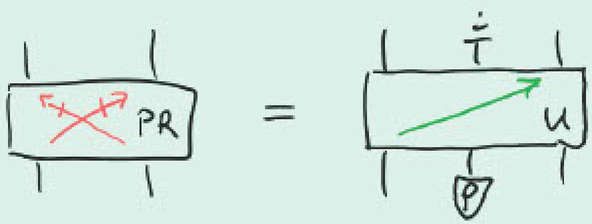
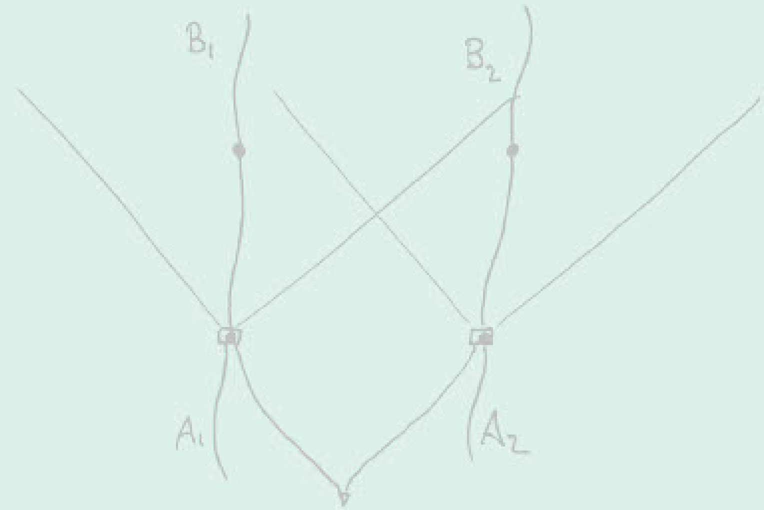
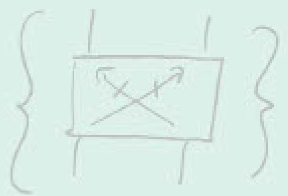


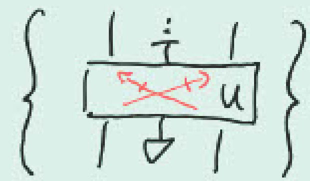
Fine-tuning!

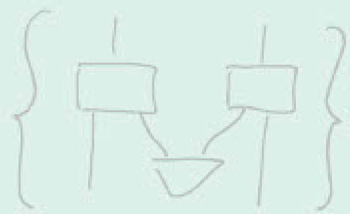
Example 2

$\mathcal{M}_2$



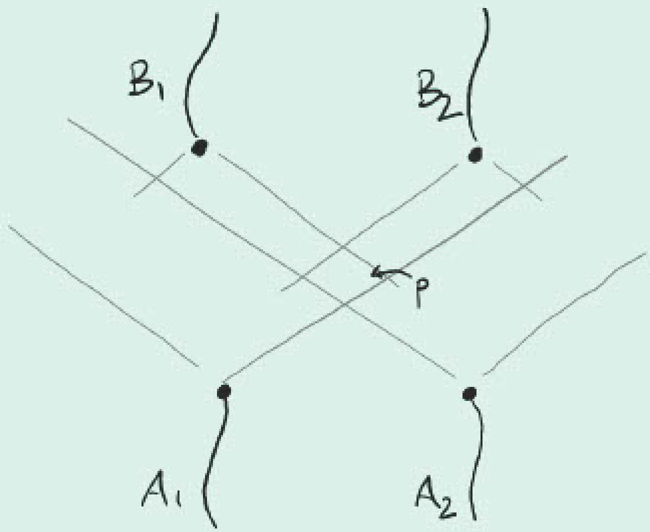
$\mathrm{Rel}_{\mathcal{M}_2} =$  ,

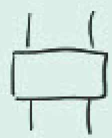
$\mathrm{RelUD}_{\mathcal{M}_2} =$ 

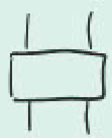
$\mathrm{Real}_{\mathcal{M}_2} =$ 

$$\mathrm{Real}_{\mathcal{M}_2} = \mathrm{RelUD}_{\mathcal{M}_2} \subsetneq \mathrm{Rel}_{\mathcal{M}_2}$$

Example 3

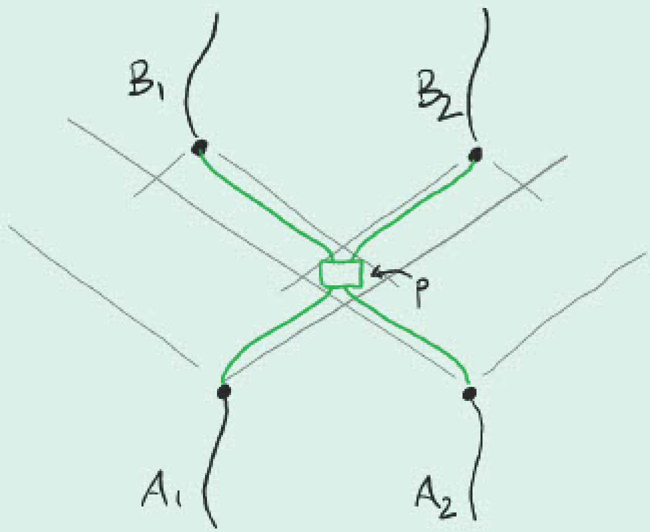


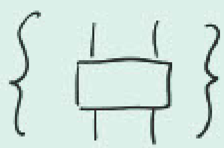
Rel = $\{$  $\}$

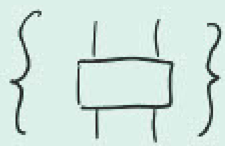
Real = $\{$  $\}$

Real = Rel

Example 3

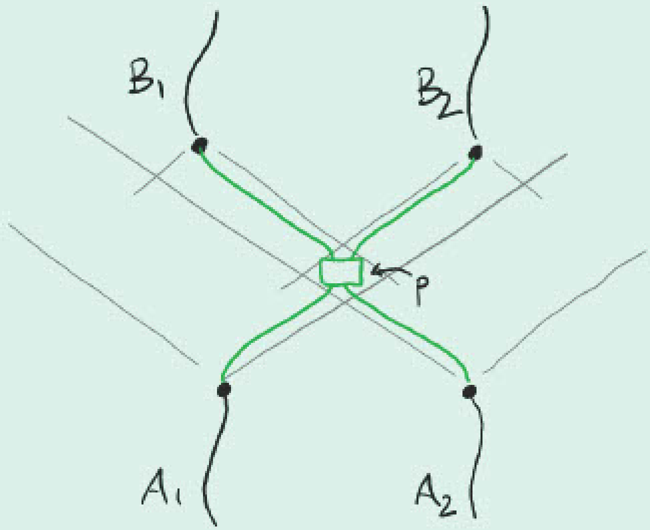


Rel = 

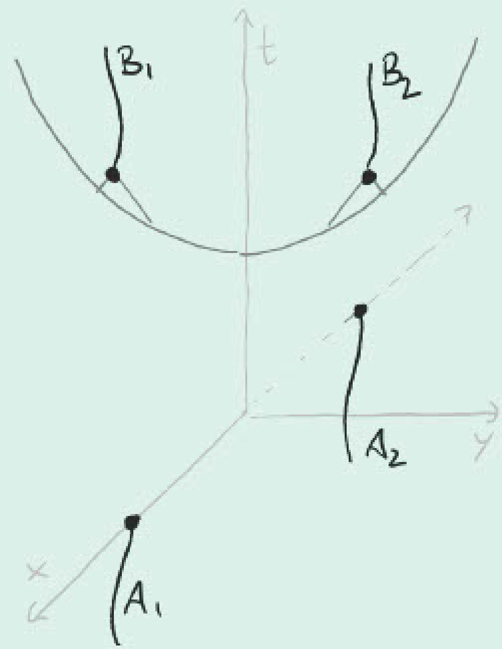
Real = 

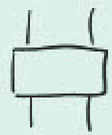
Real = Rel

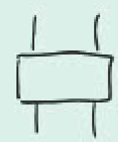
Example 3

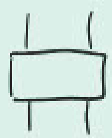


M_3 :



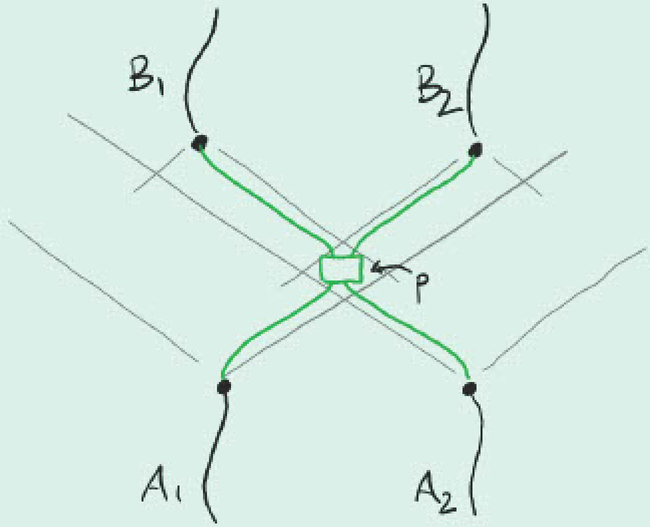
$\mathrm{Rel} = \{$  $\}$

$\mathrm{Rel}_{M_3} = \{$  $\}$

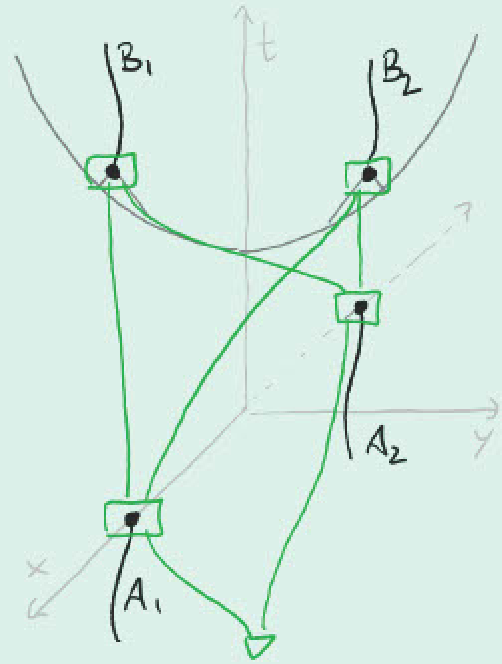
$\mathrm{Real} = \{$  $\}$

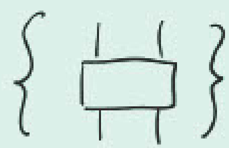
$\mathrm{Real} = \mathrm{Rel}$

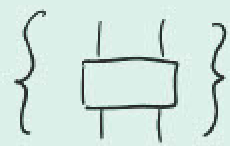
Example 3



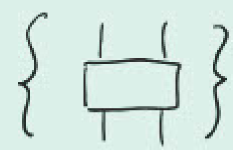
$\mathcal{M}_3$:

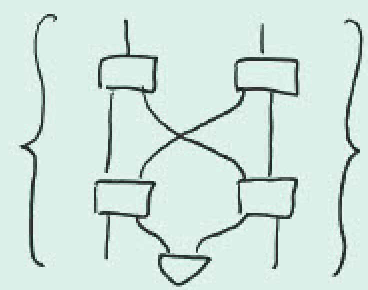


Rel = 

Real = 

Real = Rel

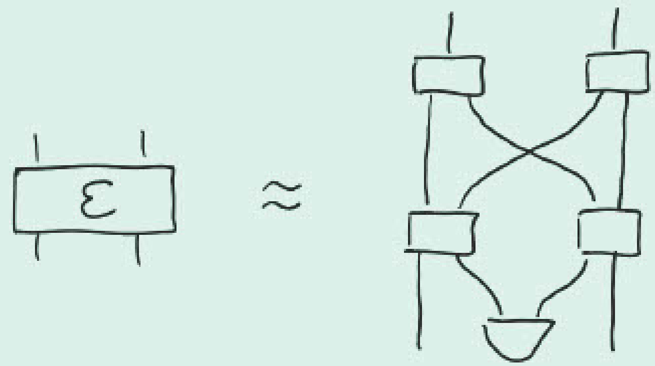
$\mathrm{Rel}_{\mathcal{M}_3}$ = 

$\mathrm{Real}_{\mathcal{M}_3}$ = 

(Information-theoretic)

$\boxed{\text{Thm}}$ (Buhrman et al., 2010/2014; Beigi, König, 2011)

$\forall \mathcal{E}$:



"(Instantaneous) nonlocal quantum computation" (NLQC)

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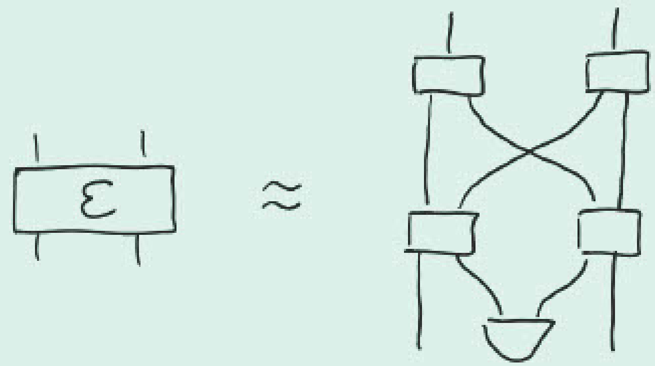

"(Instantaneous) nonlocal quantum computation" (NLQC)

Cf. Alex Pozas-Kerstjens' talk Wednesday

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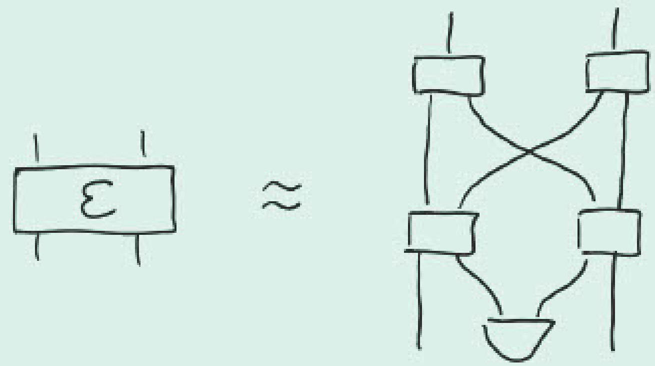


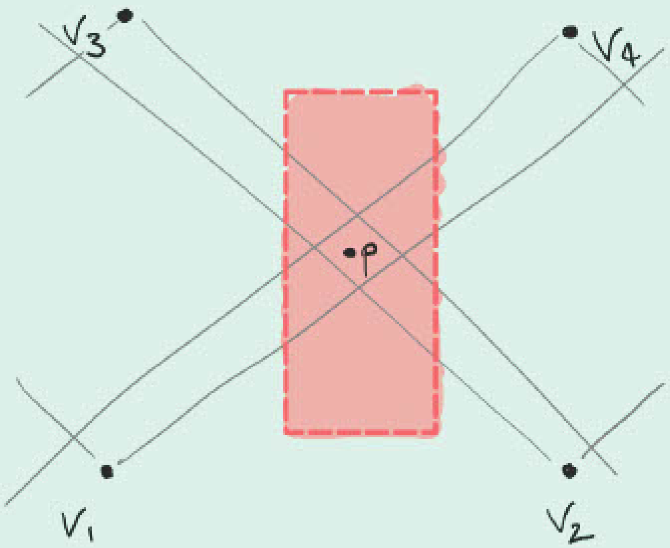
"(Instantaneous) nonlocal quantum computation" (NLQC)

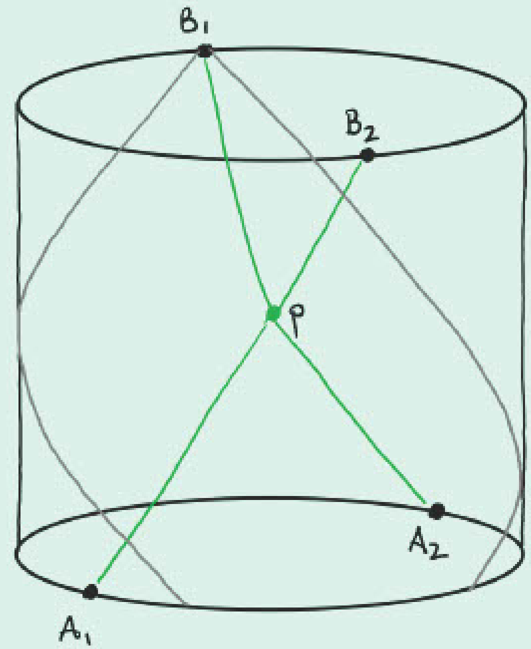
Cf. Alex Pozas-Kerstjens' talk Wednesday

Miller, Alnawakhtha, 2024 :

Exact NLQC is impossible in general

arXiv: 2406.20022



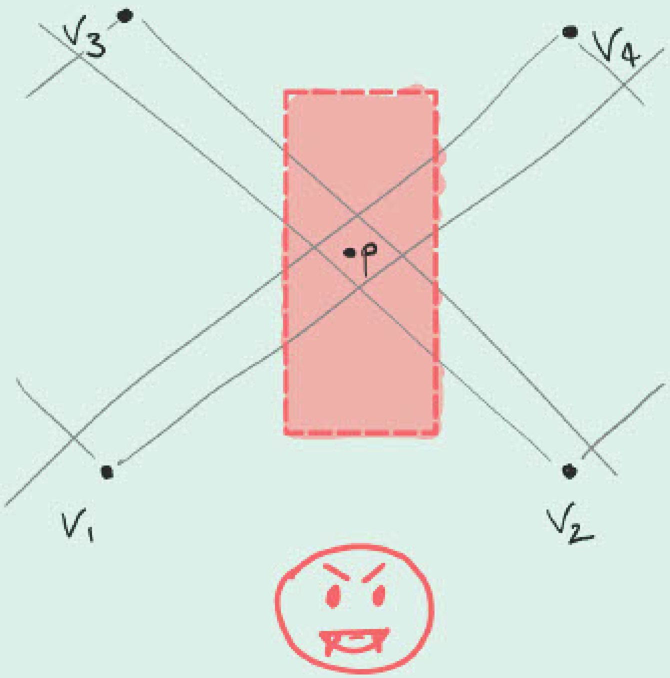


Quantum position verification

(Kent, Munro, Spiller, 2011;
Buhrman et al. 2014; etc etc)

Holographic principle

(May, 2019)



Quantum position verification

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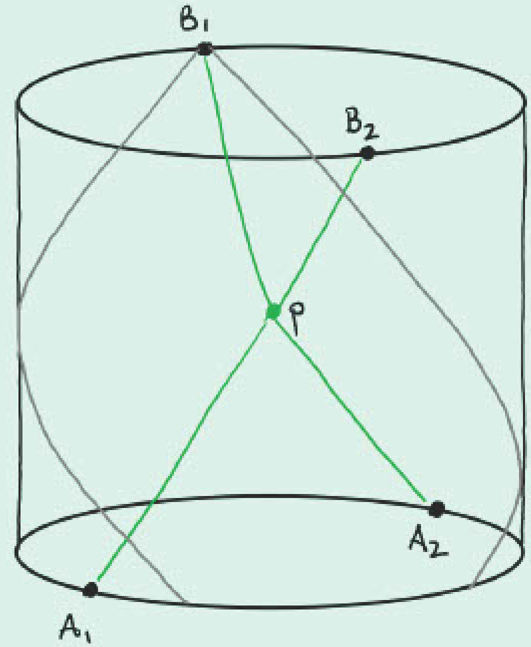
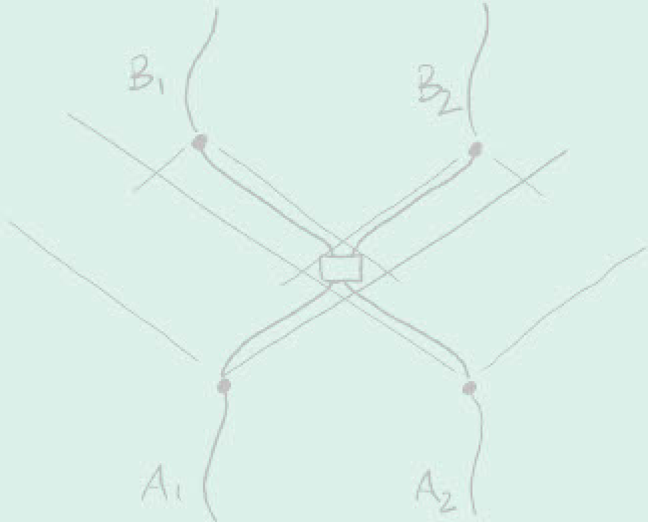
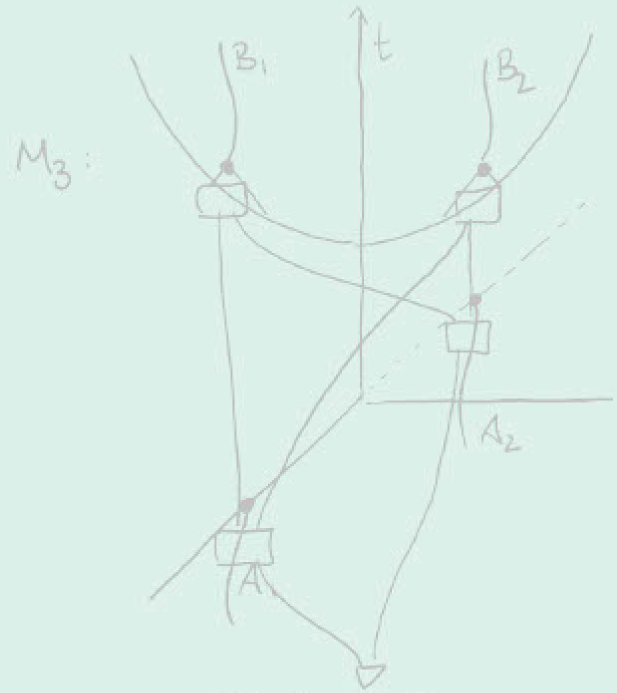


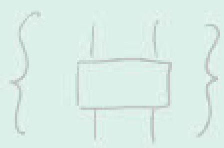
Holographic principle

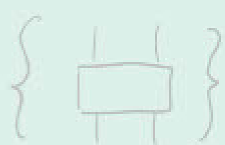
(May, 2019)

Example 3



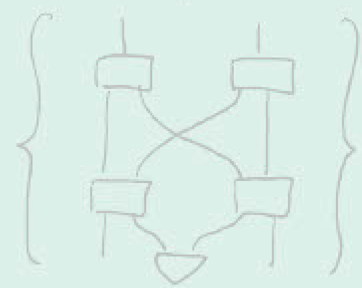


$\mathrm{Rel} =$ 

$\mathrm{Real} =$ 

$\mathrm{Real} = \mathrm{Rel}$

$\mathrm{Rel}_{M_3} =$ 

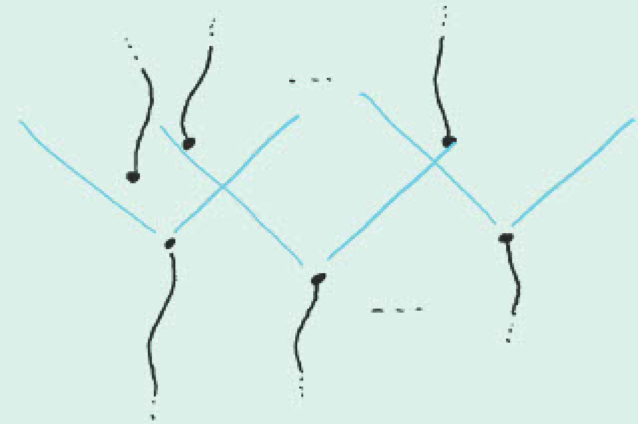
$\mathrm{Real}_{M_3} =$ 

$\mathrm{Real}_{M_3} \subsetneq \mathrm{Rel}_{M_3}$ but $\boxed{\overline{\mathrm{Real}_{M_3}} = \mathrm{Rel}_{M_3}}$

Three examples

→ **General conjecture**

Progress towards proof



A **spacetime situation** is a tuple

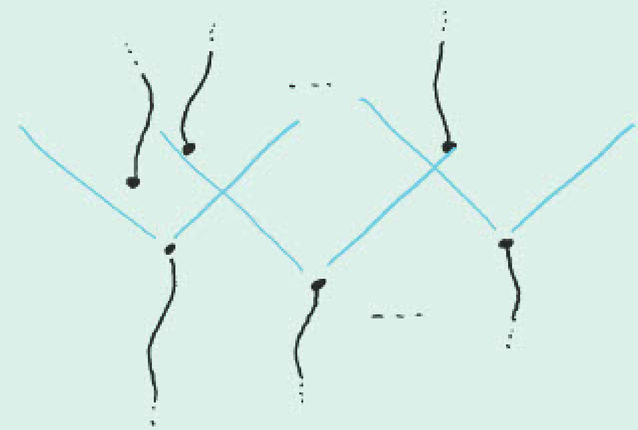
$$\mathcal{M} = \left((M,g),\ \{A_i\}_{i=1}^{m},\ \{B_j\}_{j=1}^{n},\ \{P_{A_i}\}_{i=1}^{m},\ \{P_{B_j}\}_{j=1}^{n} \right)$$

cf. quantum tasks (Kent 2012; Dolev 2019; May 2019)

$\boxed{\text{Def}}$ $\mathrm{Rel}_{\mathcal{M}}$:= set of **Rel**ativistically causal channels

$\mathrm{RelUD}_{\mathcal{M}}$:= set of channels with rel. caus. **U**nitary **D**ilation

$\overline{\mathrm{Real}_{\mathcal{M}}}$ = set of approximately relativistically **Real**isable channels



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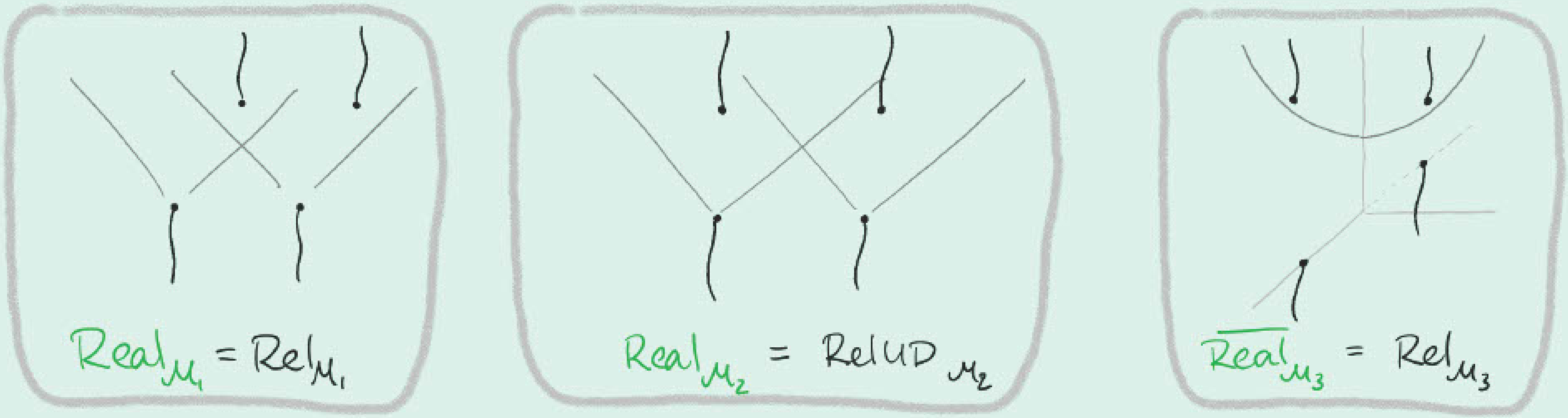
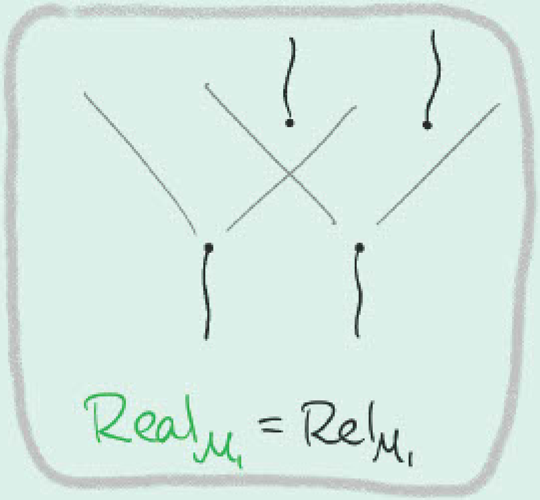


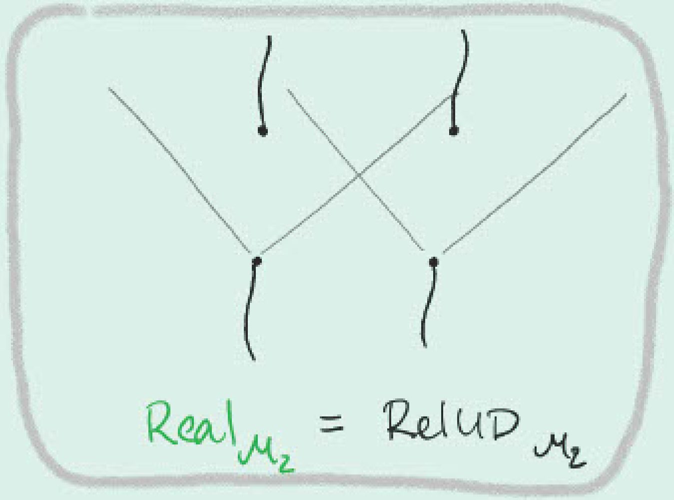
$\boxed{\text{Conj}}$ $\quad \forall \mathcal{M} : \quad \overline{\mathrm{Real}}_{\mathcal{M}} = \mathrm{RelUD}_{\mathcal{M}}$

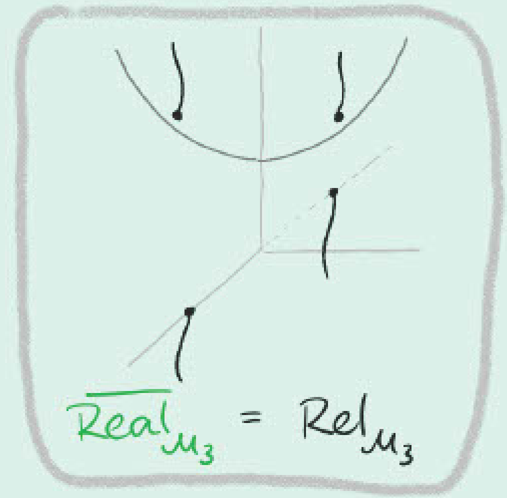
"A chanel is approximately realisable in $\mathcal{M}$

iff

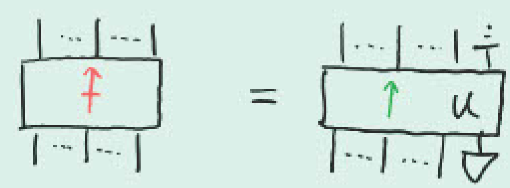
it has a relativistically causal unitary dilation"

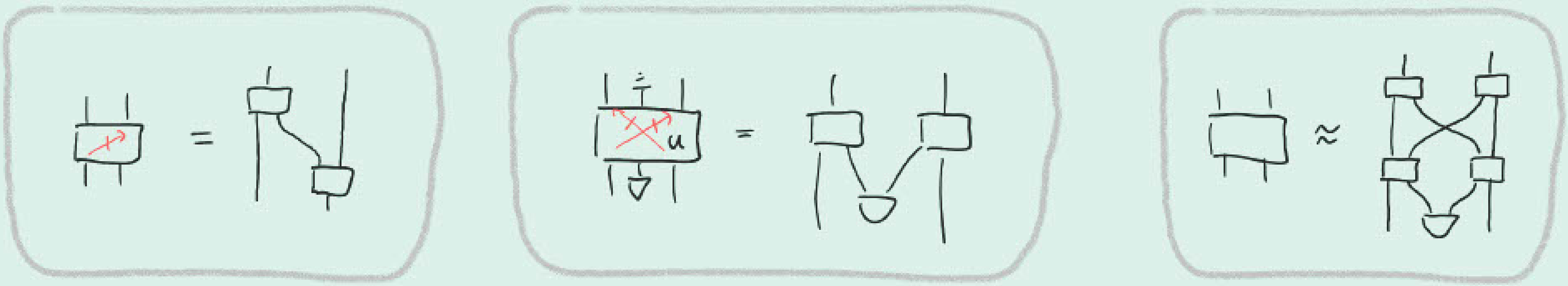


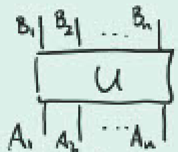


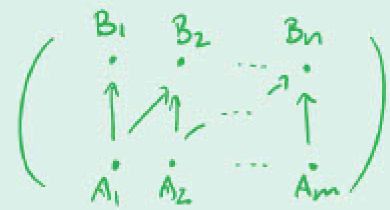


$\boxed{\text{Conj}}$ $\forall \mathcal{M}$: $\overline{\mathrm{Real}}_{\mathcal{M}} = \mathrm{RelUD}_{\mathcal{M}} \subseteq \mathrm{Rel}_{\mathcal{M}}$

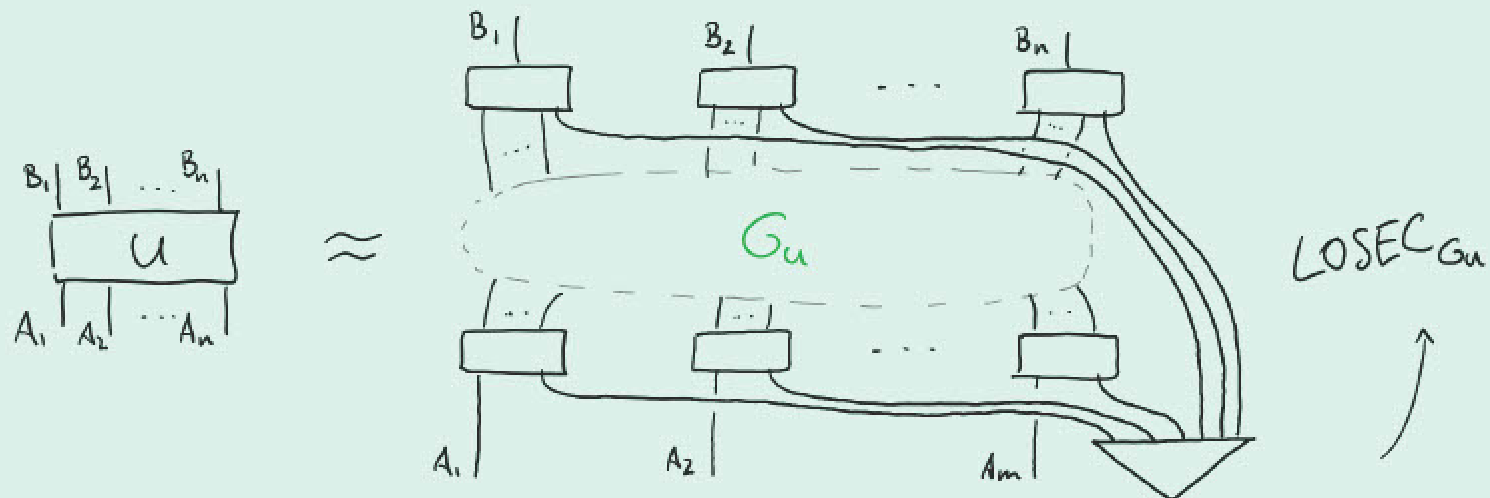
N.B. All channels in $\mathrm{Rel}_{\mathcal{M}} \setminus \mathrm{RelUD}_{\mathcal{M}}$ exhibit fine-tuning, just like the PR box. ()



Conj Any unitary 

with causal structure G_u 

can be approximated by



i.e. **L**ocal **O**perations, **S**hared **E**ntanglement, and (non-interactive, quantum) **C**ommunication along G_u

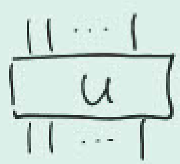
(Spacetime)

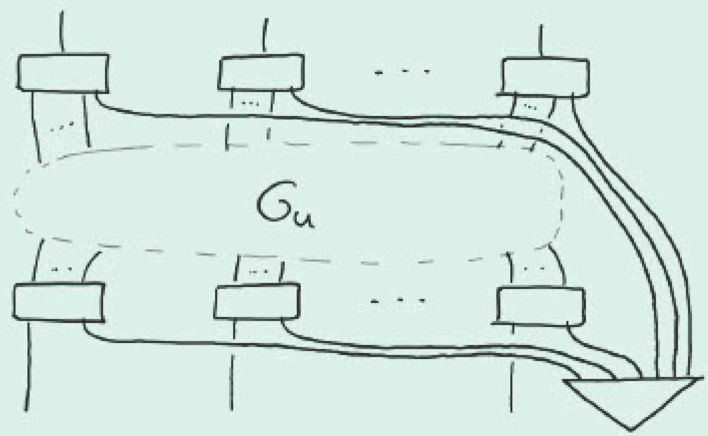
$\boxed{\text{Conj}}$ $\forall \mathcal{M}$: $\overline{\text{Real}_{\mathcal{M}}} = \text{RelUD}_{\mathcal{M}}$

$\Updownarrow$

For $\Downarrow$: $\boxed{\text{Lma}}$ Every causal structure G_U occurs in some spacetime M.
(Corollary of Anusiak, 1965)

(Info-theoretic)

$\boxed{\text{Conj}}$ $\forall U$: 

$\approx$ 

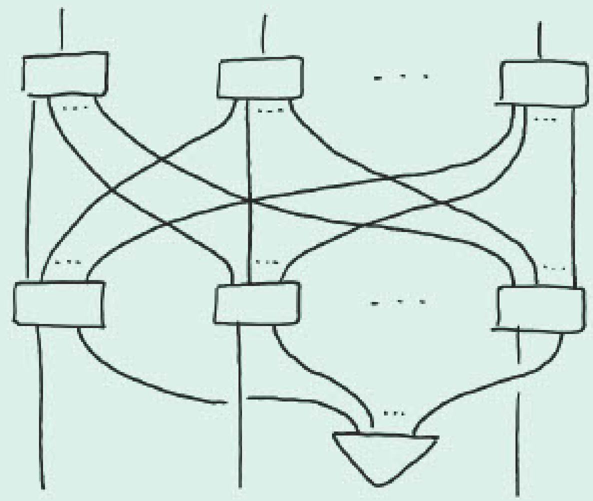
Three examples

General conjecture

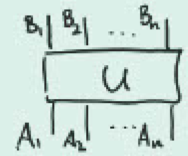
→ Progress towards proof

Case 0 : Communication everywhere

Just nonlocal quantum computation:

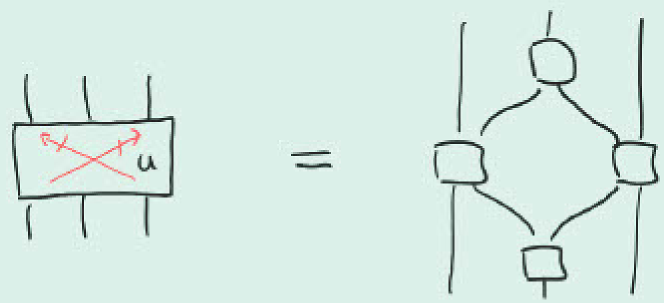
$\boxed{\text{Thm}}$ $\forall U$: $\boxed{U}$ $\approx$ 

Case 1 : Known causal decompositions

$\boxed{\text{Def}}$ A causal(ly faithful circuit) decomposition of 

is s.t.

$\exists$ (directed) path from A_i to B_j $\iff$ $A_i \to B_j$ through U.



[1] Lorenz, Barrett '21 Quantum 5, 511

[2] Augustin Vanrietvelde, PIRSA: 24090117

[3] TvdL, PIRSA: 24090118

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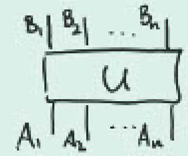
$\boxed{\text{Def}}$ A causal(ly faithful circuit) decomposition of 

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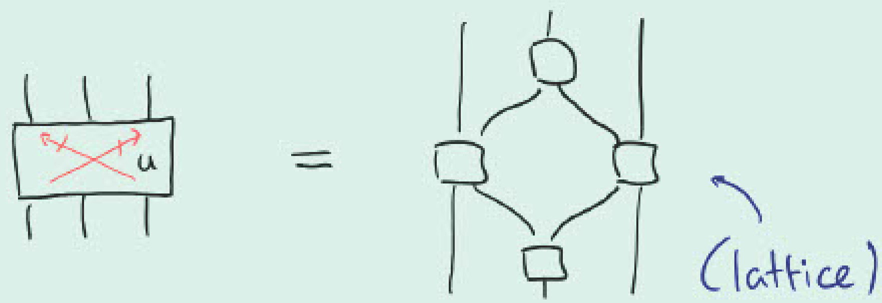

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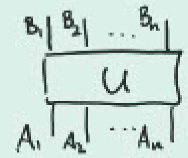
$\boxed{\text{Def}}$ A causal(ly faithful circuit) decomposition of 

is s.t.

$\exists$ (directed) path from A_i to B_j $\iff$ $A_i \to B_j$ through U.

$\boxed{\text{Conj}}$ Every unitary has a causal decomposition.

Proven for

- up to 3 inputs / up to 3 outputs [1]
- cellular automata [2]
- Parental intersection condition [3]

[1] Lorenz, Barrett '21 Quantum 5, 511

[2] Augustin Vanrietvelde, PIRSA: 24090117

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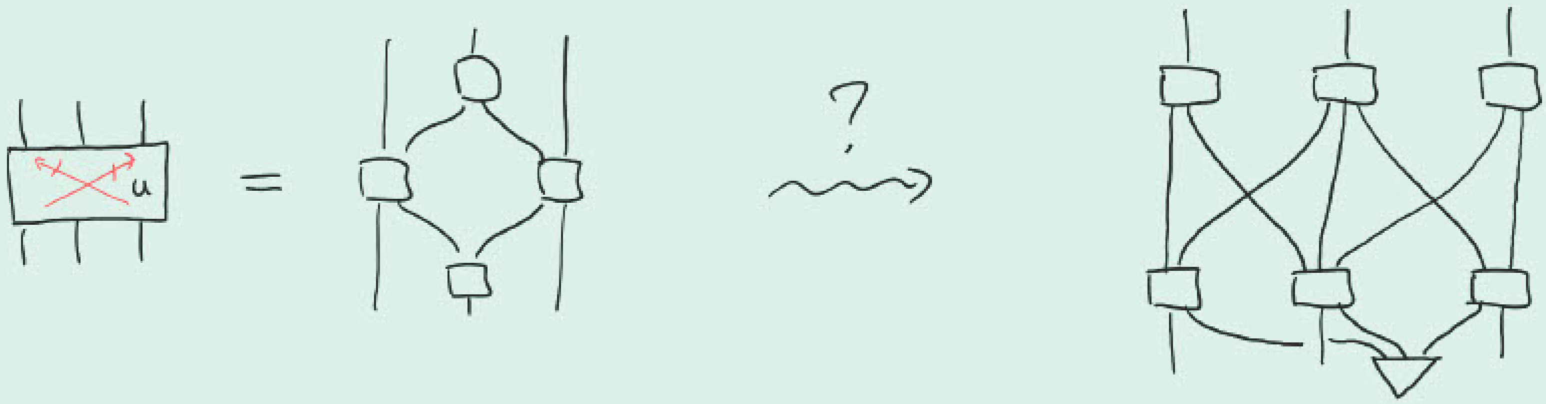

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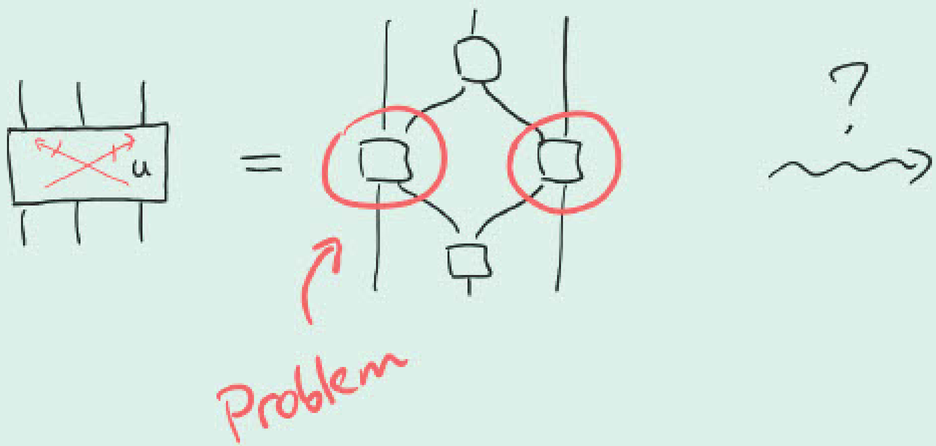
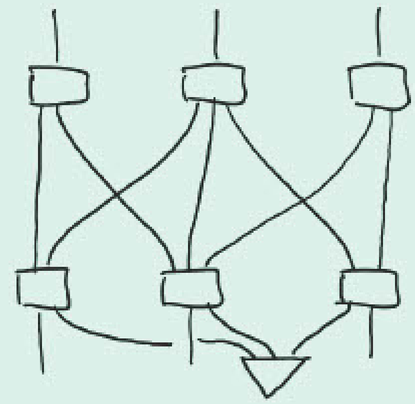
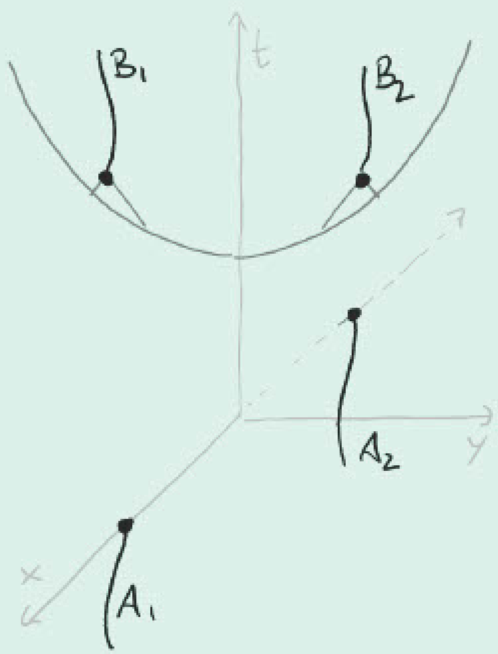
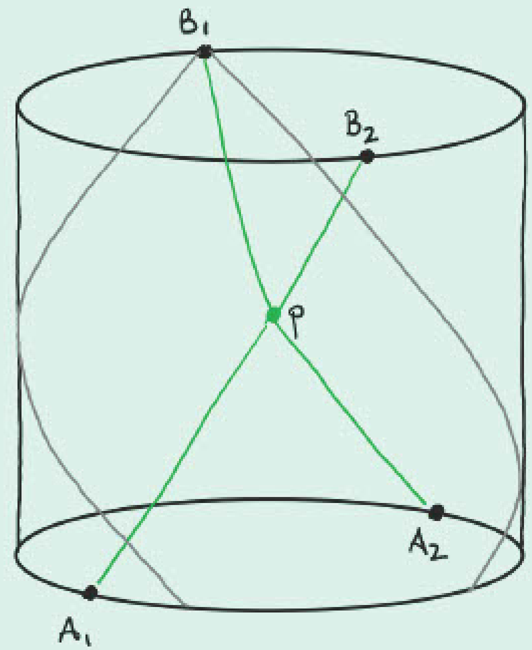





B1
B2
t
A2
y
x
A1


B1
B2
p
A2
A1

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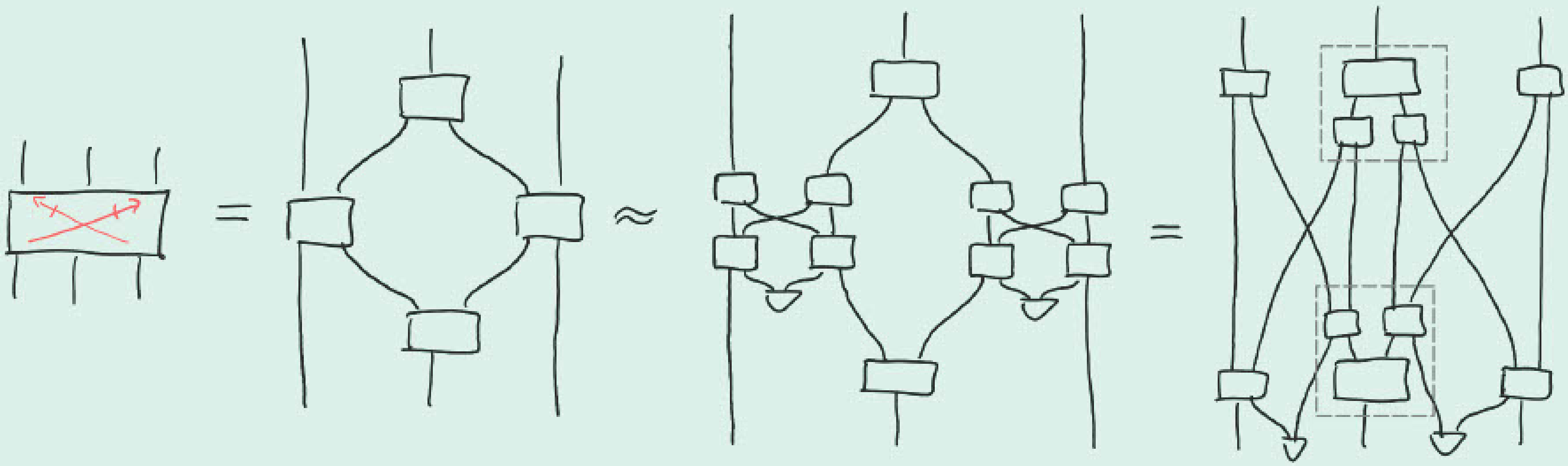
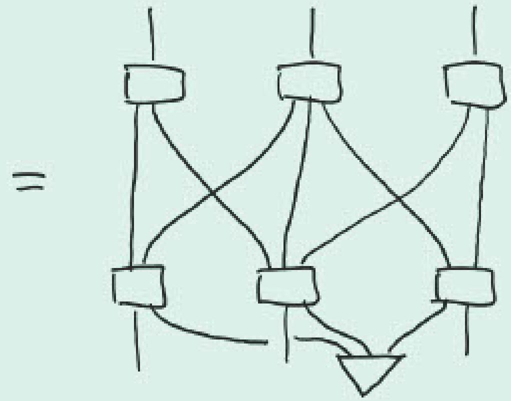
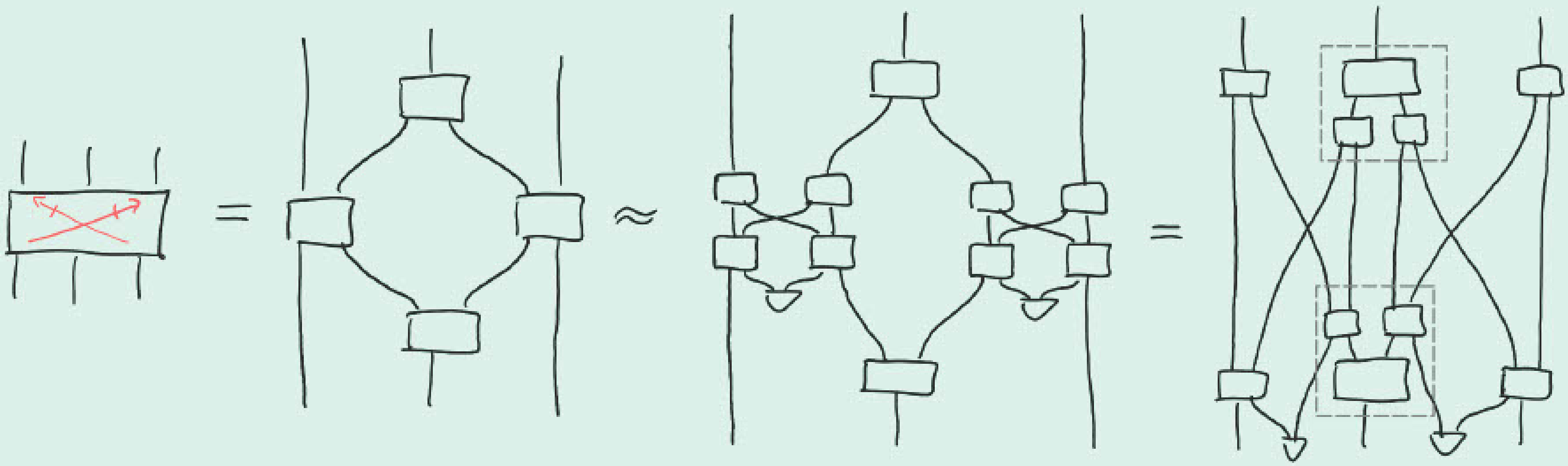
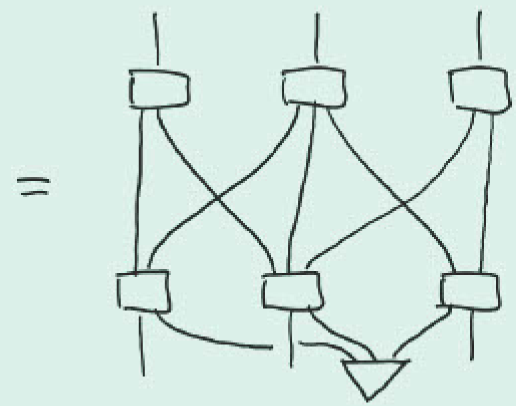


Dolev (2019) arXiv: 1909.05403



Case 1 : Known causal decompositions





Dolev (2019) arXiv: 1909.05403

Coecke, Kissinger (2017)
"Picturing Quantum Processes"

Case 2: Cliffords

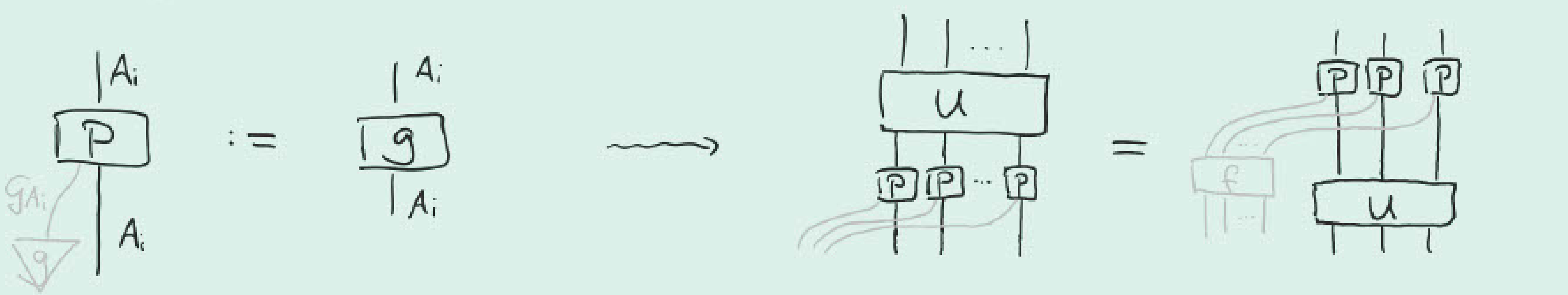
$A_i \rightsquigarrow$ generalised Pauli group $\mathcal{G}_{A_i}$

$A_1 \cdots A_m \rightsquigarrow \mathcal{G}_{A_1 \cdots A_m} := \{ g_1 \otimes \cdots \otimes g_m \mid \forall i \;\; g_i \in \mathcal{G}_{A_i} \}$

$\boxed{\text{Def}}$ $U : A_1 \cdots A_m \to B_1 \cdots B_n$ is <u>Clifford</u> iff

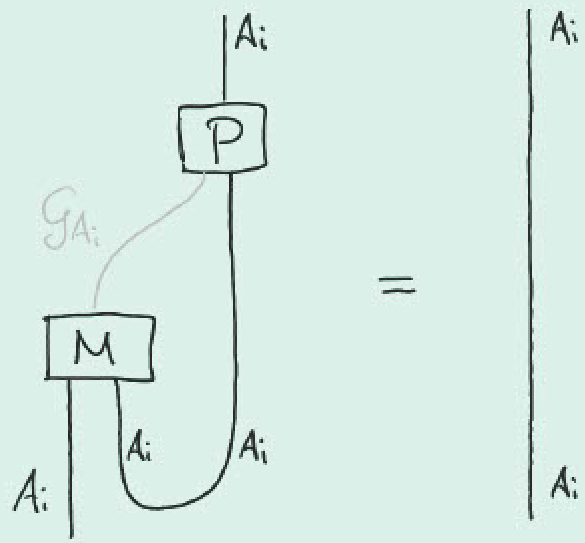
$\forall g \in \mathcal{G}_{A_1 \cdots A_m}: \quad U g U^{\dagger} = f(g)$ for some $f : \mathcal{G}_{A_1 \cdots A_m} \to \mathcal{G}_{B_1 \cdots B_n}$

Diagrammatically:

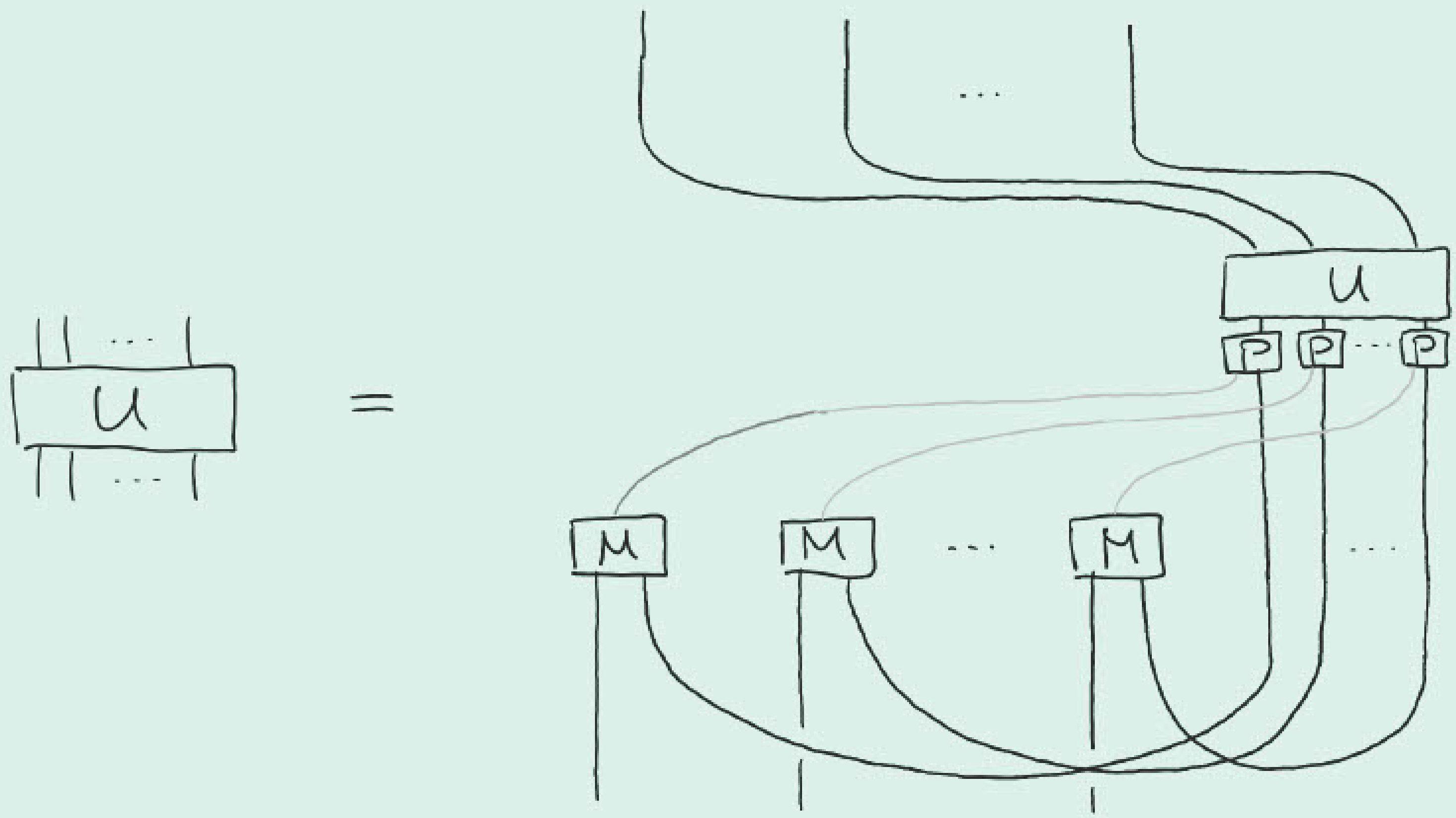


Case 2: Cliffords

Teleportation:



Case 2: Cliffords



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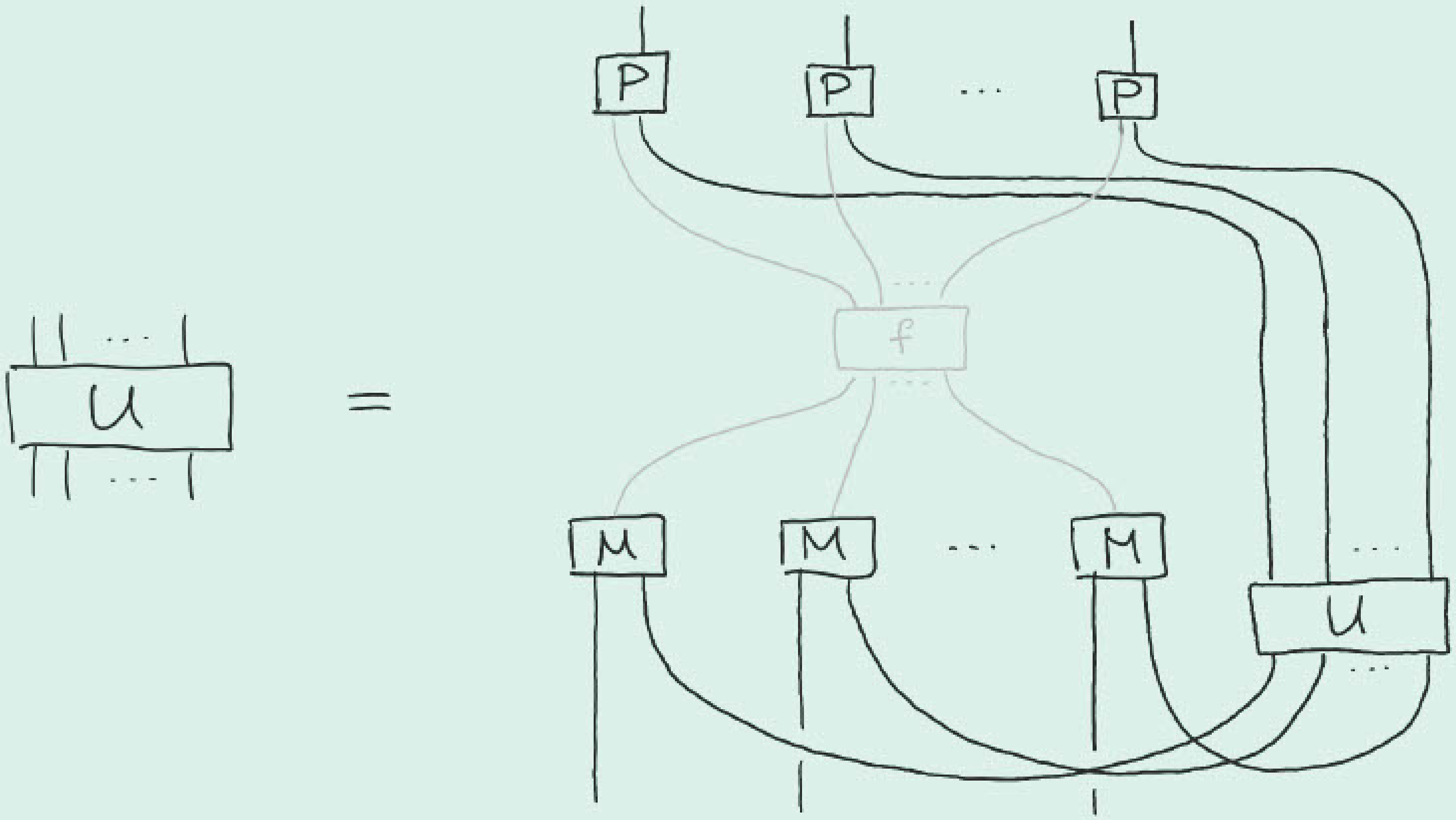

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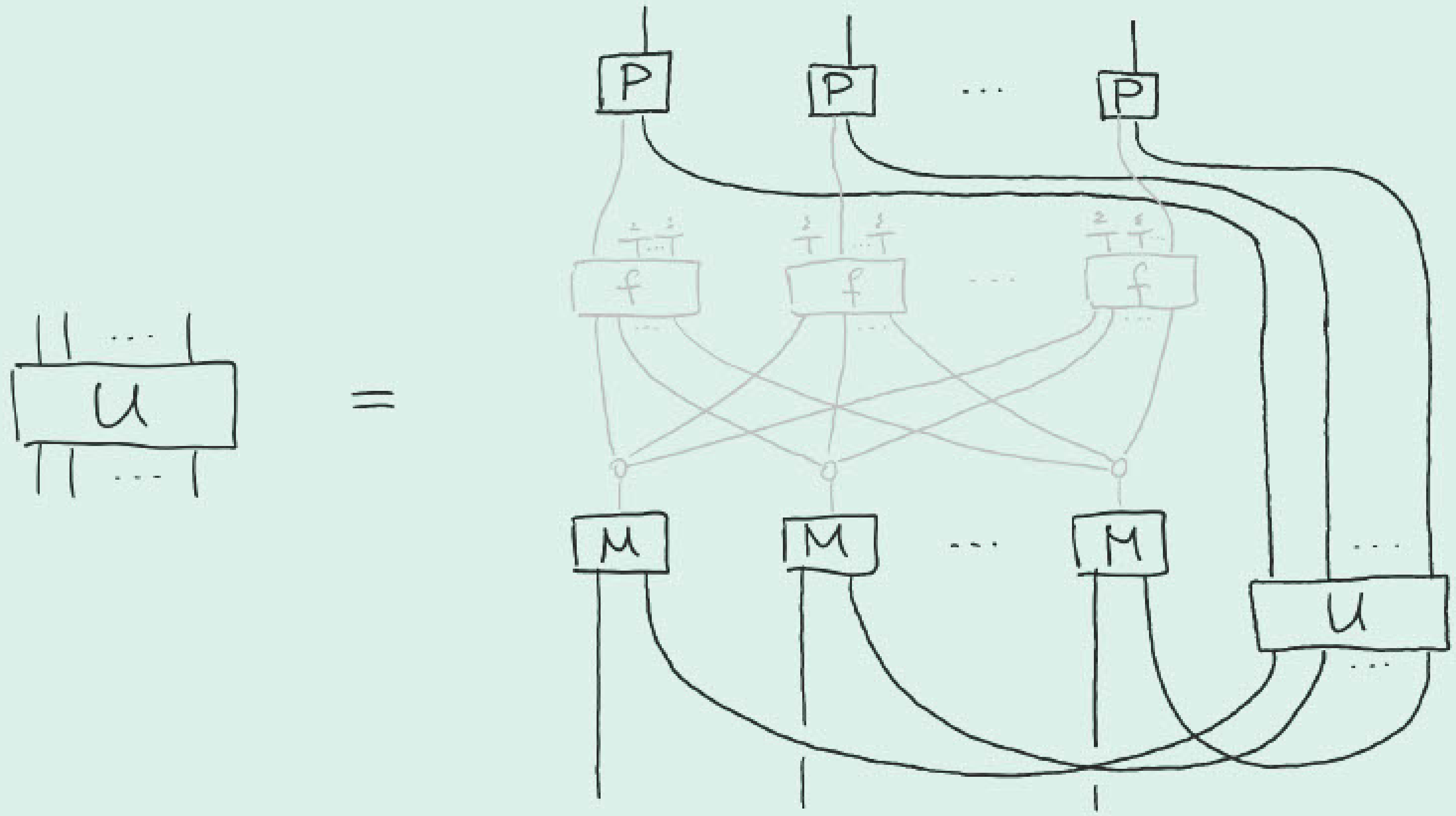

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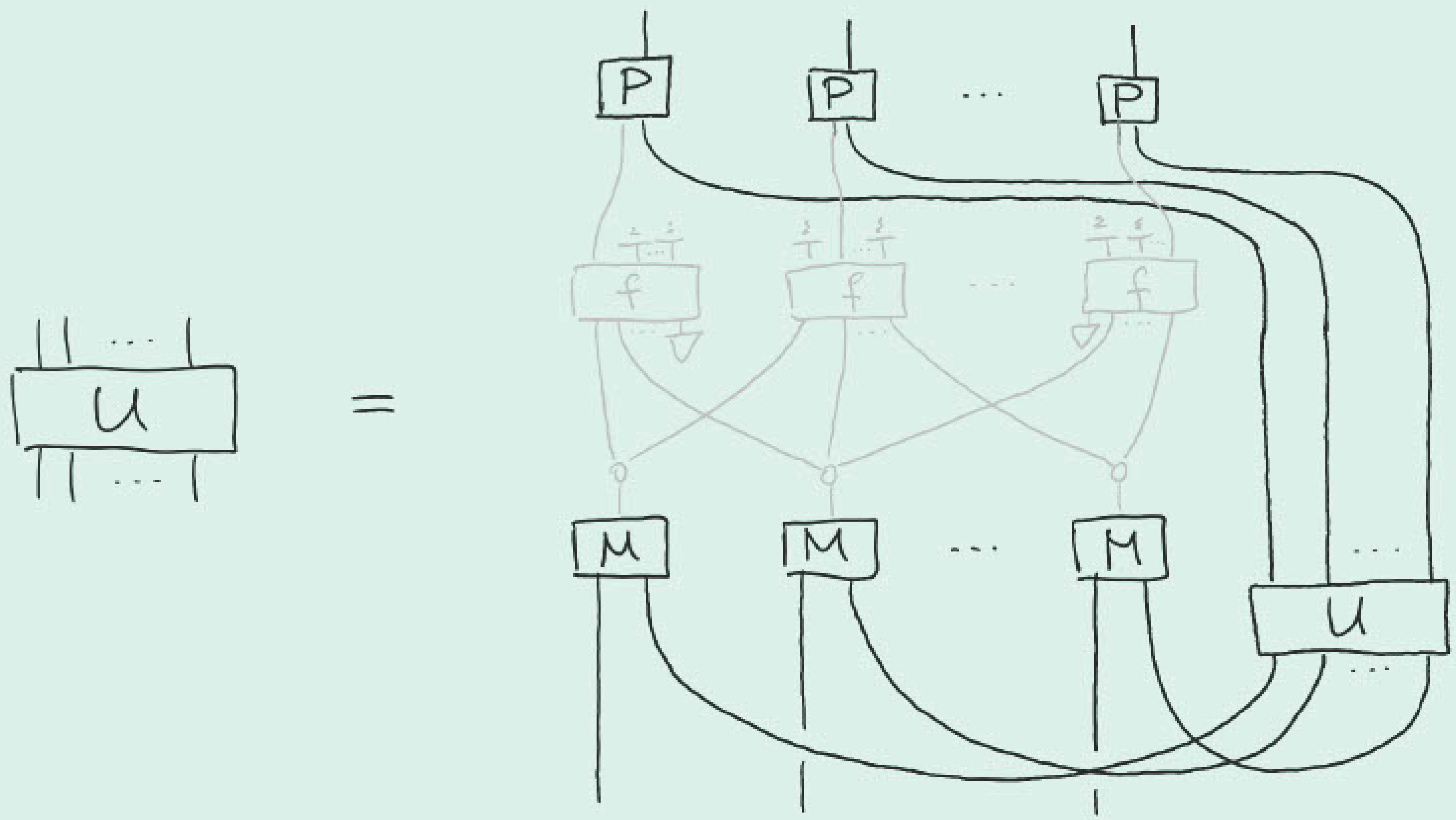

$\boxed{\text{Lma}}$ $A_i \longrightarrow B_j$ through U iff $\mathcal{G}_{A_i} \rightarrow \mathcal{G}_{B_j}$ through $f = U(\cdot)U^\dagger$.

two things

What can you do in a spacetime?

probably pretty much anything

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probably pretty much anything

10:50 - 11:30

PI/1-100 - Theatre, Perimeter Institute for Theoretical Physics

Renato Renner

Fundamental limits for realising quantum processes in spacetime

11:30 - 12:10

PI/1-100 - Theatre, Perimeter Institute for Theoretical Physics

Lunch

What can you do in a spacetime?

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Fundamental limits for realising quantum processes in spacetime

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11:30 - 12:10

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Lunch

↳ indefinite causal order

What about AQFT?

What about AQFT?

Bostelmann, Fewster, Ruep (2021):

FV-Realisable $\subseteq$ Relativistically Causal

$\swarrow$

Fewster, Verch (2020)

See also arXiv: 2108.05904

Take home

Requiring no superluminal signalling $\neq$ requiring realisability by a relativistically valid dynamics

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Requiring no superluminal signalling $\neq$ requiring realisability by a relativistically valid dynamics

causal structure $\neq$ compositional structure

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causal structure $\neq$ compositional structure

nonlocal quantum computation

+

Causal decompositions