Title: Enhancing Non-Classicality Detection with Interventions

Speakers: Rafael Chaves

Series: Quantum Foundations, Quantum Information

Date: September 19, 2024 - 11:30 AM

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Abstract: Generalizations of Bell's theorem, particularly within quantum networks, are now being analyzed through the causal inference lens. However, the use of interventions, a central concept in causality theory, remains unexplored. As will be discussed, if we are not limited to observational data and can intervene in our experimental setup, we can witness quantum violations of classical causal bounds even when no Bell-like violation is possible. Through interventions, the quantum behavior of a system, that would seem classical otherwise, can be demonstrated. We will then present a photonic experiment implementing those ideas and consider applications of this framework for measurement-based quantum computation, quantification of causality in quantum gates and quantum network protocols.

Unveiling Non-Classicality via Node and Edge Interventions in Causal Networks

CausalWorlds 2024

INTERNATIONAL INSTITUTE OF PHYSICS
Federal University of Rio Grande do Norte

Rafael Chaves

Quantum Information and Quantum Matter Group

www.iip.ufrn.br/qiqm

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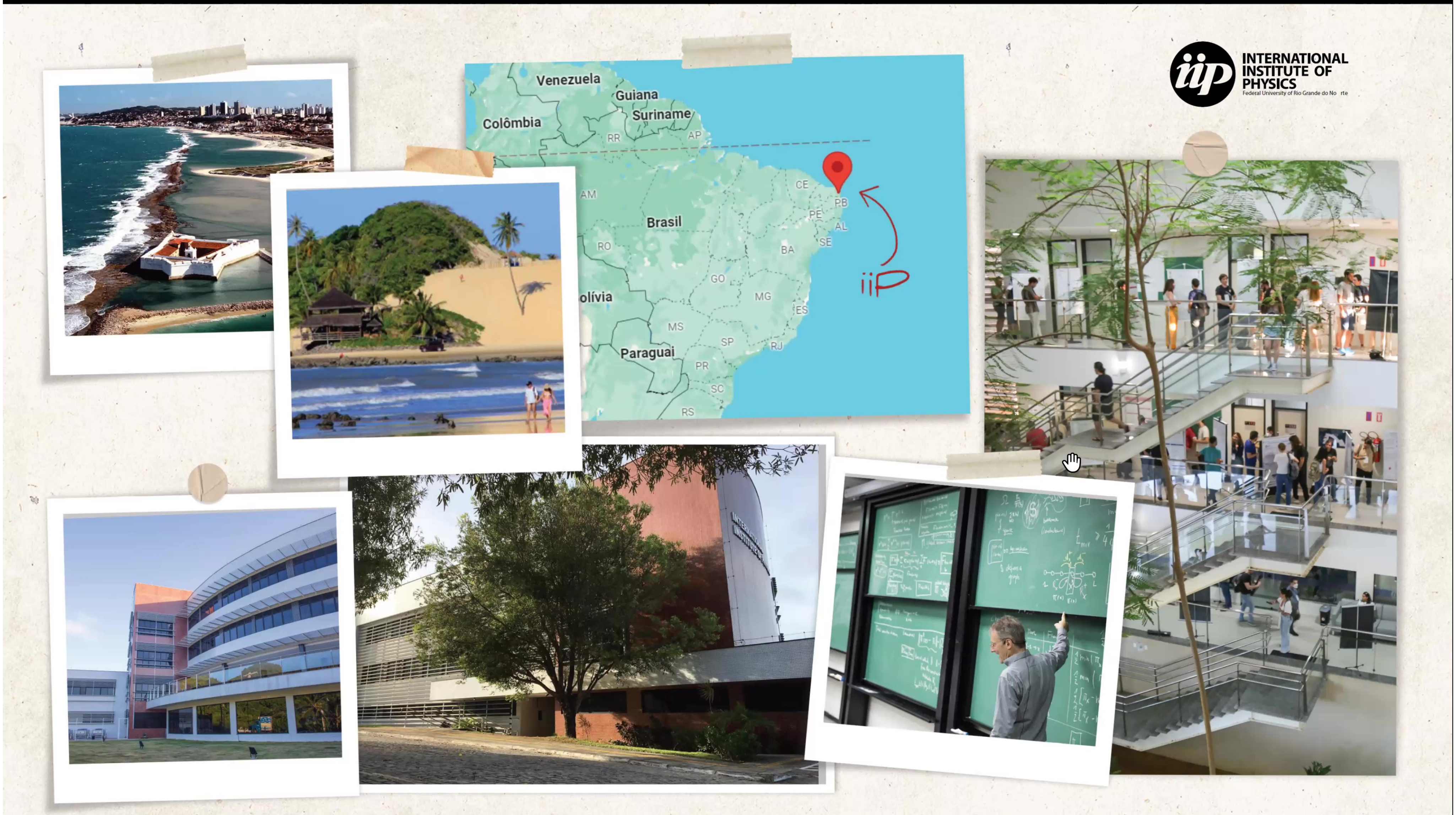
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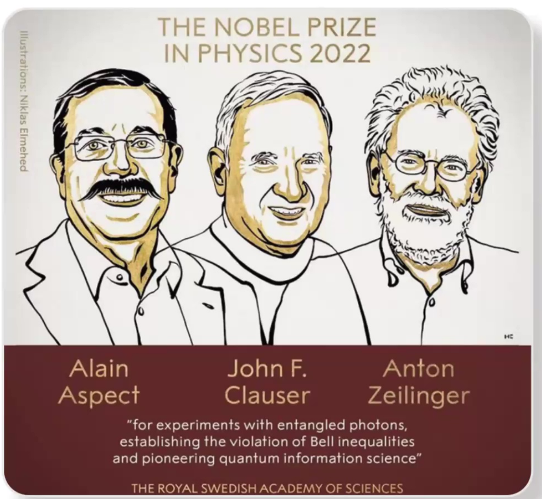
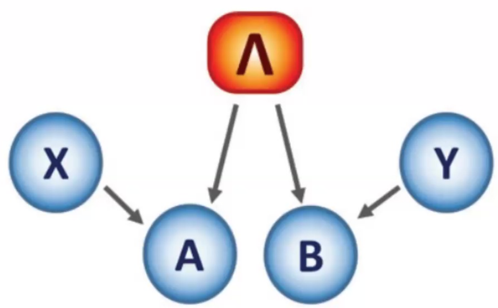
Causal discovery: **learn** causal relations from data

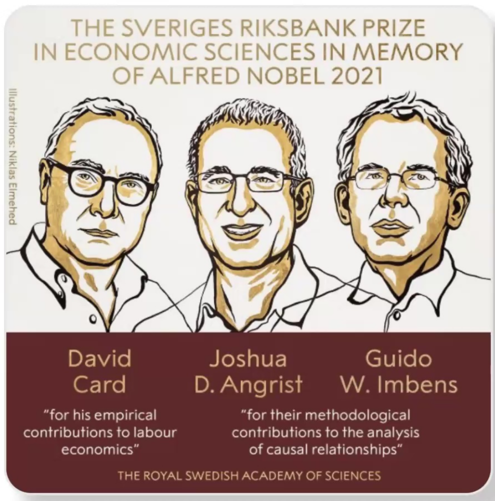
Causal compatibility: **verify/falsify** causal relations from data

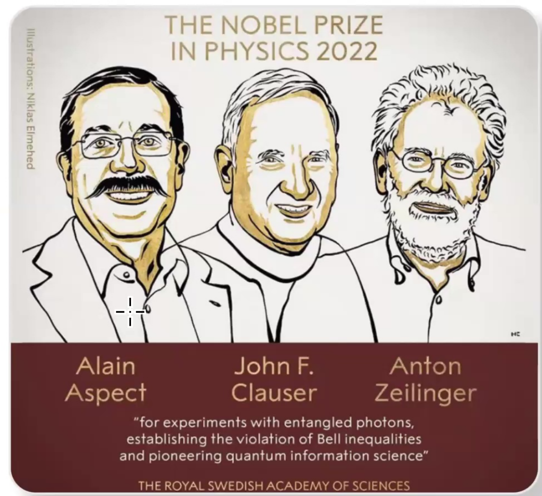


Bell's theorem is a particular example of a causal compatibility problem!

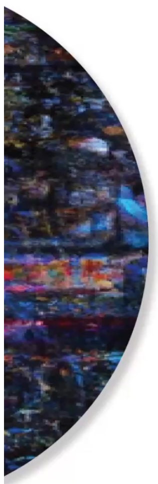


What can we learn at the interface between Causality and Quantum Theory?





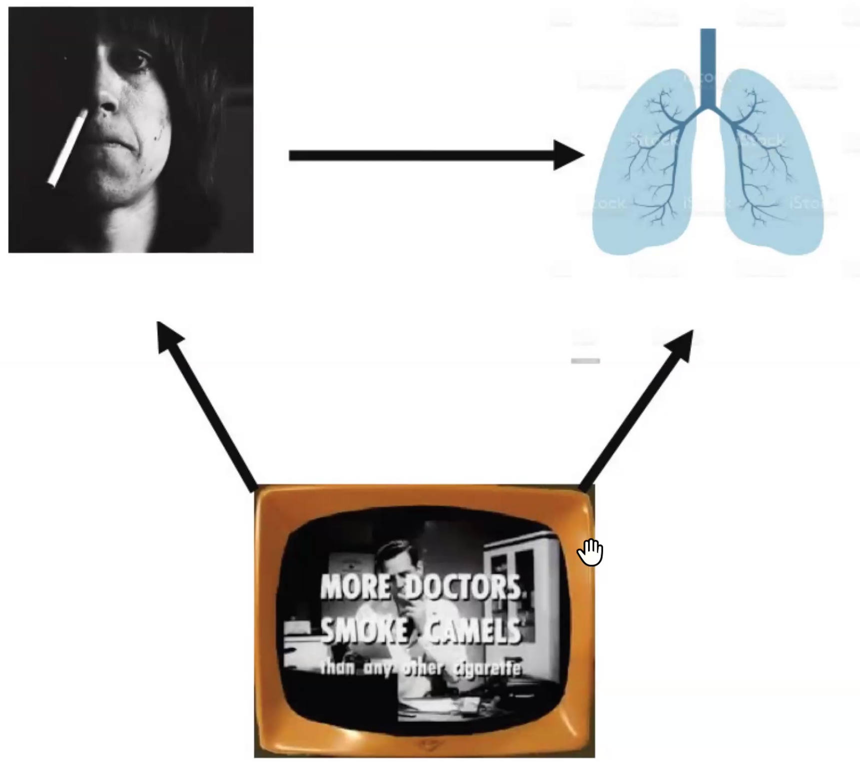
Quantum Causality!!!



Outline

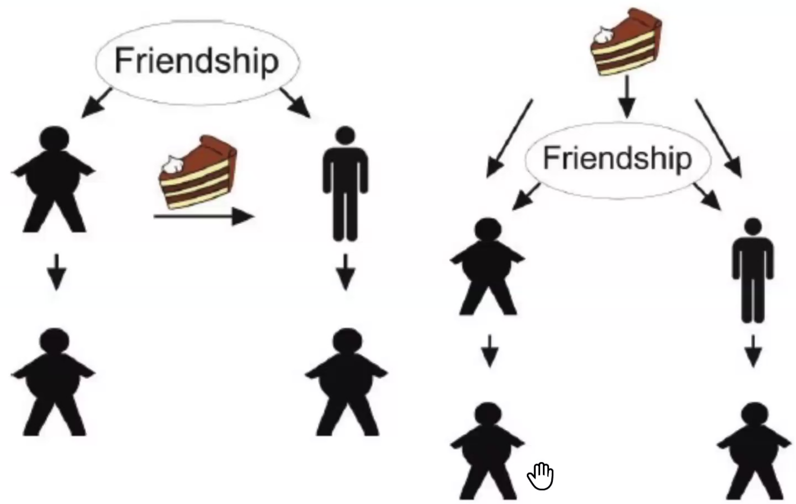
- Interventions (aka node interventions)
- Quantifying quantum causality
- Edge Interventions

Common causes
X
Causal Influences



Does smoking cause cancer?

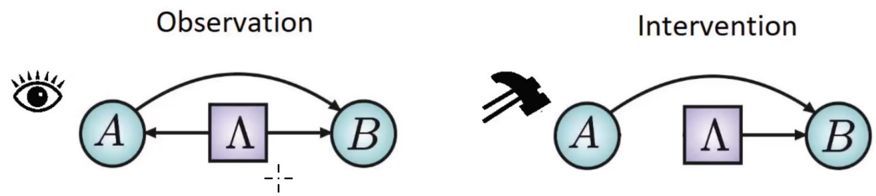
Common causes
X
Causal Influences

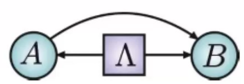


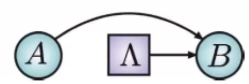
Is obesity contagious?

Direct Influences x Common Causes

Does **A** have some causal influence over **B**, or all the correlations between **A** and **B** are mediated via a common ancestor?







$$p(b|a) = \sum_{\lambda} p(b|a,\lambda)p(\lambda|a)$$

$$p(b|\mathrm{do}(a)) = \sum_{\lambda} p(b|a,\lambda)p(\lambda)$$

$$p(b|a) \neq p(b|\mathrm{do}(a))$$

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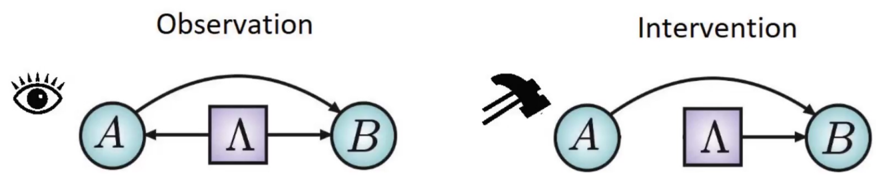
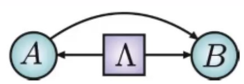
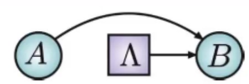




$$p(b|a) = \sum_{\lambda} p(b|a,\lambda)p(\lambda|a)$$



$$p(b|\mathrm{do}(a)) = \sum_{\lambda} p(b|a,\lambda)p(\lambda)$$

$$p(b|a) \neq p(b|\mathrm{do}(a))$$

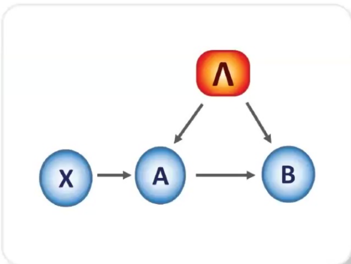
Measure of causality

$$\mathrm{ACE}_{A\to B} = \sup_{a,a',b} |p(b|\mathrm{do}(a)) - p(b|\mathrm{do}(a'))|$$

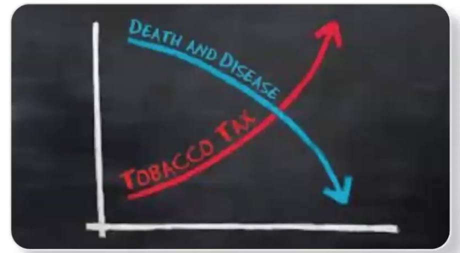
Do we really have to force people to smoke?

Or can we infer causality without interventions?

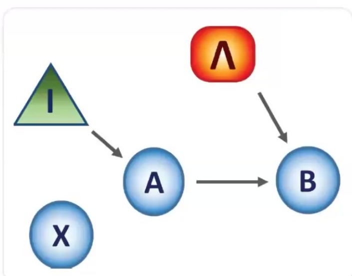
Instrumental variables



Empirical data is encoded in the distribution $p(a,b|x)$



We can estimate the average causal effect (ACE) from observations alone



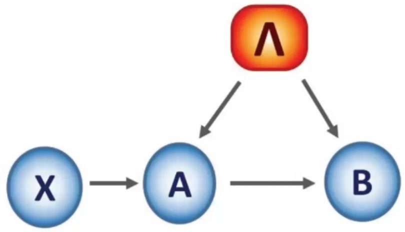
$$\mathrm{ACE}_{A\to B} \geq 2p(a=0,b=0|x=0)-2 + p(a=1,b=1|x=0)+p(b=1|x=1)$$

Balke & Pearl JASA 1997

$$\mathrm{ACE}_{A\to B} = \sup_{a,a',b} |p(b|\mathrm{do}(a)) - p(b|\mathrm{do}(a'))|$$

Instrumentality inequalities

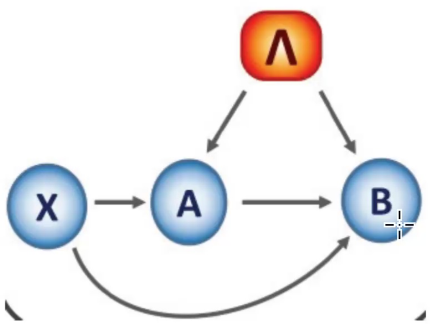
- Do we have a good instrument? Just like Bell local-realism assumption, instrumental variables impose strict constraints on which correlations are compatible with it.

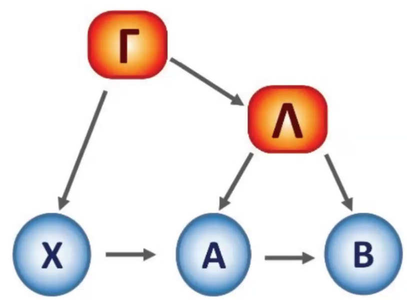


$$\max_a \sum_b \max_x p(a,b|x) \leq 1.$$

Pearl UAI 1995

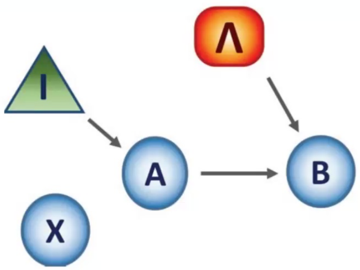
- Any violation of instrumentality inequalities would be classically interpreted as a violation of some of the instrumental assumptions.





The instrumental scenario has two pillars

- Estimation of ACE from observational data.



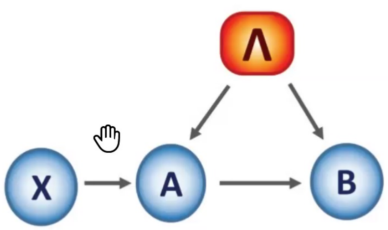
PHYSICAL REVIEW LETTERS 125, 230401 (2020)

Quantifying Causal Influences in the Presence of a Quantum Common Cause

Mariami Gachechiladze,[1,*] Nikolai Miklin,[2,†] and Rafael Chaves[3,4]

$$\text{ACE}_{A\to B} = \sup_{a,a',b} |p(b|\text{do}(a)) - p(b|\text{do}(a'))|$$

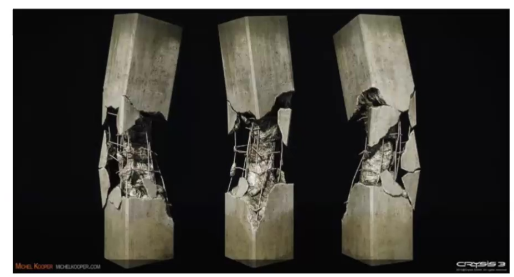
- Checking whether we have a good instrument via an instrumental test/ inequality.



nature physics | ARTICLES
https://doi.org/10.1038/s41567-017-0008-5

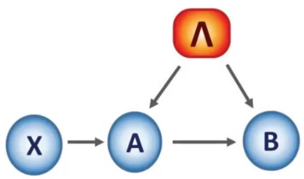
Quantum violation of an instrumental test

Rafael Chaves[1*], Gonzalo Carvacho[2], Iris Agresti[2], Valerio Di Giulio[2], Leandro Aolita[3], Sandro Giacomini[2] and Fabio Sciarrino[2*]



- Can we ***shake*** these pillars with quantum mechanics?

Quantifying classical causal influences



- In the **simplest** scenario all correlations are **classical**

$$p(a,b|x) = \sum_{\lambda} p(a|x,\lambda)p(b|a,\lambda)p(\lambda)$$

$$p(a,b|x) = \mathrm{tr}[(M_a^x \otimes N_b^a)\rho_{AB}]$$

Henson, Lal, Pusey NJP 2014

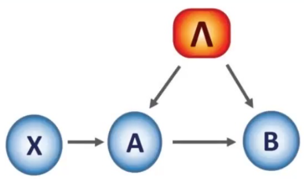
- But does the **classical bounds** on the ACE still apply?

$$p(b|do(a)) = \sum_{\lambda} p(b|a,\lambda)p(\lambda)$$

$$p(b|do(a)) = \mathrm{tr}\left[(\mathbb{1} \otimes N_b^a)\rho_{AB}\right] = \mathrm{tr}\left[N_b^a \rho_B\right]$$

$$\mathrm{ACE}_{A\to B} \geq 2p(a=0,b=0|x=0) - 2 + p(a=1,b=1|x=0) + p(b=1|x=1) \quad ($$

Quantifying classical causal influences



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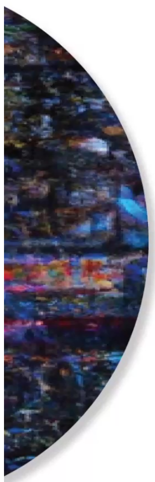

Outline

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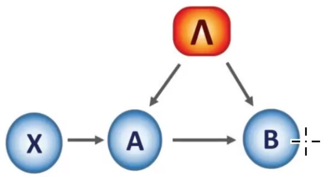
- **Quantifying quantum causality**
- Edge Interventions

Quantifying classical causal influences



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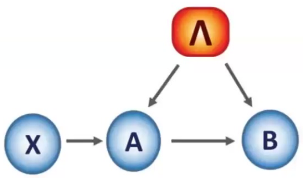
$$\mathrm{ACE}_{A\to B} \geq 2p(a=0,b=0|x=0) - 2 + p(a=1,b=1|x=0) + p(b=1|x=1)$$

Quantifying quantum causal influences

- **Quantum ACE**



$$p_Q(a,b|x) = \mathrm{Tr}\left[(M_a^x \otimes M_b^a)\varrho\right]$$

$$\varrho = v|\phi^+\rangle\langle\phi^+| + (1-v)\mathbb{1}/4$$
$$|\phi^+\rangle = (1/\sqrt{2})(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$p_Q(b|\mathrm{do}(a)) = p_Q(b|\mathrm{do}(a')) = \mathrm{Tr}\left[(M_b^a)\mathbb{1}/2\right] = 1/2$$

$$\mathrm{ACE}_{A\to B} = 0$$

- **Classical ACE**

$$\mathrm{ACE}_{A\to B} \geq 2p(a=0,b=0|x=0) - 2 \quad ($$
$$+p(a=1,b=1|x=0) + p(b=1|x=1))$$

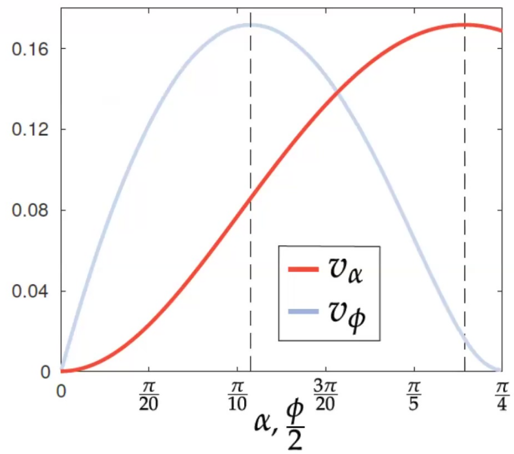
$$O^{x=0} = \sigma_Z,\ O^{x=1} = \sigma_X$$
$$O^{a=0} = -\sin(\pi/8)\sigma_X + \cos(\pi/8)\sigma_Z$$
$$O^{a=1} = (\sigma_X + \sigma_Z)/\sqrt{2}$$

$$\mathrm{ACE}_{A\to B} \gtrsim 0.91v - 0.75$$

Quantum effects can lead to an overestimation of causal influences!

Quantifying quantum causal influences

Result 1. *Every pure entangled state can generate correlations that violate the classical bound on* ACE*. Moreover, entanglement is necessary but not sufficient for such violations.*
Result 2. *Every pair of incompatible rank-*1 *projective qubit measurements can generate correlations that violate the classical bound on* ACE*. Moreover, incompatibility of both Alice's and Bob's observables is necessary but not sufficient for the violation.*



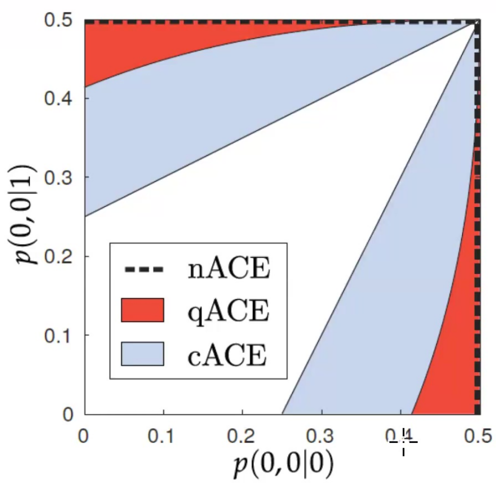
- Even though **no Bell inequality** can be violated, we still can witness the **non-classicality** of the correlations.
- A quantum common source leads to na **overestimation** of causal influences if the classical bounds are used.

Beyond quantum causal influences

Result 3. *In the instrumental scenario with dichotomic measurements* qACE *is lower bounded as*

$$\mathrm{qACE}_{A\to B} \geq \sum_{x=0,1} (p(0,0|x) + p(1,1|x)) + \zeta - 1, \quad (11)$$

$$\zeta = \max_{\pm} \sqrt{\prod_{a=0,1} (1 \pm \sum_{x=0,1} (-1)^x (p(a,0|x) - p(a,1|x)))}.$$



$$p(b|do(a)) = \mathrm{tr}\,[(\mathbb{1} \otimes N_b^a)\rho_{AB}] = \mathrm{tr}\,[N_b^a \rho_B]$$

$$\mathrm{qACE}_{A\to B} = \max_{a,a',b} (\mathrm{tr}[(N_b^a - N_b^{a'})\rho_B])$$

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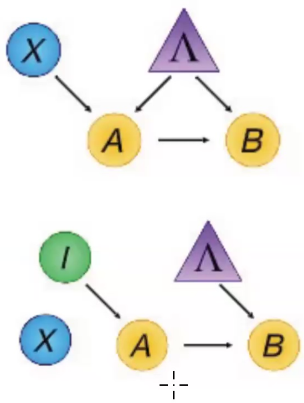
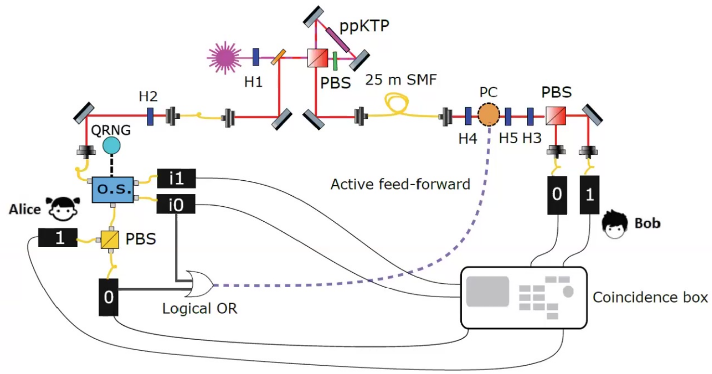
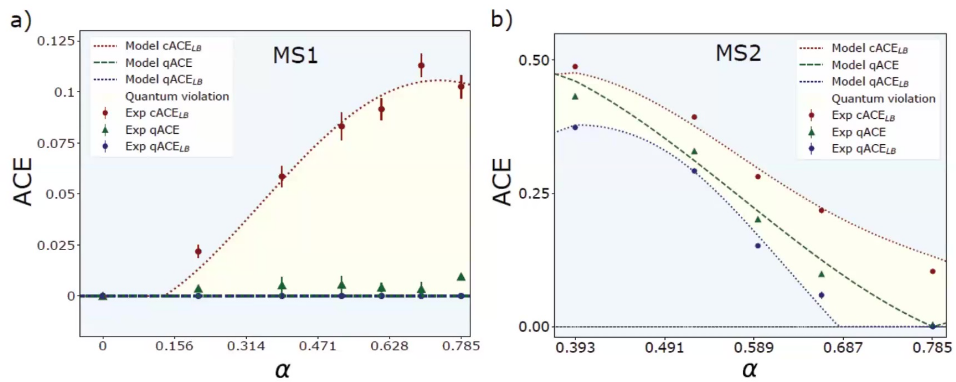
Experimental test of quantum causal influences

Iris Agresti[1], Davide Poderini[1], Beatrice Polacchi[1], Nikolai Miklin[2,3], Mariami Gachechiladze[4], Alessia Suprano[1], Emanuele Polino[1], Giorgio Milani[1], Gonzalo Carvacho[1], Rafael Chaves[5]*, Fabio Sciarrino[1]*







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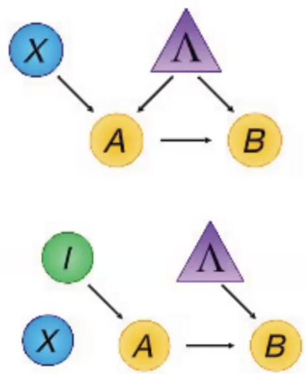

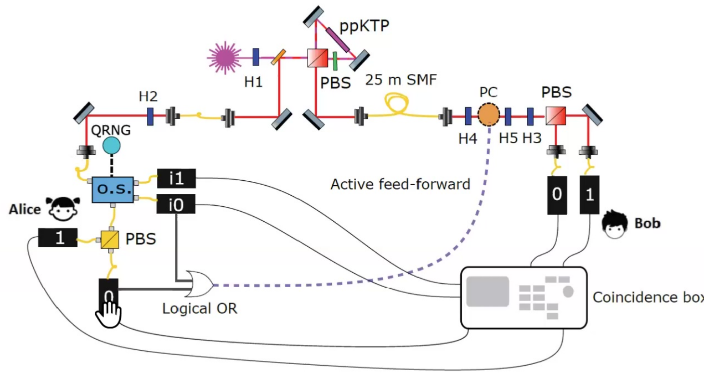

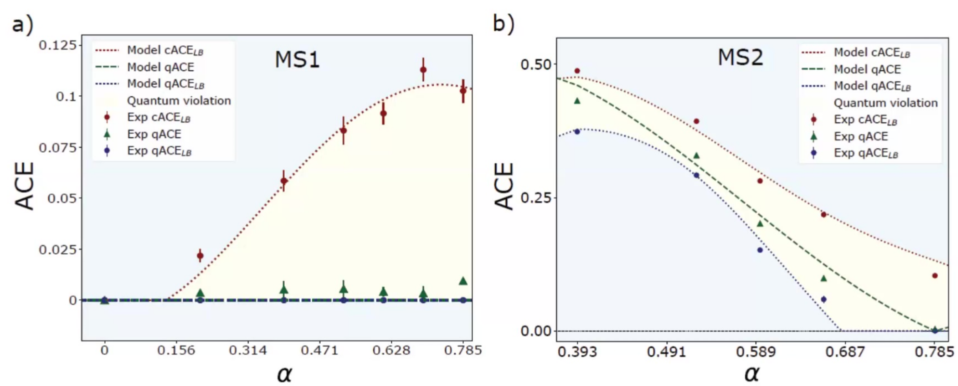

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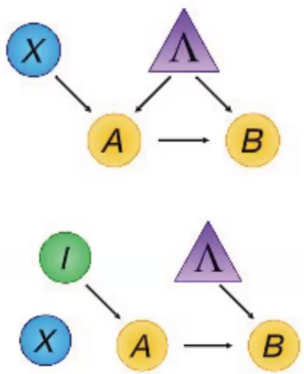

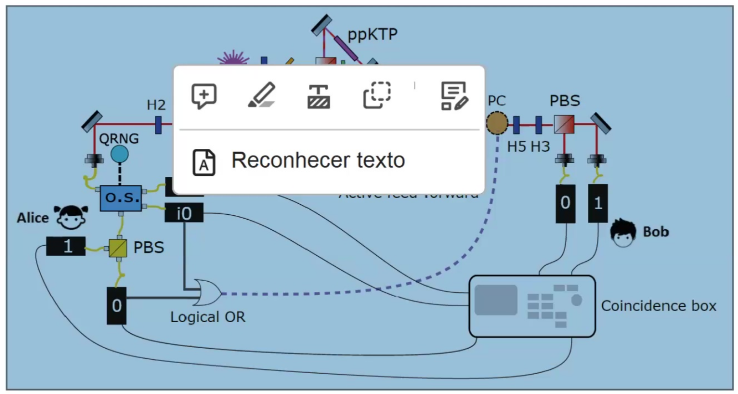

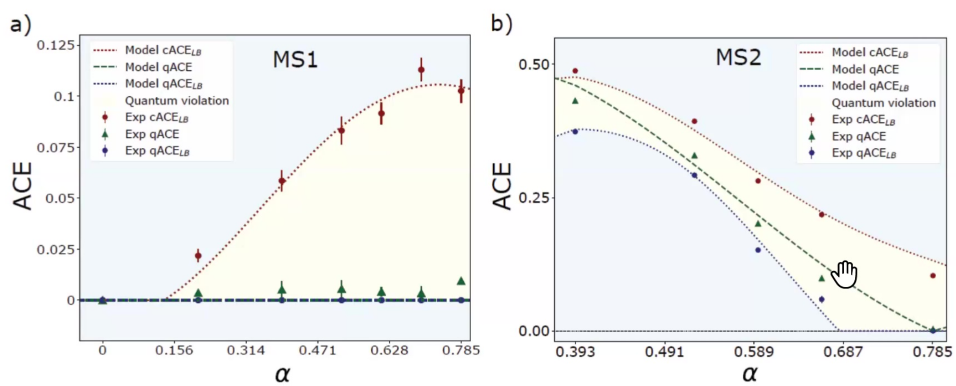

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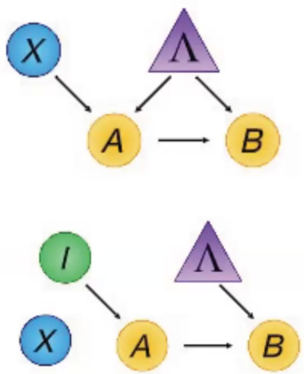
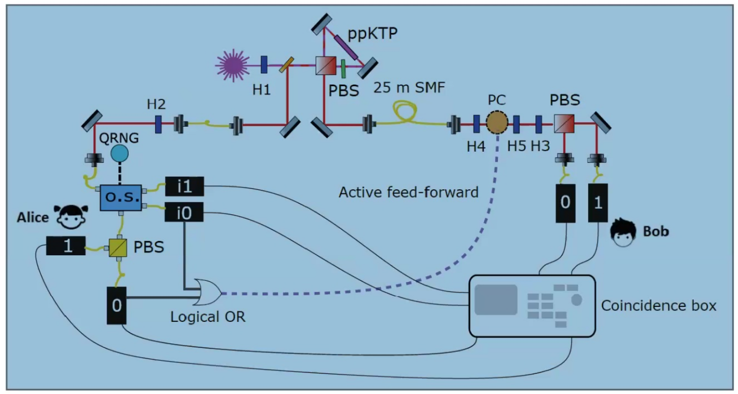
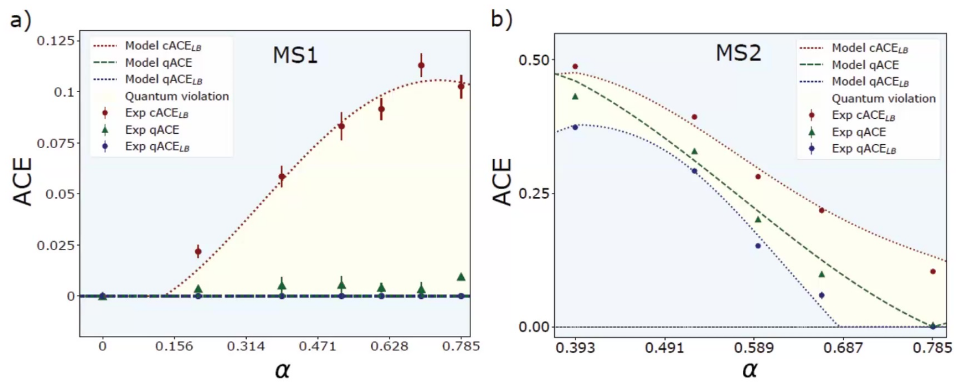






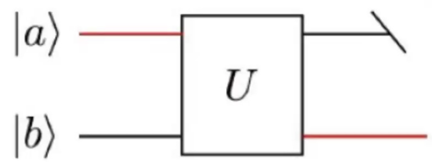
Applications in Quantum Computing

PHYSICAL REVIEW A **108**, 022222 (2023)

Quantifying quantum causal influences

Lucas Hutter,[1] Rafael Chaves,[2,3] Ranieri Vieira Nery,[2] George Moreno,[2,4] and Daniel Jost Brod[1]

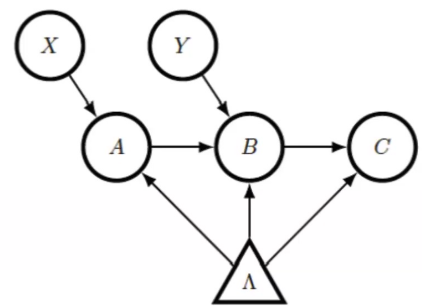
Quantum Circuits



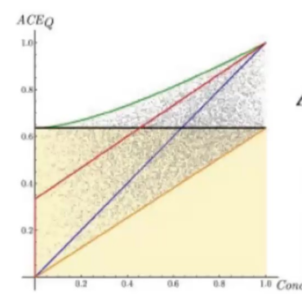
$$ACE_Q(U) = \mathop{\mathbb{E}}_{|a\rangle}\mathop{\mathbb{E}}_{|b\rangle} TD(\rho(b|do(a)), \rho(b|do(a^{\perp}))$$

$$\rho(b|do(a)) = \mathrm{tr}_A\left(U\,|a,b\rangle\langle a,b|\,U^{\dagger}\right)$$

Gate	ACE_Q
Local	0
CNOT	$\pi/8$
CZ	$\pi/8$
B gate	0.5878
$\sqrt{\text{SWAP}}$	0.6427
SWAP	1



Measurement based QC



$$ACE_Q(\rho_{sep}) \leq 2/\pi$$

$$ACE_Q(|G_2^{\epsilon}\rangle\langle G_2^{\epsilon}|) = \frac{2}{\pi}E[(1-2\epsilon)]$$

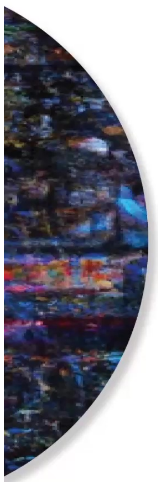
Any pure entangled state surpasses the maximum quantum ACE achievable by separable states

See Davide Poderini's talk for fresh results on interventions and its applications

arXiv > quant-ph > arXiv:2404.05015

Observational-Interventional Bell Inequalities

Davide Poderini, Ranieri Nery, George Moreno, Santiago Zamora, Pedro Lauand, Rafael Chaves



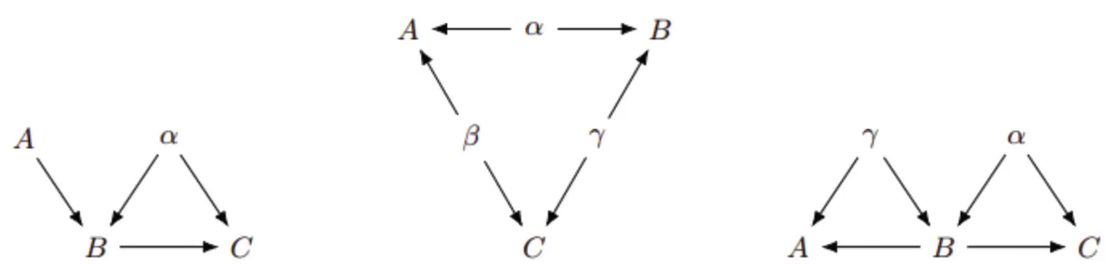
Outline

- Interventions (aka node interventions)
- Quantifying quantum causality
- Edge Interventions

In collaboration with I. Veeren, D. Poderini, S. Zamora, P. Lauand

What about other causal networks beyond the instrumental scenario?

For three observable variables, there are only three inequivalent classes of DAGs

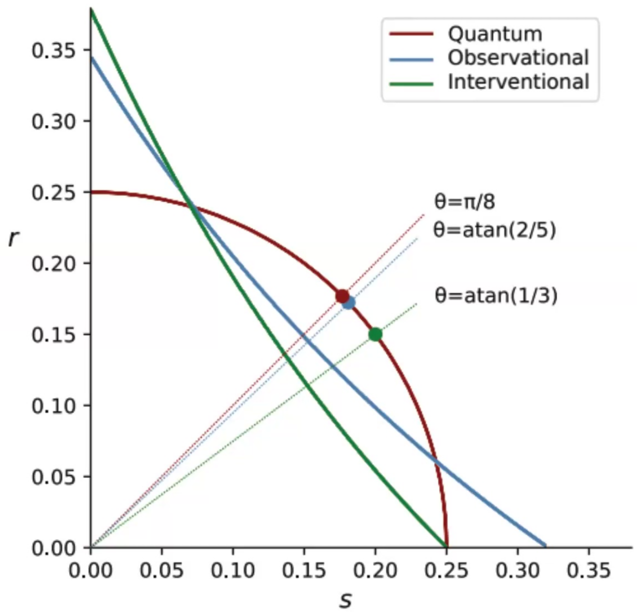


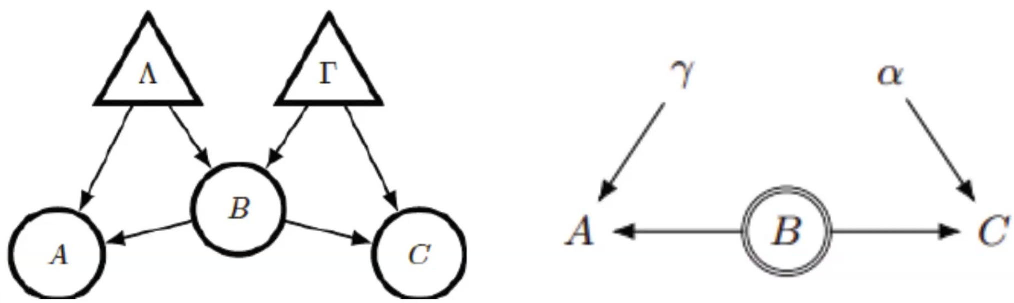
(a) The Instrumental scenario. (b) The Triangle scenario. (c) The UC scenario.

arXiv > quant-ph > arXiv:2404.12790

Quantum non-classicality in the simplest causal network

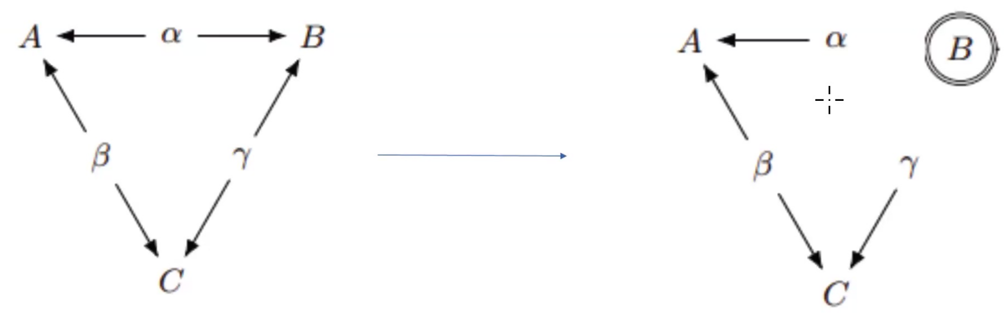
Pedro Lauand, Davide Poderini, Rafael Rabelo, Rafael Chaves



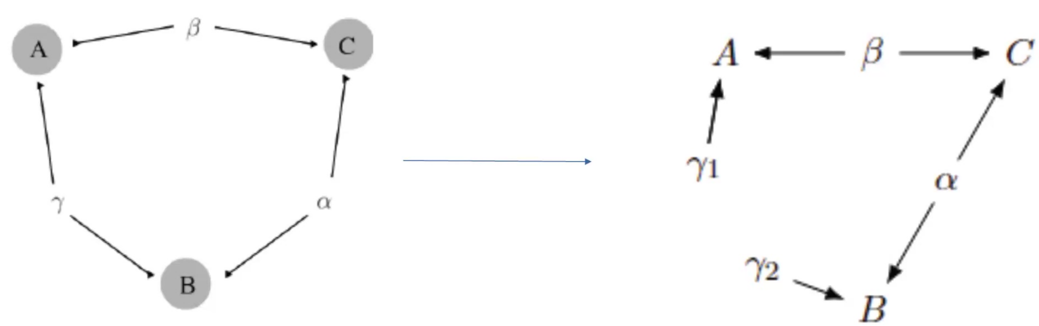


See Pedro Lauand's talk for the use of interventions on the UC scenario

For the triangle, however, node interventions are useless.

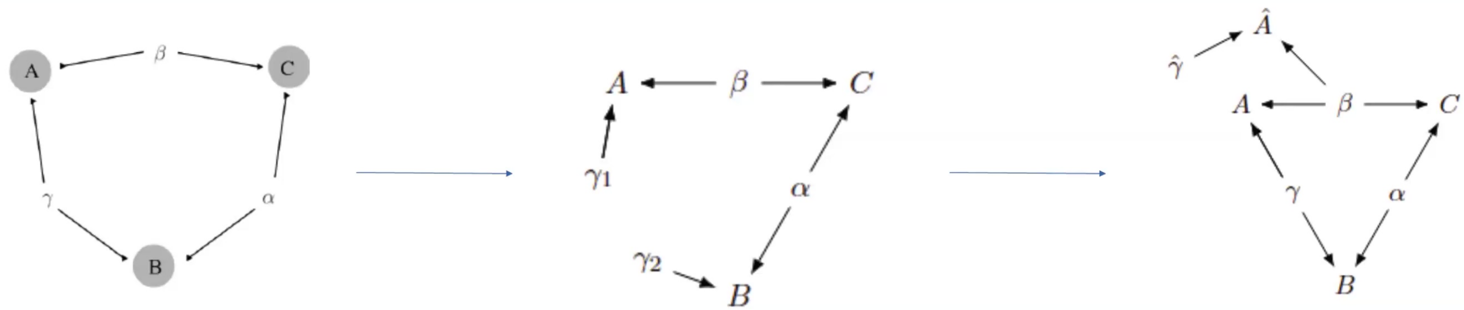


But what about edge/partial interventions?



Edge interventions?

- Classically, they are probably non-sense. In a quantum setting, however, latent nodes represent sources of quantum states that we do have control over.



$$\Psi_{\gamma_1} = \mathrm{tr}_A[|\Psi_\gamma\rangle\langle\Psi_\gamma|] = \frac{\mathbb{1}}{2}$$

$$\Psi_{\mathrm{int}} = |\Psi_{\gamma_1}\rangle\langle\Psi_{\gamma_1}| \otimes |\Psi_{\gamma_2}\rangle\langle\Psi_{\gamma_2}| \otimes |\Psi_\alpha\rangle\langle\Psi_\alpha| \otimes |\Psi_\beta\rangle\langle\Psi_\beta|$$

$$P_u(a,b,c|\gamma \not\to a) = \mathrm{tr}[(\mathcal{A}_a \otimes \mathcal{B}_b \otimes \mathcal{C}_c)\Psi_{\mathrm{int}}]$$

Edge interventions are not inputs!

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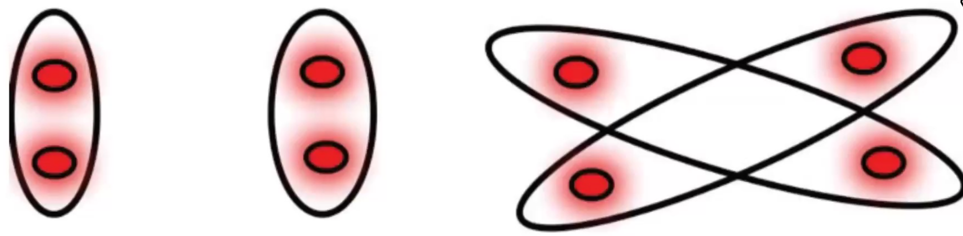

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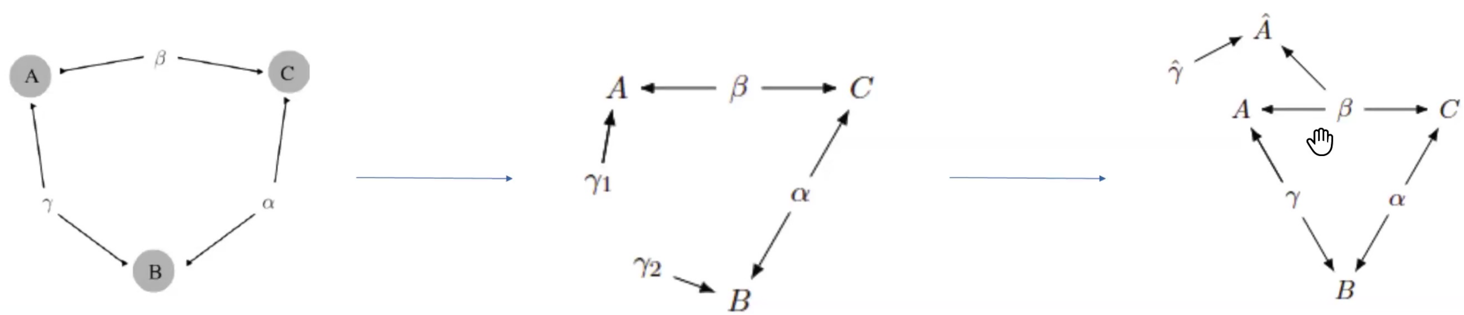

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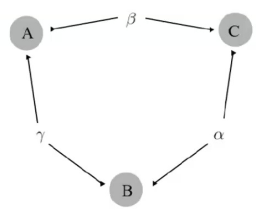
Edge interventions are not inputs!

Improving non-classicality detection with edge intervention

Let's take the distribution of [Boreiri et al PRA 107 (2023)], known to be non-classical in the range 0.785 <u2 <1. Not detected by inflation!

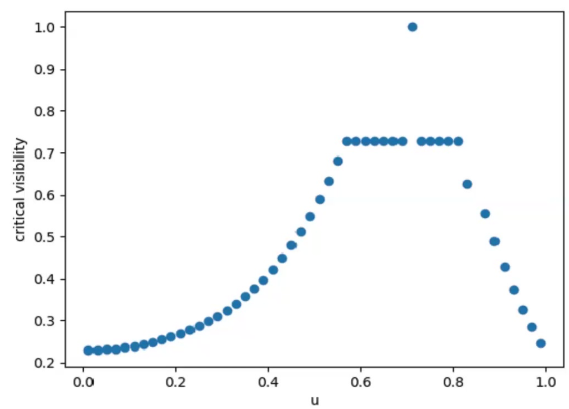


$$P_u(a,b,c) = \mathrm{tr}[(\mathcal{A}_a \otimes \mathcal{B}_b \otimes \mathcal{C}_c)\,|\Psi\rangle\langle\Psi|]$$

$$\mathcal{A}_{\bar{0}} : |00\rangle\langle 00| + |11\rangle\langle 11|\,,$$
$$\mathcal{A}_{\bar{1}_0} : |\bar{1}_0\rangle\langle\bar{1}_0|\,,$$
$$\mathcal{A}_{\bar{1}_1} : |\bar{1}_1\rangle\langle\bar{1}_1|\,,$$

$$|\bar{1}_0\rangle = u\,|01\rangle + v\,|10\rangle$$
$$|\bar{1}_1\rangle = v\,|01\rangle - u\,|10\rangle$$

- With partial interventions and inflation we can detect and expand the non-classicality of the scenario!



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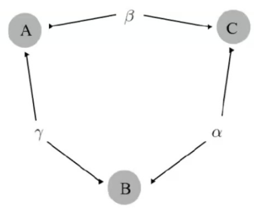

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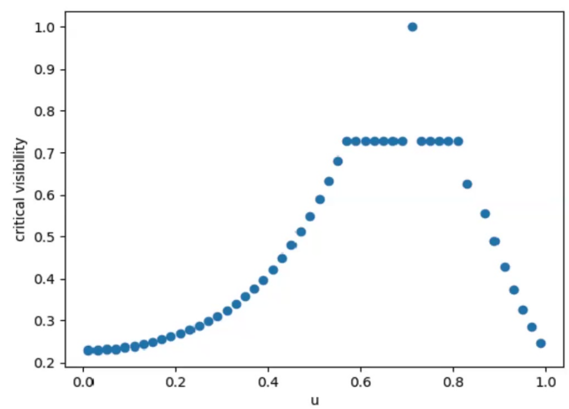
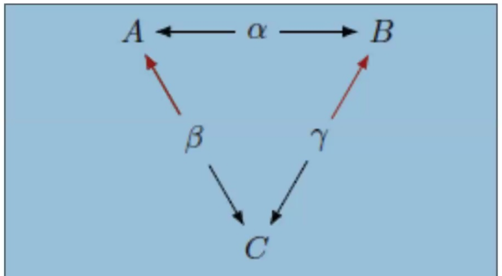
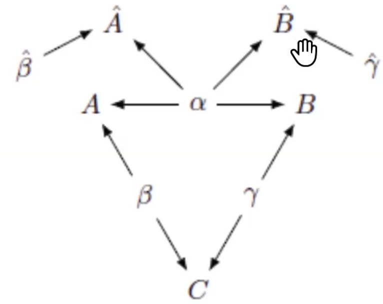


Reconhecer texto

I... classicality in the triangle



(a) **Double intervention DAG.**



(b) **Carrot DAG :).**

$$p(a,b,c) = \sum_{\hat{a},\hat{b}} P(a,\hat{a},b,\hat{b},c)$$

$$p(a,b,c|\beta \not\to a) = P(\hat{a},b,c) = \sum_{a,\hat{b}} P(a,\hat{a},b,\hat{b},c)$$

$$p(a,b,c|\gamma \not\to b), = P(a,\hat{b},c) = \sum_{\hat{a},b} P(a,\hat{a},b,\hat{b},c)$$

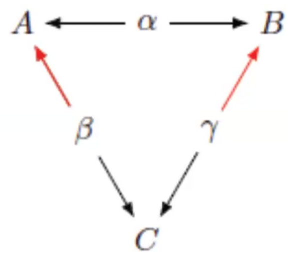
$$p(a,b,c|\beta \not\to a, \gamma \not\to b) = P(\hat{a},\hat{b},c) = \sum_{a,b} P(a,\hat{a},b,\hat{b},c)$$

$$S = \langle AB\rangle_1 + \langle \hat{A}B\rangle_1 + \langle A\hat{B}\rangle_1 - \langle \hat{A}\hat{B}\rangle \leq 2$$

$$\langle AB\rangle_c = \sum_{ab}(-1)^{a+b}P(ab|c)$$

$$\langle \hat{A}\hat{B}\rangle = \langle \hat{A}\hat{B}\rangle_c$$

Minimum non-classicality in the triangle



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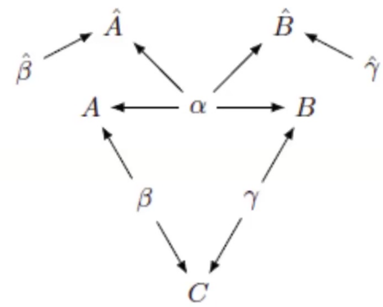


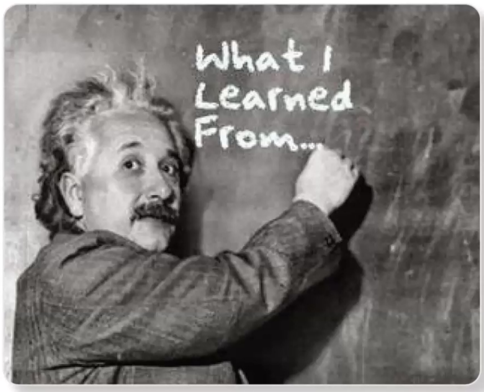
(b) **Carrot DAG :).**

$$p(\beta = 1) = p(\gamma = 1) = \epsilon$$

$$S = \langle AB \rangle_1 + \langle \hat{A}B \rangle_1 + \langle A\hat{B} \rangle_1 - \langle \hat{A}\hat{B} \rangle \leq 2$$

$$S = \frac{2\epsilon^2 + 4(1-\epsilon)}{\sqrt{2}}$$

Violated for $\epsilon \leq 1 - \sqrt{\sqrt{2} - 1} \approx 0.3564$



Take-Home Messages

- Interventions and the quantification of causal influences allow for new method to detect nonclassicality

- It is a almost unexplored new tool

- Application in foundations, quantum computing, quantum crypto and comm, quantum nets...

- Edge interventions increase the scope of applications, for example, minimum non-locality in the triangle