Title: Quantum discreteness and spacetime causality: what's in the mix?

Speakers: Sumati Surya

Series: Quantum Foundations, Quantum Information

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Abstract: The notion of causality is intimately tied to both, a transitive ordering on events, and the possibility of unrelated events. Thus, any causality structure is a partially ordered set or poset. This is the case in Lorentzian spacetime, which possesses a single time direction. In causal set quantum gravity, this spacetime causality structure is "first quantised" by discretising it. However, as with any dynamical quantum theory of spacetime, background notions of causality are insufficient. I will discuss how ordering and discreteness, as manifested in the sequential growth paradigm, provide a broad framework for quantum dynamical notions of causality.

Quantum discreteness and spacetime causality: what's in the mix?

Sumati Surya,

Raman Research Institute

Causal Worlds Conference, September 16-20th, 2024,
Perimeter Institute

Outline

- Causality and Posets
- Spacetime Causal Structure as the “essence of Lorentzian geometry”
- Causality in Quantum Gravity: the causal set approach
- Histories and the Decoherence Functional
- Sequential growth dynamics
- Interference, causality and covariance

Causality and Posets

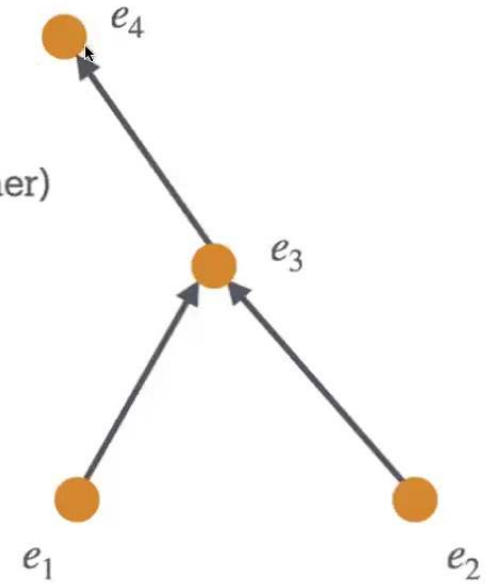
The notion of causality requires:

- transitive ordering on events: $e_1 \prec e_3, \quad e_3 \prec e_4 \quad \Rightarrow e_1 \prec e_4$
- possibility of "unrelations" : $e_1 \,||\, e_2$ (e_1 and e_2 cannot influence each other)

A **Kinematical Causal Structure** on the set of events is a pre-requisite for drawing possible causal inferences:

A possible cause at the kinematic event e_1 could:

- Give rise a potential effect at the kinematic event e_3 and/or e_4
- It can never give rise to an effect at a kinematic event e_2
- It can never give rise to an effect that will influence the cause.



Poset Structure

Causal Inference (from Wikipedia):

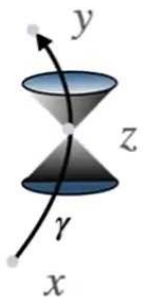
- "...reason[ing] to the conclusion that something is, or is likely to be, the cause of something else".
- "Identification of the cause or causes of a phenomenon, by establishing covariation of cause and effect, a time-order relationship with the cause preceding the effect, and the elimination of plausible alternative causes."

.... defined over a **Kinematic Causal Structure Poset**

Posets and Lorentzian Geometry

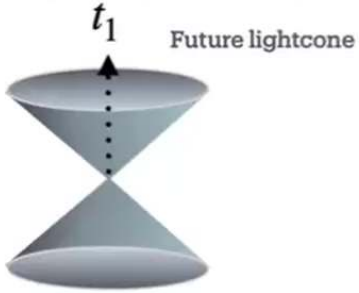
—Robb, 1914, "A theory of time and space"

- Causality on (M, g) defines an order relation on space of events $M : x \prec y$
- Principle of Causality: (M, g) is a causal spacetime $\Rightarrow \prec$ is acyclic : $x \prec y \Rightarrow y \not\prec x$

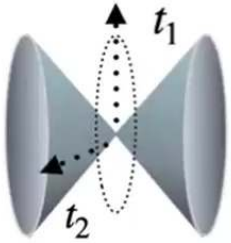


Acyclic: $x \prec y \Rightarrow y \not\prec x$
Transitive: $x \prec y, y \prec z \Rightarrow x \prec z$

$(M, \prec)$ is a partially ordered set



Poset only for $(-, +, +, +, \ldots)$



No lightcones and hence no causality

$(-, -, +, +)$ has no poset

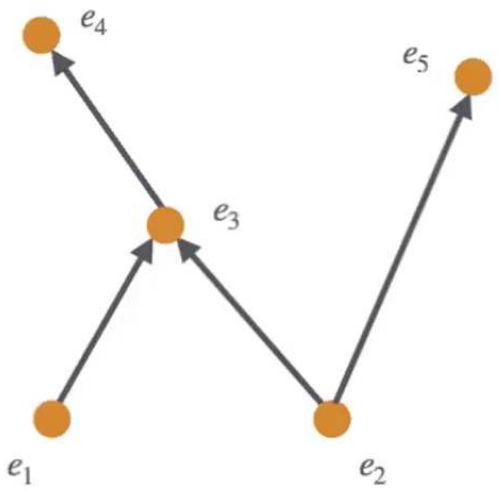
Spacetime provides a Physical, Kinematic Causal Structure

The stage $(M, \prec)$:

- M is a set of spacetime points or "events"
- $\prec$ is its intrinsic causality relation based on the existence of causal curves between spacetime points

General Relativist: Events are points in a spacetime

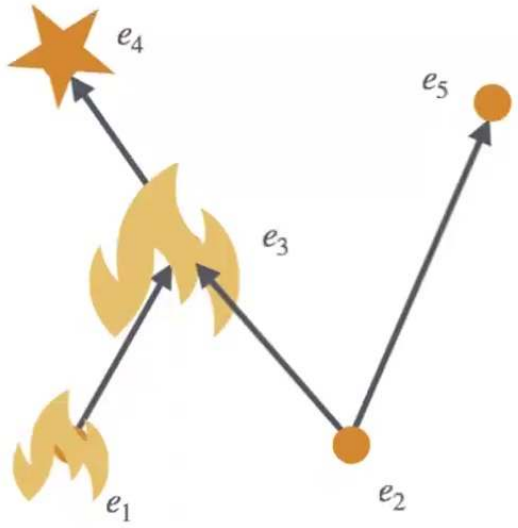
Others: Events are that things that happen.



Something happens :

$e_1 \prec e_3 \prec e_4$

Fire at event e_1 develops into a conflagration at event e_3 and subsequently an explosion at event e_4



Nothing happens:

$e_2 \prec e_5$

Fire at e_1 cannot lead to a conflagration at e_2

$e_1 \,||\, e_2$

Spacetime Geometry and Causality

"Extract from (M, g) its causal essence"

Kronheimer and Penrose 1967

For every causal Lorentzian spacetime (M, g), there is a causal structure poset $(M, \prec)$

Theorem:

If a chronological bijection exists between two d-dimensional spacetimes with d>2, then they are conformally isometric .

—Kronheimer, Penrose , 1967
—Hawking, King, McCarthy, 1976
—Malament, 1977

"HKMMKP theorem": $(M, g) = (M, \prec) + \epsilon$

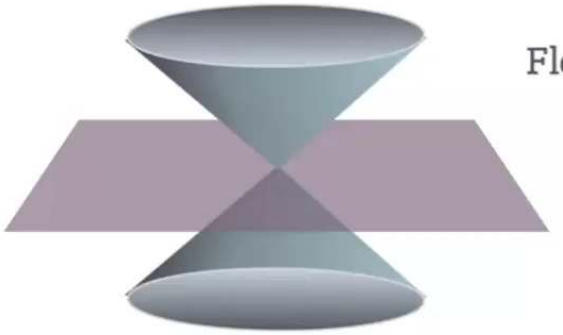
"Spacetime is its causal structure $(M, \prec)$ plus a local volume element ϵ"

Causal Structure is Global/Teleological

Penrose, 1972
Hawking and Ellis, 1973

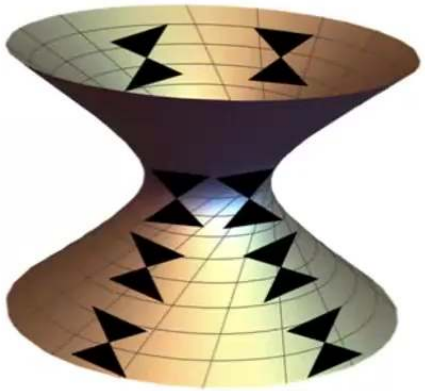
- A single spacetime is a **HISTORY**
- A given spacetime has **ONE** causal structure

Special Relativity $(\mathbb{R}^d, \eta)$



Flat light cones

General Relativity (M, g)



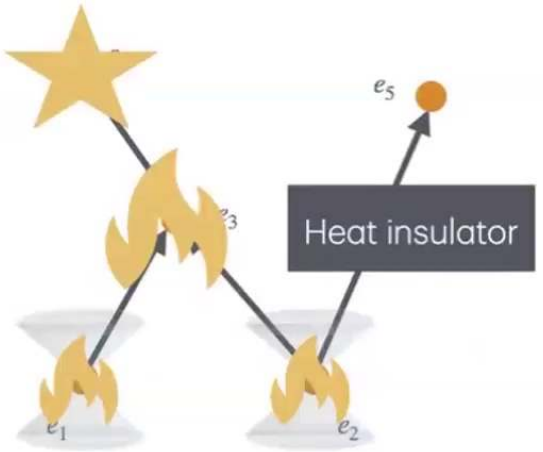
Curved light cones

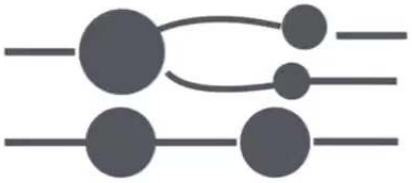
- Not an "indefinite" causality structure.
- Spacetime Solutions are "dynamically generated" but each solution has a single global causality structure.

Can you Generate Dynamical Causality in the Lab?

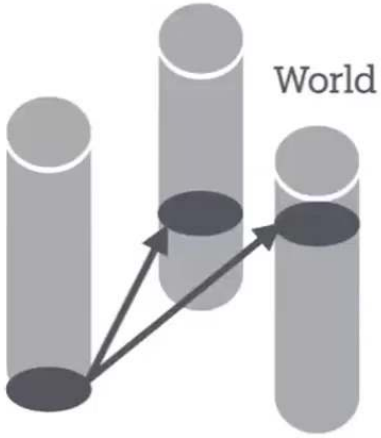
Salzger and V. Vilasini,2024

- Background IS Minkowski spacetime
- Blocking Causation doesn't "modify" causal structure
- Circuits are histories
- Modify Potential → Realisation





Quantum Circuit



World "tubes" of a Quantum Circuit

We only measure distances through causal processes. Space is reconstructed through causal processes.

Kinematic Causality Structure

- Fixed: General Relativity, Special Relativity
- Stochastic : Stochastic Geometry
- Quantum: Quantum Gravity

Combining causal structures

- Indefinite causal structures requires combining spacetimes in some way
- Only physically meaningful theory for this is a **quantum theory of gravity**

Causality in Quantum Theories of Gravity

$$Z = \int \mathcal{D}g e^{\frac{i}{\hbar}S(g)}$$

Should the ontology of quantum gravity include causality at a fundamental level?

- Euclidean approaches: $Z_{Euc} = \int \mathcal{D}g e^{\frac{1}{\hbar}S_E(g)}$, g is Euclidean $(+,+,+,+)$
- Lorentzian Approaches: g is a Lorentzian spacetime, $(-,+,+,+)$

Euclidean calculations may be relatively easier, but physics needs causality!

Causal Sets,

Causal Dynamical Triangulations,

Causal Spin Foams, ..

Causality is built into the fundamental ontology of geometry

Causal Set Theory: Quantising the Causal Structure

"HKMMKP theorem": $(M, g) = (M, \prec) + \epsilon$

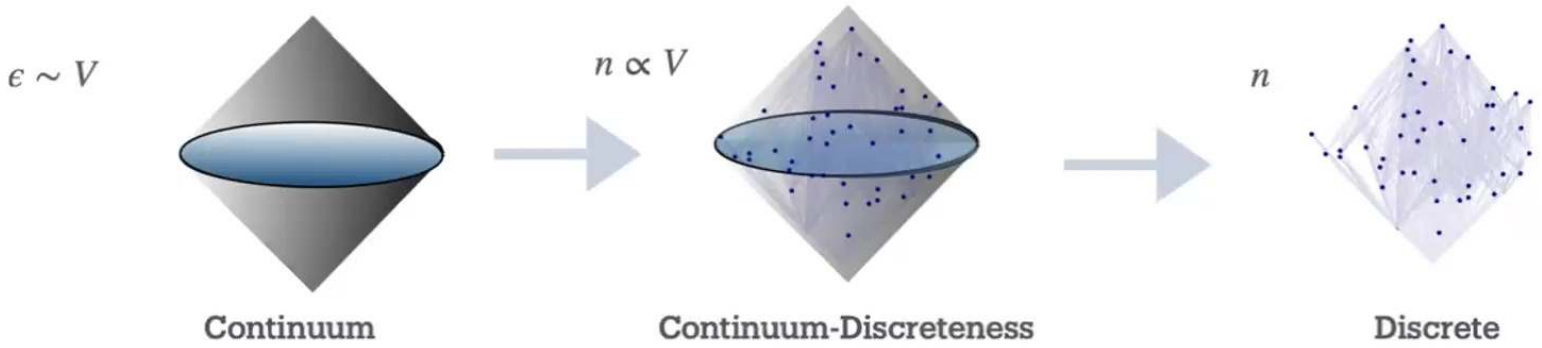
Acyclic: $x \prec y \Rightarrow y \not\prec x$

Transitive: $x \prec y, y \prec z \Rightarrow x \prec z$

+

Discretise $(M, \prec)$:

Local Finiteness: $|\,\mathrm{Fut}(x) \cap \mathrm{Past}(y)\,| < \infty$



The Causal Set Hypothesis

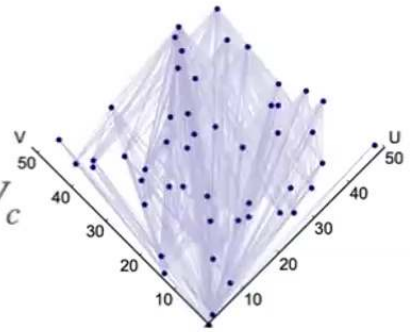
-- Myrheim, 1978
-- Bombelli, Lee, Meyer and Sorkin,1987

1. Causal Sets are the fine grained structure of spacetime

2. Continuum Spacetime is an approximation of underlying causal sets

Order ↔ Causal Order
Number ↔ Spacetime Volume

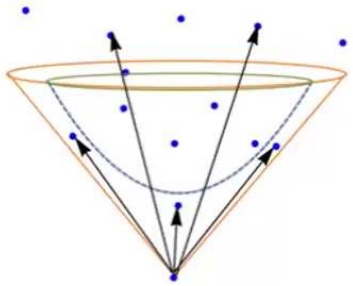
$\langle n \rangle \propto V$, via a Poisson point process: $P_V(n) = \frac{(\rho V)^n}{n!} e^{-\rho V}, \quad \rho^{-1} = V_c$



Continuumlike causal set ~ "random lattice"

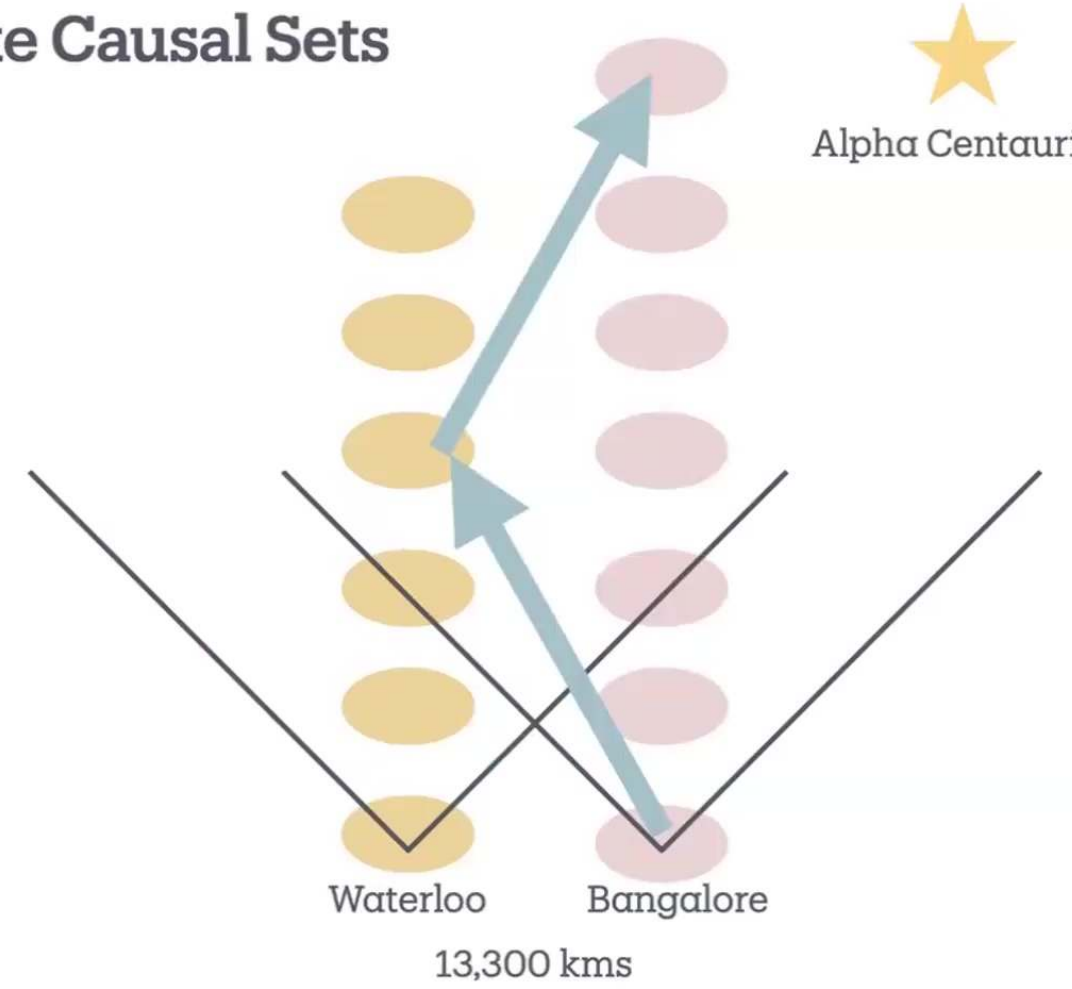
Properties of Continuumlike Causal Sets

- **Non-locality**

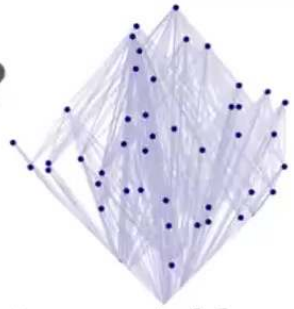


- **Lorentz Invariance**

Bombelli, Henson and Sorkin,



Combining causal structures: which causality?



- Each causal set is a **history**
- Non-locality of causal sets means that Hamiltonian/state quantisation is not viable
- Sum over causal sets:

$$Z = \sum_{c \in \Omega} e^{\frac{i}{\hbar} S(c)}, \quad \langle O \rangle \equiv \frac{1}{Z} \sum_{c \in \Omega} O(c) e^{\frac{i}{\hbar} S(c)}$$

- What are the rules for dynamics/quantum interference?

Quantum theory as a Generalisation of Stochastic Theory

Decoherence functional cool.. 😎

Measures on Spaces of Histories

- Ω — sample space of histories
- $\mathcal{S}$ — sigma algebra of "events"
- $\mu : \mathcal{S} \to [0,1]$ — Probability measure

General Relativist: Events are points in a spacetime

Others: Events are that things that happen.

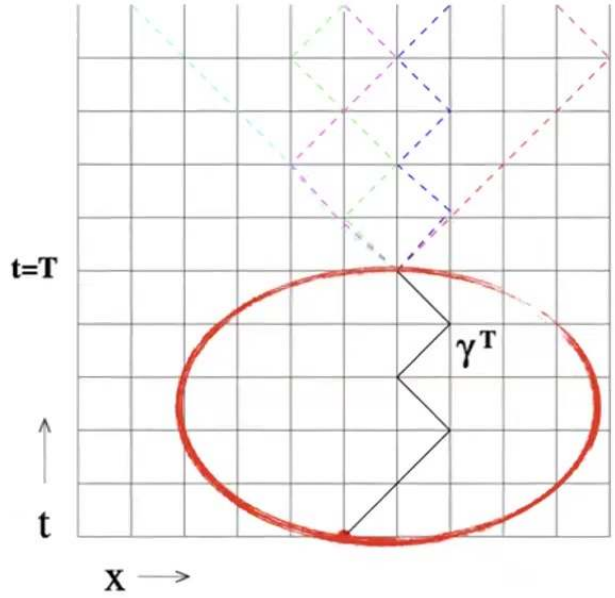
Measure Theorists: Events are measurable sets

Example: The Random Walk

Finite time event algebra $\mathcal{A}$ generated by:

$$\mathrm{cyl}(\gamma^T) = \{\gamma \in \Omega \,|\, \gamma|_T = \gamma^T\}$$

Extension: $(\Omega, \mathcal{A}, \mu) \to (\Omega, \mathcal{S}, \mu)$



The Decoherence Functional

Griffiths 1984;
Omnès 1988,
Gell-Mann and Hartle 1990
Sorkin, 1994

- Quantum Measure Theory $(\Omega, \mathcal{S}, D)$, $\quad D : \mathcal{S} \times \mathcal{S} \to \mathbb{C}$
  - $D(\alpha, \beta) = D^*(\beta, \alpha)$: Hermitian
  - $D(\alpha \sqcup \alpha', \beta) = D(\alpha, \beta) + D(\alpha', \beta)$: Biadditive
  - $D(\Omega, \Omega) = 1$: Normalisation
  - $\mu(\alpha) \equiv D(\alpha, \alpha) \geq 0$: Positive but non-additive
  - $\psi^\dagger D \psi \geq 0$ on any finite set $\{\alpha\}$: Strong Positivity

Away from unitary:

Conservation of Probability does not imply unitarity

- Vector Measure $(\Omega, \mathcal{A}, |\,.\,\rangle)$, $\quad |\alpha\rangle \in \mathcal{H}$, **Histories Hilbert Space**
- State $|\alpha\rangle$ is over a set of histories, not at a moment of time

—Dowker, Johnston and Sorkin, 2010
—Dowker, Johnston and Surya, 2010

Quantum Interpretation

Was the world wave function waiting to jump for thousands of millions of years .... for some ..highly qualified measurer ... with a PhD?

– J.S Bell, 'Quantum Mechanics for Cosmologists', 1981

- Quantum Measure Theory $(\Omega, \mathcal{S}, D)$
- Observables are Bell's be-ables or the **events** $\alpha \in \mathcal{S}$
- **Principle of Preclusion:** $\langle \alpha \,|\, \alpha \rangle = 0 \Rightarrow \alpha$ doesn't happen

Coevent Interpretation

Sorkin, 1994
Sorkin, 2007
Dowker, Ghazi-Tabatabai , 2008

- Intrinsic Causality in Histories Based Quantum Theory: —*Dowker and Sorkin, 2023*
- Measuring a "Probability" >1 —*Chakraborty, Sinha and Sorkin, 2024*

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Covariance and Teleology in Quantum Gravity

Covariance or Observer independence: wait for the full spacetime to evolve!

Teleological events:

- The return event in the Random Walk
- Black Holes
- Features of Causal Structure

Is there a single connected universe?

Is the causal structure continuous or discontinuous?

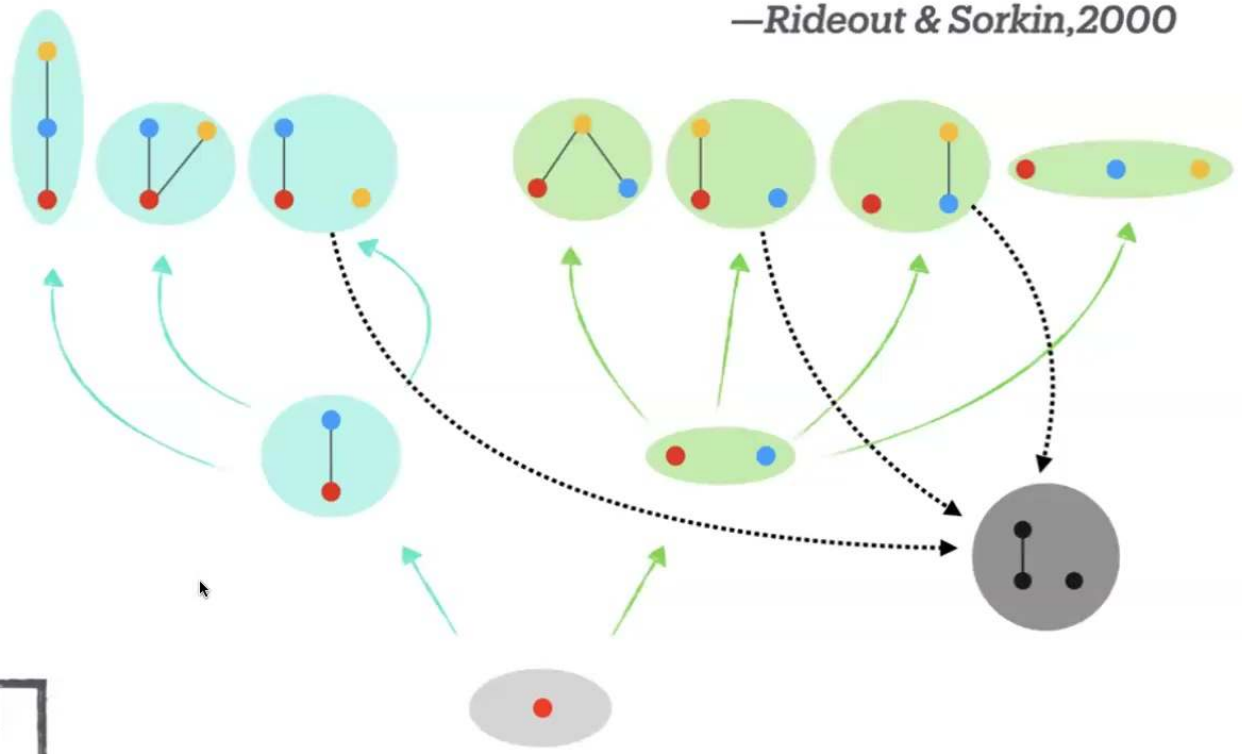
Is the spacetime globally hyperbolic?

Is the spacetime causally simple?

The Sequential Growth Paradigm: growing a universe causally

- **Causal Growth**: new element cannot be added to the past of an existing element
- **Covariance** or Label Independence
- **Markovian**
- **Bell Causality**: Spectator independence

—Rideout & Sorkin,2000

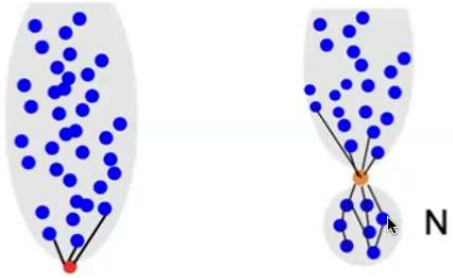


Stochastic Dynamics is specified by a single transition probability per stage of the growth $\{q_n\}$

Stochastic Growth: Causality in an Unpinned Universe
Huw Price's talk

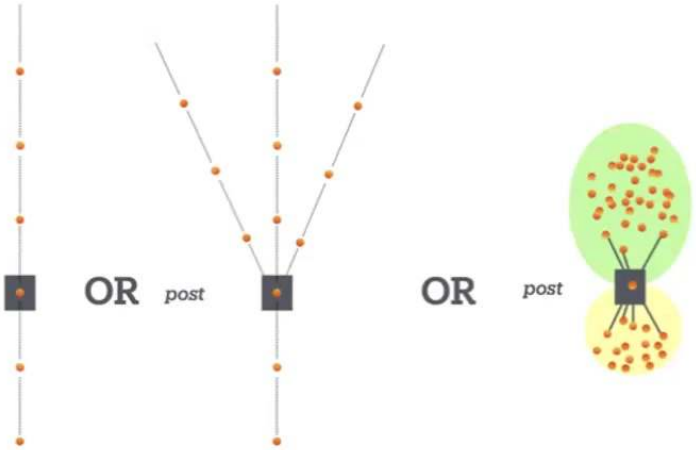
Covariant Events/Observables

- Examples:
  - Originary event $\alpha_{\text{orig}} \equiv \{c \in \Omega_{pf} | \exists e, \forall e' \neq e \Rightarrow e \prec e'\}$
  - Post event $\alpha_{\text{post}} \equiv \{c \in \Omega_{pf} | \exists e, \forall e' \neq e, e' \prec eore \prec e'\}$



- Each such event is a union of different causal sets each with a different causal structure:

$\alpha_{\text{post}} =$ post OR post OR post OR .....



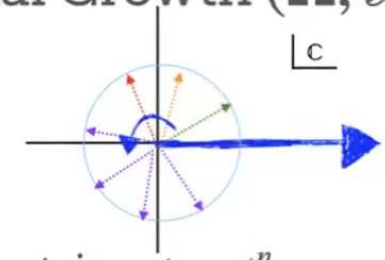
Whither causality?
- Dynamics
- Induced causal structure

The post event is causally defined and needs no external time!

Quantum Sequential Growth $(\Omega, \mathscr{A}, |\ .\ \rangle)$: An example

— *Dowker, Johnston & Surya, 2010*
— *Surya & Zalel, 2020*
— *Surya, 2016 (unpublished)*

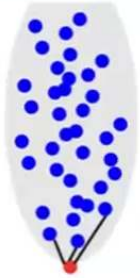
CSG Dynamics: $\mathscr{H} \simeq \mathbb{C}$



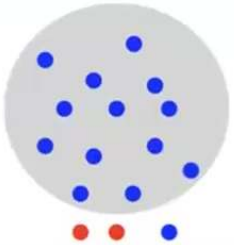
Complex Percolation: $t = x + iy, \quad t_n = t^n$

Is the originary event precluded?

- $|\alpha_{\text{orig}}\rangle = \varphi(q)\,|\Omega\rangle, \quad \varphi(q)$ is the Euler function
- $|\text{stem}(a_2)\rangle = |\Omega\rangle - |\alpha_{\text{orig}}\rangle = 1 - \varphi(q)$



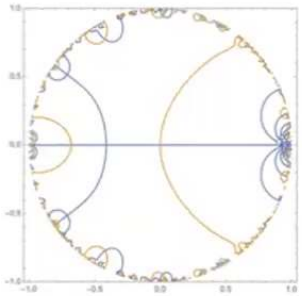
Originary

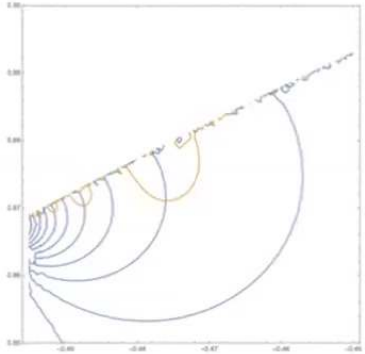


Stem(a_2)

No zeros for $|\alpha_{\text{orig}}\rangle$: Originary event is never precluded

There are zeros for $|\text{stem}(a_2)\rangle$, i.e., this event is precluded.





This is an example of a "quantum reality" for a covariant, teleological and causally defined event which is itself a superposition of different causal structures.

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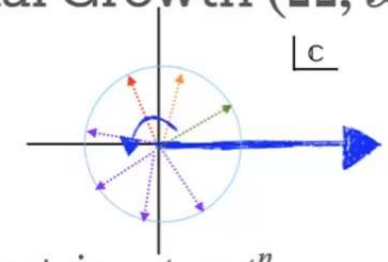

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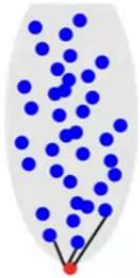

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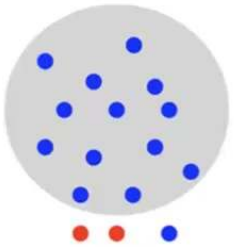

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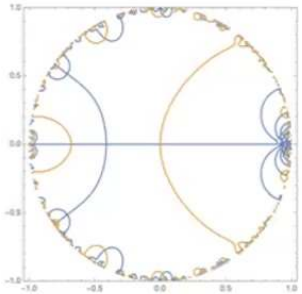

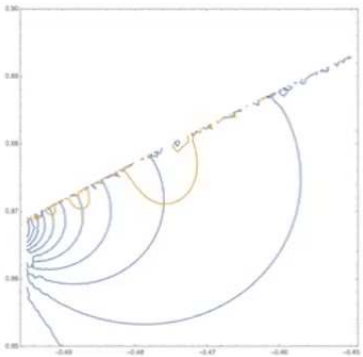

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Summary

- Spacetime provides a kinematic background causal structure
- Causal Structure is Globally/Teleologically defined
- Physical indefiniteness in the causal structure arises from a quantum theory of gravity
- Histories based approach for Observer/Measurement Independence
- Example of causal set theory: a covariant formulation of causally interesting questions

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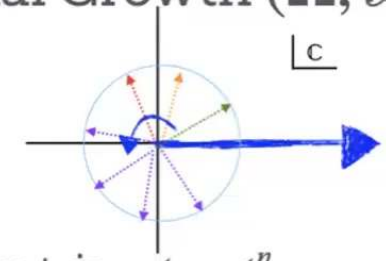

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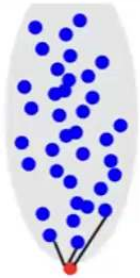
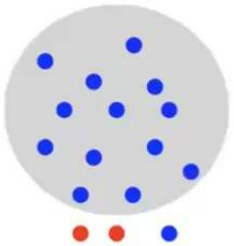
Is the originary event precluded?

- $|\alpha_{\text{orig}}\rangle = \varphi(q)\,|\Omega\rangle, \quad \varphi(q)$ is the Euler function
- $|\text{stem}(a_2)\rangle = |\Omega\rangle - |\alpha_{\text{orig}}\rangle = 1 - \varphi(q)$



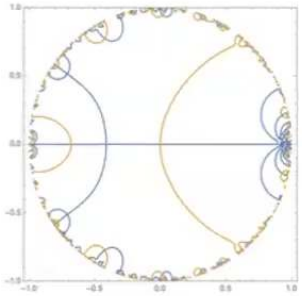
Originary

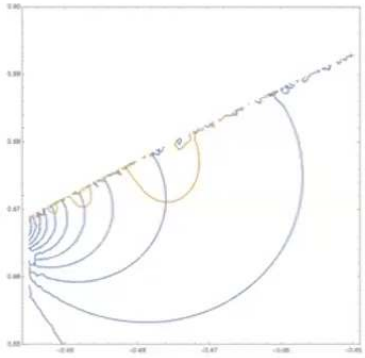


Stem(a_2)

No zeros for $|\alpha_{\text{orig}}\rangle$: Originary event is never precluded

There are zeros for $|\text{stem}(a_2)\rangle$, i.e., this event is precluded.





This is an example of a "quantum reality" for a covariant, teleological and causally defined event which is itself a superposition of different causal structures.