

Title: Quantum non-causality in spacetime may be not exclusively quantum

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Abstract: There are several non-causal effects that have been attributed to quantum physics. These include the analogues of "closed timelike curve effects" in quantum circuits proposed by David Deutsch (D-CTC), and the "impossible measurements" in relativistic quantum field theory discussed by Raphael Sorkin. Based on previous work, it will be pointed out in the talk that the alleged non-causality features arise not only in quantum systems, but in the very same manner in systems that are described in the framework of classical (non-quantum) statistical mechanics or classical field theory. Therefore, although the said non-causality scenarios have been portrayed as pertaining to quantum systems or quantum fields, they are in fact not based on, nor characteristic of, the quantum nature of physical systems.

Quantum Non-Causality in Spacetime May Be Not Exclusively Quantum

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This talk is based on the following articles:

- ① J. Tolksdorf, RV, *Quantum physics, fields and closed timelike curves: The D-CTC condition in quantum field theory*
Commun. Math. Phys. **357**, 319-351 (2018)
- ② J. Tolksdorf, RV, *The D-CTC condition is generically fulfilled in classical (non-quantum) statistical systems*
Found. Phys. **51**, 93 (2018)
- ③ A. Much, RV, *Superluminal local operations in quantum field theory: A ping-pong ball test*
Universe **9**, 447 (2023)

Statistical (Physical) Theories

Set of **random variables** $X \in \mathcal{X}$

Describe observations and measurements by **correlation functions** $C \in \mathcal{C}$

$$C(X_1, X_2, \dots, X_n) \in \mathbb{C} \quad (X_j \in \mathcal{X})$$

E.g. **expectation values, (conditional) probabilities**

Typically:

$$\mathcal{X} \leftrightarrow \text{set of observables} \quad \mathcal{C} \leftrightarrow \text{set of states}$$

Positive probabilities and multi-linearity in the $X_j \implies$

$$\mathcal{X} \simeq \mathcal{A} \text{ a }^*\text{-algebra}$$

$$\mathcal{C} \simeq \mathcal{S} \subset \mathcal{A}_+^* \text{ positive linear functionals on } \mathcal{A}$$

Example 1 – classical statistical theories

$\mathcal{A} = C^0(\mathcal{T})$ commutative algebra, $\mathcal{T} =$ topological space

$$X \leftrightarrow f: \mathcal{T} \rightarrow \mathbb{C}$$

$$\begin{aligned} C(X_1, X_2, \dots, X_n) &= C(f_1, f_2, \dots, f_n) = \langle f_1 \cdot f_2 \cdots f_n \rangle_\mu \\ &= \int_{\mathcal{T}} f_1(\xi_1) f_2(\xi_2) \cdots f_n(\xi_n) d\mu(\xi_1, \xi_2, \dots, \xi_n) \end{aligned}$$

$\mu =$ any probability measure on (the Borel sets of) \mathcal{T}

$$\langle f \rangle_\mu = \int_{\mathcal{T}} f d\mu \text{ is a state on } \mathcal{A} = C^0(\mathcal{T})$$

$$\langle f^* f \rangle_\mu \geq 0 \text{ with } f^* = \bar{f}, \quad \langle 1 \rangle_\mu = 1$$

set of states $\mathcal{S} \leftrightarrow$ set of probability measures on (Borel sets of) \mathcal{T}

Example 2 – quantum statistical theories

\mathcal{A} = a non-commutative $*$ -algebra (or C^* -algebra), with unit $\mathbf{1}$

$$X \leftrightarrow A \in \mathcal{A}$$

$$\begin{aligned} C(X_1, X_2, \dots, X_n) &= C(A_1, A_2, \dots, A_n) = \langle A_1 \cdot A_2 \cdots A_n \rangle_\omega \\ &= \omega(A_1 \cdot A_2 \cdots A_n) \end{aligned}$$

ω = any (sufficiently regular) state on \mathcal{A} , where a **state** is a linear functional $\omega : \mathcal{A} \rightarrow \mathbb{C}$ with

$$\omega(A^*A) \geq 0 \quad \text{and} \quad \omega(\mathbf{1}) = 1$$

Standard example in quantum physics:

$$\mathcal{A} \subset B(\mathcal{H}), \quad \omega(A) = \langle A \rangle_\varrho = \text{Tr}(\varrho A), \quad \varrho = \text{any density matrix}$$

$$\text{set of states } \mathcal{S} \leftrightarrow \text{set of density matrices on } \mathcal{H}$$

Some key points:

- In both classical and quantum case:

The set of states \mathcal{S} is closed under **convex combinations** and **(suitable types of) limits**.

A **convex combination** of states $\omega_1, \dots, \omega_n$ in \mathcal{S} with weights $\lambda_1, \dots, \lambda_n \geq 0$ is the state

$$\lambda_1 \omega_1 + \lambda_2 \omega_2 + \dots + \lambda_n \omega_n \in \mathcal{S} \quad (\lambda_1 + \dots + \lambda_n = 1)$$

- The theorems by Gelfand, Naimark, Segal and Wightman establish correspondences

(1) set of all states on \mathcal{A} \leftrightarrow set of all Hilbert space representations of \mathcal{A} for C^* -algebras

(2) $\mathcal{A} = C^0(\mathcal{T})$ for commutative C^* -algebra \mathcal{A}
 then: set of pure states on \mathcal{A} \leftrightarrow set of points $\xi \in \mathcal{T}$

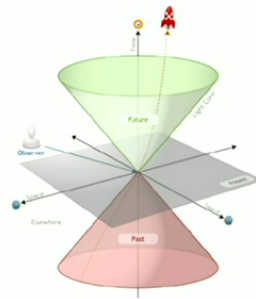
Algebraic /quantum/ field theory on a fixed spacetime manifold

M = a (4-dim) spacetime manifold, e.g. Minkowski spacetime
(or any globally hyperbolic spacetime)

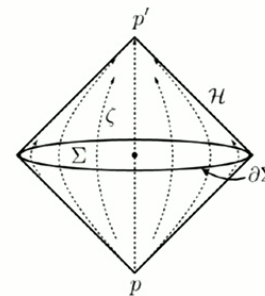
$J^\pm(p)$ = set of all $q \in M$ on any future(+)/past(-) directed worldline
emanating from $p \in M$

$$J^\pm(S) = \bigcup_{p \in S} J^\pm(p) \quad \text{for } S \subset M$$

O = open interior of $J^+(p) \cap J^-(p')$ for $p' \in J^+(p)$ “double cone”



picture source: F. Bellaïche,
www.quantum-bits.org



picture source: T. Jacobson, M. Visser,
arXiv: 1812.01598v1

In **algebraic quantum field theory** (or algebraic *classical* field theory), there is a **local structure** for the observables:

\mathcal{A} = *-algebra of (or: generated by) observables,

formed by *-subalgebras

$\mathcal{A}(O)$ = algebra of observables that can be
measured in the spacetime region O

with the properties:



- $O_1 \subset O_2 \implies \mathcal{A}(O_1) \subset \mathcal{A}(O_2)$
- $O_2 \cap J^\pm(O_1) = \emptyset \implies [A_1, A_2] = 0$ for all $A_j \in \mathcal{A}(O_j)$
- For every symmetry (isometry) $L : M \rightarrow M$ of the spacetime, there is an automorphism $\alpha_L : \mathcal{A} \rightarrow \mathcal{A}$ so that

$$\alpha_L(\mathcal{A}(O)) = \mathcal{A}(L(O)) \quad \text{and} \quad \alpha_{L_1} \circ \alpha_{L_2} = \alpha_{L_1 L_2}$$

The algebra \mathcal{A} may be non-commutative (quantum case) or commutative (classical case)

Typical situation in QFT:

- $\mathcal{A}(O)$ are weakly closed $*$ -subalgebras of $B(\mathcal{H})$ (“von Neumann algebras”)
- Set of (physical) states $\omega \in \mathcal{S}$ given by density matrices ϱ on \mathcal{H} :

$$\omega(A) = \langle A \rangle_{\varrho} = \text{Tr}(\varrho A)$$

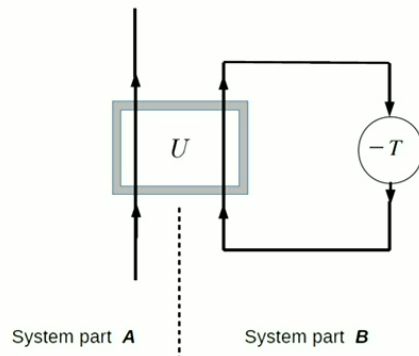


- $\alpha_L(A) = U_L A U_L^*$ with continuous unitary group repr $L \mapsto U_L$
- There is a unit vector $\psi_0 \in \mathcal{H}$ with $U_L \psi_0 = \psi_0$
- static and geodesic time-translations have positive generators: I.e. if $U_t = e^{itH}$ implements time-shifts of an inertial time-coordinate, then $H \geq 0$.

This is the setting we will adopt in the following, mainly for $M = \text{Minkowski spacetime}$.

2 — The D-CTC Condition by David Deutsch, 1991 (2.1)

A very simple quantum circuit



View this as bipartite quantum system with Hilbert spaces \mathcal{H}_A and \mathcal{H}_B

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$U : \mathcal{H} \rightarrow \mathcal{H} \text{ unitary}$$

$-T$ symbolizes “step backward in time”, meaning that partial state of full system after applying U is the same as before applying U on system part B

Given a unitary U on \mathcal{H} and a partial state (density matrix) ρ_A on system part A , a state (density matrix) ρ of the full system is said to **fulfill the D-CTC condition** if the restriction of ρ to system part A coincides with ρ_A and if $U\rho U^*$ and ρ agree when restricted to system part B .

Given a unitary U on \mathcal{H} and a partial state (density matrix) ϱ_A on system part A , a state (density matrix) ϱ of the full system is said to **fulfill the D-CTC condition** if the restriction of ϱ to system part A coincides with ϱ_A and if $U\varrho U^*$ and ϱ agree when restricted to system part B .

- Given: U unitary on \mathcal{H} , ϱ_A density matrix on \mathcal{H}_A

A density matrix ϱ on \mathcal{H} **fulfills the D-CTC condition** if

- $\text{Tr}_B \varrho = \varrho_A \Leftrightarrow \text{Tr}(\varrho(\mathbf{a} \otimes \mathbf{1})) = \text{Tr}_{\mathcal{H}_A}(\varrho_A \mathbf{a})$
- $\text{Tr}_A U\varrho U^* = \text{Tr}_A \varrho \Leftrightarrow \text{Tr}(\varrho(\mathbf{1} \otimes \mathbf{b})) = \text{Tr}(U\varrho U^*(\mathbf{1} \otimes \mathbf{b}))$

David Deutsch has shown: If \mathcal{H}_A and \mathcal{H}_B are *finite dimensional*, then for any given U and ϱ_A there is a ϱ fulfilling the D-CTC condition.

His argument rests on compactness of the state space = set of density matrices for finite-dimensional Hilbert spaces. This permits to employ a fixed-point argument.

Some (including David Deutsch) have claimed that D-CTC provides a form (or analog) of a **time travel scenario** —

“...quantum mechanics therefore allows for causality violation without paradoxes whilst remaining consistent with relativity”

Ringbauer et al., Nature Communications **5** (2014) 4145

...it has also recently gained popularity in pop culture...



Tony Stark – aka Iron Man

dixit:

“Quantum fluctuation messes with the Planck scale, which then triggers the

Deutsch Proposition”

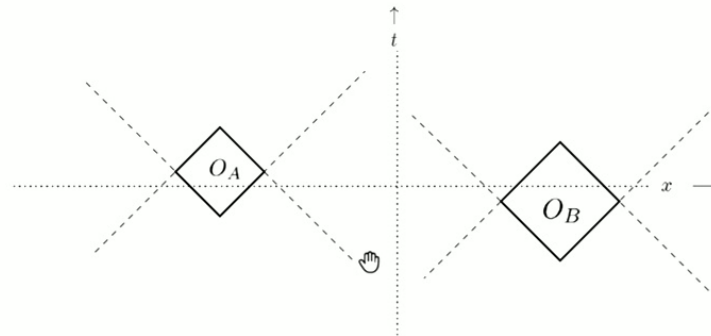
Questions:

- Is the D-CTC condition characteristic for quantum processes involving CTCs?
- Or is it merely an analogy of certain aspects of CTCs?
- Can the claim by Ringbauer et al. be substantiated or refuted?

The original version of the D-CTC condition makes no reference to spacetime structure (deliberately). To check on the previous questions, translate the setting into algebraic quantum field theory on Minkowski spacetime M with its built-in local and causal structure of the local observable algebras $\mathcal{A}(O)$.

In algebraic QFT:

Bipartite systems are represented by operator algebras $\mathcal{A}(O_A)$ and $\mathcal{A}(O_B)$ for causally separated spacetime regions O_A and O_B



D-CTC Problem: Given a unitary U on \mathcal{H} and a density matrix state

$$\omega_A(\mathbf{a}) = \text{Tr}(\rho_A \mathbf{a}) \quad \text{on } \mathcal{A}(O_A),$$

is there a density matrix state $\omega(\mathbf{c}) = \text{Tr}(\rho \mathbf{c})$ on $B(\mathcal{H})$ whose partial state on $\mathcal{A}(O_A)$ agrees with ω_A and which is U -invariant in restriction to $\mathcal{A}(O_B)$, i.e.

$$\omega(\mathbf{a}) = \omega_A(\mathbf{a}) \text{ on } \mathcal{A}(O_A) \quad \text{and} \quad \omega(U^* \mathbf{b} U) = \omega(\mathbf{b}) \text{ on } \mathcal{A}(O_B) \quad ?$$

Theorem 1 (JT & RV, CMP 357)

Assume that the QFT fulfills the split property (\Leftrightarrow density matrix states ω_A and ω_B are always restrictions of a density matrix state on \mathcal{H} without correlations across $\mathcal{A}(O_A)$ and $\mathcal{A}(O_B)$).

Then, given any unitary U in \mathcal{H} and any density matrix state $\omega_A(\mathbf{a}) = \text{Tr}(\rho_A \mathbf{a})$ on $\mathcal{A}(O_A)$, there is an *approximate* solution to the D-CTC problem in the following sense:

Given arbitrary $R > 0$ (large) and $\epsilon > 0$ (small), there is a density matrix state $\omega = \omega_{R,\epsilon}$ on $B(\mathcal{H})$ such that

- $\omega(\mathbf{a}) = \omega_A(\mathbf{a}) \quad (\mathbf{a} \in \mathcal{A}(O_A))$
- $|\omega(U^* \mathbf{b} U) - \omega(\mathbf{b})| < \epsilon \quad (\mathbf{b} \in \mathcal{A}(O_B), \|\mathbf{b}\| < R)$

This indicates that the D-CTC condition is **not** characteristic for occurrence of CTCs since it can be fulfilled to arbitrary precision in QFT on Minkowski spacetime. The proof rests on convexity and approximate completeness (relates to insisting on density matrix states) of the state space in QFT.

Operations

Given: Statistical theory, with observable algebra \mathcal{A} , set of states \mathcal{S}

An **operation** is a convex (and weak*-continuous) map $\tau : \mathcal{S} \rightarrow \mathcal{S}$

Typical example: If $U \in \mathcal{A}$ is **unitary**, then

$$\tau_U : \omega \mapsto \omega_U, \quad \omega_U(\mathbf{a}) = \omega(U^* \mathbf{a} U)$$

is an operation (unitary operation). 

- Definition of operation applies both for non-commutative or commutative \mathcal{A}
- If \mathcal{A} is commutative, then unitary operations are trivial: $\omega_U = \omega$ for every unitary $U \in \mathcal{A}$.
- Concept of operation defined here is *non-selective*, or *probability preserving*. Could generalize to selective operations. That would include measurements.

Formulating the **D-CTC condition** for classical statistical systems, with the following given data:

- (i) $\mathcal{T} = \mathcal{T}_A \times \mathcal{T}_B$ with locally compact Hausdorff spaces $\mathcal{T}_A, \mathcal{T}_B$;
 $\mathcal{A} = C_b(\mathcal{T})$, $\mathcal{A}_A = C_b(\mathcal{T}_A)$, $\mathcal{A}_B = C_b(\mathcal{T}_B)$
- (ii) An operation $\tau : \mathcal{S} \rightarrow \mathcal{S}$ ($\mathcal{S} = \text{states}(\mathcal{A})$)

We say that the **D-CTC condition can be fulfilled** in the system if for any given τ and for any given $\omega_A \in \mathcal{S}_A$ there is some $\omega \in \mathcal{S}$ so that

$$\begin{aligned}\omega(\mathbf{f}_A \otimes \mathbf{1}_B) &= \omega_A(\mathbf{f}_A) \quad (\mathbf{f}_A \in \mathcal{A}_A = C_b(\mathcal{T}_A)) \\ \tau(\omega)(\mathbf{1}_A \otimes \mathbf{f}_B) &= \omega(\mathbf{1}_A \otimes \mathbf{f}_B) \quad (\mathbf{f}_B \in \mathcal{A}_B = C_b(\mathcal{T}_B))\end{aligned}$$

Also want: ω is a probability measure if ω_A is prob. measure and if τ maps prob. measures to prob. measures (always fulfilled if \mathcal{T} is compact).

Theorem 2 (JT & RV, Found. Phys. 51)

Assumptions:

- ★ \mathcal{T}_A and \mathcal{T}_B are locally compact **metric spaces**,
- ★ τ maps probability measures to probability measures,
- ★ $\omega_A = \mu_A$ is a tight probability measure,
- ★ there is a probability measure μ_B° on \mathcal{T}_B so that

$$\tau^n(\mu_A \times \mu_B^\circ), \quad \overset{!}{n} \in \mathbb{N}, \quad \text{is tight}$$

($\mu_A \times \mu_B^\circ$ is the product measure)

Then the D-CTC condition can be fulfilled for the given ω_A and τ and the state ω fulfilling it is given by a Borel probability measure μ .

A sequence of Borel probability measures $\{\mu_n\}_{n \in \mathbb{N}}$ is called **tight** if:

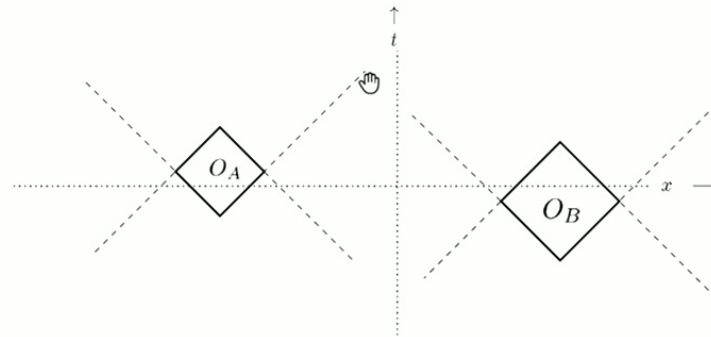
For any $\varepsilon > 0$ there is a compact set $\mathcal{K} \subset \mathcal{T}$ so that

$$\mu_n(\mathcal{T} \setminus \mathcal{K}) \leq \varepsilon \quad (n \in \mathbb{N})$$

For a QFT with local observable algebras $\mathcal{A}(O)$:

If O_1 and O_2 are causally separated ($O_2 \cap J^\pm(O_1) = \emptyset$) then any unitary operation τ_U with $U \in \mathcal{A}(O_1)$ has no effect on $\mathcal{A}(O_2)$:

$$\omega_U(\mathbf{a}_2) = \omega(U^* \mathbf{a}_2 U) = \omega(U^* U \mathbf{a}_2) = \omega(\mathbf{a}_2) \quad (\mathbf{a}_2 \in \mathcal{A}(O_2))$$

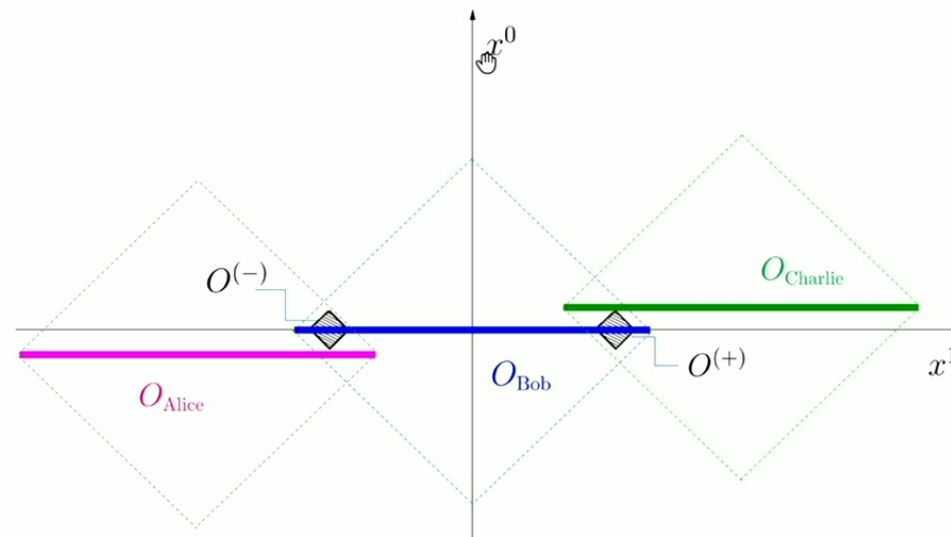


Therefore, such τ_U is called a **local operation**, *localized in O* .

Can all such local operations be physically performed?

If they could – for any unitaries in the local observable algebras – that may lead to **superluminal signalling** (a violation of causality) as pointed out by Raphael Sorkin (1993):

Consider 3 spacetime regions, named after experimenters carrying out measurements/operations therein:



3 — Impossible measurements/operations in QFT (3.3)

Since O_{Alice} and O_{Charlie} are causally separated, Charlie cannot know by measuring in O_{Charlie} if Alice has carried out a unitary operation $\tau_{U_{\text{Alice}}}$ with $U_{\text{Alice}} \in \mathcal{A}(O_{\text{Alice}})$:

$$\tau_{U_{\text{Alice}}}(\omega)(\mathbf{c}) = \omega(U_{\text{Alice}}^* \mathbf{c} U_{\text{Alice}}) = \omega(\mathbf{c}) \quad \text{for all } \mathbf{c} \in \mathcal{A}(O_{\text{Charlie}}), \omega \in \mathcal{S}$$

But if first Alice carries out a unitary operation, and then Bob, we have:

$$\tau_{U_{\text{Bob}}} \circ \tau_{U_{\text{Alice}}}(\omega)(\mathbf{c}) = \omega(U_{\text{Alice}}^* U_{\text{Bob}}^* \mathbf{c} U_{\text{Bob}} U_{\text{Alice}}) \quad \text{for all } \mathbf{c} \in \mathcal{A}(O_{\text{Charlie}})$$

In general, $U_{\text{Bob}} \in \mathcal{A}(O_{\text{Bob}})$ won't commute with all $\mathbf{c} \in \mathcal{A}(O_{\text{Charlie}})$ nor with all $U_{\text{Alice}} \in \mathcal{A}(O_{\text{Alice}})$ since

O_{Alice} causally overlaps with O_{Bob} and O_{Bob} causally overlaps with O_{Charlie}

3 — Impossible measurements/operations in QFT (3.4)

Hence, one can choose $U_{\text{Alice}}, U_{\text{Bob}}$ \mathbf{c} and ω such that

$$\tau_{U_{\text{Bob}}} \circ \tau_{U_{\text{Alice}}}(\omega)(\mathbf{c}) \neq \tau_{U_{\text{Bob}}}(\omega)(\mathbf{c})$$

This means, Charlie can determine by measuring the observable \mathbf{c} in O_{Charlie} if Alice has carried out an operation $\tau_{U_{\text{Alice}}}$ in O_{Alice} , if Bob carries out a suitable operation $\tau_{U_{\text{Bob}}}$ in O_{Bob} .

This would mean a superluminal transfer of information since O_{Alice} and O_{Charlie} are causally separated.

Examples are given in: R. Sorkin (1993); L. Bosten, I. Jubb, G. Kells, PRD 104 (2021); I. Jubb, PRD 105 (2022).

The issue is that $\tau_{U_{\text{Bob}}}$ amounts to a superluminal communication channel between O_{Alice} and O_{Charlie} which is unphysical.

But such superluminal communication channels arise also in [classical field theory](#), e.g. by local, kinematical symmetries.

Theorem 3 (AM & RV, Universe (2023))

Let $\mathcal{A}(O)$ be the local observable algebras of the **classical** or the **quantized** Klein-Gordon field on Minkowski spacetime M , with field equation $(\square + m^2)\varphi = 0$.

Then there are states ω and operations τ_{Alice} and τ_{Bob} together with observables $\mathbf{c} \in \mathcal{A}(O_{\text{Charlie}})$ so that

$$\tau_{\text{Bob}} \circ \tau_{\text{Alice}}(\omega)(\mathbf{c}) \neq \tau_{\text{Bob}}(\omega)(\mathbf{c})$$

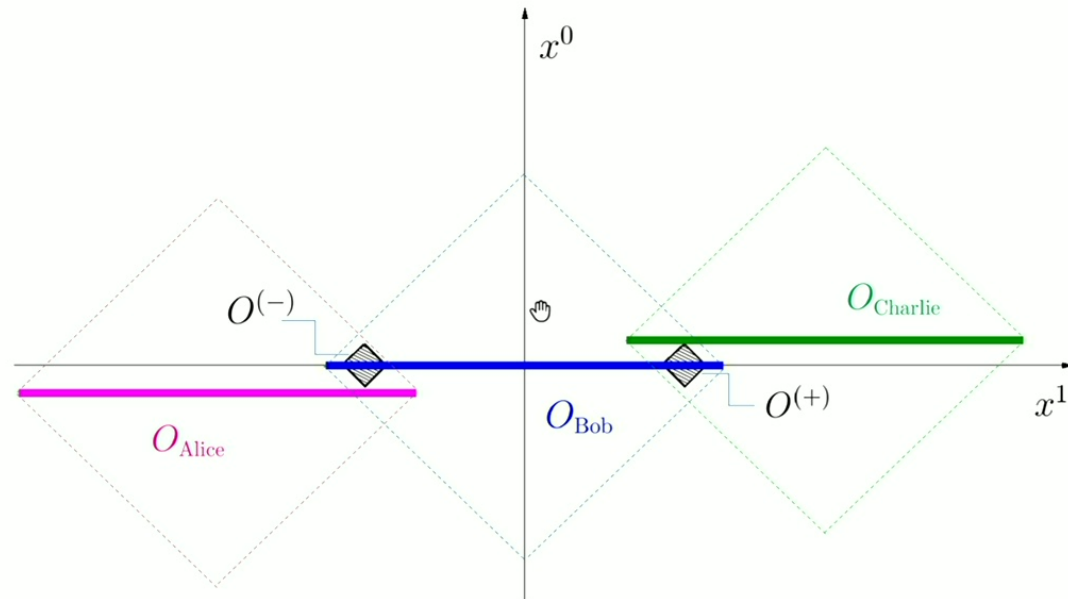
τ_{Alice} and τ_{Bob} are localized in O_{Alice} and O_{Bob} , i.e. $\tau_{\text{Bob}}(\tilde{\omega})(\mathbf{d}) = \tilde{\omega}(\mathbf{d})$ if $\mathbf{d} \in \mathcal{A}(O_d)$ with O_d causally separated from O_{Bob} .

Specifically, τ_{Bob} can be chosen so that it corresponds to an **instantaneous rotation around the x^3 -axis by 180 degrees**, flipping $O^{(-)} \leftrightarrow O^{(+)}$ (local kinematical symmetry).

For the quantized Klein-Gordon field, there is a unitary $U_{\text{Bob}} \in \mathcal{A}(O_{\text{Bob}})$ so that

$$\tau_{\text{Bob}}(\omega)(\cdot) = \omega(U_{\text{Bob}}^* \cdot U_{\text{Bob}})$$

3 — Impossible measurements/operations in QFT (3.6)



τ_{Bob} has the effect of flipping $O^{(-)}$ instantaneously to $O^{(+)}$ and vice versa.

Remarks

- The approach of describing classical field theory in terms of a local algebra framework has been developed by Brunetti, Duetsch, Fredenhagen and Rejzner (and co-authors). See:
K. Rejzner: *Perturbative Algebraic Quantum Field Theory*, Springer, 2016
M. Duetsch: *From Classical Field Theory to Perturbative Quantum Field Theory*, Birkhäuser, 2019
- In the *classical* case, τ_{Bob} and τ_{Alice} are not implemented by unitaries in the local algebras since the local algebras are commutative — they are formed by (certain) functions on the phase space.
The generator of τ_{Bob} can be obtained with the help of a *Peierls bracket*, generalising the Poisson bracket of Hamiltonian mechanics.

- Original setting for D-CTC condition does not refer to spacetime structure; does not relate to CTCs in the sense of GR
- Thm 1 shows that D-CTC is not characteristic for occurrence of CTCs in the sense of GR.
- Thm 2 shows that **D-CTC can always be fulfilled in classical (non-quantum) statistical systems** – it is more a generalized ergodic theorem than related to quantum mechanics.



Put bluntly: **The D-CTC condition has nothing to with quantum mechanics** (uncertainty relations, interference, entanglement) but only relates to the basic statistical setting of quantum mechanics.

The *impossible measurements/impossible operations scenario* does not only arise in QFT, but also in classical field theory.

There are “superluminal” local operations also in classical field theory, e.g. by local kinematical symmetries. Not all local operations in quantum or classical field theory can be “actively” carried out.

We have carried out the **ping-pong ball test*** on “D-CTC” and “impossible operations/measurements” (and it failed in both cases) —

When someone presents a paradox as being rooted in quantum physics, replace the term quantum mechanical particle by ping-pong ball everywhere.

If the paradox persists, it is unrelated to quantum physics.

But this does not mean that “D-CTC” and “impossible operations/measurements” are not interesting. They point to issues that need to be better understood in QFT.

The “impossible measurements scenario” can be avoided in more recent approaches towards QFT measurements:

C.J. Fewster, RV, Comm. Math. Phys. 378 (2020);
H. Bostelmann, C.J. Fewster, M. Rued, PRD 103 (2021);
M. Papageorgiou, D. Fraser, Found. Phys. 53 (2024)

* Due to Reinhard Werner (oral version)