Title: Joint measurements on distant physical systems

Speakers: Alejandro Pozas Kerstjens

Series: Quantum Foundations, Quantum Information

Date: September 18, 2024 - 10:50 AM

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Abstract: It is not explicitly obvious that relativity and quantum mechanics are consistent with each other. Extensive research has shown that quantum states are consistent with relativity, in that they do not allow for faster-than-light transferring of information. In contrast, much less research has been done in quantum measurements, and in fact, naive attempts to put together relativity and quantum measurements lead to signaling between space-like separated regions. In this talk I will describe how this same problem arises in non-relativistic quantum physics, where measurements on systems kept spatially separated in general lead to signalling. By giving away the projection postulate, it is possible to alleviate this problem and measure non-local variables without signaling by exploiting pre-shared entanglement as a resource. I will describe a protocol for implementing any joint measurement in a non-signaling manner, and argue that this leads to a complete classification of all joint quantum measurements, based on the required amount of entanglement necessary to measure them.

Joint (quantum) measurements on distant physical systems

Alejandro Pozas-Kerstjens

Université de Genève

Why study joint quantum measurements?

Because it is the fair thing to do (and it may be profitable to do it)

In the 1980s there were two main challenges in the foundations of quantum physics:

1. Quantum nonlocality (i.e., violations of Bell inequalities)
2. The quantum measurement problem

> *The Schrödinger equation is interrupted whenever the system encounters a bunch of atoms with a sticker saying "measurement apparatus"*

We worked a lot in the first one. It led to (a large part of) quantum information science as we know it.

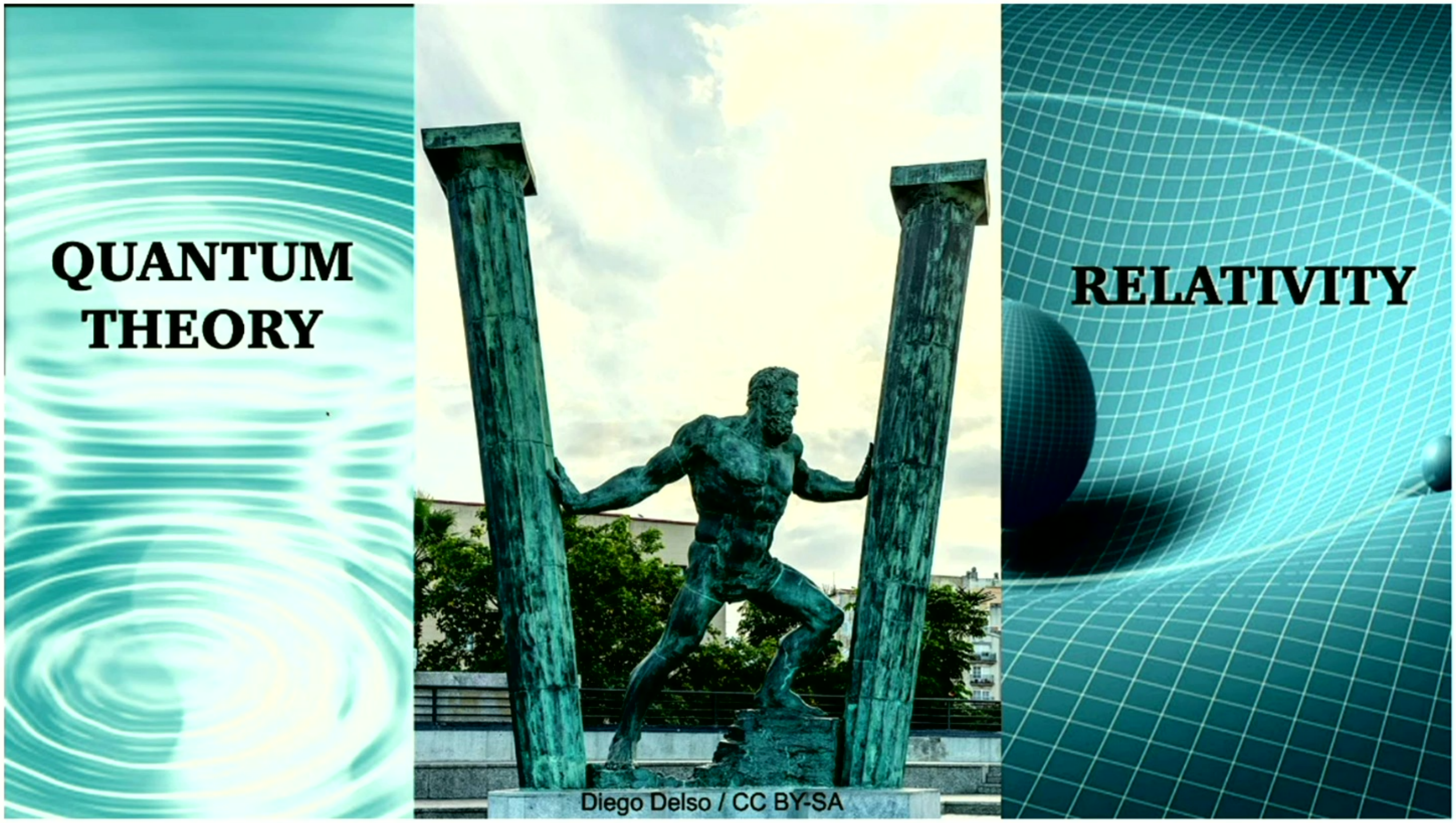
Along the way to understanding quantum nonlocality, we advanced in

Mathematics	Connes' embedding prob., Grothendieck constants, NPO...
Theoretical physics	Communication cost for quantum correlations, PR boxes...
Mathematical physics	Local polytopes, network non-locality, entanglement theory...
Experimental physics	Loophole-free Bell tests, measurement-dependent locality, tests of superluminal influences...
Applied physics	DIQIP, QRNG, QKD...

After more of 30 research on quantum information, we know a lot about quantum correlations and, in particular, about entangled quantum states.

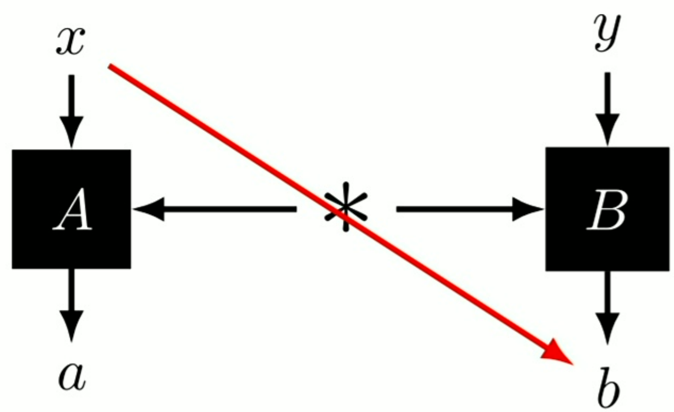
But what about joint quantum measurements (measurements with entangled eigenstates)?

In this talk:
How do joint quantum measurements combine with relativistic causality?



If you have paid attention at the conference, you will recognize this image

Quantum statistics are consistent with relativistic causality



$$p(a,b|x,y) = \mathsf{Tr}\left(\rho_{AB} E_{a|x} \otimes F_{b|y}\right)$$
$$\rho_{AB} \succeq 0,\ \mathsf{Tr}\left(\rho_{AB}\right) = 1$$
$$E_{a|x} \succeq 0,\ \sum_a E_{a|x} = \mathbb{1}\ \forall x$$
$$F_{b|y} \succeq 0,\ \sum_b F_{b|y} = \mathbb{1}\ \forall y$$

Signaling: $\sum_a p(a,b|x,y) \neq \sum_a p(a,b|x',y)$

$$\sum_a \mathsf{Tr}\left(\rho_{AB} E_{a|x} \otimes F_{b|y}\right) = \mathsf{Tr}\left[\rho_{AB}\left(\sum_a E_{a|x}\right) \otimes F_{b|y}\right] = \mathsf{Tr}\left(\rho_{AB} \mathbb{1} \otimes F_{b|y}\right) = p(b|y)\ \forall x$$

Born's rule accounts for the impossibility of instantaneous transfer of information

But relativistic causality is not enough to limit quantum correlations

Quantum Nonlocality as an Axiom

Sandu Popescu[1] and Daniel Rohrlich[2]

Received July 2, 1993; revised July 19, 1993

In the conventional approach to quantum mechanics, indeterminism is an axiom and nonlocality is a theorem. We consider inverting the logical order, making nonlocality an axiom and indeterminism a theorem. Nonlocal "superquantum" correlations, preserving relativistic causality, can violate the CHSH inequality more strongly than any quantum correlations.

Found. Phys. 24, 379-385 (1994)

$$\mathsf{CHSH}(p) = \sum_{a,b,x,y} (-1)^{a+b+xy} p(a,b|x,y) \leq \begin{cases} 2 & \mathsf{C} \\ 2\sqrt{2} & \mathsf{Q} \end{cases}$$

$$p_{PR}(a,b|x,y) = \tfrac{1}{4}\left[1 + (-1)^{a+b+xy}\right]$$

$$\sum_a p_{PR}(a,b|x,y) = \tfrac{1}{2} \;\forall\, b,x,y$$

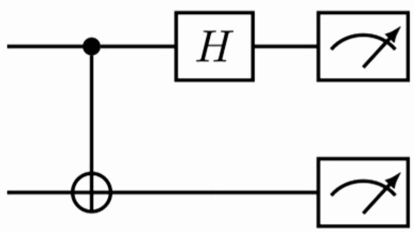
$$\mathsf{CHSH}(p_{PR}) = 4$$

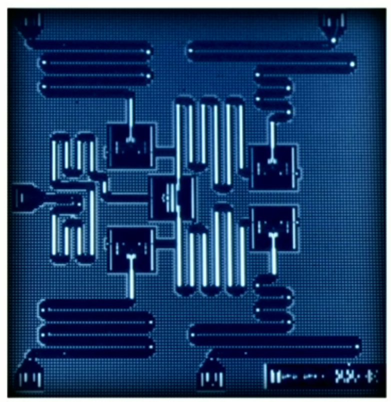
Are quantum measurements consistent with relativistic causality?

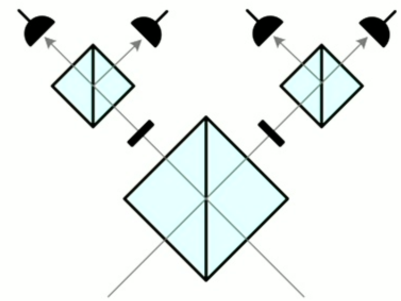
In quantum mechanics, we can perform joint measurements over several systems

For example, the Bell state measurement

$$E_a = \{|\phi^+\rangle\langle\phi^+|, |\phi^-\rangle\langle\phi^-|, |\psi^+\rangle\langle\psi^+|, |\psi^-\rangle\langle\psi^-|\}$$
$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \; |\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$







Credit: Peter Rohde

Are quantum measurements consistent with relativistic causality?

Erweiterung des Unbestimmtheitsprinzips für die relativistische Quantentheorie.

Von **L. Landau** und **R. Peierls** in Zürich.

(Eingegangen am 3. März 1931.)

Durch Betrachtung der möglichen Meßmethoden wird gezeigt, daß alle in der Wellenmechanik auftretenden physikalischen Größen im relativistischen Gebiet im allgemeinen nicht mehr definierbar sind. Damit hängt das bekannte Versagen der wellenmechanischen Methoden in diesem Gebiet zusammen.

Zeitschrift fur Physik 1, 56-59

Are quantum measurements consistent with relativistic causality?

Impossible Measurements on Quantum Fields

RAFAEL D. SORKIN

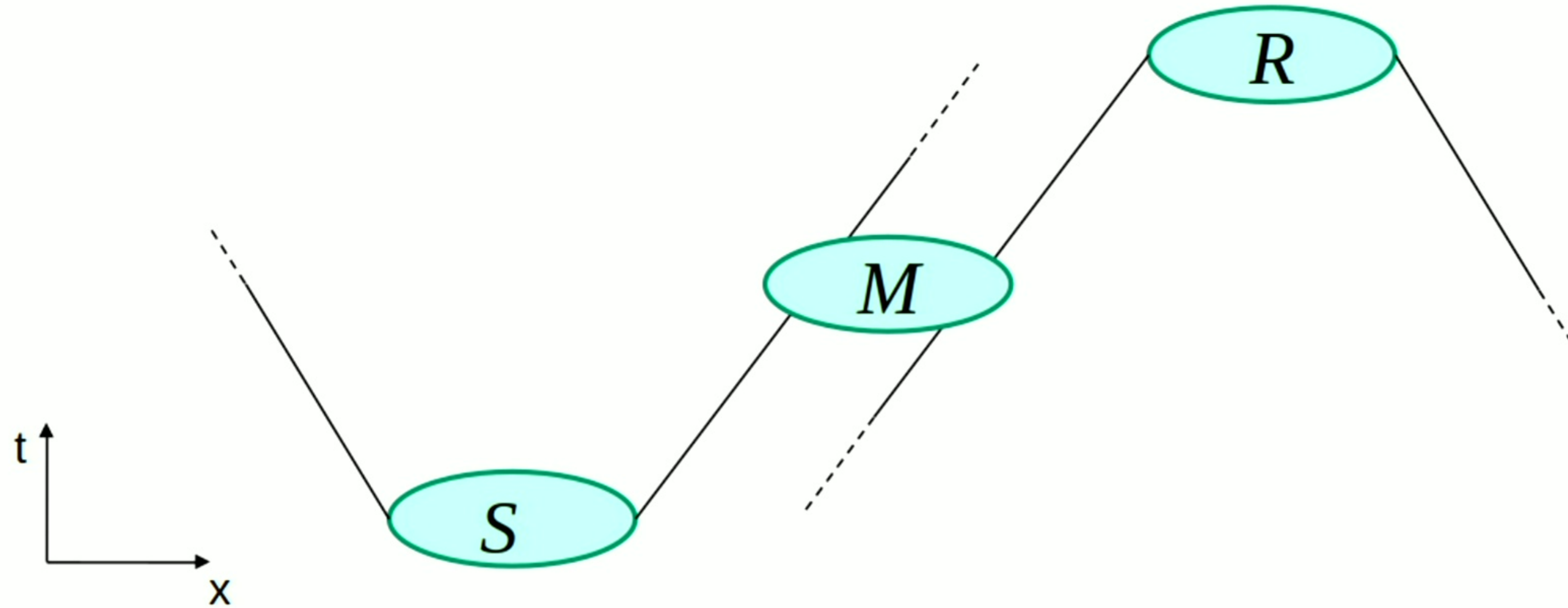
Department of Physics, Syracuse University, Syracuse NY 13244-1130

Abstract

It is shown that the attempt to extend the notion of ideal measurement to quantum field theory leads to a conflict with locality, because (for most observables) the state vector reduction associated with an ideal measurement acts to transmit information faster than light. Two examples of such information-transfer are given, first in the quantum mechanics of a pair of coupled subsystems, and then for the free scalar field in flat spacetime. It is argued that this problem leaves the Hilbert space formulation of quantum field theory with no definite measurement theory, removing whatever advantages it may have seemed to possess vis a vis the sum-over-histories approach, and reinforcing the view that a sum-over-histories framework is the most promising one for quantum gravity.

arXiv:gr-qc/9302018

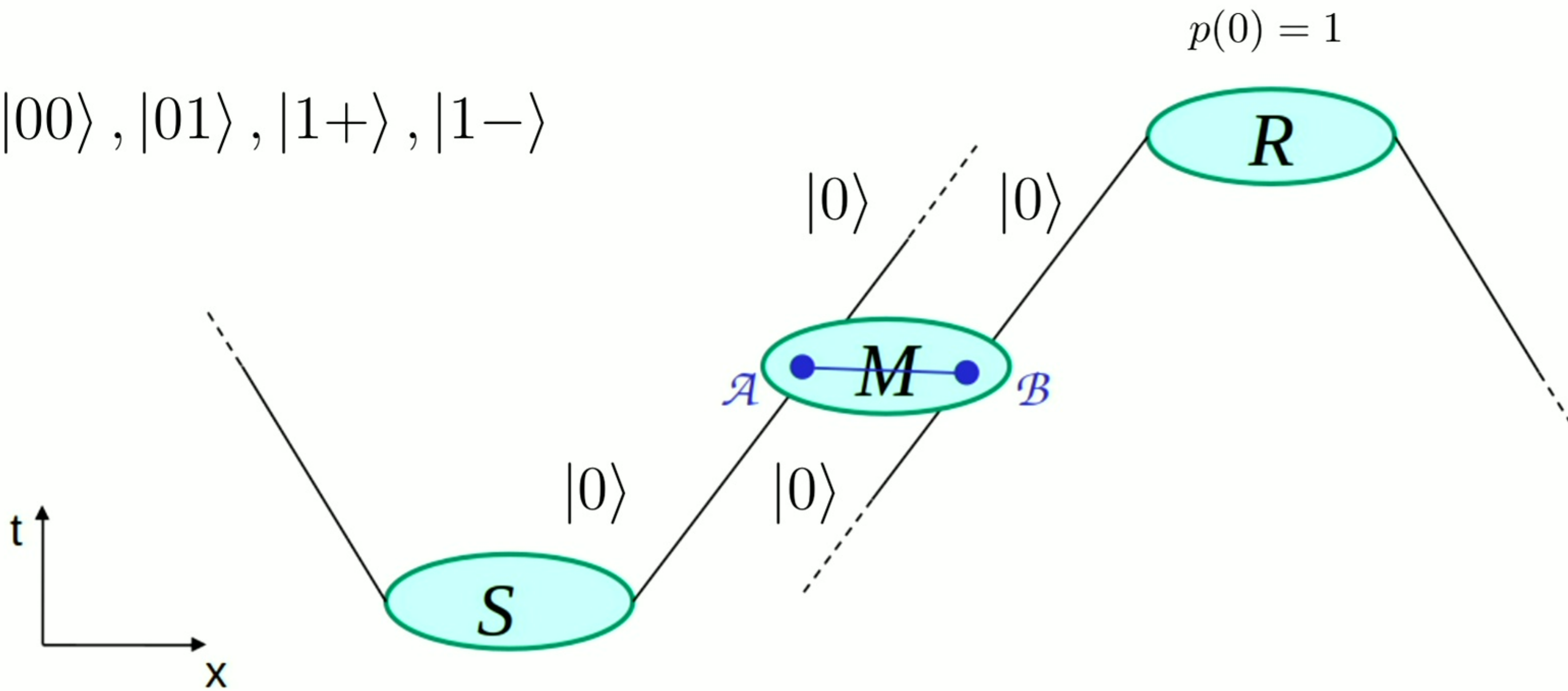
Superluminal signaling in QFT



Borsten *et al.*, PRD 2019

Superluminal signaling in nonrelativistic QM!

$$|00\rangle\,, |01\rangle\,, |1+\rangle\,, |1-\rangle$$



Superluminal signaling in nonrelativistic QM!

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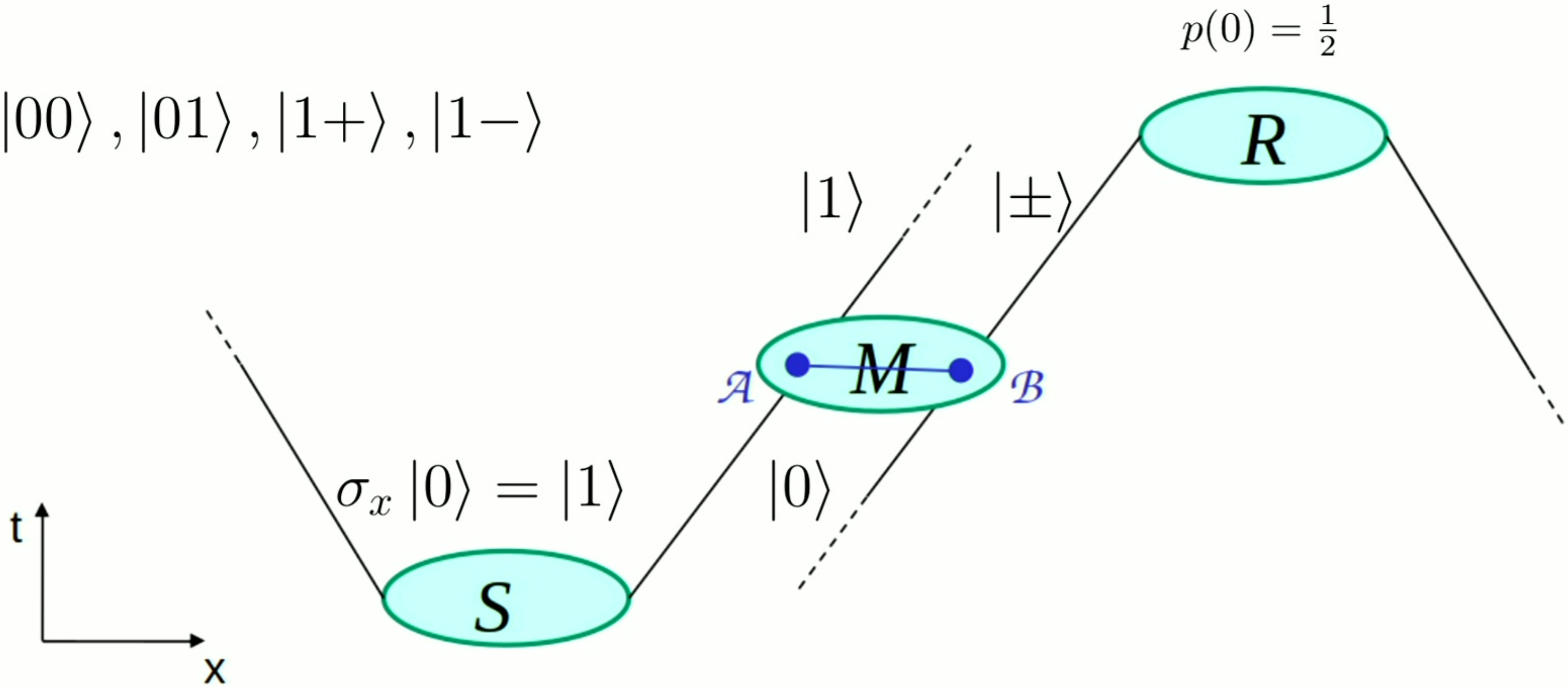

Signaling is a very big problem

There is a (real) lot of measurements that lead to signaling, even in non-relativistic QM

PHYSICAL REVIEW A VOLUME 49, NUMBER 6 JUNE 1994

Causality constraints on nonlocal quantum measurements

Sandu Popescu
Service de Physique Théorique, Université Libre de Bruxelles, Campus Plaine CP 225, Boulevard du Triomphe, 1050 Bruxelles, Belgium

Lev Vaidman
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv, 69978 Israel

(Received 2 April 1993)

Consequences of relativistic causality for measurements of nonlocal characteristics of composite quantum systems are investigated. It is proved that verification measurements of entangled states necessarily erase local information. A complete analysis of measurability of nondegenerate spin operators of a system of two spin-$\frac{1}{2}$ particles is presented. It is shown that measurability of certain projection operators which play an important role in axiomatic quantum theory contradicts the causality principle.

Joint measurements that do not lead to signaling erase all local information
$\Rightarrow$ for two qubits, the BSM is **the only** measurement that does not lead to signaling
$\Rightarrow$ the BSM is not a typical measurement, it is exceptional

The problem may be with *ideal* measurements

$A = \sum_i a_i P_i, \quad P_i \succeq 0 \ \forall i, \quad \sum_i P_i = \mathbb{1}$

Born's rule: $p(a_i|\rho) = \mathrm{Tr}\,(\rho P_i)$

Lüders' rule: $\rho \xrightarrow{a_i} \rho_i = P_i \rho P_i / \mathrm{Tr}\,(\rho P_i)$

Impossible Measurements on Quantum Fields

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Department of Physics, Syracuse University, Syracuse NY 13244-1130

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It is shown that the attempt to extend the notion of ideal measurement to quantum field theory leads to a conflict with locality, because (for most observables) the state vector reduction associated with an ideal measurement acts to transmit information faster than light. Two examples of such information-transfer are given, first in the quantum mechanics of a pair of coupled subsystems, and then for the free scalar field in flat spacetime. It is argued that this problem leaves the Hilbert space formulation of quantum field theory with no definite measurement theory, removing whatever advantages it may have seemed to possess vis a vis the sum-over-histories approach, and reinforcing the view that a sum-over-histories framework is the most promising one for quantum gravity.

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PHYSICAL REVIEW A **66**, 022110 (2002)

Measurements of semilocal and nonmaximally entangled states

Berry Groisman and Benni Reznik
School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel
(Received 15 November 2001; published 16 August 2002)

Consistency with relativistic causality narrows down dramatically the class of measurable observables. We argue that, by weakening the preparation role of ideal measurements, many of these observables become measurable. In particular, we show by applying entanglement assisted remote operations that all Hermitian observables of a (2×2)-dimensional bipartite system are measurable.

VOLUME 90, NUMBER 1 PHYSICAL REVIEW LETTERS week ending 10 JANUARY 2003

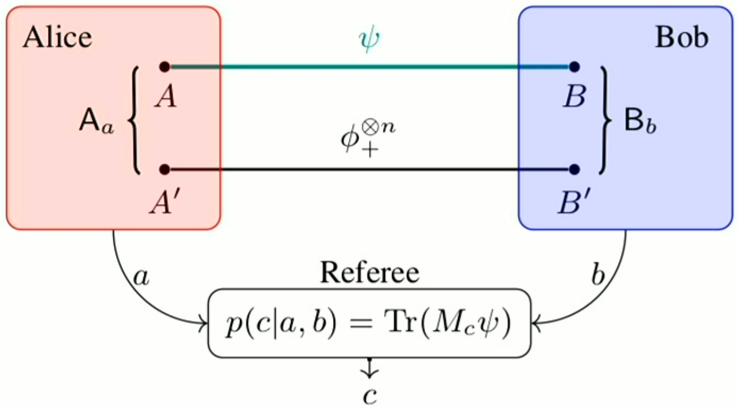
Instantaneous Measurement of Nonlocal Variables

Lev Vaidman
[1]*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel*
(Received 14 December 2001; revised manuscript received 15 April 2002; published 2 January 2003)

It is shown, under the assumption of the possibility to perform an arbitrary local operation, that all nonlocal variables related to two or more separate sites can be measured instantaneously, except for a finite time required for bringing to one location the classical records from these sites which yield the result of the measurement. It is a verification measurement: it yields reliably the eigenvalues of the nonlocal variables, but it does not prepare the eigenstates of the system.

Performing (non-ideal) measurements in a way consistent with relativity

Definition (localized measurement): the quantum-to-classical transition occurs locally

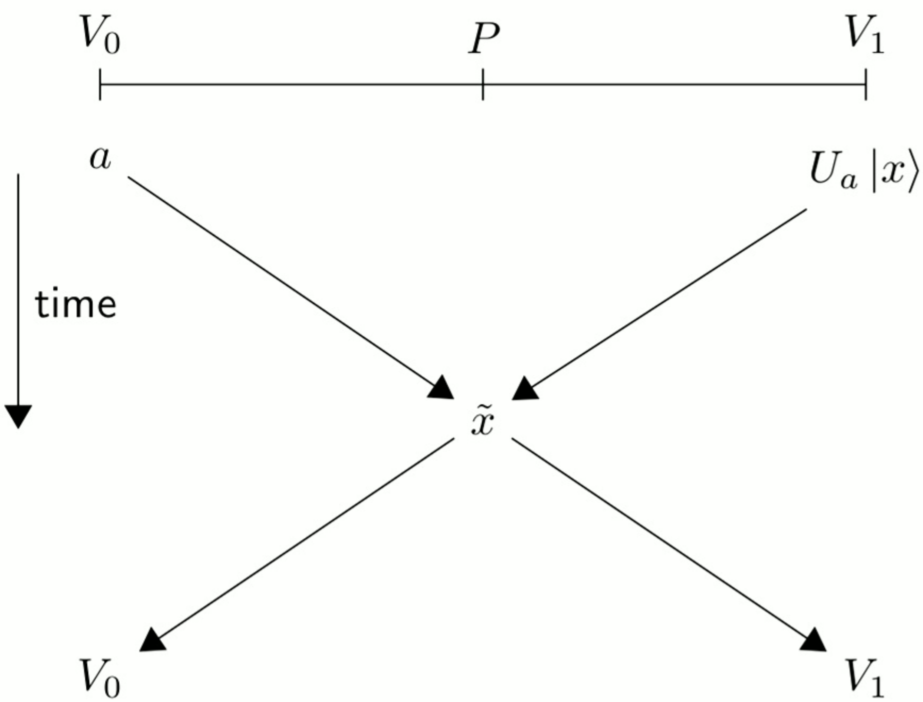


A quantum measurement $\{M_c\}_c$ is n-ebit localizable if there exist local measurements $\{\mathsf{A}_a\}_a \subset \mathcal{H}_{AA'}$ and $\{\mathsf{B}_b\}_b \subset \mathcal{H}_{BB'}$, and distributions $p(c|a,b)$ such that

$$M_c = \sum_{a,b} p(c|a,b) \mathsf{Tr}_{A'B'} \left[(\mathsf{A}_a \otimes \mathsf{B}_b) \left(\mathbb{1}_{AB} \otimes {\phi^+}^{\otimes n}_{A'B'} \right) \right]$$

Localization is connected to quantum position verification

Goal: convince two verifiers (V_0 and V_1) that I'm at P



$t = 0$: V_0 sends a, V_1 sends $U_a |x\rangle$

$t = 1$: P receives both pieces of information, applies $U_a^\dagger$ and measures in the computational basis, transmitting the result back to V_0 and V_1

$t = 2$: V_0 and V_1 accept if they receive x, and they receive it on time

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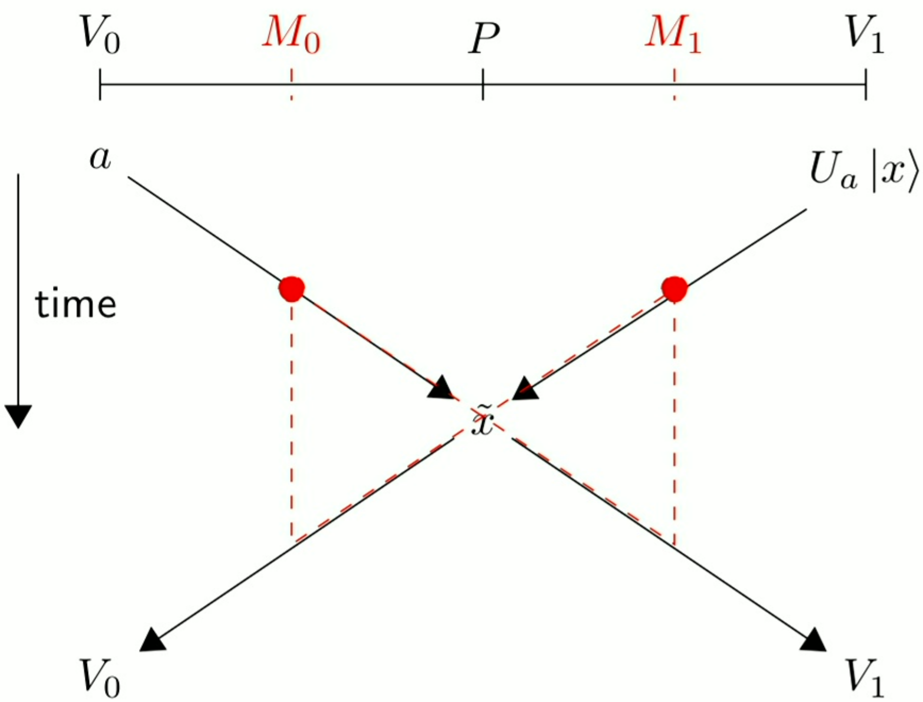


$t = 0$: V_0 sends a, V_1 sends $U_a |x\rangle$

$t < 1$: two coordinated adversaries intercept the information and run a localization protocol

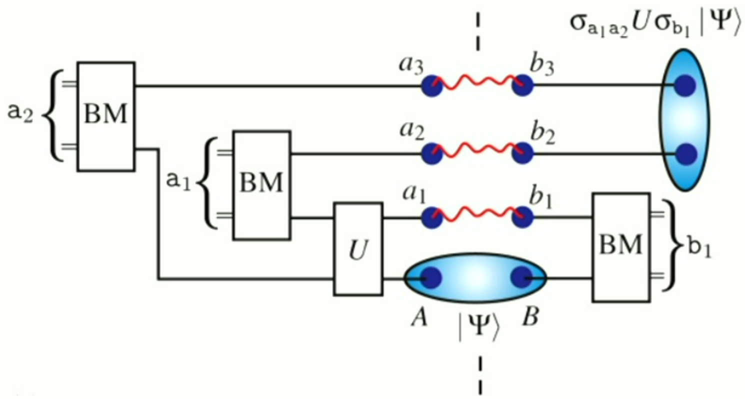
$t = 1$: P receives both pieces of information, applies $U_a^\dagger$ and measures in the computational basis, transmitting the result back to V_0 and V_1

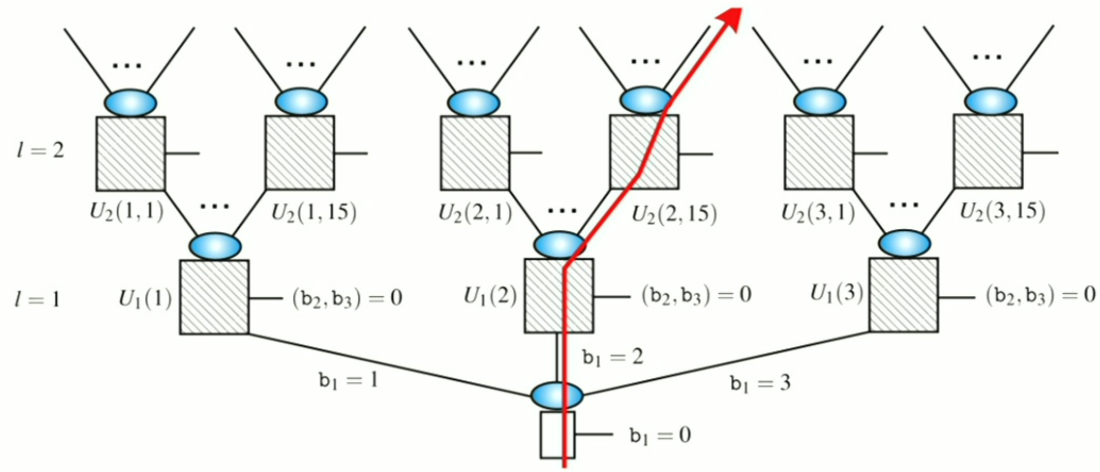
$t = 2$: V_0 and V_1 accept if they receive x, and they receive it on time

Localization of measurements: blind ping-pong teleportation

Rationale: "move" the full state to one party and measure it $\rightarrow$ teleportation.

Problem: teleportation induces Pauli distortions on the states.





Figures from Clark *et al.*, NJP 2010

Known localization protocols need infinite entanglement either always (Vaidman, PRL 2003) or in the worst case (Clark *et al.*, NJP 2010)

arXiv:2408.00831

Classification of joint quantum measurements based on entanglement cost of localization

Jef Pauwels, Alejandro Pozas-Kerstjens, Flavio Del Santo, and Nicolas Gisin
Group of Applied Physics, University of Geneva, 1211 Geneva, Switzerland and
Constructor University, 1211 Geneva, Switzerland

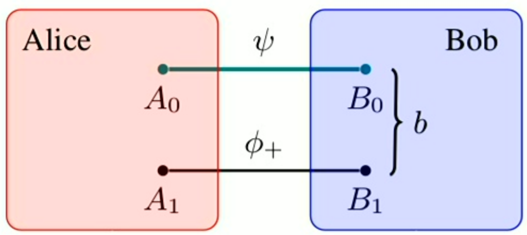
Despite their importance in quantum theory, joint quantum measurements remain poorly understood. An intriguing conceptual and practical question is whether joint quantum measurements on separated systems can be performed without bringing them together. Remarkably, by using shared entanglement, this can be achieved perfectly when disregarding the post-measurement state. However, existing localization protocols typically require unbounded entanglement. In this work, we address the fundamental question: "Which joint measurements can be localized with a finite amount of entanglement?" We develop finite-resource versions of teleportation-based schemes and analytically classify all two-qubit measurements that can be localized in the first steps of these hierarchies. These include several measurements with exceptional properties and symmetries, such as the Bell state measurement and the elegant joint measurement. This leads us to propose a systematic classification of joint measurements based on entanglement cost, which we argue directly connects to the complexity of implementing those measurements. We illustrate how to numerically explore higher levels and construct generalizations to higher dimensions and multipartite settings.

Which measurements can we localize with a **fixed** amount of entanglement?
(reproduce their statistics on any state)

Simplest case: 2-qubit state, 1 shared ebit

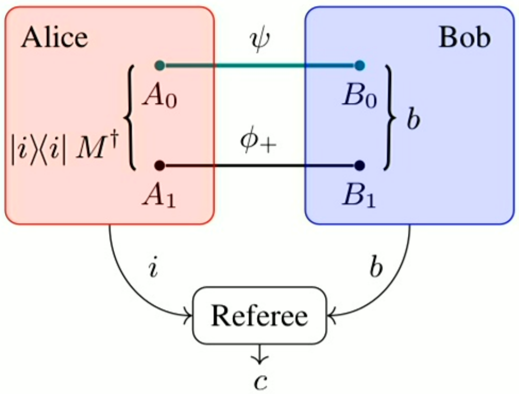
Step 1: Bob teleports to Alice using the ebit

$$|\psi\rangle \to \mathbb{1} \otimes \sigma_b |\psi\rangle$$



J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831

Simplest case: 2-qubit state, 1 shared ebit



Step 1: Bob teleports to Alice using the ebit

$$|\psi\rangle \rightarrow \mathbb{1} \otimes \sigma_b \, |\psi\rangle$$

Step 2: Alice applies $M^\dagger$ to rotate the measurement basis to the computational basis

$$M = (|v_1\rangle, |v_2\rangle, |v_3\rangle, |v_4\rangle), \quad O = \sum_i \lambda_i |v_i\rangle \langle v_i|$$

Step 3: Alice measures in the computational basis

$$\left|\langle i, j|\, M^\dagger \cdot (\mathbb{1} \otimes \sigma_b)\, |\psi\rangle\right|^2 = \left| \quad \langle \quad i, j \;| M^\dagger \, |\psi\rangle \right|^2$$

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831

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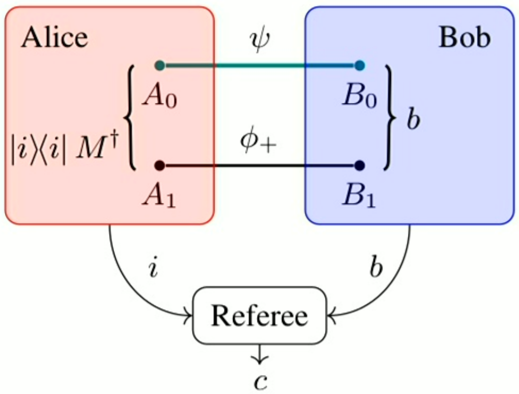

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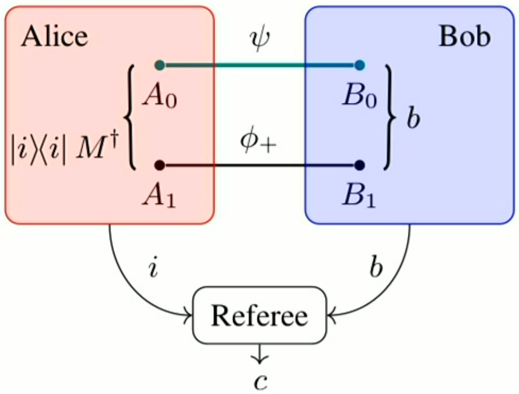
$$\left|\langle i,j|\, M^\dagger \cdot (\mathbb{1} \otimes \sigma_b)\, |\psi\rangle\right|^2 = \left|e^{\mathrm{i}\phi_b(i,j)}\,\langle \pi_b(i,j)|\, M^\dagger\, |\psi\rangle\right|^2$$

$$M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) \cdot M = P_b \cdot \Phi_b$$

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831

Simplest case: 2-qubit state, 1 shared ebit



$$M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) \cdot M = P_b \cdot \Phi_b$$

The solutions are intertwiners between (red.) representations of $SU(2)$

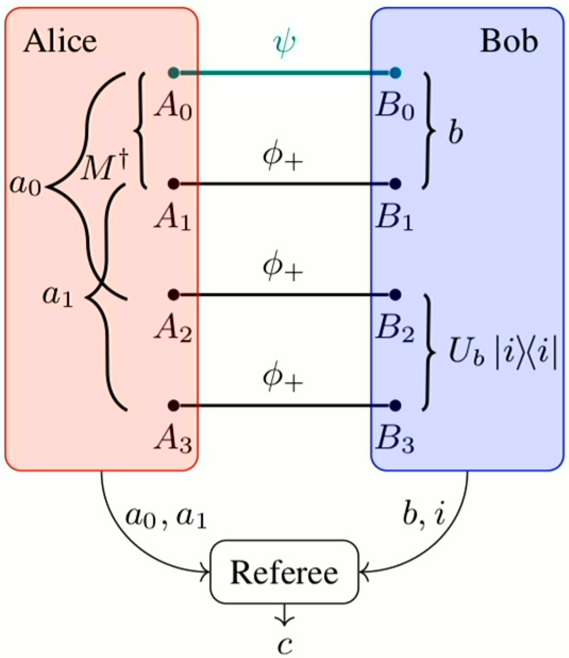
$$\{\mathbb{1} \otimes \sigma_b\}_b,\ \{P_b \cdot \Phi_b\}_b$$

Only two nontrivial solutions:

1. Bell state measurement (σ_b relabels the outcomes)
2. $\pi/2$-twisted (BB84) basis: $\{|00\rangle, |01\rangle, |1+\rangle, |1-\rangle\}$

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831

A bit less simple: 2-qubit state, 3 shared ebits



Bob teleports, Alice rotates *and teleports back*, Bob amends knowing his previous outcome

$$M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) \cdot M \cdot (\sigma_{a_1} \otimes \sigma_{a_2}) \cdot M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) \cdot M = P_{a_1,a_2,b} \cdot \Phi_{a_1,a_2,b}$$

If we define $\mathcal{M}_b = M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) \cdot M$:

$$\mathcal{M}_b^\dagger \cdot (\sigma_{a_1} \otimes \sigma_{a_2}) \cdot \mathcal{M}_b = P_{a_1,a_2,b} \cdot \Phi_{a_1,a_2,b}$$

Five new solutions:

1. (Linear-optical) partial BSM: $\{|00\rangle, |11\rangle, |\psi^+\rangle, |\psi^-\rangle\}$
2. Elegant Joint Measurement (Gisin, 2019)
3. $\pi/2$-twisted BSM: $\{|0+\rangle \pm |11\rangle, |0-\rangle \pm |10\rangle\}$
4. Two more iso-entangled measurements

$$\{|\psi^-\rangle \pm |00\rangle, |\psi^+\rangle \pm |11\rangle\}$$
$$\{|1-\rangle \pm |01\rangle, |1+\rangle \pm \mathrm{i}\,|00\rangle\}$$

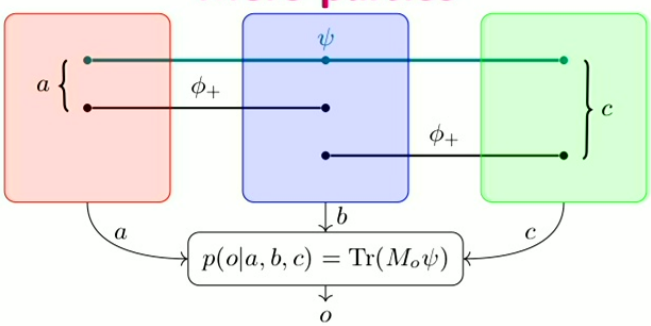
(which cannot be localized with fewer ebits)

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831

Generalizations

$$\mathcal{V}_n^{d,m} \equiv \{M \in \mathcal{U}(d^m) \,|\, M^\dagger \cdot (\mathbb{1} \otimes \mathcal{P}_{m-1}) \cdot M \in \bar{\mathcal{V}}_{n-1}^{d,m}\}$$
$$\bar{\mathcal{V}}_n^{d,m} \equiv \{M \in \mathcal{U}(d^m) \,|\, M^\dagger \cdot \mathcal{P}_m \cdot M \in \bar{\mathcal{V}}_{n-1}^{d,m}\}$$

More parties



1st level: (at least) 8 solutions, nothing surprising
2nd level: (at least) 64 solutions, 2 generalizations of EJM

Higher levels

2 qubits, 9 ebits: (at least) 27 new bases.
In all cases, all entangled states in the basis are iso-entangled.

Higher dimensions

Recall 2-qubit, 2nd level equation:
$\mathcal{M}_b^\dagger \cdot (\sigma_{a_1} \otimes \sigma_{a_2}) \cdot \mathcal{M}_b = P_{a_1,a_2,b} \cdot \Phi_{a_1,a_2,b}$

Approach: write $\{P_{a_1,a_2,b} \cdot \Phi_{a_1,a_2,b}\}_{a_1,a_2,b}$ in dimension-free form

Goal: generalization of EJM to dimension d

Our analytical methods explode combinatorially for all cases

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831

Conclusions

- Joint quantum measurements deserve attention. We have only scratched the surface.
- If we only care about measurement outcomes, it is possible to perform any joint measurement locally, if given enough entanglement.
- Entanglement cost of localization is a sound and physically motivated measure of measurement complexity.

- Improve methods for analytical/numerical characterization.
- Applications: network nonlocality, cryptography...
- Foundations: is relativistic causality sufficient to describe joint quantm measurements? Do we need new update rules for the post-measurement states?



Thanks for your attention

Questions? Comments?

2408.00831

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apozas/localizable-measurements

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