Title: Tutorial - Indefinite quantum causality

Speakers: Cyril Branciard

Series: Quantum Foundations, Quantum Information

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Abstract: Recent advances in quantum foundations have unveiled the idea that the causal order between quantum events may not always be fixed or even well-defined, allowing for some form of *indefinite quantum causality*. This tutorial will introduce the key concepts and motivations behind this rapidly developing area of research. Focusing on one of the main frameworks developed to explore indefinite quantum causalityâ€"the process matrix formalismâ€"I will present key theoretical results, highlight the potential of indefinite causal orders as a resource for quantum information processing, and discuss experimental implementations as well as the physical interpretation of indefinite causal structures.

Tutorial:

Indefinite Quantum Causality

Cyril Branciard

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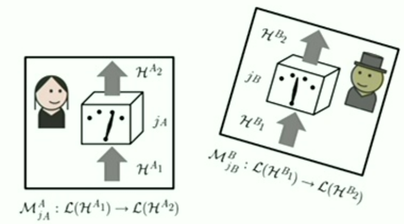
Motivation

- **Foundational:**
  - “Quantum theory is a **probabilistic** theory with **fixed** causal structure. General relativity is a **deterministic** theory but where the causal structure is **dynamic**.”

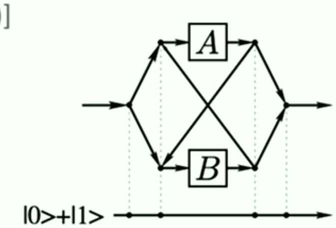
    [Hardy, arXiv:gr-qc/0509120]

  - Could quantum indefiniteness be extended to causal structures? Could we have “superpositions” of causal structures?
    - The “process matrix” framework

      [Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]



- **Practical:**
  - How can quantum operations be transformed into other operations?
    - “Quantum supermaps” [Chiribella *et al.*, EPL **83**, 30004 (2008)]
  - What if we apply quantum operations in some indefinite causal order? What can we do?
    - “Beyond causally ordered quantum computers”: e.g., the “Quantum Switch”

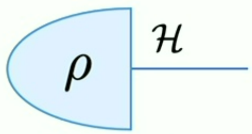


[Chiribella *et al.*, arXiv:0912.0195; Phys. Rev. A **88**, 022318 (2013)]

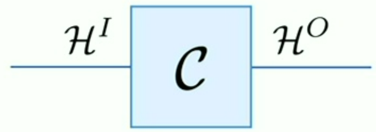
Outline

- The process matrix framework / Quantum supermaps
- Indefinite quantum causality ≡ "causal nonseparability"
- Examples – incl. the "Quantum switch"
- Certifying causal nonseparability
- Applications
- Experiments
- Indefinite causality, really?
- Related frameworks

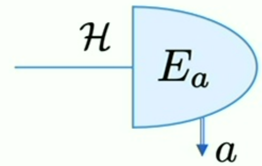
Quantum operations



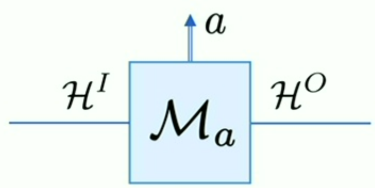
States



Channels (CPTP maps)



Measurements

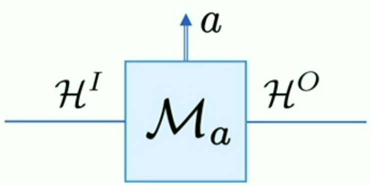


"Quantum instruments"

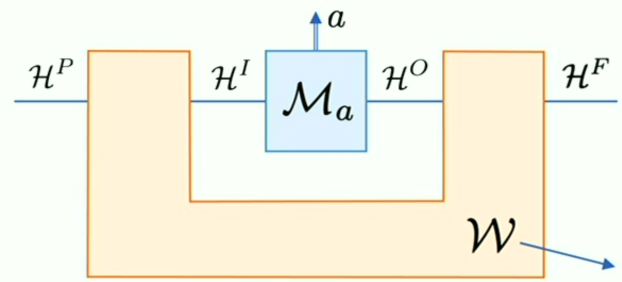
(sets of CP maps that sum up to a CPTP map)

Transforming quantum operations

"Quantum instruments"



$$\mathcal{M}_a : \mathcal{L}(\mathcal{H}^I) \to \mathcal{L}(\mathcal{H}^O)$$

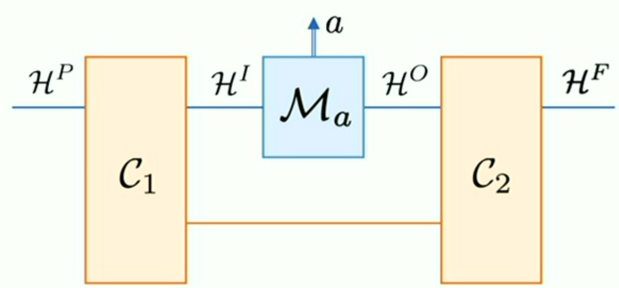


$$\text{“}\mathcal{W} \circ \mathcal{M}_a\text{”} : \mathcal{L}(\mathcal{H}^P) \to \mathcal{L}(\mathcal{H}^F)$$

a "quantum supermap"
(linear, completely
CPTP-preserving)

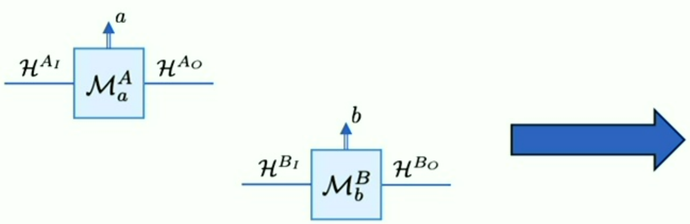
[Chiribella *et al.*, EPL **83**, 30004 (2008),
Phys. Rev. Lett. **101**, 060401 (2008),
Phys. Rev. A **80**, 022339 (2009)]

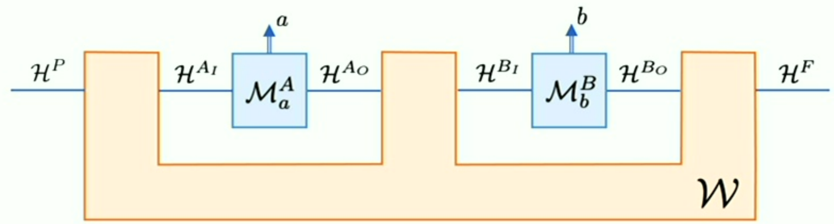
$$\equiv$$



Transforming quantum operations

"Quantum instruments"

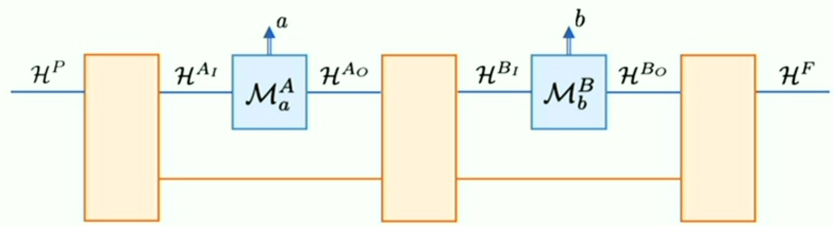




"$\mathcal{W} \circ (\mathcal{M}_a^A \otimes \mathcal{M}_b^B)$" : $\mathcal{L}(\mathcal{H}_P) \to \mathcal{L}(\mathcal{H}_F)$

If fixed causal order
– no backwards in time influence –

$\equiv$ [Chiribella *et al.*, EPL **83**, 30004 (2008), Phys. Rev. Lett. **101**, 060401 (2008), Phys. Rev. A **80**, 022339 (2009)]



"Channels w/ memory" [Kretschmann & Werner, Phys. Rev. A **72**, 062323 (2005)],
"Quantum strategy" [Gutoski & Watrous, Proc. 39th ACM STOC (2008)], or "Quantum comb" [Chiribella *et al.*]

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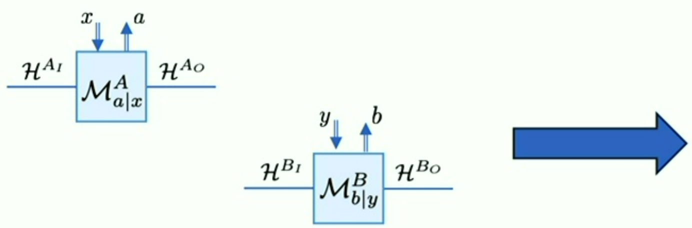
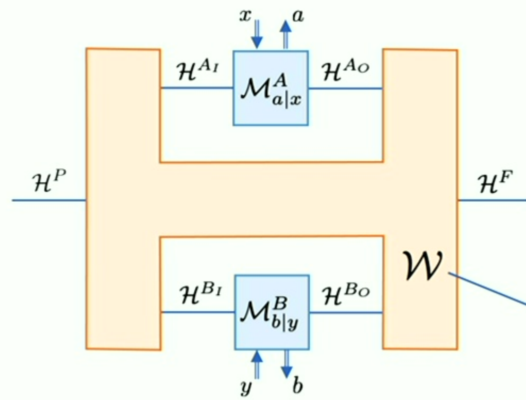
Transforming quantum operations

"Quantum instruments"





$$\mathcal{G}_{ab|xy} = \text{“}\mathcal{W} \circ (\mathcal{M}^A_{a|x} \otimes \mathcal{M}^B_{b|y})\text{”} : \mathcal{L}(\mathcal{H}^P) \to \mathcal{L}(\mathcal{H}^F)$$

General "Quantum supermap"

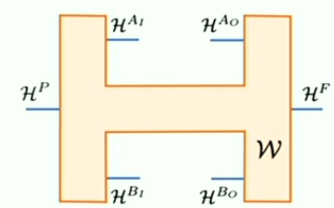
[Chiribella *et al.*, EPL **83**, 30004 (2008), Phys. Rev. Lett. **101**, 060401 (2008), Phys. Rev. A **80**, 022339 (2009)]

Choi representation:

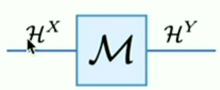
$$G_{ab|xy} = \mathrm{Tr}_{AB}\left[(M^A_{a|x} \otimes M^B_{b|y} \otimes \mathbb{1}^{PF})^T\, W\right]$$

"Process matrix"

[Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]



Choi isomorphism:



$\mathcal{M} : \mathcal{L}(\mathcal{H}^X) \to \mathcal{L}(\mathcal{H}^Y)$ ⟺ $M = \sum_{i,j} |i\rangle\langle j| \otimes \mathcal{M}(|i\rangle\langle j|) \in \mathcal{L}(\mathcal{H}^X) \otimes \mathcal{L}(\mathcal{H}^Y)$

$$W \geq 0,\ \ W \in \mathcal{L}_V(\mathcal{H}^{PA_IA_OB_IB_OF})$$

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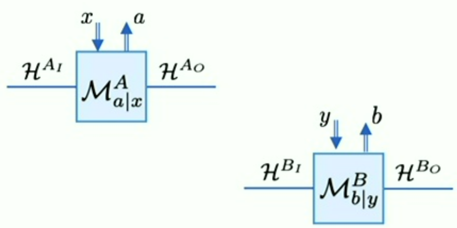
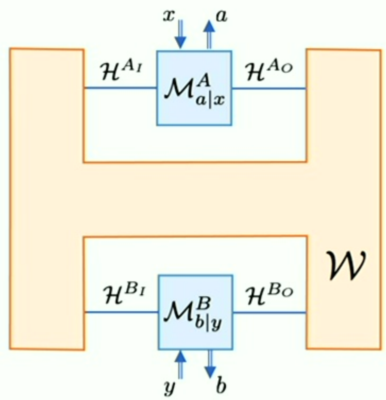
Transforming quantum operations

“Quantum instruments”





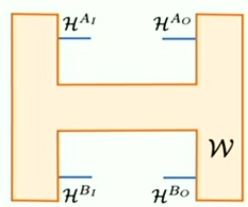
with no open $\mathcal{H}^P, \mathcal{H}^F$:

$$p(a,b|x,y) = \mathrm{Tr}\big[(M^A_{a|x} \otimes M^B_{b|y})^T\, W\big]$$

“Generalized Born rule”

“Process matrix”

[Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]



$$W \geq 0,\ \ W \in \mathcal{L}_V(\mathcal{H}^{A_I A_O B_I B_O})$$

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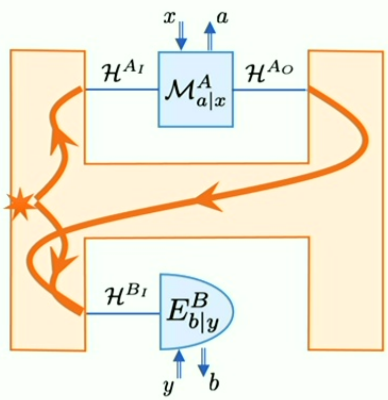
Process matrices – first examples



"Generalized Born rule"

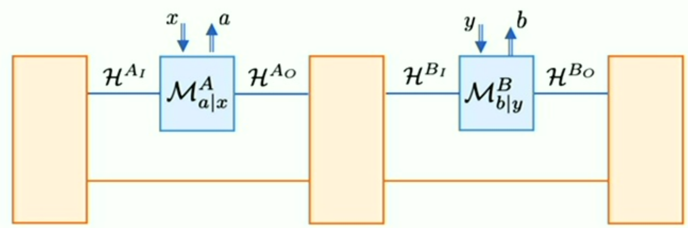
$$p(a,b|x,y) = \mathrm{Tr}\big[(M^A_{a|x} \otimes M^B_{b|y})^T\, W\big]$$

✓ Quantum state: $p(a,b|x,y) = \mathrm{Tr}[(E^A_{a|x} \otimes E^B_{b|y})\,\rho]$

✓ Quantum channel: $p(b|x,y) = \mathrm{Tr}[E^B_{b|y}\, \mathcal{C}_{A\to B}(\rho_x)]$

$$= \mathrm{Tr}[(\rho_x^T \otimes E^B_{b|y})\, C^{\mathrm{Choi}}_{A\to B}]$$

✓ Quantum channel with memory / quantum comb

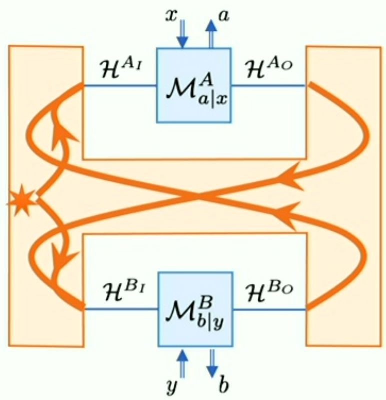


Causal order:

$A \prec B$

(no signaling *B*↛*A*)

Process matrices – first examples



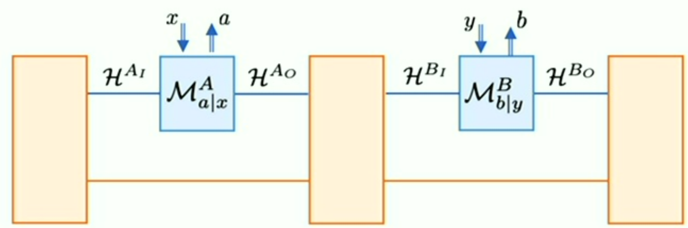
"Generalized Born rule"

$$p(a,b|x,y) = \mathrm{Tr}\big[(M^A_{a|x} \otimes M^B_{b|y})^T\, W\big]$$

✓ Quantum state: $p(a,b|x,y) = \mathrm{Tr}[(E^A_{a|x} \otimes E^B_{b|y})\,\rho]$

✓ Quantum channel: $p(b|x,y) = \mathrm{Tr}[E^B_{b|y}\,\mathcal{C}_{A\to B}(\rho_x)]$
$= \mathrm{Tr}[(\rho_x^T \otimes E^B_{b|y})\,C^{\mathrm{Choi}}_{A\to B}]$

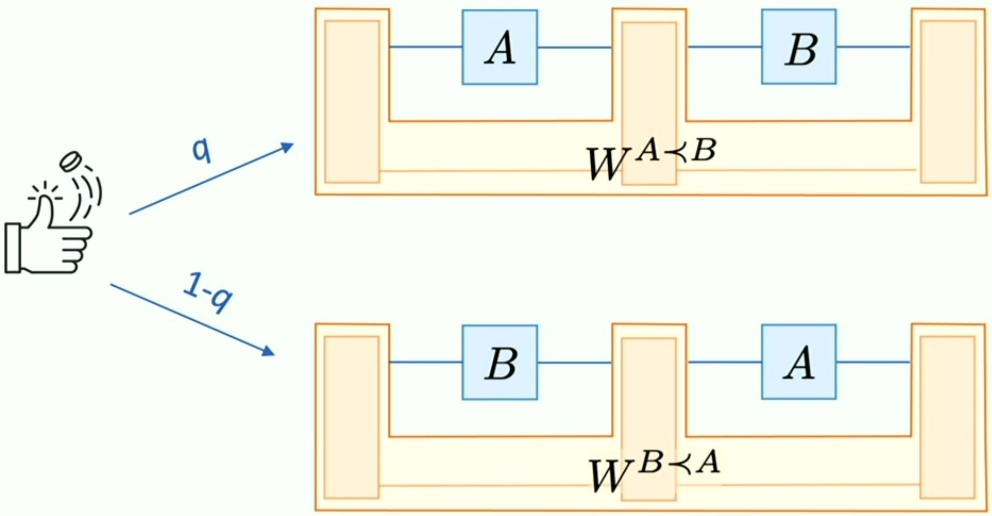
✓ Quantum channel with memory / quantum comb



Causal order:
$A \prec B$
(no signaling *B*↛*A*)

✓ More general process matrices? No causal order?

Causal (non)separability



$W^{A\prec B}$: compatible with *A* causally preceding *B*

$$W^{\text{c-sep}} = q\,W^{A\prec B} + (1-q)\,W^{B\prec A}$$

is "causally separable"

(it is compatible a well-defined causal order, be it probabilistic)

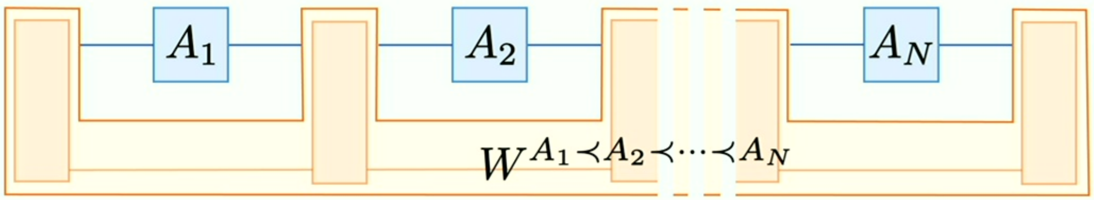
[Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]

$$W^{\text{c-nonsep}} \neq q\,W^{A\prec B} + (1-q)\,W^{B\prec A}$$ is **"causally nonseparable"**

(for any $q, W^{A\prec B}, W^{B\prec A}$)

➢ It has no well-defined causal order

Multipartite causal (non)separability



$W^{A_1 \prec A_2 \prec \cdots \prec A_N}$:

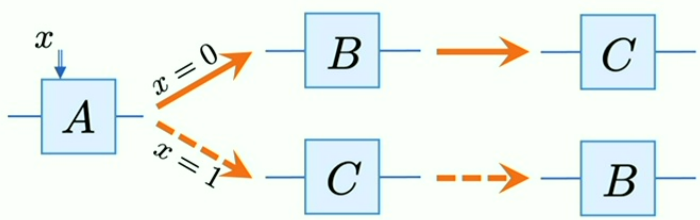
Compatible with a *fixed* causal order

(no signaling from "future operations" to "past operations")

Naïve multipartite generalisation: $$W^{\text{c-sep}} \stackrel{?}{=} \sum_{\substack{(k_1,k_2,\ldots,k_N): \\ \text{permutations} \\ \text{of } \{1,2,\ldots,N\}}} q_{(k_1,k_2,\ldots,k_N)} \, W^{A_{k_1} \prec A_{k_2} \prec \cdots \prec A_{k_N}}$$

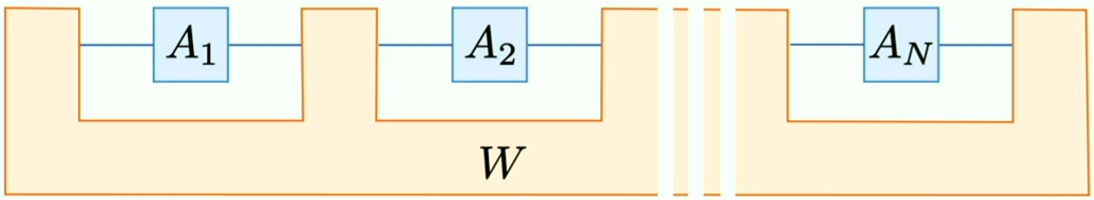
➢ Not enough: we want to allow for *dynamical* – still well-defined – causal orders

e.g.



[on dynamical orders: see Raphaël Mothe's talk this afternoon]

Multipartite causal (non)separability

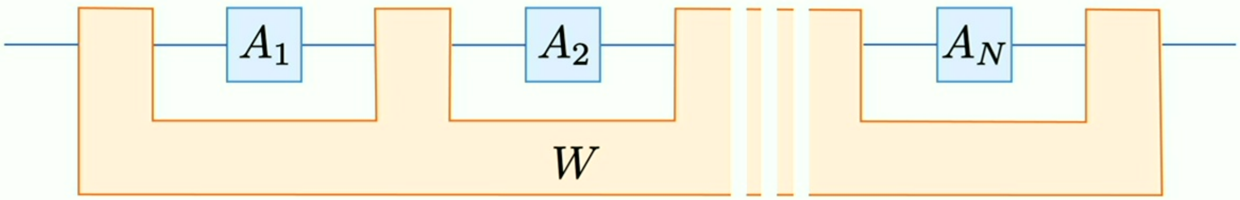


- Recursive definition: $W^{\text{c-sep}} = \sum_{k_1} q_{k_1} W^{(k_1)}$

with $W^{(k_1)}$
- compatible with A_{k_1} being first
- such that the "conditional process"
$W^{(k_1)}_{|A_{k_1}} := \mathrm{Tr}_{k_1}[(A_{k_1} \otimes \mathbb{1}^{\text{rest}})\, W^{(k_1)}]$ is itself causally separable $(\forall A_{k_1})$

[Oreshkov & Giarmatzi, New J. Phys. **18**, 093020 (2016)]

Multipartite causal (non)separability



- Recursive definition: $W^{\text{c-sep}} = \sum_{k_1} q_{k_1} W^{(k_1)}$

  with $W^{(k_1)}$
  - compatible with A_{k_1} being first
  - such that the "conditional process"

    $W^{(k_1)}_{|A_{k_1}} := \mathrm{Tr}_{k_1}[(A_{k_1} \otimes \mathbb{1}^{\text{rest}})\, W^{(k_1)}]$ is itself causally separable ($\forall A_{k_1}$)

[Oreshkov & Giarmatzi, New J. Phys. **18**, 093020 (2016)]

- Otherwise, W is **causally nonseparable**

(also applies to process matrices with nontrivial $\mathcal{H}^P, \mathcal{H}^F$)

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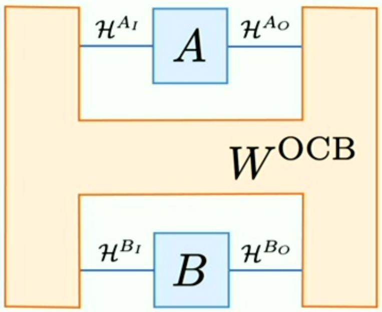
OCB's process matrix

[Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]



$$W^{\text{OCB}} = \frac{1}{4}\left[\mathbb{1}^{\otimes 4} + \frac{\mathbb{1}^{A_I} Z^{A_O} Z^{B_I} \mathbb{1}^{B_O} + Z^{A_I} \mathbb{1}^{A_O} X^{B_I} Z^{B_O}}{\sqrt{2}}\right]$$

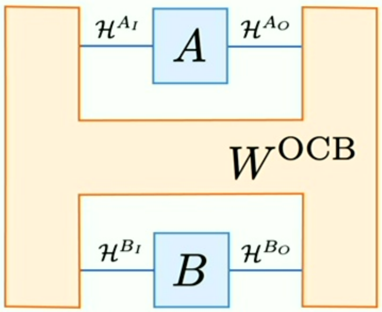
$W^{A \prec B} = \frac{1}{4}\left[\mathbb{1}^{\otimes 4} + \mathbb{1}^{A_I} Z^{A_O} Z^{B_I} \mathbb{1}^{B_O}\right]$ is a channel from *A* to *B*

$W^{B \prec A} = \frac{1}{4}\left[\mathbb{1}^{\otimes 4} + Z^{A_I} \mathbb{1}^{A_O} X^{B_I} Z^{B_O}\right]$ is a channel from *B* to *A*

$W = \frac{1}{2} W^{A \prec B} + \frac{1}{2} W^{B \prec A} = \frac{1}{4}\left[\mathbb{1}^{\otimes 4} + \frac{\mathbb{1}^{A_I} Z^{A_O} Z^{B_I} \mathbb{1}^{B_O} + Z^{A_I} \mathbb{1}^{A_O} X^{B_I} Z^{B_O}}{2}\right]$ would be causally separable

OCB's process matrix

[Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]



$$W^{\mathrm{OCB}} = \frac{1}{4}\left[\mathbb{1}^{\otimes 4} + \frac{\mathbb{1}^{A_I} Z^{A_O} Z^{B_I} \mathbb{1}^{B_O} + Z^{A_I} \mathbb{1}^{A_O} X^{B_I} Z^{B_O}}{\sqrt{2}}\right]$$

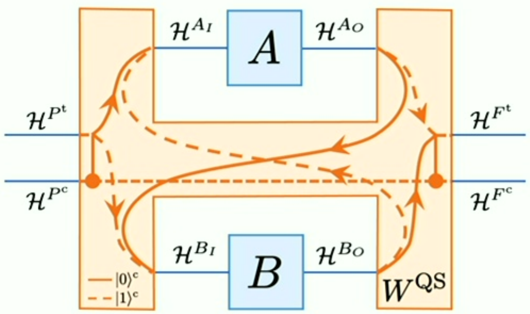
is causally **non**separable

$W^{A \prec B} = \frac{1}{4}\left[\mathbb{1}^{\otimes 4} + \mathbb{1}^{A_I} Z^{A_O} Z^{B_I} \mathbb{1}^{B_O}\right]$ is a channel from *A* to *B*

$W^{B \prec A} = \frac{1}{4}\left[\mathbb{1}^{\otimes 4} + Z^{A_I} \mathbb{1}^{A_O} X^{B_I} Z^{B_O}\right]$ is a channel from *B* to *A*

$W = \frac{1}{2} W^{A \prec B} + \frac{1}{2} W^{B \prec A} = \frac{1}{4}\left[\mathbb{1}^{\otimes 4} + \frac{\mathbb{1}^{A_I} Z^{A_O} Z^{B_I} \mathbb{1}^{B_O} + Z^{A_I} \mathbb{1}^{A_O} X^{B_I} Z^{B_O}}{2}\right]$ would be causally separable

The “Quantum Switch”



(for unitaries:)

$$|\psi\rangle^{t} \otimes |0\rangle^{c} \mapsto BA|\psi\rangle^{t} \otimes |0\rangle^{c}$$

$$|\psi\rangle^{t} \otimes |1\rangle^{c} \mapsto AB|\psi\rangle^{t} \otimes |1\rangle^{c}$$

$$|\psi\rangle^{t} \otimes |+\rangle^{c} \mapsto \tfrac{1}{\sqrt{2}}\big(BA|\psi\rangle^{t} \otimes |0\rangle^{c} + AB|\psi\rangle^{t} \otimes |1\rangle^{c}\big)$$

As a supermap:

$$(A, B) \mapsto BA \otimes |0\rangle\langle 0|^{c} + AB \otimes |1\rangle\langle 1|^{c}$$

$$(\mathcal{A}, \mathcal{B}) \mapsto \mathcal{G}$$

Kraus operators $\{A_i\}_i$ $\{B_j\}_j$ $\{G_{ij} = B_j A_i \otimes |0\rangle\langle 0|^{c} + A_i B_j \otimes |1\rangle\langle 1|^{c}\}_{ij}$

As a process matrix:

[Oreshkov & Giarmatzi, New J. Phys. **18**, 093020 (2016); Araújo *et al.*, New J. Phys. **17**, 102001 (2015)]

$$W^{QS} = |w^{QS}\rangle\langle w^{QS}|$$

$$|w^{QS}\rangle = |0\rangle^{P^c}|\mathbb{1}\rangle\rangle^{P^t A_I}|\mathbb{1}\rangle\rangle^{A_O B_I}|\mathbb{1}\rangle\rangle^{B_O F^t}|0\rangle^{F^c} + |1\rangle^{P^c}|\mathbb{1}\rangle\rangle^{P^t B_I}|\mathbb{1}\rangle\rangle^{B_O A_I}|\mathbb{1}\rangle\rangle^{A_O F^t}|1\rangle^{F^c}$$

$$|\mathbb{1}\rangle\rangle^{XY} = \sum_i |i\rangle^X |i\rangle^Y$$ (identity channel)

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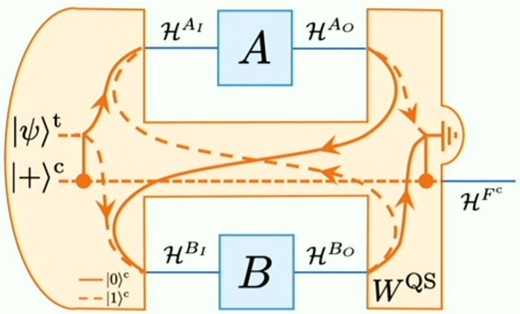
The "Quantum Switch" [Chiribella *et al.*, arXiv:0912.0195; Phys. Rev. A **88**, 022318 (2013)]



As a process matrix:

$$W^{\mathrm{QS}} = \mathrm{Tr}_{F^{\mathrm{t}}} |w^{\mathrm{QS}}\rangle\!\langle w^{\mathrm{QS}}|$$

$$|w^{\mathrm{QS}}\rangle = \frac{1}{\sqrt{2}} |\psi\rangle^{A_I} |\mathbb{1}\rangle\!\rangle^{A_O B_I} |\mathbb{1}\rangle\!\rangle^{B_O F^{\mathrm{t}}} |0\rangle^{F^{\mathrm{c}}} + \frac{1}{\sqrt{2}} |\psi\rangle^{B_I} |\mathbb{1}\rangle\!\rangle^{B_O A_I} |\mathbb{1}\rangle\!\rangle^{A_O F^{\mathrm{t}}} |1\rangle^{F^{\mathrm{c}}}$$

[Oreshkov & Giarmatzi, New J. Phys. **18**, 093020 (2016);
Araújo *et al.*, New J. Phys. **17**, 102001 (2015)]

$$W^{\mathrm{QS}} \neq q\, W^{A \prec B} + (1-q)\, W^{B \prec A}$$

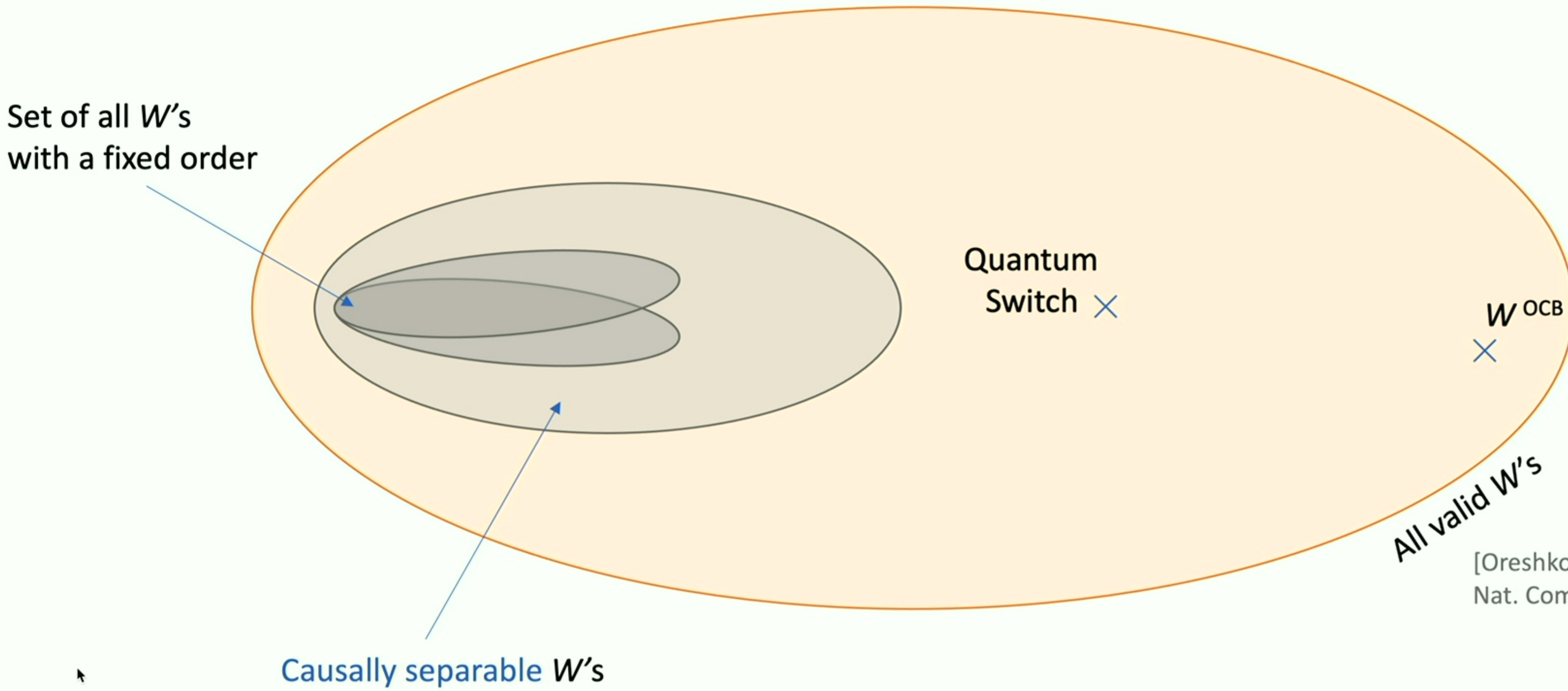
➢ Causally nonseparable

➢ QS can be generalized to the *"N-Quantum Switch"* [Araújo *et al.*, Phys. Rev. Lett. **113**, 250402 (2014)]

or to even more general *Q. circuits with Q. control of the causal order*

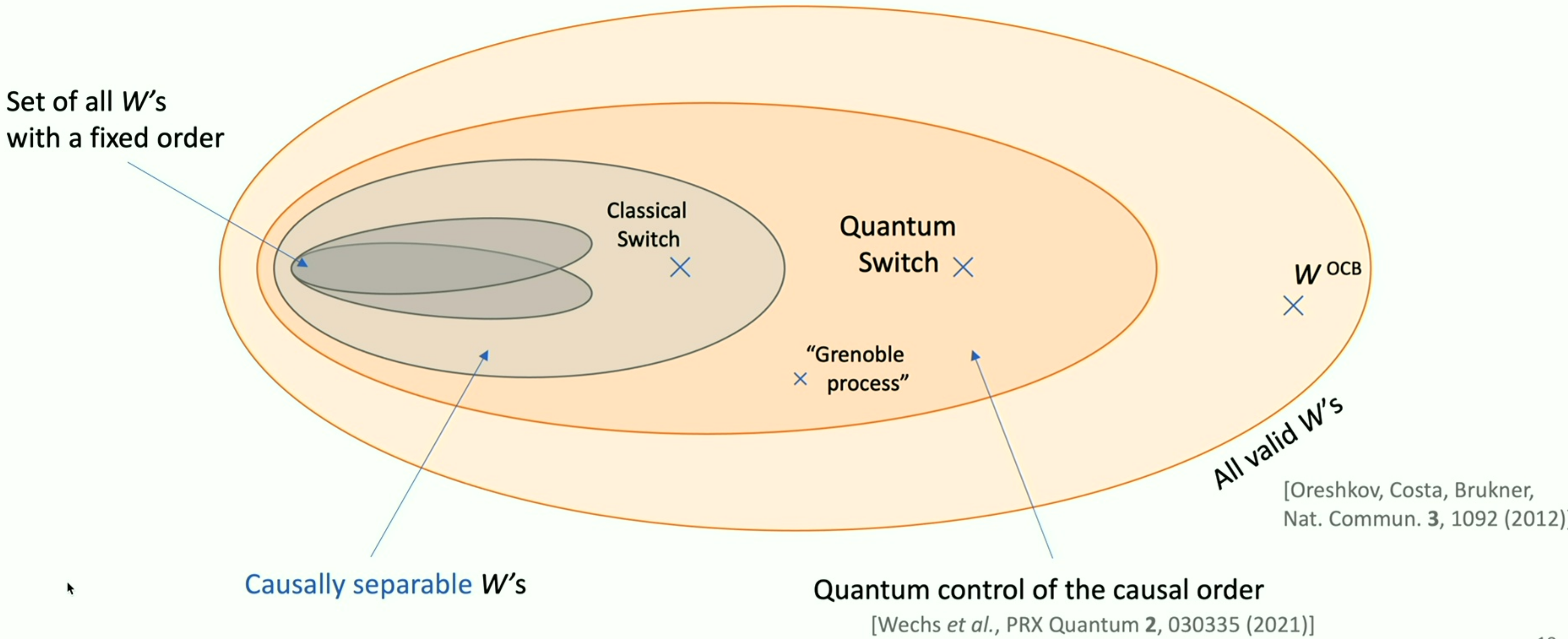
[Wechs *et al.*, PRX Quantum **2**, 030335 (2021)]

Landscape of supermaps / process matrices



[Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]

Landscape of supermaps / process matrices



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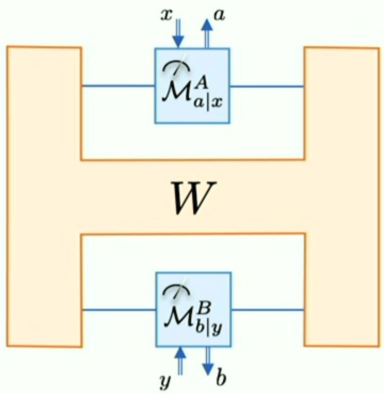
Outline

- The process matrix framework / Quantum supermaps
- Indefinite quantum causality ≡ “causal nonseparability”
- Examples – incl. the “Quantum switch”
- **Certifying causal nonseparability**
- Applications
- Experiments
- Indefinite causality, really?
- Related frameworks

Certifying causal nonseparability: Device-Dep^ly



"Generalized Born rule"

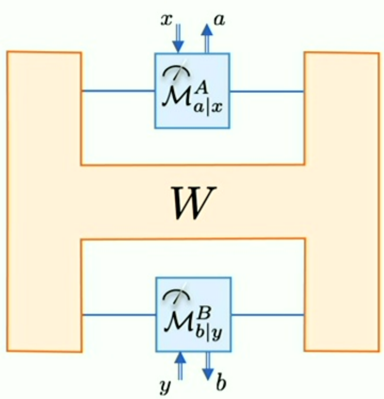
$$p(a,b|x,y) = \mathrm{Tr}\big[(M^A_{a|x} \otimes M^B_{b|y})^T\, W\big]$$

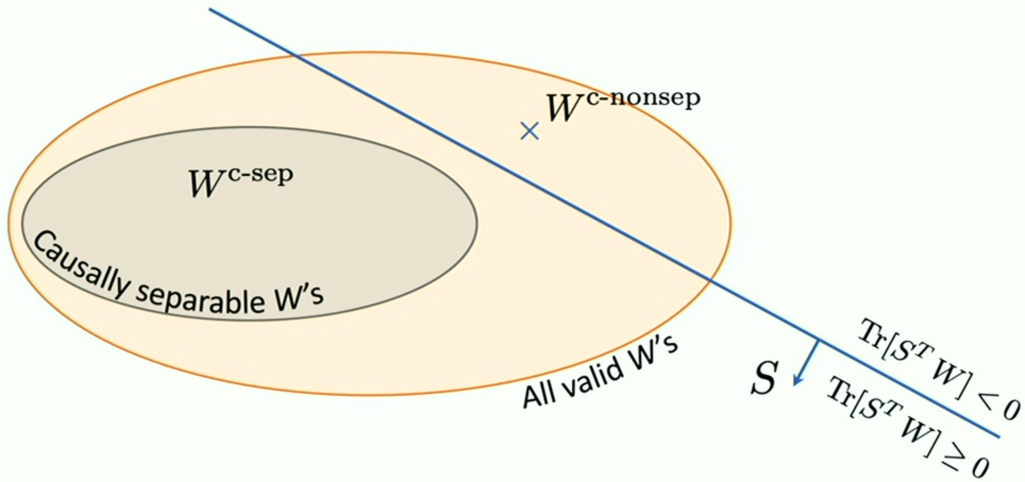
- Can perform process tomography *[Device-Dependent]*
  - Reconstruct W, check for decomposition

$$W \stackrel{?}{=} q\, W^{A\prec B} + (1-q)\, W^{B\prec A}$$

✓ that's a SemiDefinite Programming (SDP) problem ☺

Certifying causal nonseparability: Device-Dep[ly]





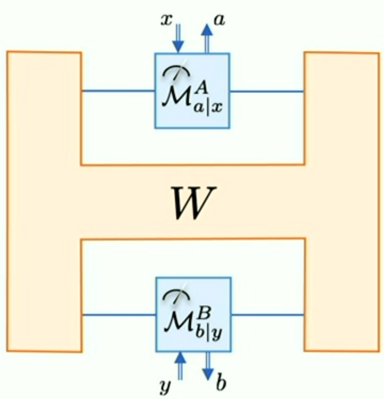
"Causal witness":

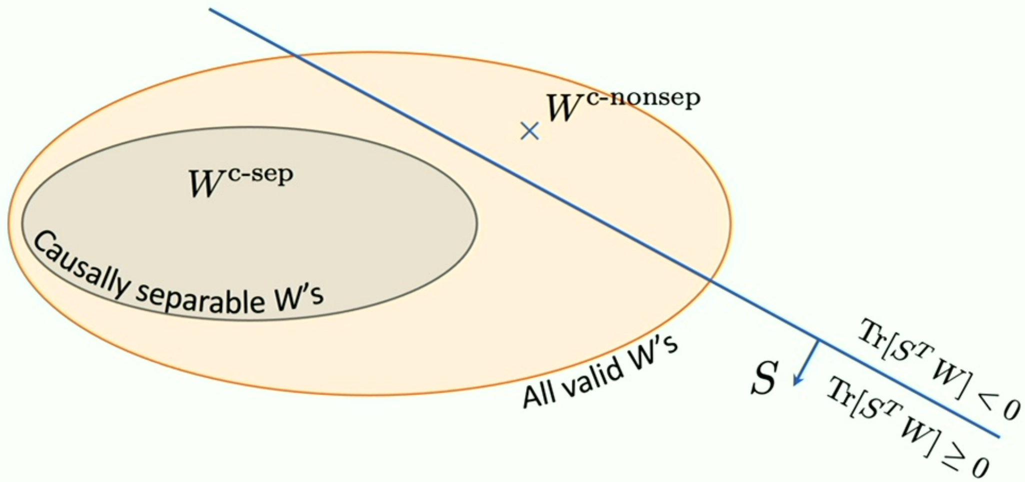
$\forall\, W^{\text{c-nonsep}},\ \exists\, S,$

- $\mathrm{Tr}[S^T\, W^{\text{c-nonsep}}] < 0$
- $\mathrm{Tr}[S^T\, W^{\text{c-sep}}] \geq 0 \quad \forall\, W^{\text{c-sep}}$

[Araújo *et al.*, New J. Phys. **17**, 102001 (2015); Branciard, Sci. Rep. **6**, 26018 (2016)]

Certifying causal nonseparability: Device-Dep^{ly}





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- $\mathrm{Tr}[S^T\, W^{\text{c-nonsep}}] < 0$
- $\mathrm{Tr}[S^T\, W^{\text{c-sep}}] \geq 0 \quad \forall\, W^{\text{c-sep}}$

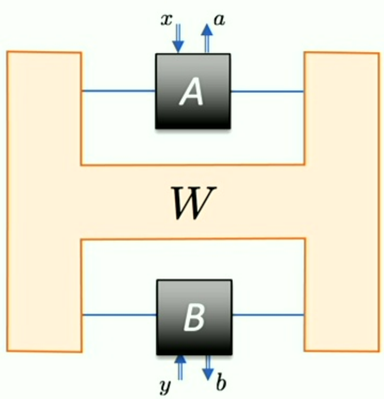
- Constructing a causal witness is a SDP
- Measuring a causal witness: $p(a,b|x,y) = \mathrm{Tr}\big[(M^A_{a|x} \otimes M^B_{b|y})^T\, W\big]$

$$S = \textstyle\sum_{abxy} \gamma_{abxy}\, M^A_{a|x} \otimes M^B_{b|y}$$

$$\implies \mathrm{Tr}[S^T\, W] = \textstyle\sum_{abxy} \gamma_{abxy}\, p(a,b|x,y)$$

[Araújo *et al.*, New J. Phys. **17**, 102001 (2015); Branciard, Sci. Rep. **6**, 26018 (2016)]

Certifying causal nonseparability: Device-Indep^{ly}



$$p(a,b|x,y) = \mathrm{Tr}\big[(M^A_{a|x} \otimes M^B_{b|y})^T\, W\big]$$

$$W^{\text{c-sep}} = q\, W^{A\prec B} + (1-q)\, W^{B\prec A}$$

$$\Longrightarrow\ p(a,b|x,y) = q\, p^{A\prec B}(a,b|x,y) + (1-q)\, p^{B\prec A}(a,b|x,y)$$

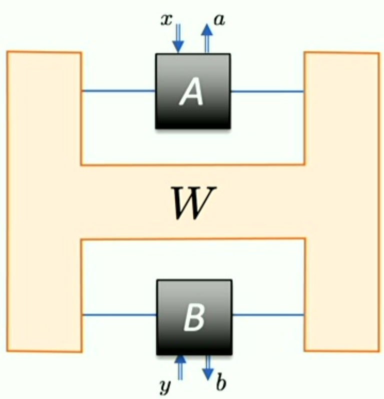
"Causal correlations" satisfy "Causal inequalities"

[Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]

Violation of causal inequality → noncausal correlations → causally nonseparable *W*

[Device-Independent]

Certifying causal nonseparability: Device-Indep^{ly}



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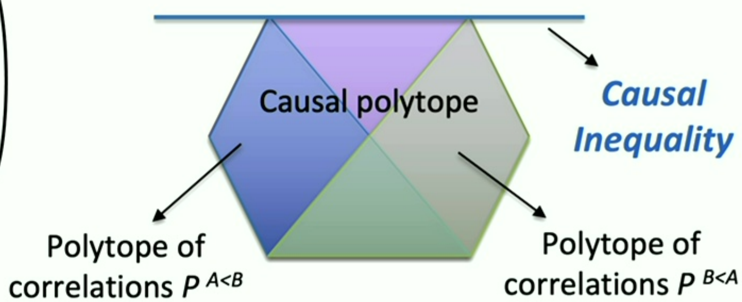
"Causal correlations" satisfy "Causal inequalities"

[Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]

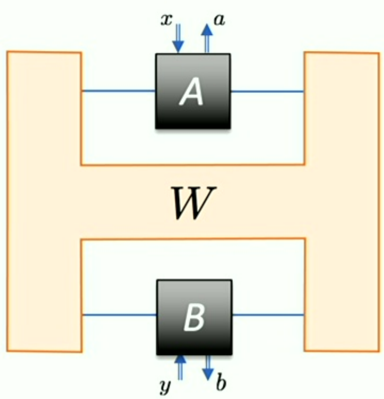
Causal correlations form a convex polytope

$$\vec{P} = \begin{pmatrix} P(0,0|0,0) \\ \vdots \\ P(a,b|x,y) \\ \vdots \\ P(1,1|1,1) \end{pmatrix}$$



[Branciard *et al.*, New J. Phys. **18**, 013008 (2016)]

Certifying causal nonseparability: Device-Indep^{ly}



$$W^{\text{c-sep}} = q\,W^{A\prec B} + (1-q)\,W^{B\prec A}$$

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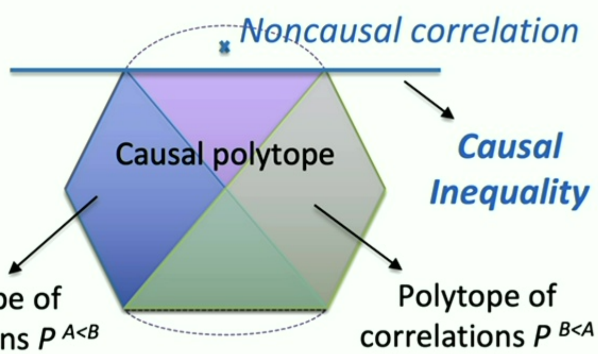

E.g. "Guess Your Neighbor's Input":

$$x, y, a, b = 0, 1$$

$$p(a=y, b=x) \overset{\text{causal}}{\leq} \frac{3}{4}$$

[Branciard *et al.*, New J. Phys. **18**, 013008 (2016)]

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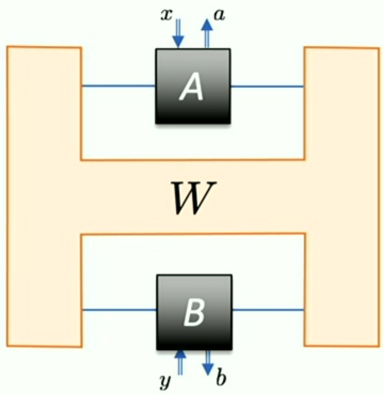
Certifying causal nonseparability: Device-Indep$^{\text{ly}}$



$$W^{\text{c-sep}} = q\, W^{A \prec B} + (1-q)\, W^{B \prec A}$$

$$\Longrightarrow \quad p(a,b|x,y) = q\, p^{A \prec B}(a,b|x,y) + (1-q)\, p^{B \prec A}(a,b|x,y)$$

"Causal correlations" satisfy "Causal inequalities"

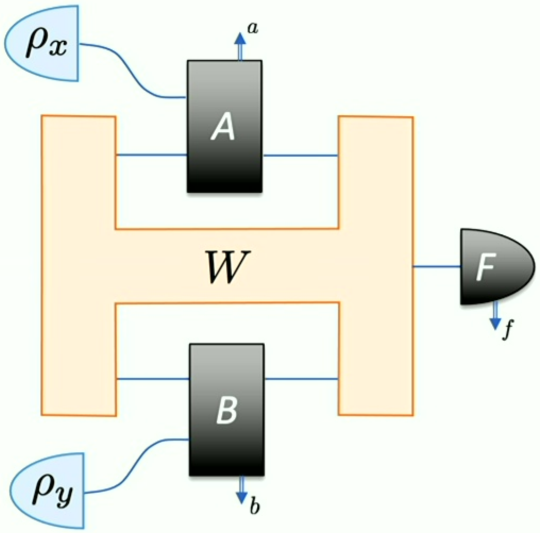
[Oreshkov, Costa, Brukner, Nat. Commun. **3**, 1092 (2012)]

☺ Some causally nonseparable *W*'s indeed violate causal inequalities. E.g., W^{OCB}

☹ But some don't. E.g., the Quantum Switch (and its generalisations)

[Araújo *et al.*, New J. Phys. **17**, 102001 (2015); Oreshkov & Giarmatzi, New J. Phys. **18**, 093020 (2016); Wechs *et al.*, PRX Quantum **2**, 030335 (2021); Purves & Short, Phys. Rev. Lett. **127**, 110402 (2021)]

Certifying causal nonseparability: Semi-Dev.-Indep^{ly}



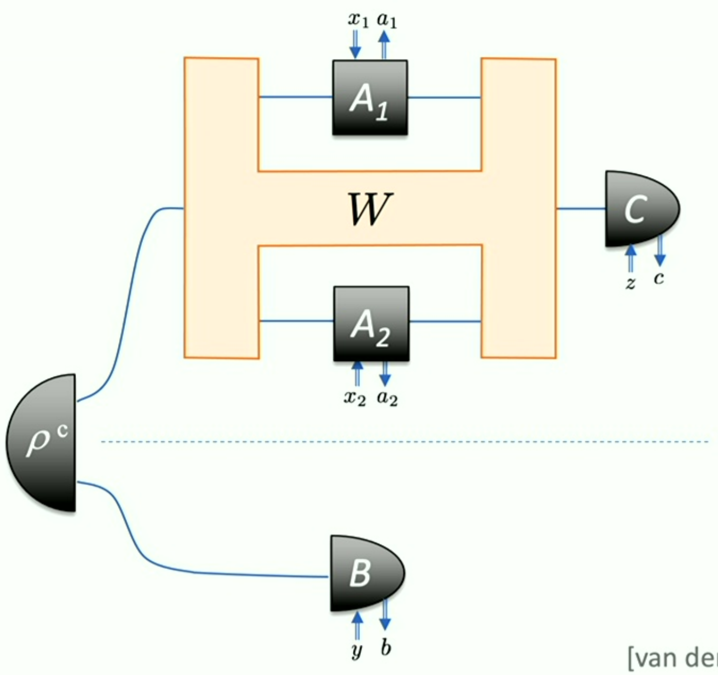
- Some trusted, some untrusted operations
  - ➢ the Quantum Switch can be certified with just *A* or *B* being trusted

    [Bavaresco *et al.*, Quantum **3**, 176 (2019)]

- Using trusted quantum inputs (*à la* Buscemi nonlocality) [Buscemi, Phys. Rev. Lett. **108**, 200401 (2012)]
  - ➢ (we conjecture:) not all causally nonseparable *W*'s can be certified
  - ➢ But the Quantum Switch can ☺

    [Dourdent *et al.*, Phys. Rev. Lett. **129**, 090402 (2022)]

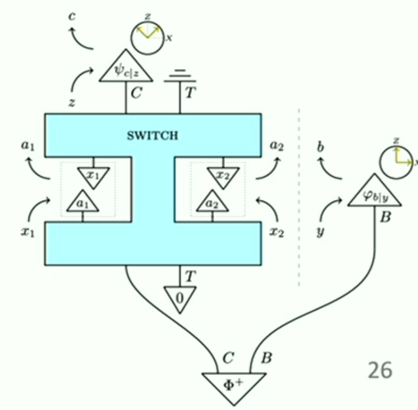
Certifying causal nonseparability:
Device-Indep^{ly}, but w/ additional assumptions



Assuming **D**efinite Causal Order,
Relativistic Causality ("parameter independence")
& **F**ree Interventions (DRF)

$$\implies p(b=0, a_2=x_1|y=0) + p(b=1, a_1=x_2|y=0) + p(b \oplus c = yz|x_1=x_2=0) \;\leq\; \frac{7}{4}$$

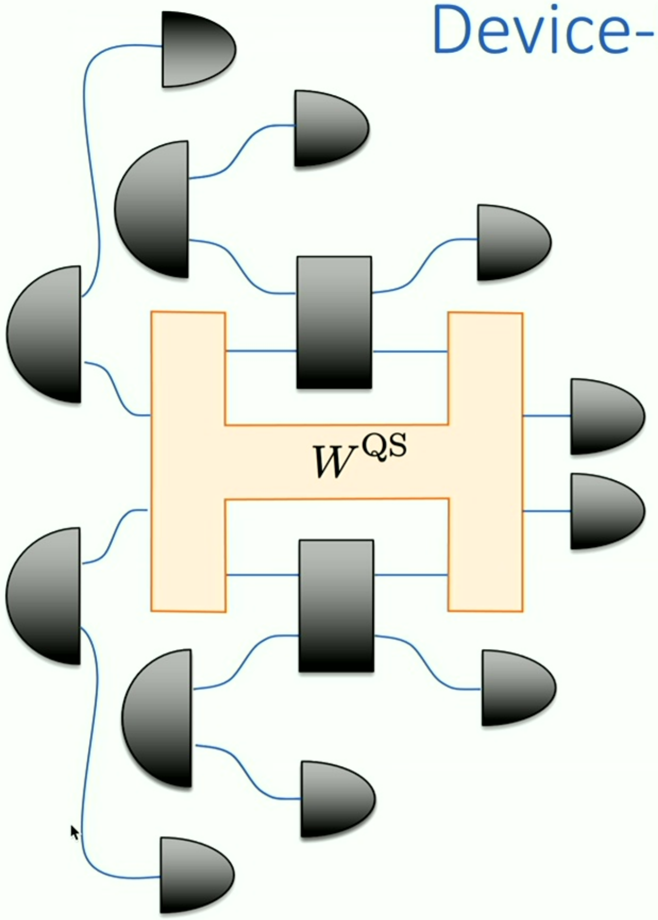
"DRF inequality"



[van der Lugt *et al.*, Nat. Commun. **14**, 5811 (2023);
see also Gogioso & Pinzani, arXiv:2206.08911;
van der Lugt & Ormrod, arXiv:2311.00557]

Certifying causal nonseparability:

Device-Indep^{ly}, but w/ additional assumptions



Quantum inputs can be "self-tested": $\mathrm{CHSH}_{AA'} = 2\sqrt{2} \implies \rho_x \simeq \Phi^+$

(similarly to the case of Buscemi's nonlocality)

[Bowles *et al.*, Phys. Rev. Lett. **121**, 180503 (2018); Phys. Rev. A **98**, 042336 (2018)]

- No need to trust them anymore:
  "Network-Device-Independent" (th^{y}-dependent) certification

  [Dourdent *et al.*, arXiv:2308.12760]

- Even stronger Dev.-Indep. statement: self-testing the QSwitch is possible

  [see Alastair Abbott's flash talk / poster]

Outline

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- **Applications**
- Experiments
- Indefinite causality, really?
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Applications

- Processes with indefinite causal order don't fit in the standard quantum circuit framework

New types of quantum circuits → new possibilities for Quantum Information Processing!

[Chiribella *et al.*, arXiv:0912.0195, "Beyond causally ordered quantum computers"]

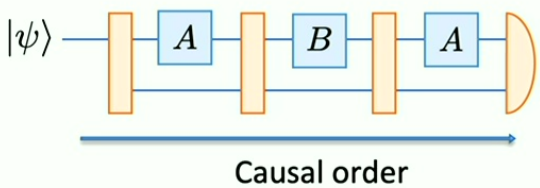
- Processes ≡ "supermaps" ≡ "higher-order" operations
  - Typically, applications for "higher-order" QIP tasks (to calculate functions of Q. operations)

Applications – *1st example*

- Discriminate between commuting vs anti-commuting operations [Chiribella, Phys. Rev. A **86**, 040301(R) (2012); *see also:* Andersson *et al.*, J. Mod. Opt. **52**, 1485 (2005)]

Task: Given $\boxed{A}$ and $\boxed{B}$ as black boxes (a single copy of each), which either commute or anti-commute: determine which of the 2 commuting properties they have

- **Cannot** be done in a standard causally ordered quantum circuit



- Can be done **in a single shot** using the quantum switch

$$\frac{1}{\sqrt{2}}(BA|\psi\rangle^t \otimes |0\rangle^c + AB|\psi\rangle^t \otimes |1\rangle^c)$$
$$= \tfrac{1}{2}\{A,B\}|\psi\rangle^t \otimes |+\rangle^c - \tfrac{1}{2}[A,B]|\psi\rangle^t \otimes |-\rangle^c$$

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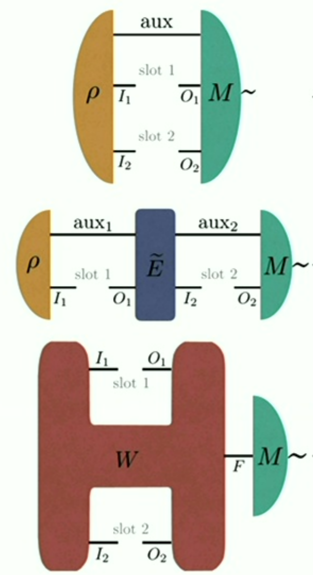
Applications – *Discrimination / Classification tasks*

- *N*-partite generalisation of the commuting vs anti-commuting task
  - ➢ Polynomial advantage

    [Araújo *et al.*, Phys. Rev. Lett. **113**, 250402 (2014); Taddei *et al.*, PRX Quantum **2**, 010320 (2021); Renner & Brukner, Phys. Rev. Research **3**, 043012 (2021); Renner & Brukner, Phys. Rev. Lett. **128**, 230503 (2022)]



- Various other discrimination / classification tasks:
  - E.g., given *N* channels, determine in which set they are
  - E.g., given $U_1, \ldots, U_N$, V as black boxes, determine which U_i is equal to V
    - ➢ P^{succ}(Parallel) < P^{succ}(fixed-order) < P^{succ}(Q. control) < P^{succ}(general W)

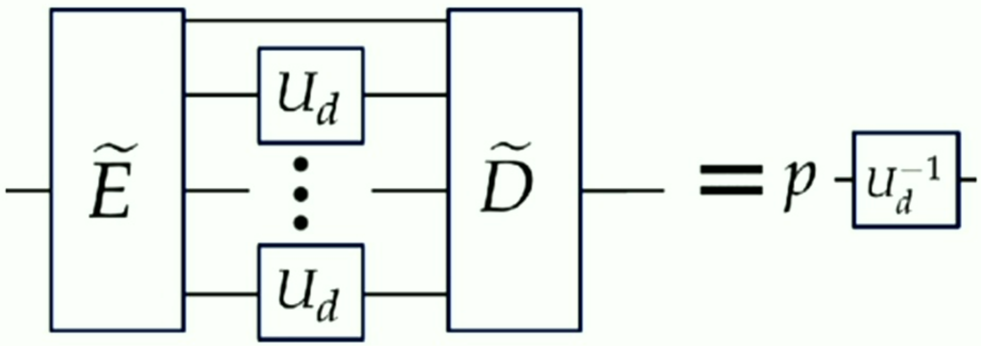
[Bavaresco *et al.*, Phys. Rev. Lett. **127**, 200504 (2021)]

[Simbo *et al.*, arXiv:1803.11414; Wechs *et al.*, PRX Quantum **2**, 030335 (2021); Bavaresco *et al.*, J. Math. Phys. **63**, 042203 (2022)]

Applications – *Transforming quantum operations*

- E.g.: transform *N* copies of *U* into U^T or U^{-1}

(or transform $U_1, \ldots, U_N$ into some $f(U_1, \ldots, U_N)$)



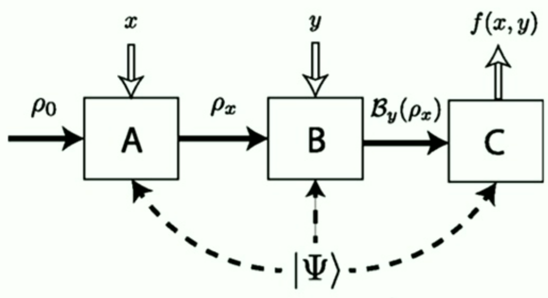
(optimize probability of success, or fidelity)

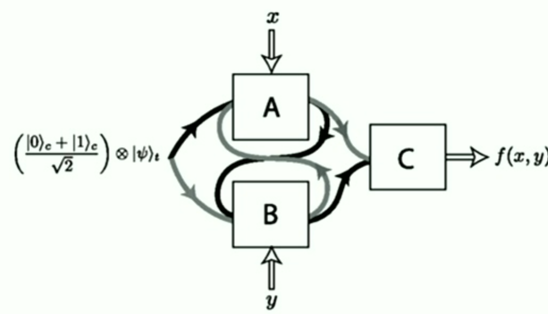
[Quintino *et al.*, Phys. Rev. Lett. **123**, 210502 (2019)]

[Chiribella & Ebler, New J. Phys. **18**, 093053 (2016); Quintino *et al.*, Phys. Rev. A **100**, 062339 (2019); Quintino & Ebler, Quantum **6**, 679 (2022)]

Applications – *Communication complexity, Computation*

- Communication complexity reduction





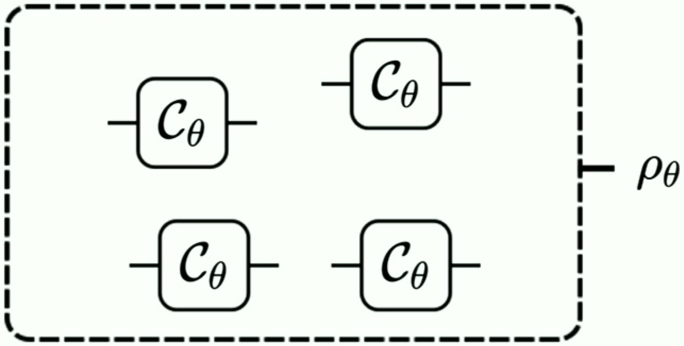
➢ Can obtain an Exponential advantage

[Feix *et al.*, Phys. Rev. A **92**, 052326 (2015); Guérin *et al.*, Phys. Rev. Lett. **117**, 100502 (2016)]

- Computation with indefinite causal structures

[Araújo *et al.*, Phys. Rev. A **96**, 052315 (2017); Baumeler & Wolf, Proc. R. Soc. A. **474**, 20170698 (2018), Apadula *et al.*, Quantum **8**, 1241 (2024); Abbott *et al.*, Phys. Rev. Research **6**, L032020 (2024)]

Applications – *Metrology*



- Advantages of the Quantum Switch
  vs Sequential use of channels…

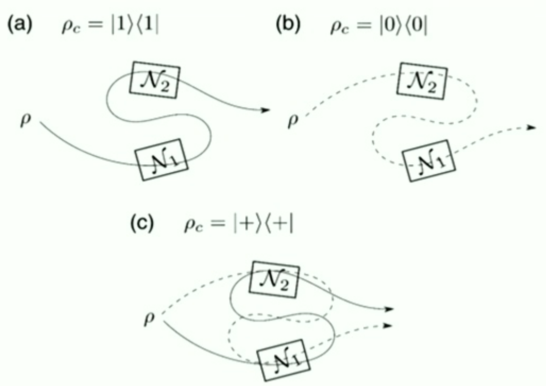
[Mukhopadhyay *et al.*, arXiv:1812.07508; Frey, Quantum Inf Process **18**, 96 (2019); Frey, Quantum Inf Process **20**, 13 (2021); Chapeau-Blondeau, Phys. Rev. A **103**, 032615 (2021); Chapeau-Blondeau, Phys. Rev. A **104**, 032214 (2021); Goldberg *et al.*, Phys. Rev. Research **5**, 033198 (2023); An *et al.*, Phys. Rev. A **109**, 012603 (2024)]

➢ …But this requires a more careful analysis to claim
  an advantage of indefinite vs definite causal orders

[Liu *et al.*, Phys. Rev. Lett. **130**, 070803 (2023); Kurdzialek *et al.*, Phys. Rev. Lett. **131**, 090801 (2023); Mothe *et al.*, Phys. Rev. A **109**, 062435 (2024)]

Applications – *Communication assisted by the Q. Switch*

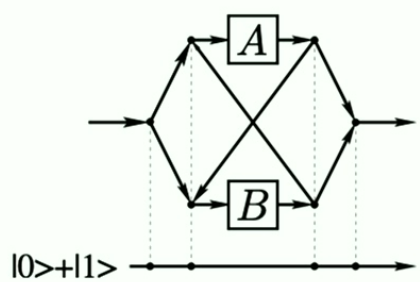
- Putting noisy channels in a superposition of orders can increase their (classical and quantum) communication capacity

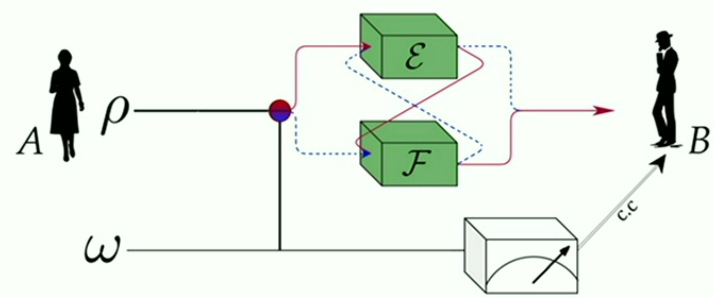


[Ebler *et al.*, Phys. Rev. Lett. **120**, 120502 (2018);
Salek *et al.*, arXiv:1809.06655]

- Fully noisy channels can transmit information!

[Goswami *et al.*, Phys. Rev. Research **2**, 033292 (2020); Caleffi & Cacciapuoti, IEEE J. Sel. Areas Commun. **38**, 575 (2020); Gupta & Sen, arXiv:1909.13125; Procopio *et al.*, Entropy **21**, 1012 (2019), Phys. Rev. A **101**, 012346 (2020); Sazim *et al.*, Phys. Rev. A **103**, 062610 (2021); Bhattacharya *et al.*, PRX Quantum **2**, 020350 (2021); Chiribella *et al.*, Phys. Rev. Lett. **127**, 190502 (2021); Kechrimparis *et al.*, arXiv:2406.19373]

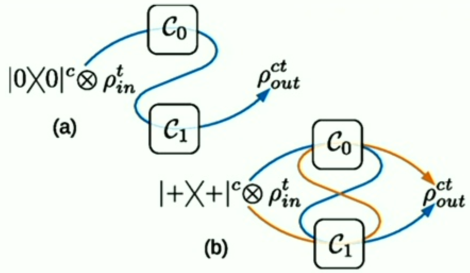




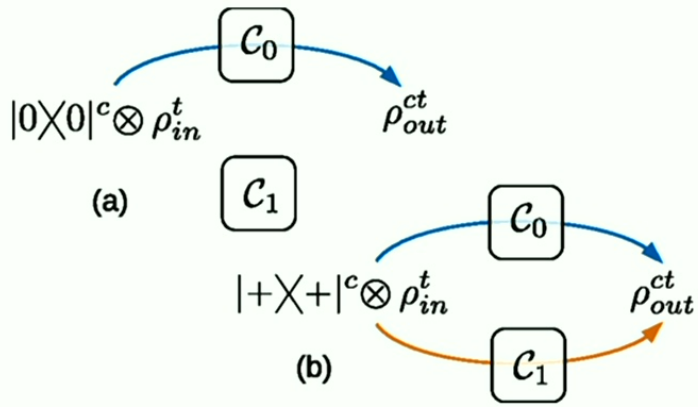
[Chiribella *et al.*, New J. Phys. **23**, 033039 (2021)]

Applications – *Communication assisted by the Q. Switch*

or by coherently controlled channels, more generally



[Gisin *et al.*, Phys. Rev. A **72**, 012338 (2005);
Abbott *et al.*, Quantum **4**, 333 (2020);
Chiribella & Kristjánsson, Proc. R. Soc. A **475**, 20180903 (2019);
Guérin *et al.*, Phys. Rev. A **99**, 062317 (2019);
Kristjánsson *et al.*, New J. Phys. **22**, 073014 (2020);
Rubino *et al.*, Phys. Rev. Research **3**, 013093 (2021);
Pang *et al.*, Quantum **7**, 1125 (2023);
Lee *et al.*, Phys. Rev. Lett. **131**, 190601 (2023);
Miguel-Ramiro *et al.*, Phys. Rev. Lett. **131**, (2023), Phys. Rev. A **108**, 062604 (2023)]



[Loizeau & Grinbaum, Phys. Rev. A **101**, 012340 (2020)]

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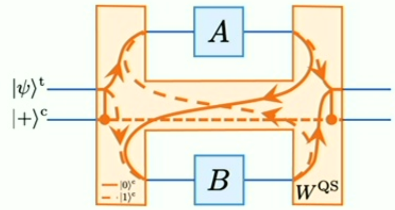
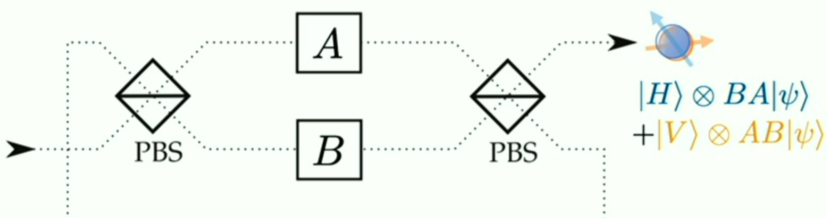
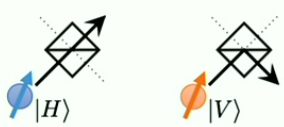
Realisations of the Quantum switch?



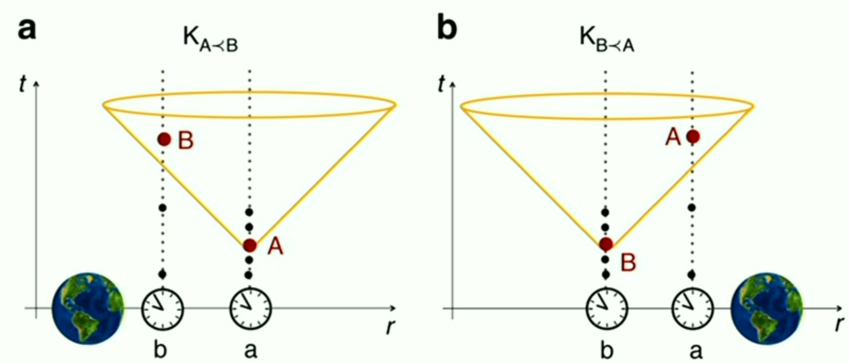
The "Photonic Quantum Switch"



[Araújo *et al.*, Phys. Rev. Lett. **113**, 250402 (2014)]



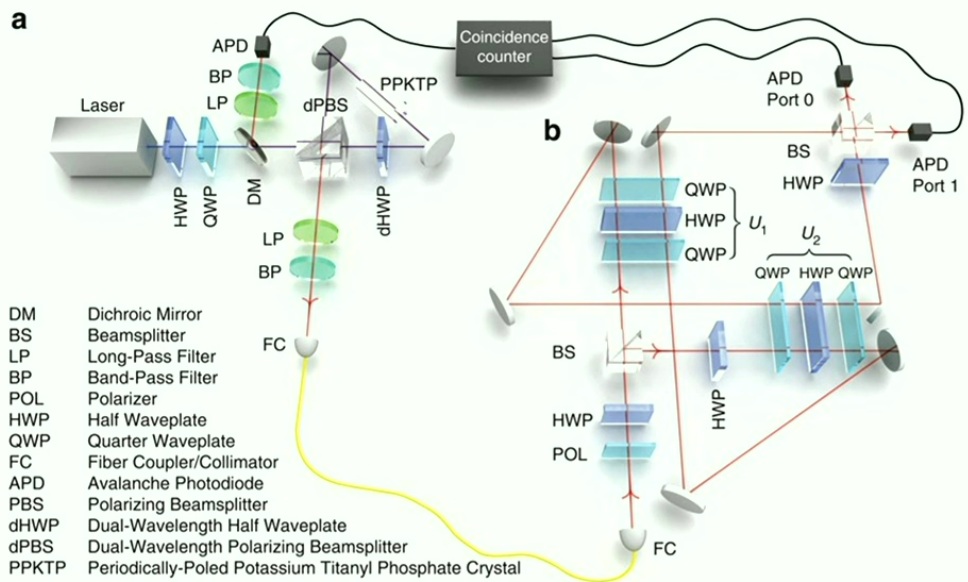
The "Gravitational Quantum Switch"



[Zych *et al.*, Nat. Commun. **10**, 3772 (2019)]

[Paunković & Vojinović, Quantum **4**, 275 (2020); Castro-Ruiz *et al.*, Nat. Commun. **11**, 2672 (2020); Rubino *et al.*, Quantum **6**, 621 (2022); Móller *et al.*, Phys. Rev. A **104**, 042414 (2021)]

Experiments



[Reviews: Goswami, Romero, AVS Quantum Sci. **2**, 037101 (2020);
Rozema *et al.*, Nat. Rev. Phys. **6**, 483 (2024)]

➡ [Procopio *et al.*, Nat. Commun. **6**, 7913 (2015)]

[Rubino *et al.*, Sci. Adv. **3**, e1602589 (2017)]

[Rubino *et al.*, Quantum **6**, 621 (2022)]

[Goswami *et al.*, Phys. Rev. Lett. **121**, 090503 (2018)]

[Guo *et al.*, Phys. Rev. Lett. **124**, 030502 (2020)]

[Goswami *et al.*, Phys. Rev. Research **2**, 033292 (2020)]

[Wei *et al.*, Phys. Rev. Lett. **122**, 120504 (2019)]

[Taddei *et al.*, PRX Quantum **2**, 010320 (2021)]

[Rubino *et al.*, Phys. Rev. Research **3**, 013093 (2021)]

[Cao *et al.*, Phys. Rev. Research **4**, L032029 (2022)]

[Nie *et al.*, Phys. Rev. Lett. **129**, 100603 (2022)]

[Felce *et al.*, arXiv:2107.12413 [quant-ph]]

[Cao *et al.*, Optica **10**, 561 (2023)]

[Strömberg *et al.*, Phys. Rev. Lett. **131**, 060803 (2023)]

[Yin *et al.*, Nat. Phys. **19**, 1122 (2023)]

[Liu *et al.*, arXiv:2305.05416]

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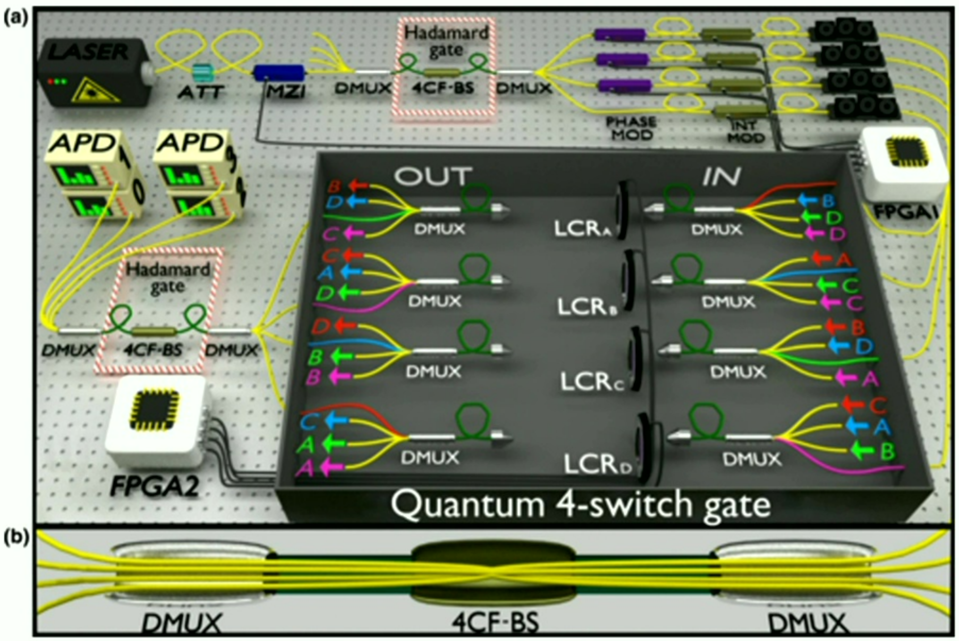
[An *et al.*, Phys. Rev. A **109**, 012603 (2024)]

[Tang *et al.*, arXiv:2406.02236]

Experiments



[Reviews: Goswami, Romero, AVS Quantum Sci. **2**, 037101 (2020);
Rozema *et al.*, Nat. Rev. Phys. **6**, 483 (2024)]

[Procopio *et al.*, Nat. Commun. **6**, 7913 (2015)]
[Rubino *et al.*, Sci. Adv. **3**, e1602589 (2017)]
[Rubino *et al.*, Quantum **6**, 621 (2022)]
[Goswami *et al.*, Phys. Rev. Lett. **121**, 090503 (2018)]
[Guo *et al.*, Phys. Rev. Lett. **124**, 030502 (2020)]
[Goswami *et al.*, Phys. Rev. Research **2**, 033292 (2020)]
[Wei *et al.*, Phys. Rev. Lett. **122**, 120504 (2019)]
➡ [Taddei *et al.*, PRX Quantum **2**, 010320 (2021)]
[Rubino *et al.*, Phys. Rev. Research **3**, 013093 (2021)]
[Cao *et al.*, Phys. Rev. Research **4**, L032029 (2022)]
[Nie *et al.*, Phys. Rev. Lett. **129**, 100603 (2022)]
[Felce *et al.*, arXiv:2107.12413 [quant-ph]]
[Cao *et al.*, Optica **10**, 561 (2023)]
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Outline

- The process matrix framework / Quantum supermaps
- Indefinite quantum causality ≡ “causal nonseparability”
- Examples – incl. the “Quantum switch”
- Certifying causal nonseparability
- Applications
- Experiments
- **Indefinite causality, really?**
- Related frameworks

Indefinite causality, really?

- The process matrix formalism provides a well-defined theoretical framework
  - ➢ proper definition of what "causally (non)separable" processes are

- Is this terminology justified? What does it have to do with causality?

- In the process matrix formalism: causality ~ possibility of signaling from the choice of an operation to the outcome of another one

If there is no possible signaling from *B* to *A*, then the process is *compatible* with the causal order "*A* < *B*"

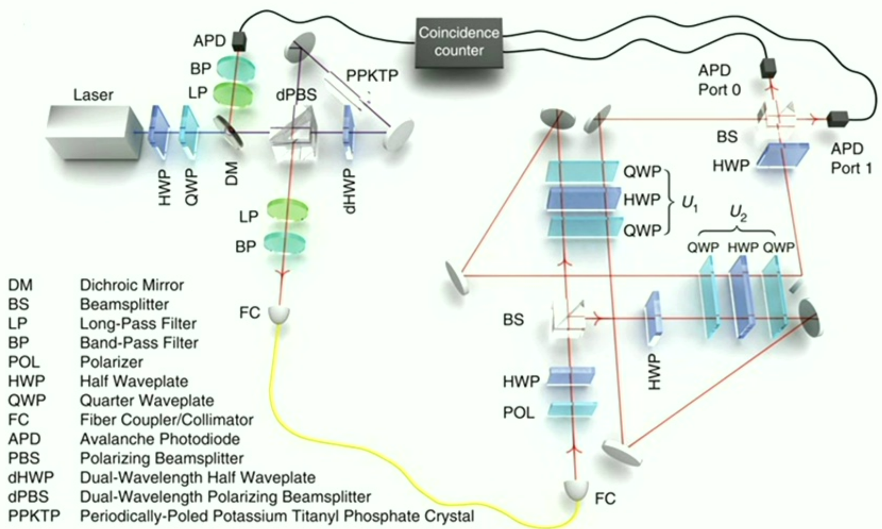
- Here the causal relata ("events") are the quantum operations *A* and *B*

The definition of causal nonseparability formalizes the notion of indefinite causality btw quantum operations

- No debate about that (?)

Indefinite causality, really?

- But does this theoretical formalism faithfully describe our experiments?
- What are the events at play in those experiments?



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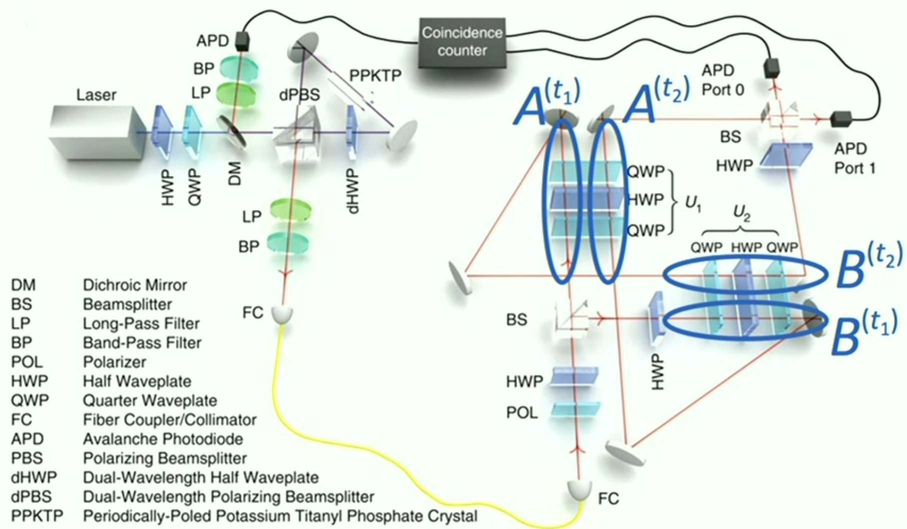

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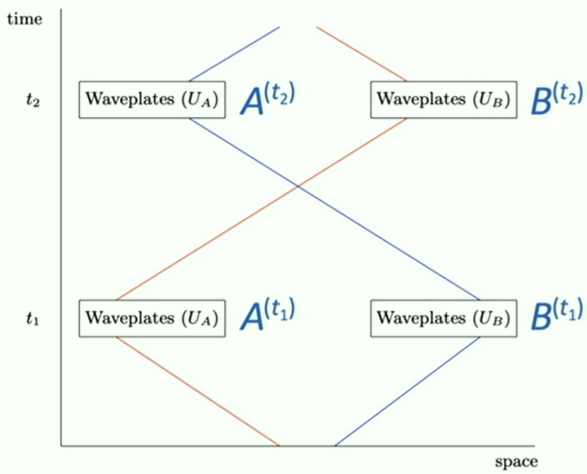

[Ormrod *et al.*, Quantum **7**, 1028 (2023)]

2 camps

2 events
(each in a superposition of 2 space-time loc.)
$A^{(t_1)} \equiv A^{(t_2)}, B^{(t_1)} \equiv B^{(t_2)}$

4 events
$A^{(t_1)}, A^{(t_2)}, B^{(t_1)}, B^{(t_2)}$

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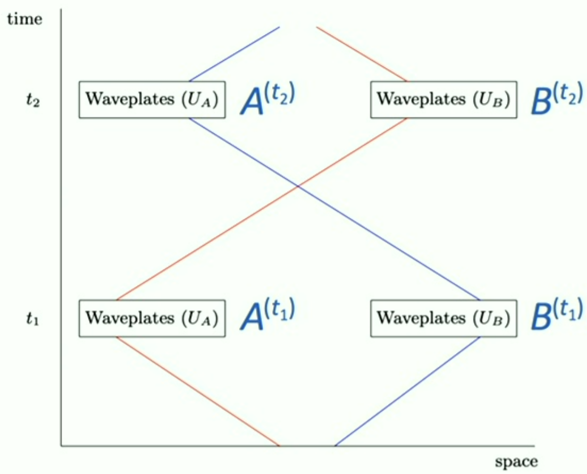

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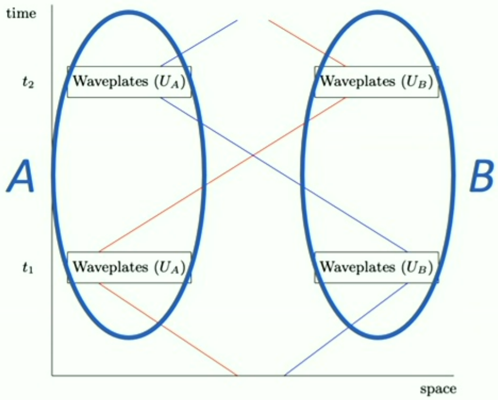
2 camps

2 events	4 events
(each in a superposition of 2 space-time loc.)	
$A^{(t_1)} \equiv A^{(t_2)}$, $B^{(t_1)} \equiv B^{(t_2)}$	$A^{(t_1)}$, $A^{(t_2)}$, $B^{(t_1)}$, $B^{(t_2)}$
"dynamical view"	"spatiotemporal view"
(events are the Q. operations)	(events are space-time points)
➢ Realisation of the QSwitch	➢ Simulation of the QSwitch

2 events – the dynamical view



[Ormrod *et al.*, Quantum **7**, 1028 (2023)]

- 2 events, each in a superposition of 2 space-time locations
  - "time-delocalised operations", acting on "time-delocalised subsystems" [Oreshkov, Quantum **3**, 206 (2019)]
- *A* and *B* could include "counting flags" (?) [Araújo *et al.*, Phys. Rev. Lett. **113**, 250402 (2014)]

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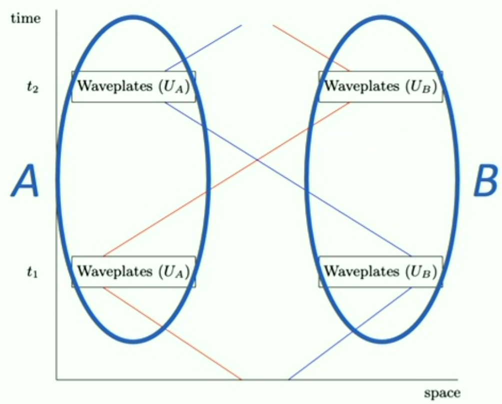

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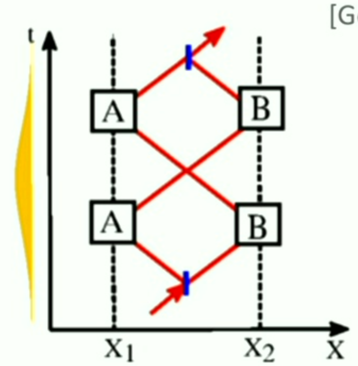

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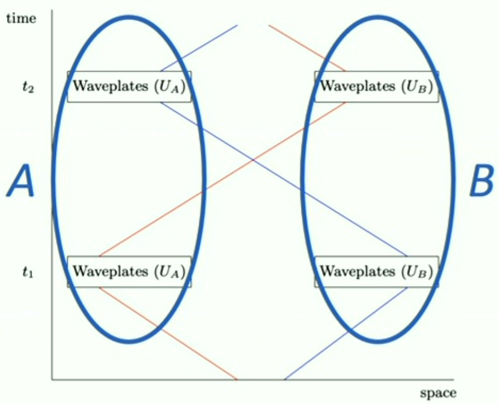

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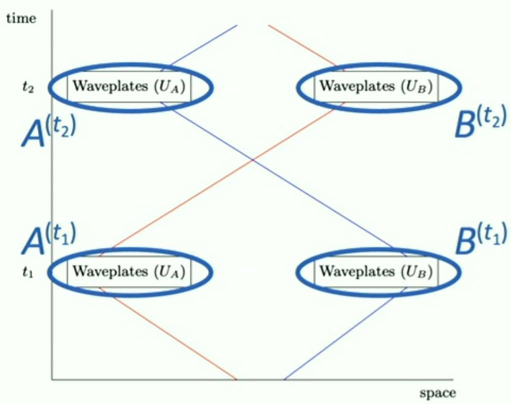
- Identifying 2 events for each operation would break the superposition

  Properties of events should be accessible without destroying phenomena of interest

  Space-time points have no physical meaning – not invariant under change of quantum reference frame

  [de la Hamette *et al.*, arXiv:2402.10267; de la Hamette *et al.*, arXiv:2404.00159]

4 events – the spatiotemporal view



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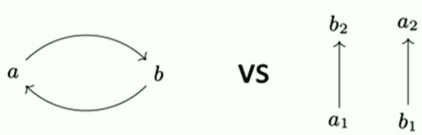

"weak indefinite causal order"

- Events ≡ space-time points in GR, we shouldn't change that!
  - ➢ We must count 2*2 events [Paunković & Vojinović, Quantum **4**, 275 (2020)]

- $A^{(t_1)}$, $A^{(t_2)}$, $B^{(t_1)}$ & $B^{(t_2)}$ have nontrivial vacuum extensions [Chiribella & Kristjánsson, Proc. R. Soc. A **475**, 20180903 (2019)]
  Also, $A^{(t_1)}$ & $A^{(t_2)}$ ($B^{(t_1)}$ & $B^{(t_2)}$) could be different (*"routed QSwitch"*)
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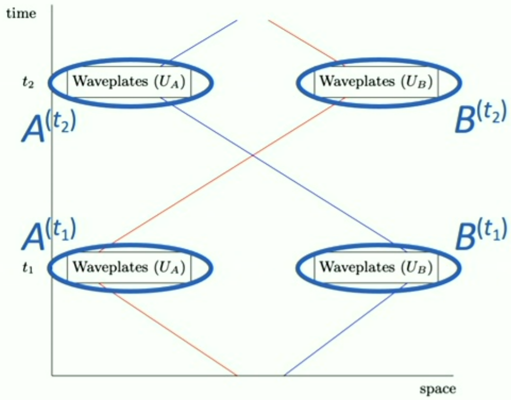
- Experiments are in Minkowski ST (equiv. to unfolded interferometers), clearly there is a definite causal order given by light-cone structure
  [MacLean *et al.*, Nat. Commun. **8**, 15149 (2017)]

No-go theorems tell us that no process with indefinite causal order can be embedded in a classical space-time

➢ There must always be a more fine-grained description w/ definite order
[Vilasini & Renner, arXiv:2203.11245 [quant-ph], Phys. Rev. Lett. **133**, 080201 (2024)]

48

4 events – the spatiotemporal view



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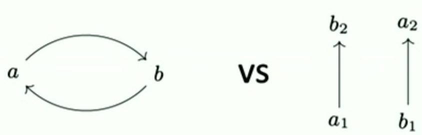

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- Only a gravitational switch (with a single proper time for each event) could genuinely realise an indefinite causal structure? [Zych *et al.*, Nat. Commun. **10**, 3772 (2019); Paunković & Vojinović, Quantum **4**, 275 (2020)]

Indefinite causality, really?

- Specify first which events you're looking at,
  which level of description – fine-grained or coarse-grained – you're interested in!

A new useful resource, really?

- Less clear?
  Probably not with the current photonic implementations?
  Need to come up with new architectures?
  (physically separate control & target systems,
  physically guarantee that each operation is applied once,
  make it all scalable…)

Other related frameworks / models (just a selection!)

Causaloïd framework

[Hardy, arXiv:gr-qc/0509120, J. Phys. A: Math. Theor. **40**, 3081 (2007), arXiv:quant-ph/0701019; Sakharwade & Hardy, arXiv:2407.01522]

Causal boxes

[Portmann *et al.*, IEEE Trans. Inf. Theory **63**, 3277 (2017); Vilasini, PhD thesis (2021); Salzger & Vilasini, arXiv:2102.02393]

Other situations with coherent control, w/ superpositions of orders, of trajectories, of direction…

[Abbott *et al.*, Quantum **4**, 333 (2020); Guérin *et al.*, Phys. Rev. A **99**, 062317 (2019); Chiribella & Kristjánsson, Proc. R. Soc. A **475**, 20180903 (2019); Kristjánsson *et al.*, New J. Phys. **22**, 073014 (2020); Vanrietvelde & Chiribella, QIC **21**, 1320 (2021); Chiribella & Liu, Commun. Phys. **5**, 190 (2022); …]

The "quantum time-flip"

[Chiribella & Liu, Commun. Phys. **5**, 190 (2022); Rubino *et al.*, Commun. Phys. **4**, 251 (2021); Rubino *et al.*, Phys. Rev. Research **4**, 013208 (2022); Liu *et al.*, New J. Phys. **25**, 043017 (2023); Guo *et al.*, Phys. Rev. Lett. **132**, 160201 (2024); Strömberg *et al.*, Phys. Rev. Research **6**, 023071 (2024); Mrini & Hardy, arXiv:2406.18489]

Quantum causal models

[Leifer & Spekkens, Phys. Rev. A **88**, 052130 (2013); Henson *et al.*, New J. Phys. **16**, 113043 (2014); Costa & Shrapnel, New J. Phys. **18**, 063032 (2016); Allen *et al.*, Phys. Rev. X **7**, 031021 (2017); Barrett *et al.*, arXiv:1906.10726; Nat. Commun. **12**, 885 (2021)]

Routed quantum circuits

[Vanrietvelde *et al.*, Quantum **5**, 503 (2021); Ormrod *et al.*, Quantum **7**, 1028 (2023); Vanrietvelde *et al.*, arXiv:2206.10042]

Addressable Quantum Gates

[Arrighi *et al.*, ACM Trans. Quantum Comput. (2023)]

PBS-, LOv-, Many-Worlds-calculus

[Clément *et al.*, MFCS 2020; MFCS 2021; MFCS 2022; Chardonnet *et al.*, arXiv:2206.10234]

Multi-round process matrices

[Hoffreumon & Oreshkov, Quantum **5**, 384 (2021)]

. . .

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