Title: Quantum simulation in the presence of errors

Speakers: Ignacio Cirac

Collection/Series: Waterloo-Munich Joint Workshop

Subject: Quantum Information

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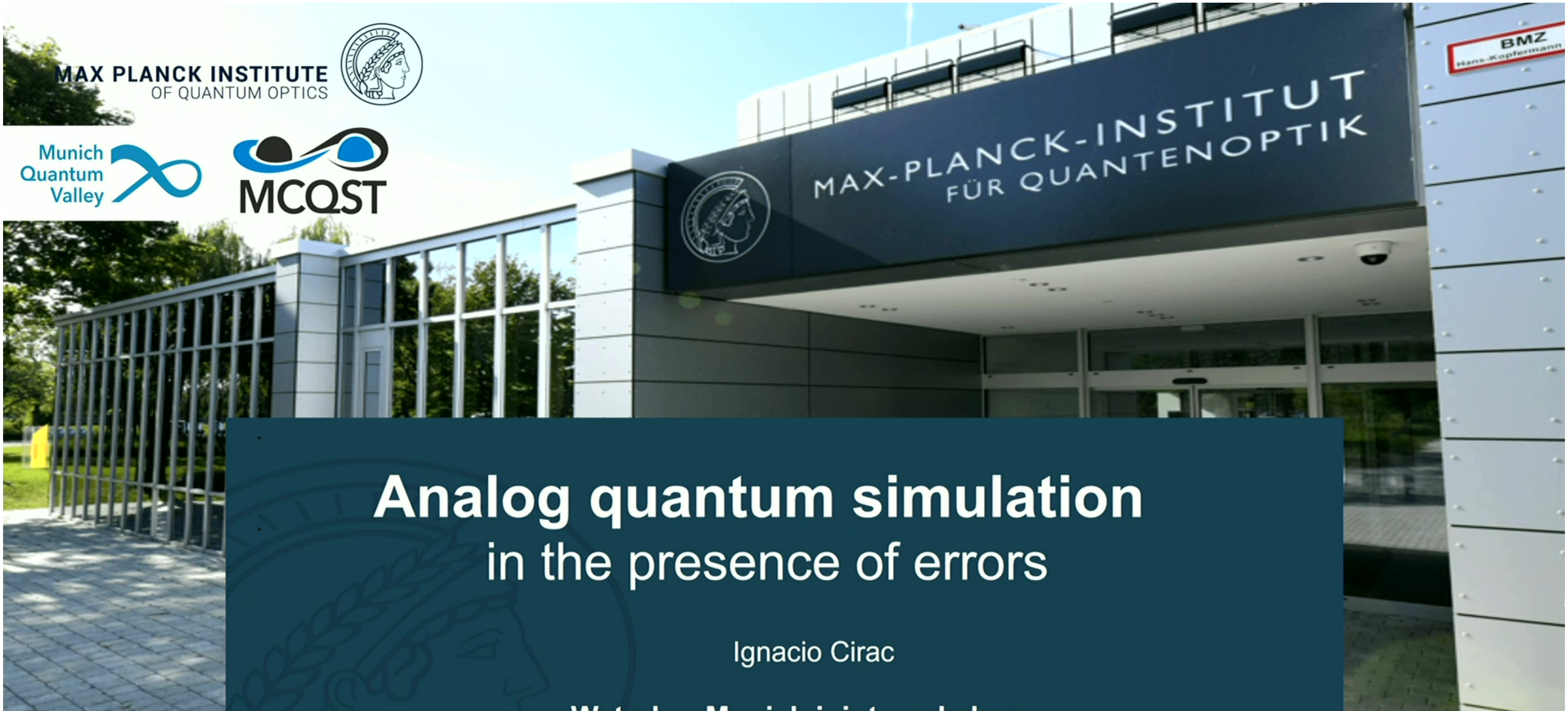


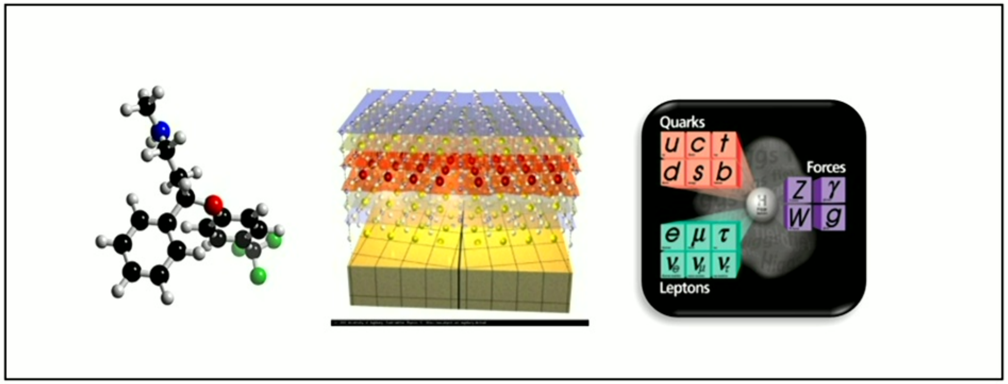
Analog quantum simulation
in the presence of errors

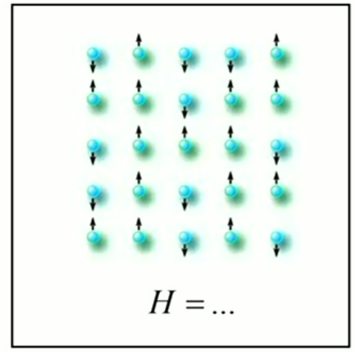
Ignacio Cirac

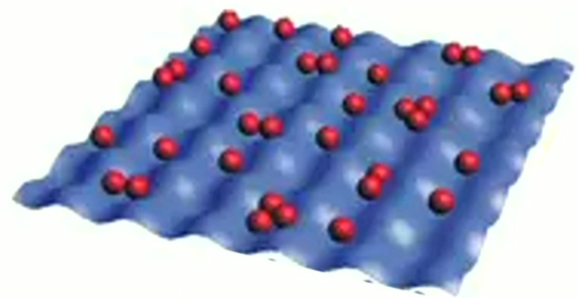
Waterloo-Munich joint workshop,

Perimeter Institute, September 30, 2024

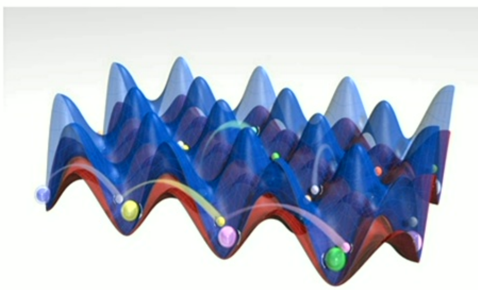
Quantum simulation





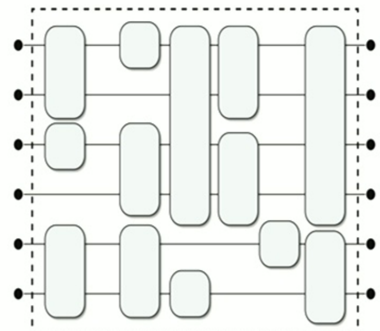


Analog



- Easier to build
- More natural for fermions

Digital



- Universal
- Error correction

Quantum Advantage

Question: is quantum advantage possible in analog quantum simulation?

- Wrt best known classical algorithms
- In terms of complexity

- Non-standard scenario (eg, analog, thermodynamic limit,…)
- Fair comparison: with „certified“ methods

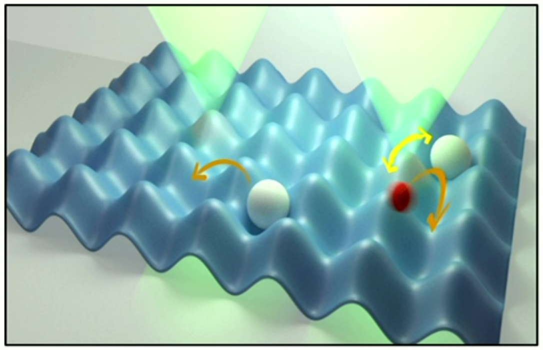
Question: is quantum advantage possible in the presence of errors?

- No error correction
- What is quantum advantage?

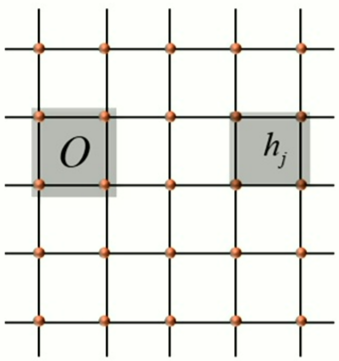
Quantum Advantage I

R. Trivedi, A. Franco, IC, Nat. Comm. 6507 (2024)

Quantum Simulation: Goal



$$H = \sum_j h_j$$



Dynamics	$i d_t \mid \Psi\rangle = H \mid \Psi\rangle$
Ground State	$H \mid \Psi\rangle = E_0 \mid \Psi\rangle$
Thermal equilibrium	$\rho = e^{-H/kT}$

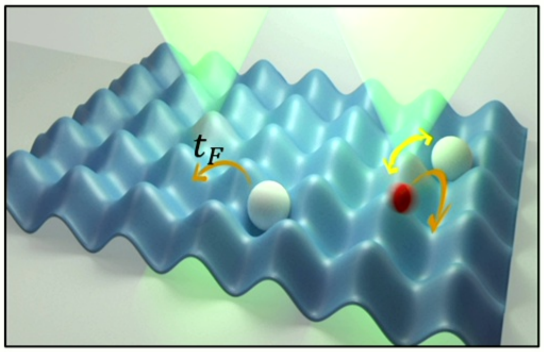
Observable

$\langle O \rangle$

Errors

Errors are extensive

$$H_0 = \sum_j h_j$$

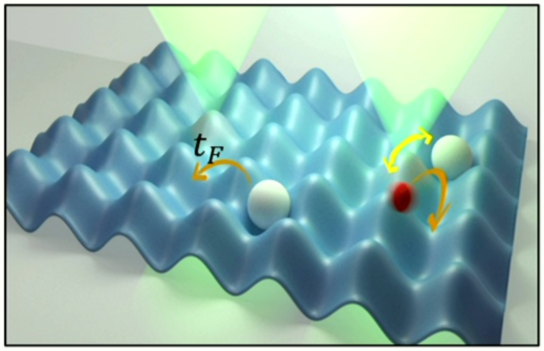


$$H = \sum_j h_j + \varepsilon \underbrace{\sum_j v_j}$$

$$V \sim n\,\varepsilon$$

Errors

$$H_0 = \sum_j h_j$$



Errors are extensive

$$H = \sum_j h_j + \varepsilon \sum_j v_j$$

$$V \sim n\,\varepsilon$$

- Ideal $|\Psi_0(t)\rangle = e^{-iH_0 t}\,|\Psi(0)\rangle$
- Real $|\Psi(t)\rangle = e^{-i(H_0+V)t}\,|\Psi(0)\rangle$

$$n\,\varepsilon\,t \approx 1 \;\Rightarrow\; |\Psi\rangle \perp |\Psi_0\rangle$$

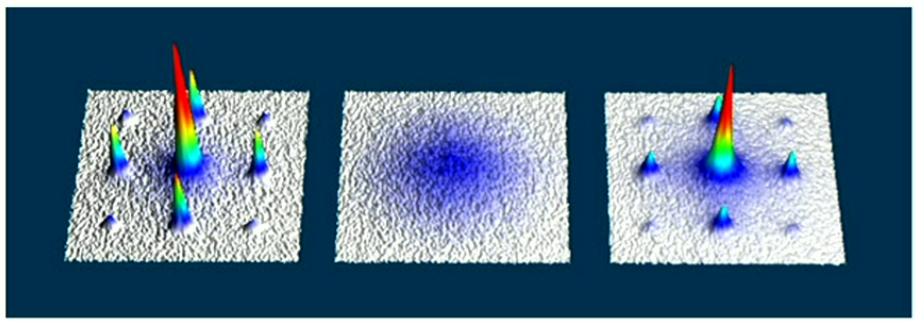
Simulation is limited to $t \approx 1/\varepsilon n$

Errors are extensive

$$H = \sum_j h_j + \varepsilon \sum_j v_j$$

Observables are intensive

$$m = \frac{1}{n}\sum_{j=1}^{n} \langle s_j^z \rangle$$

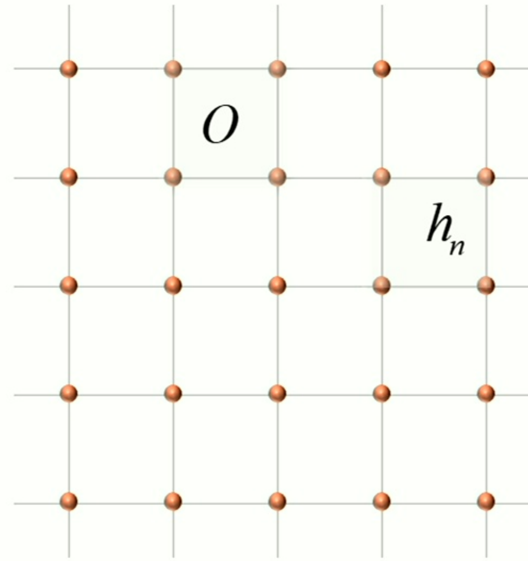


Bloch, Esslinger, Greiner, Hänsch, Nature (2002)

- Model: $H_0 = \sum_j h_j$
- Q. Simulator: $H = \sum_j h_j + \varepsilon \sum_j v_j$
- Observable: $o = \frac{1}{n} \sum_{j=1}^{n} \langle o_j \rangle$

Stable: if $| o_{H_0} - o_H | \leq f(\varepsilon)$ independent of n

- Stable problems:
  - Dynamics
  - Ground states of gapped Hamiltonians*
  - Thermal states (non critical)
  - Critical Gaussian states



Quantum Simulation **without** Errors

- Q. Simulator: $H = \sum_j h_j$
- Thermodynamic limit: $o^* = \lim_{n\to\infty} \langle o_n \rangle$

Does not fit the standard scenario of complexity

Aharonov and Irani (2022)
Wathson and Cubitt (2022)

We have a:

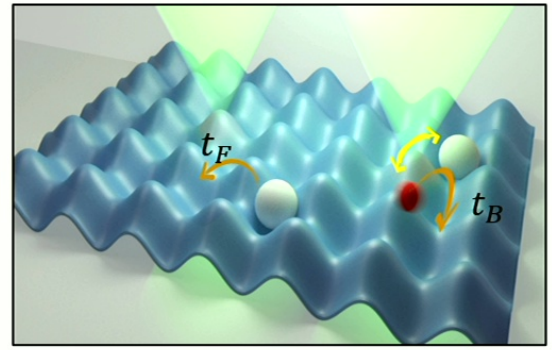
- Classical algorithm: o^{cl}
- Quantum algorithm: o^{q}

- Question: Given $\delta > 0$, what is computational time to obtain $| o^{\mathrm{cl,q}} - o^* | < \delta$

T^{q} = polynomial in $1/\delta$ $\begin{cases} \text{Dynamics} \\ \text{Ground state (1/poly}(N)\text{) gap} \end{cases}$

T^{cl} = superpolynomial (dynamics) or exponential (ground state) in $1/\delta$

Quantum Simulation with Errors



- Q. Simulator: $H = \sum_j h_j + \varepsilon \sum_j v_j$

- Quantum advantage:
  - With a quantum simulator, the error will be $\delta = f(\varepsilon)$
  - What is the time in a classical computer to reach an error $\delta = f(\varepsilon)$?
  - How does that time scale with ε?

  T^{q} = polynomial in $1/\varepsilon$

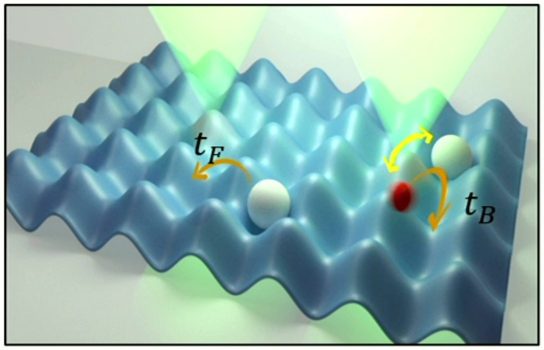
  T^{cl} = superpolynomial (dynamics) or exponential (ground state) in $1/\varepsilon$

- Example: if hardware error is reduced by a factor of 10, i.e. $\varepsilon \to \varepsilon/10$
  the classical computational time (depth) $T^{\mathrm{cl}} \to \left(T^{\mathrm{cl}}\right)^{10}$

Quantum Advantage II

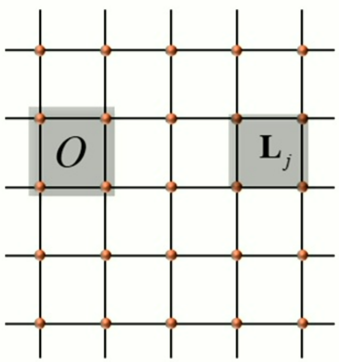
V. Kashyap, G. Styliaris, S. Mouradian, IC, R. Trivedi, arXiv:2404.11081

Simulating dissipative Quantum Systems: Goal



~~$H = \sum_j H_j$~~

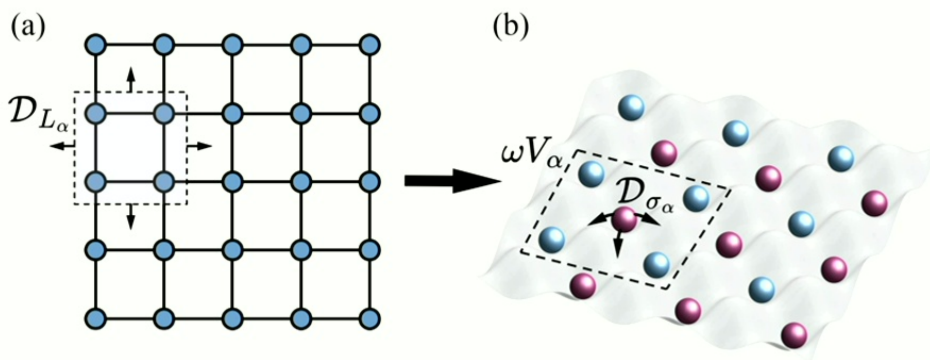
$$\mathbf{L} = \sum_j \mathbf{L}_j$$



$d_t \rho(t) = \mathbf{L}\rho(t)$	Dynamics	➡	Observable $\langle O \rangle$
$\mathbf{L}\rho = 0$	Steady state (Fixed point)		

Quantum simulation of dissipative systems

Simulating dissipative systems:



Pastawski, Clemente, IC, Phys. Rev. A (2011)

Quantum Simulation without Errors

Rapid Mixing: A local observable O (with $\|O\|<1$) in a spatially local Lindbladdian L with unique steady state, σ, is rapidly mixing if it converges exponentially fast to $\mathrm{Tr}(O\sigma)$

$$\left|\mathrm{Tr}\left[Oe^{\mathbf{L}t}\rho(0)\right]-\mathrm{Tr}\left(O\sigma\right)\right|<ke^{-\gamma t}$$

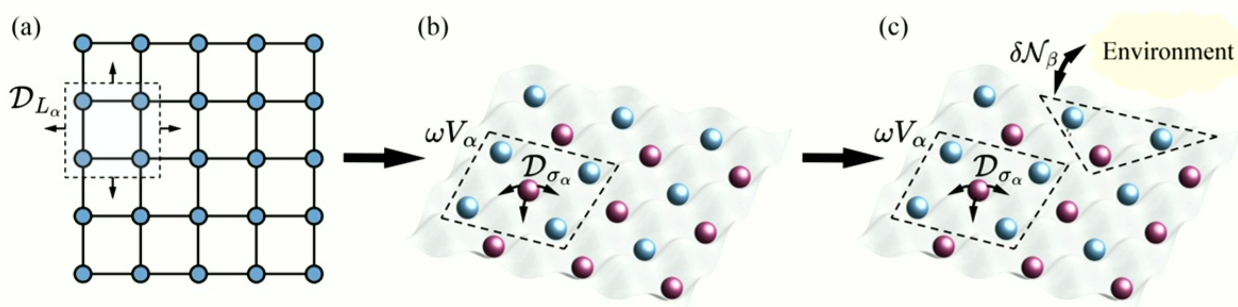
Toby Cubit, Angelo Lucia, Spyridon Michelakis, David Perez-Garcia, CMP (2015)

Simulation: To achieve additive error δ in $\langle O \rangle$ in steady state requires an analog Quantum Simulator with

$$T^q=\Theta\left[\gamma^{-k}\delta^{-1}\log(\delta^{-1})\right]$$

Advantage: No classical algorithm in 2D that achieves that in time poly(γ^{-1}, δ^{-1}) unless BQP=BPP

Errors



(a)
$\mathcal{D}_{L_\alpha}$
(b)
ωV_α
$\mathcal{D}_{\sigma_\alpha}$
(c)
$\delta\mathcal{N}_\beta$
Environment
ωV_α
$\mathcal{D}_{\sigma_\alpha}$

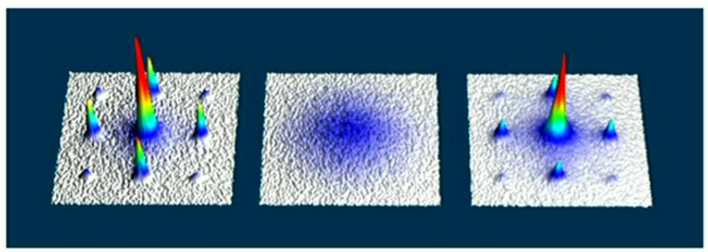
Errors are extensive

$$\mathbf{L} = \sum_j \mathbf{L}_j + \varepsilon \sum_j \mathbf{N}_j$$

$$\left\| \sum_j \mathbf{N}_j \right\| = O(n)$$

Observables are intensive

$$m = \frac{1}{n} \sum_{j=1}^{n} \langle s_j^z \rangle$$



Bloch, Esslinger, Greiner, Hänsch (2002)

Quantum Simulation with Errors

Q. Simulator: $\mathbf{L} = \sum_j \mathbf{L}_j + \varepsilon \sum_j \mathbf{N}_j$

Simulation: In the presence of error rate ε, one can obtain <*O*> in steady state with an error δ and time t_{sim}

$$\delta = O\left[\sqrt{\varepsilon}\right] \qquad T^Q = O\left[\varepsilon^{-1/2}\log(\varepsilon^{-1/2})\right]$$

Advantage: There cannot exist a randomized classical algorithm in 2D that achieves that with the same precision in time $T^{\mathrm{cl}} = \mathrm{poly}(\varepsilon^{-1})$ unless BQP=BPP

Example: if hardware error is reduced by a factor of 10, $\delta \to \delta / 10$

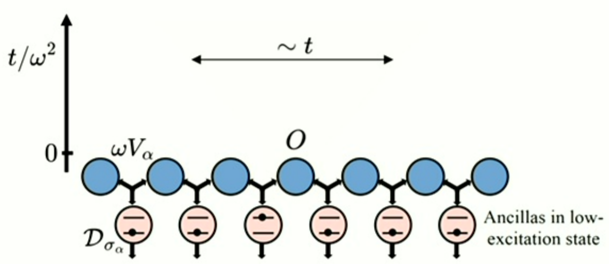
the classical computational time (depth) $T^{\mathrm{cl}} \to \left(T^{\mathrm{cl}}\right)^{10}$

Techniques

Rahul Trived, Adrian Franco-Rubio, IC, Nat. Comm. 6507 (2024)
Vikram Kashyap, Georgios Styliaris, Sara Mouradian, IC, Rahul Trivedi, arXiv:2404.11081

R. Trivedi

Engineered dissipation



Pastawski, Clemente, Cirac (2011)

Quantum advantage

Adapt to a 2D geometry

Verstraete, Wolf, Cirac (2009)
Aharonov, van Dam, Kempe, Landau, Lloyd, Regev (2008)

Stability with respect to errors in all qubits both in time and steady state

Quantum simulation is (arguably) the most suitable application for quantum computers

Currently, analog and digital quantum computers are very well suited for quantum simulation despite errors

Errors in Quantum Simulation and Computing

The quantum adiabatic algorithm suppresses the proliferation of errors

Benjamin F. Schiffer,[1,2,*] Adrian Franco Rubio,[1,2] Rahul Trivedi,[3,1,2] and J. Ignacio Cirac[1,2]
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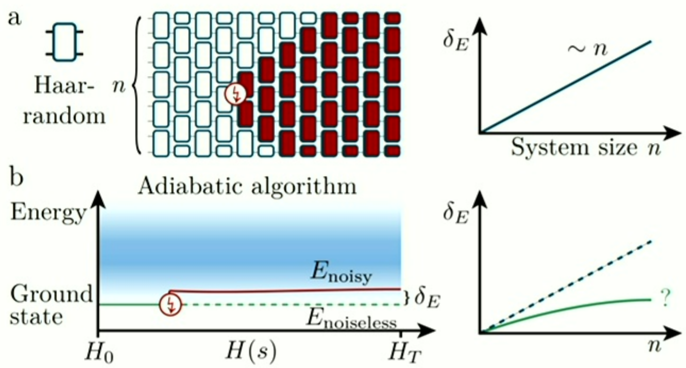
[2] *Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, D-80799 Munich, Germany*
[3] *Electrical and Computer Engineering, University of Washington, Seattle, WA 98195, USA*
(Dated: April 25, 2024)



Pauli path simulations of noisy quantum circuits beyond average case

Guillermo González-García[1,2], J. Ignacio Cirac[1,2], and Rahul Trivedi[1,3]

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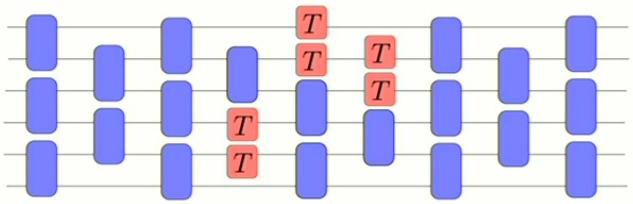

Quantum Computing

Efficient Simulation of Quantum Chemistry Problems in an Enlarged Basis Set

Maxine Luo[1,2] and J. Ignacio Cirac[1,2]

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[2] *Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, D-80799 München, Germany*

(Dated: July 8, 2024)

Standard Hamiltonian:

$$\begin{aligned} H =& h + V \\ =& \sum_{ij} h_{ij} a_i^\dagger a_j + \frac{1}{2} \sum_{ijkl} V_{ijkl} a_i^\dagger a_k^\dagger a_l a_j, \end{aligned}$$

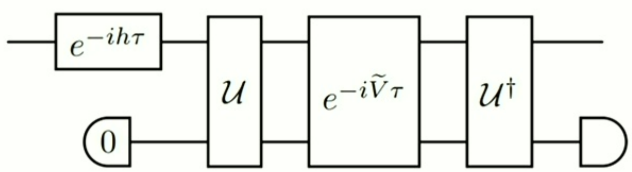
add N-M fictitous modes:

$$c_\alpha = \sum_i u_{i\alpha} a_i + \sum_m v_{m\alpha} b_m$$

New interaction Hamiltonian:

$$\widetilde{V} = \sum_{\alpha \neq \beta} \widetilde{V}_{\alpha\beta} n_\alpha n_\beta$$

$$V = {}_b\langle 0 | \widetilde{V} | 0 \rangle_b$$



Efficient preparation of MPS

Preparation of matrix product states with log-depth quantum circuits

Daniel Malz,[1,*] Georgios Styliaris,[2,3,*] Zhi-Yuan Wei,[2,3,*] and J. Ignacio Cirac[2,3]
[1] *Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, 2200 Copenhagen, Denmark*
[2] *Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, 85748 Garching, Germany*
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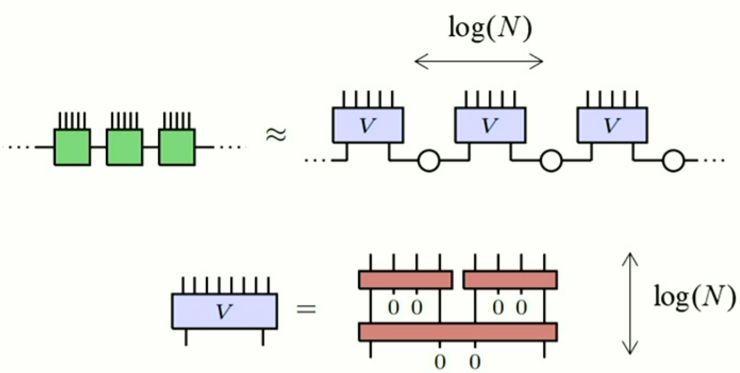
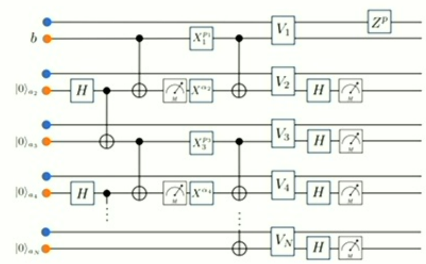
(Dated: July 6, 2023)



Circuit depth: $T = O(\log N)$

Approximating many-body quantum states with quantum circuits and measurements

Lorenzo Piroli,[1] Georgios Styliaris,[2,3] and J. Ignacio Cirac[2,3]
[1] *Dipartimento di Fisica e Astronomia, Università di Bologna and INFN, Sezione di Bologna, via Irnerio 46, I-40126 Bologna, Italy*
[2] *Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, 85748 Garching, Germany*
[3] *Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, D-80799 München, Germany*



Tensor Network Theory

Matrix-product unitaries: Beyond quantum cellular automata

Georgios Styliaris,[1,2] Rahul Trivedi,[3,1,2] David Pérez-García,[4,5] and J. Ignacio Cirac[1,2]
[1] *Max Planck Institute of Quantum Optics, Hans-Kopfermann-Str. 1, Garching 85748, Germany*
[2] *Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, 80799 München, Germany*
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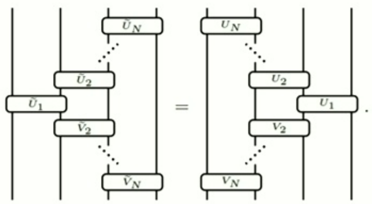
(Dated: June 17, 2024)



Dual-isometric Projected Entangled Pair States

Xie-Hang Yu,[1,2] J. Ignacio Cirac,[1,2] Pavel Kos,[1,2,*] and Georgios Styliaris[1,2,*]
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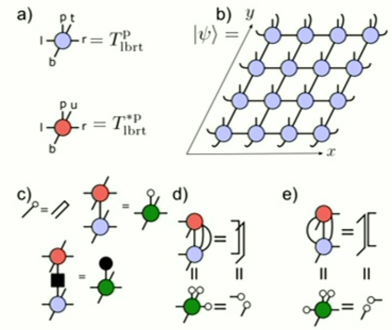

Regular language quantum states

Marta Florido-Llinàs,[1,2,*] Álvaro M. Alhambra,[3,†] David Pérez-García,[4,‡] and J. Ignacio Cirac[1,2,§]
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[2] *Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, 80799 München, Germany*
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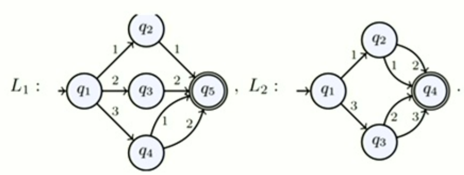
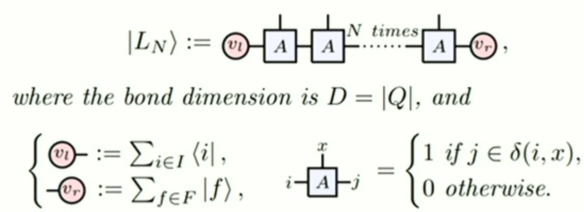
(Dated: July 26, 2024)





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Collaborators: Demler (Zürich), Lukin (Harvard), Perez (Madrid), Polzik (Cophenhagen), Schuch (Vienna), Shi (Beijing), Vuckovik (Standord), Verstraete (Cambridge), Zoller (Innsbruck)

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