Title: Lecture - Quantum Theory, PHYS 605

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Subject: Quantum Foundations

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Feynman Diagrams

$$\langle f|S|i\rangle \xleftrightarrow{\text{LSZ}} \langle\Omega|T\varphi_1\varphi_2\cdots\varphi_n|\Omega\rangle \xleftrightarrow{\text{Dyson}} \langle 0|T\varphi_1\cdots\varphi_n|0\rangle \xleftrightarrow{\text{Wick}} \Delta_F$$

start today finish Wednesday

today

each term $\leftrightarrow$ diagram

Feyman rules determine which diagram + analytic expressions for each diagram

line = Δ_F

point x = spacetime point x

$\Delta_F(x_1 - x_2) \equiv \Delta_{12}$

Wick

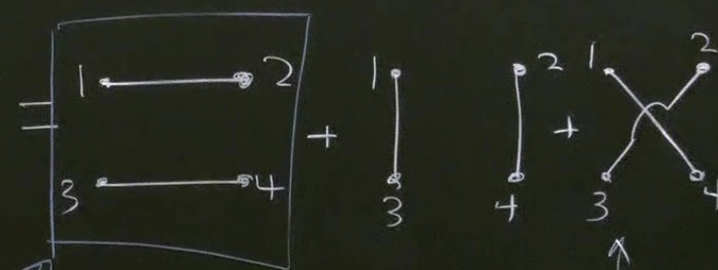
$0\rangle \longleftrightarrow \Delta_F$

$\bullet\!\!-\!\!\bullet \; = \Delta_F$

$\bullet \; =$ spacetime point x

$(x_1 - x_2) \equiv \Delta_{12}$

$$\langle 0 | T\, \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}$$

$=$ 

Best practice:
spacetime points
at same location
in each diagram

Single diagram
with 2 connected
components

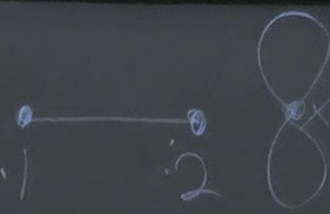
no intersection
of two lines
$\to$ no dot

Example: $\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$ (ignore renormalization for now)

$$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi_2\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}$$

$$\text{numerator} = \langle 0|T\varphi_1\varphi_2|0\rangle + \underbrace{\langle 0|T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4x\,\varphi_x^4|0\rangle}_{O(\lambda)\text{ term}} + \cdots$$

$\mathcal{O}(\lambda)$ term =



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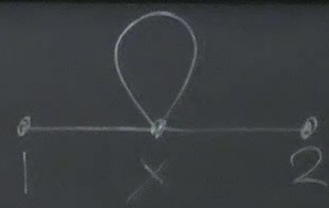


coefficient is $\frac{-i\lambda}{4!} \cdot$ # Wick contractions

1 2

$O(\lambda)$ term

Feynman rules for the numerator $\langle 0|T\varphi_1 \cdots \varphi_n \exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle$

numerator = sum of all diagrams with n external points

point that is fixed (not integrated over)

1. x —— y $= \Delta_{xy}$

2. x —— $= 1$

3. $\times_x$ $= -i\lambda\int d^4x$ (depends on theory)

for the numerator $\langle 0|T\varphi_1 \quad \varphi_n \exp[i\int d^4x \mathcal{L}_{int}]|0\rangle$

sum of all diagrams with n $\underbrace{\text{external points}}_{\text{point that is fixed (not integrated over)}}$

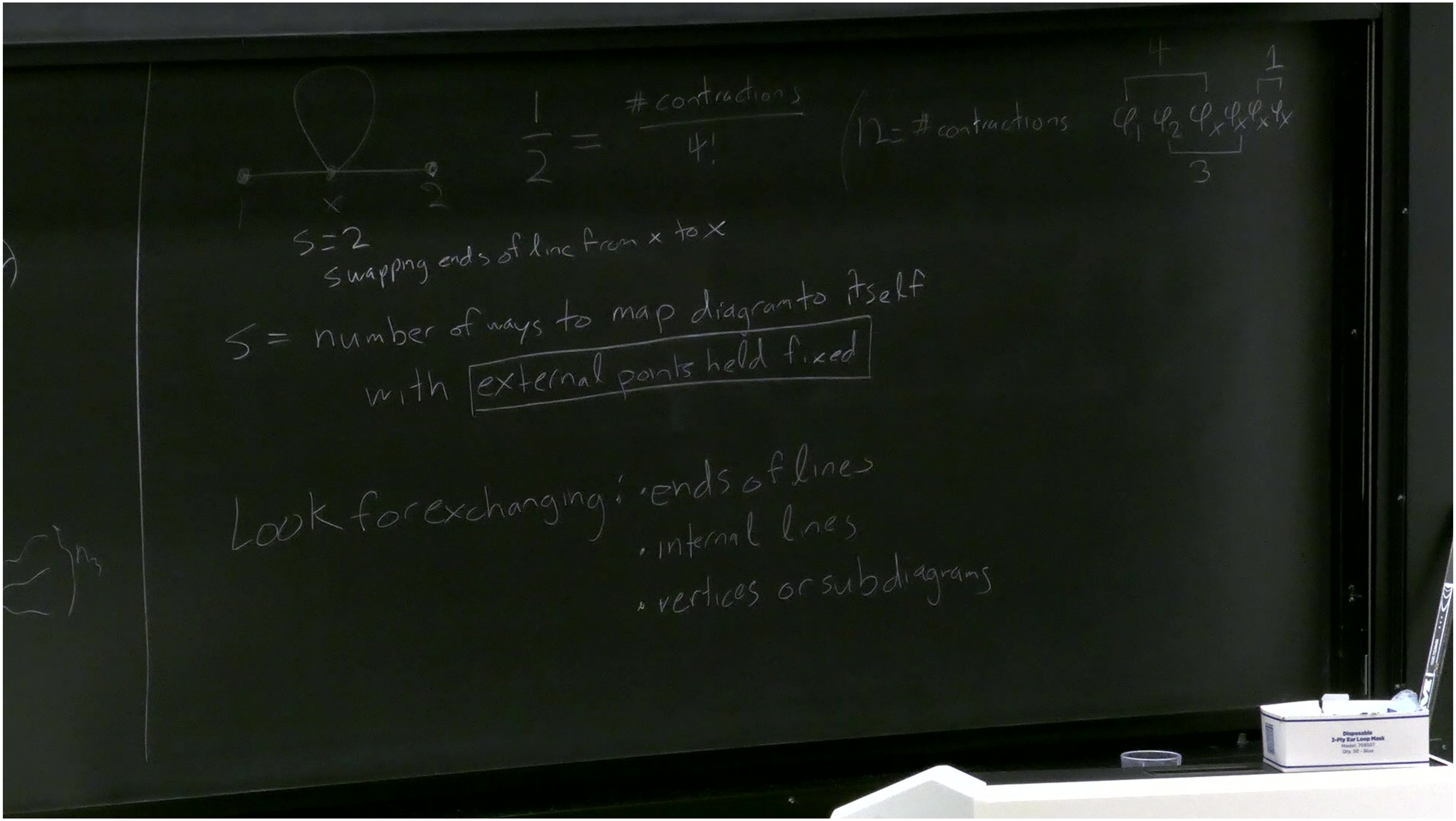
$= \Delta_{xy}$

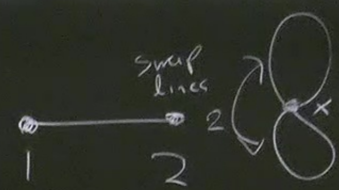
$= 1$

$-i\lambda\int d^4x$ (depends on theory)

$$\left(\frac{-g}{(n_1)!(n_2)!(n_3)!}\varphi^{n_1}\Phi^{n_2}\psi^{n_3}\right)$$

by symmetry factor S:


1
x
2
$\frac{1}{2} = \frac{\#\text{ contractions}}{4!}$
$12 = \#$ contractions
$\varphi_1 \varphi_2 \varphi_x \varphi_x \varphi_x \varphi_x$
4
1
3
S=2
swapping ends of line from x to x
S = number of ways to map diagram to itself
with external points held fixed
Look for exchanging: · ends of lines
· internal lines
· vertices or subdiagrams



2 swapping ends of lines

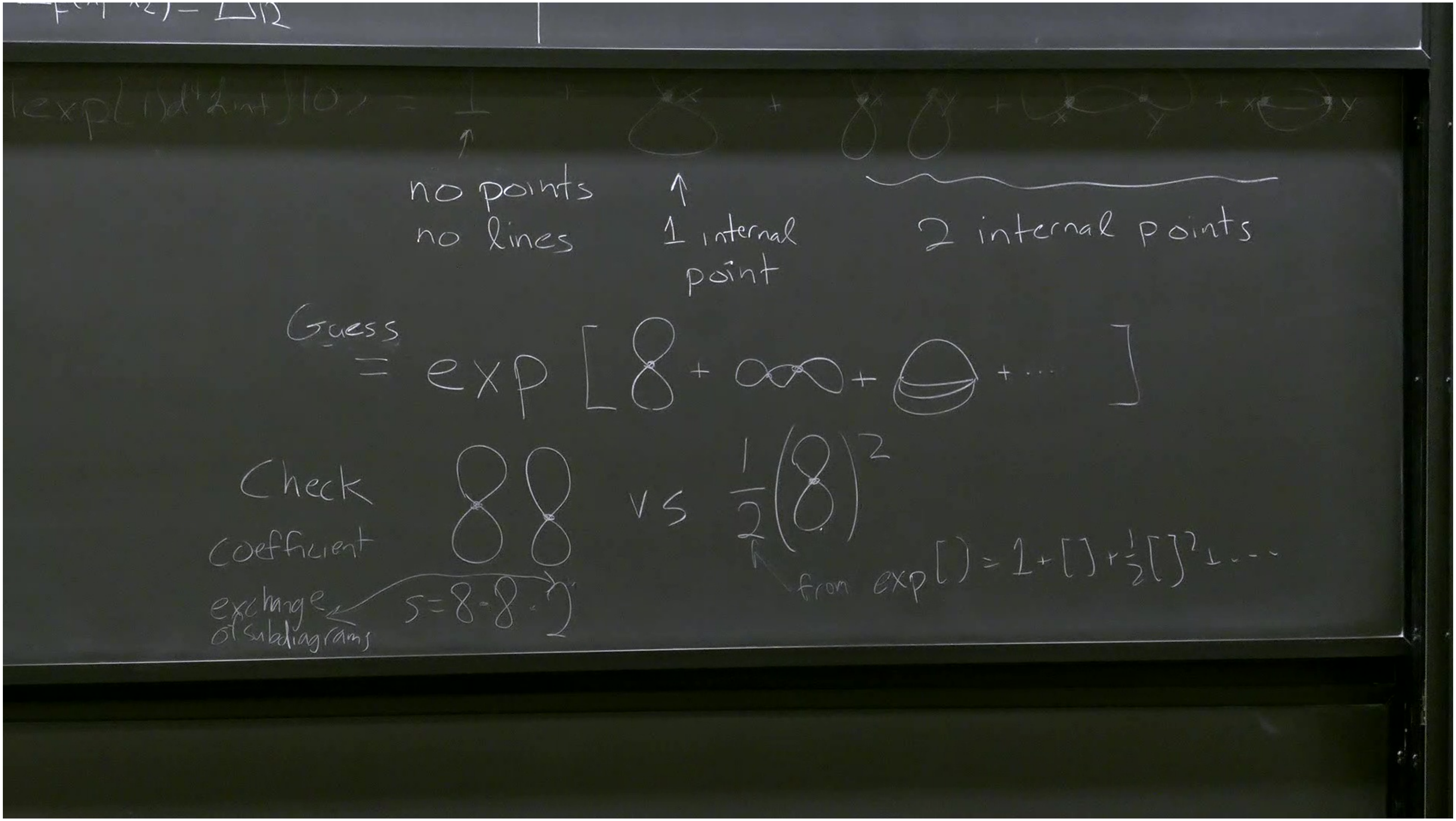
2 swapping ends of lines

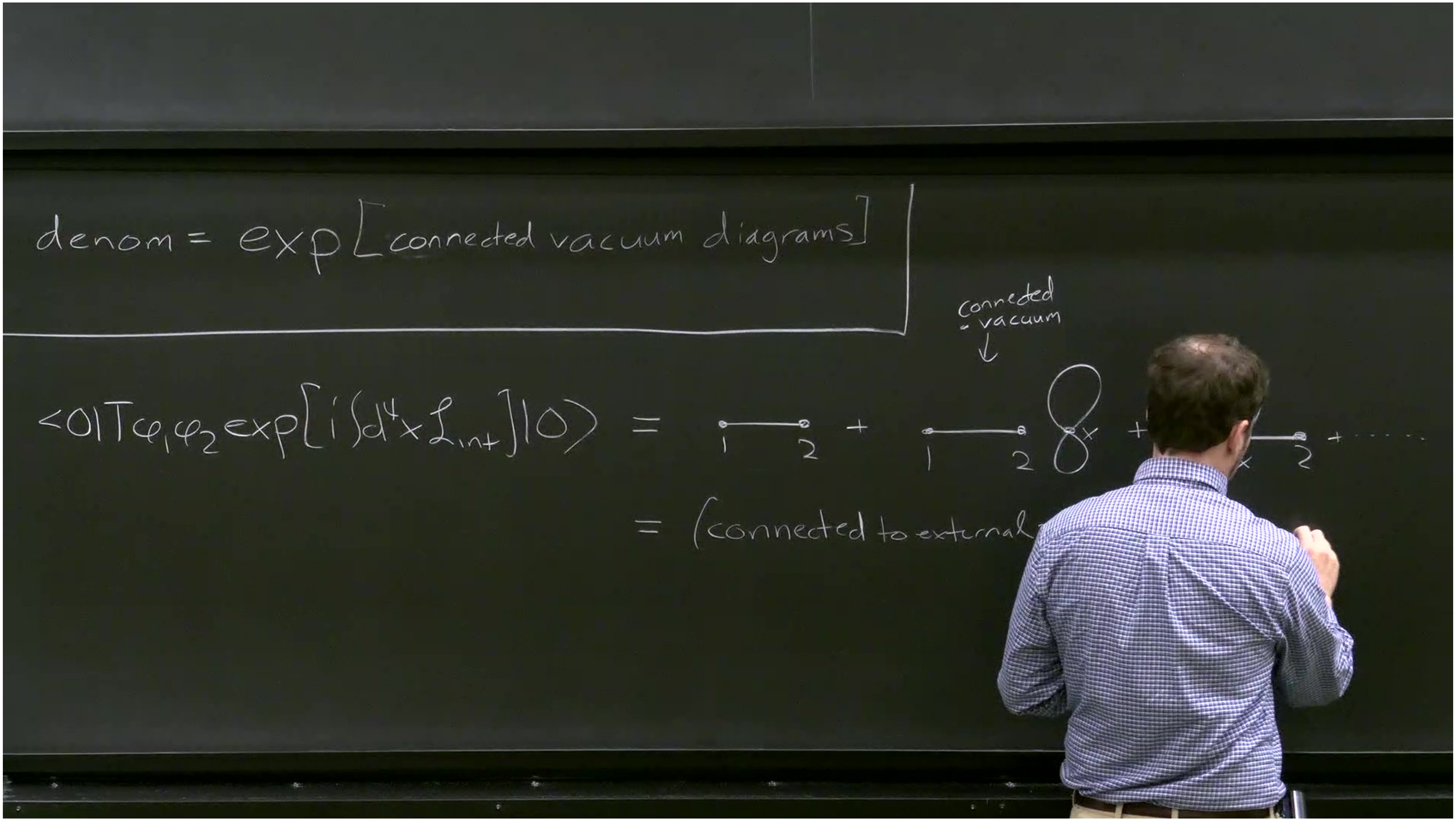
$$\frac{1}{8} = \frac{3}{4!}$$

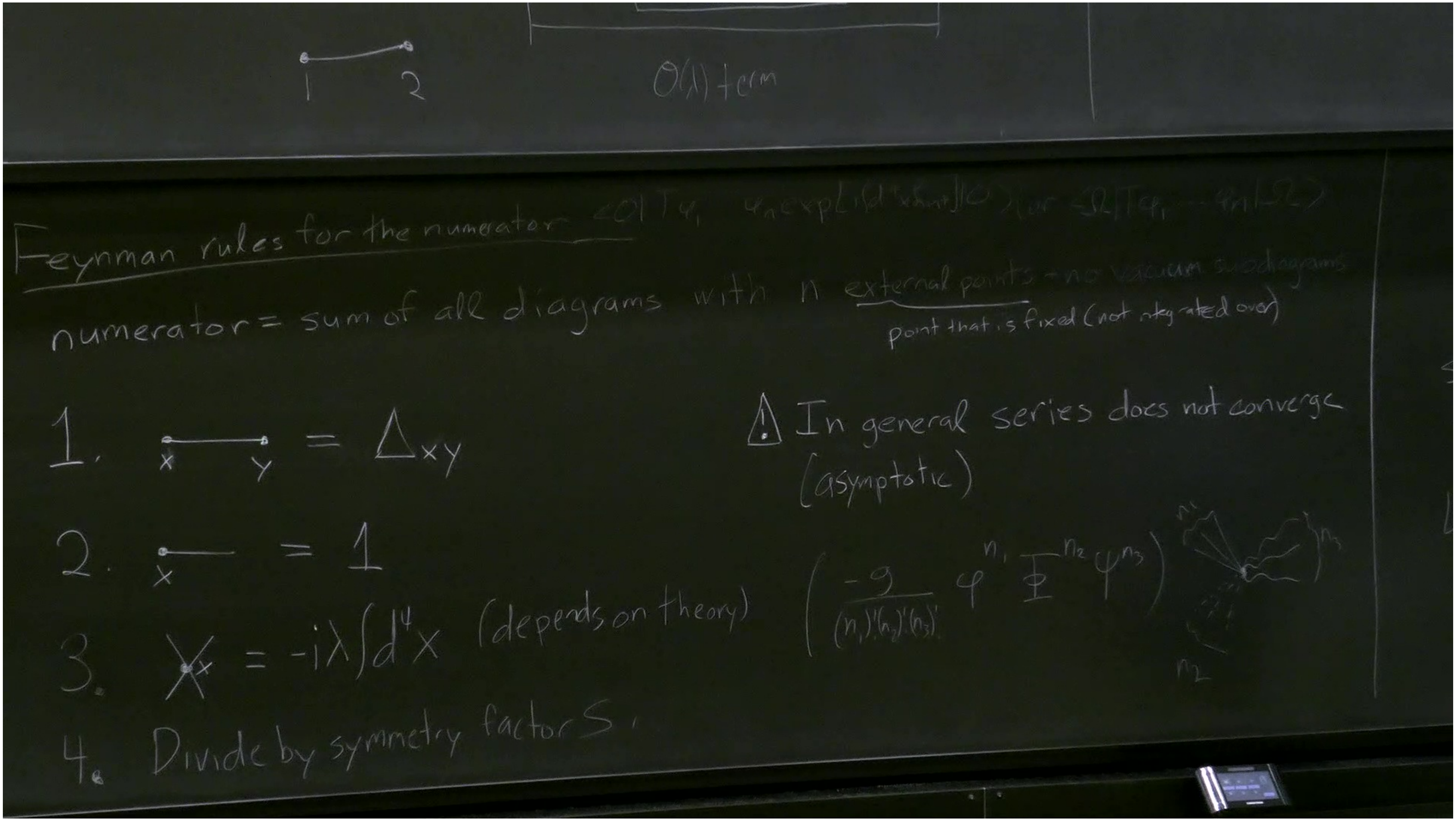
Denom

$S = 8$

$$\overbrace{\varphi_1 \varphi_2}^{1}\,\overbrace{\varphi_x \varphi_x}^{3}\,\overbrace{\varphi_x \varphi_x}^{1}$$


no points
no lines
1 internal point
2 internal points
Guess
= exp [+ + + ...]
Check
coefficient
vs 1/2 ( )^2
exchange of subdiagrams
S = 8·8·2
from exp[ ] = 1 + [ ] + 1/2 [ ]^2 + ...


denom = exp[connected vacuum diagrams]
connected vacuum
$\langle 0|T\varphi_1\varphi_2 \exp[i\int d^4x\, \mathcal{L}_{int}]|0\rangle$ =
1 2 + 1 2 + 2 + · · · ·
= (connected to external



1 2
O(λ) term
Feynman rules for the numerator ⟨0| T φ_1 … φ_n exp[i∫d^4x L_int] |0⟩ (or ⟨Ω| T φ_1 … φ_n |Ω⟩)
numerator = sum of all diagrams with n external points + no vacuum subdiagrams
point that is fixed (not integrated over)
1. x y = Δ_xy
2. x = 1
3. x = -iλ∫d^4x (depends on theory)
4. Divide by symmetry factor S.
In general series does not converge
(asymptotic)
(-g / (n_1)!(n_2)!(n_3)! φ^{n_1} Φ^{n_2} ψ^{n_3})
n_1 n_2 n_3

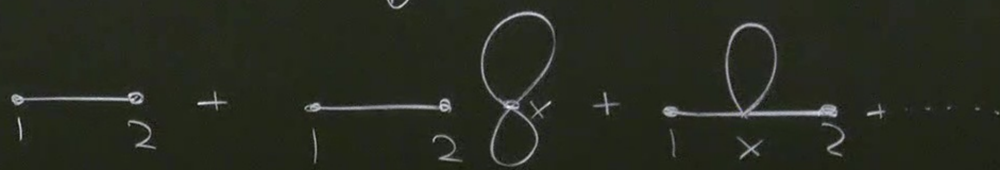
exchange of subdiagrams $S = 8 \cdot 8 \cdot 1)$

…cted vacuum diagrams]

1 or 0 — external

× or • — internal

connected vacuum

…]|0⟩ = 

= (connected to external points)(1 + one connected vacuum diagram + …)

= (connected to external points) exp[connected vacuum diagrams]

Feynman rules for the numerator $\langle 0|T\varphi_1 \cdots \varphi_n \exp[i\int d^4x\, \mathcal{L}_{int}]|0\rangle$ (or $\langle\Omega|T\varphi_1 \cdots \varphi_n|\Omega\rangle$)

numerator = sum of all diagrams with n external points + no vacuum subdiagrams

external point: point that is fixed (not integrated over)

1. x —— y $= \Delta_{xy}$
2. x —— $= 1$
3. $\times_x = -i\lambda\int d^4x$ (depends on theory)
4. Divide by symmetry factor S.

⚠ In general series does not converge (asymptotic)

$$\left(\frac{-g}{(n_1)!(n_2)!(n_3)!}\varphi^{n_1}\Phi^{n_2}\psi^{n_3}\right)$$

4. Divide by symmetry factor S.

Feynman diagrams for $\langle f|S|i\rangle$ or $i\mathcal{M}$

matrix element

Expect

$$\langle f|S|i\rangle = (2\pi)^4 \, \delta\left(\sum p_i - \sum p_f\right) \boxed{i\mathcal{M}_{i\to f}}$$

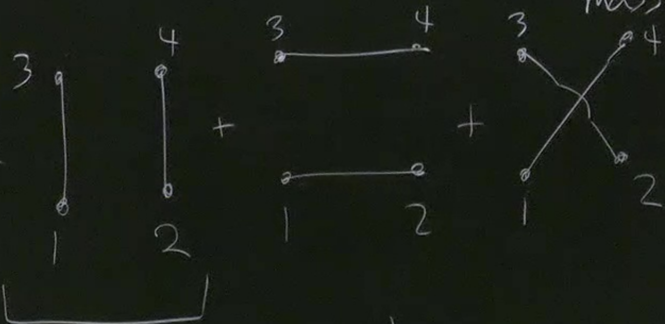
LSZ:

can compute using Feynman diagrams

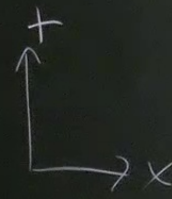
$$\langle f|S|i\rangle = \prod \int dx_i \, e^{-i\lambda_i p_i \cdot x_i} (\partial_i^2 + m^2) \langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle$$

physical mass

$$\langle \Omega | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | \Omega \rangle =$$



$+ \, \mathcal{O}(\lambda)$



$\Delta_{13}\Delta_{24}$ factorizes!

exchange of subdiagrams $S = 8 \cdot 8 \cdot 2$

from $\exp(\ ) = 1 + (\ ) + \frac{1}{2}(\ ) \ldots$

exteral

internal

...

nected

m diagram + ...

d vacuum diagrams]

(13) factor of || contribution to $\langle f|S|i\rangle$

$$F(x_1 - x_3) = (\partial_1^2 + m^2)(\partial_3^2 + m^2)\,\Delta_{13}$$

$$\Delta_{13} = \Delta(x_1 - x_3)$$

$$\int d^4x_1\, d^4x_3\, e^{-ip_1\cdot x_1 + ip_3\cdot x_3}\, F(x_1 - x_3)$$

change variables to $x_{13} = x_1 + x_3$ $\qquad P_{13} = \frac{p_1 + p_3}{2}$

$\bar{x}_{13} = x_1 - x_3$ $\qquad \bar{P}_{13} = \frac{p_1 - p_3}{2}$

independent of $x_{13} \to \delta(\bar{P}_{13})$

$\langle f|S|i\rangle_{11} \propto \delta(p_1 - p_3)\,\delta(p_2 - p_4)$

Extra delta!

1 Delta per connected component

$\rightarrow$ 4-momentum conserved separately!