Title: Effective cuscuton theory

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Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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Abstract:

Cuscuton field theory is an extension of general relativity that does not introduce additional propagating degrees of freedom, or violate relativistic causality. We construct a general geometric description of the cuscuton field theory by introducing curvature corrections to both the volume (potential) and the surface (kinetic) terms in the original cuscuton action. Our assumptions involve a stack of spacelike branes, separated by 4-dimensional bulks. We conjecture that the cuscuton, initially a discrete field, becomes continuous in the limit, there are many such transitions. From this we derive an effective action for the cuscuton theory and show that at the quadratic level our theory propagates only the two tensorial degrees of freedom.

Effective Cuscuton Theory

2312.06066[hep-th]

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50 Years of Horndeski Gravity
19 July 2024

Non-propagating fields?

- QH systems: Maxwell (2-form) fields

How the QH state samples the topology of the space the system is in

- SLED models: (4-form) fields

Tell the EFT that flux in the extra dimensions is quantized

[Burgess, Diener, Williams (2015)]

- Low energy 4-dimensional EFT of string compactifications

Non-propagating topological (4-form) fields

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The cuscuton

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_P^2}{2}R^{(4)} + \mu^2\sqrt{X} - V(\phi)\right), \quad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \quad X > 0$$

- Does not propagate scalar dof [Afshordi, Chung, Doran, Geshnizjani, (2007)]
- Applications to modified gravity (no need for screening!)
  [Gao et al (2011); Hiramatsu & Kobayashi (2022); K. Aoki, A. De Felice, C. Lin, S. Mukohyama (2019)]
- Applications for VSL theories – i.e., UV limit of anti-DBI $\mathcal{L}_{aDBI} \sim \frac{1}{B(\phi)}\sqrt{1+2B(\phi)X}\Big|_{X\gg 1} \to \sim \sqrt{X}$
  [D. Bessada, W. H. Kinney, D. Stojkovic and J. Wang (2010), Afshordi & Magueijo (2016), MM, Moschou, Afshordi & Magueijo, (2021)]
- In flat spacetimes the cuscuton possesses a scalarless symmetry [Tasinato, Chagoya (2017), Tasinato (2020)]
- Stable bouncing cosmologies! [see Amir's talk!]
- The cuscuton is the low-energy limit to Horava-Liftzing gravity [see Sergey's talk]
- [Afshordi (2009)]

The cuscuton

Afshordi, Chung, Doran
Geshnizjani, (2007)

Action:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \mu^2\sqrt{X} - V(\phi)\right), \quad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \quad X > 0$$

EOM in FLRW:

$$H^2 = -\frac{1}{3}V(\phi)$$

$$\dot{H} = -\frac{3H^2}{2} + \frac{1}{2}\mu^2\sqrt{\dot{\phi}^2} - \frac{1}{2}V(\phi)$$

$$3\mu^2 H = -V'(\phi)$$

Pure constraint system

$$\frac{M_P^2}{3\mu^4}V'^2(\phi) - V(\phi) = \rho_m$$

- Non-dynamical auxiliary field, provides constraint equations which modify the dynamics of the fields it couples to.
- If we fix ρ_m this consequently fixes ϕ.

Superluminal field

$$c_s = \infty$$

Afshordi, Chung, Doran
Geshnizjani (2007)
Afshordi (2009)

- Infinite sound speed – but does not propagate information outside the light cone
  (it has no internal dynamics – no phase space). $(\mathrm{d}\Pi \wedge \mathrm{d}\phi = 0)$

Requires: restricted set of boundary conditions

- Enforce constraints at spatial infinity
- Well posed in the frame the cuscuton is uniform, which lead to elliptic equations.

- Geometric perspective: CMC surfaces that do not intersect in the bulk.

Enlarged set of symmetries - protected from radiative corrections

Pajer, Stefanyszyn (2019)
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Geometric picture of the cuscuton

Afshordi, Chung, Doran Geshnizjani 2007

Scalar field equ:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}\frac{g^{\mu\nu}\partial_\nu\phi}{\sqrt{X}}\right]+V'(\phi)=0$$

Unit normal vectors for constant ϕ hypersurfaces:

$$n^\mu \equiv \frac{\partial^\mu\phi}{\sqrt{X}}$$

From the extrinsic curvature $K=\nabla_\mu n^\mu$:

$$K(\phi)=-\frac{V'(\phi)}{\mu^2}$$

- The mean curvature on constant ϕ hypersurfaces is only a function of ϕ and hence constant (CMC)

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Geometric picture of the cuscuton

Afshordi, Chung, Doran
Geshnizjani 2007

Think of soap bubbles and films!

CMC: in Euclidean space can be seen as a surface where the exterior pressure and surface tension forces balance

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left[\mu^2|n^\mu\partial_\mu\phi| - V(\phi)\right]$$
$$= \mu^2\int_\phi \mathrm{d}\phi\,\Sigma(\phi) - \int \mathrm{d}^4x\sqrt{-g}V(\phi)$$

Pressure difference across the surface

- Solve to find the surface of the bubble

$$-\mu^2\Delta\phi K_{dis} = \Delta V$$

surface tension

Mean extrinsic curvature of the constant-ϕ surface

Extending the cuscuton theory

Will the theory be cuscuton-like if we add higher-order curvature terms?

i.e., at certain energy scales gravity may need to be supplemented by higher-order operators.

What makes a theory cuscuton-like?

How to build an EFT for a non-propagating degree of freedom?

- ~~Fundamental symmetries.~~
- ~~Available dof's.~~

- **Theory that generalizes the surface and volume terms**

Extending the geometric picture

- Replace 2d surfaces or soap films with 3d spatial hypersurfaces of constant ϕ
- Replace Euclidean space with 4d Lorentzian spacetime

S-branes

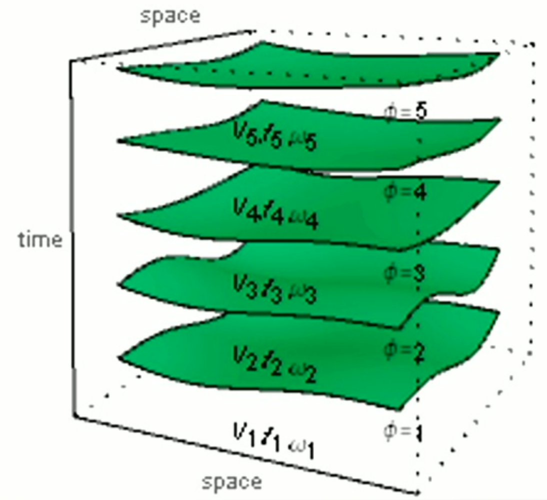
(spacelike branes)

Topological defects localized on a spatial hypersurface representing an instant in time.

- corrections to the cuscuton kinetic term live on the boundaries or S-brane
- corrections to the potential live in the 4-dimensional bulk

Extending the geometric picture

- Consider a stack of spatial (3+0d) branes living in a 3+1d bulk. Surfaces do not talk to each other.
- Assume cuscuton is a discrete field (labelling transitions by $\phi = 1, \ldots, n$. It becomes continuous in the limit of many such transitions.
- Discontinuous jumps in spacetime (deformed branes interface between different phases/vacua).
- We expect, bulk terms (E.H., Lovelock, e.t.c.) will have associated boundary terms.
- Cuscuton action: sum and take the continuous limit of the discrete transitions.



Sum over geometric invariants of the boundaries and bulks, ordered in powers of curvature

Approx. continuous limit

- Approximate the sum as an integral over geometric invariants
- Unitary gauge (homogeneous scalar field ϕ=ϕ(t), gradient points in the direction of time).

Discrete action:

$$S_{\rm disc} = \sum_\phi \mathcal{V}(\phi, \phi + \Delta\phi) \times V(\phi) + \sum_\phi \mathcal{S}(\phi) \times c_1(\phi)\Delta\phi$$

- $\mathcal{V}(\phi, \phi + \Delta\phi)$ is the volume enclosed between ϕ and $\phi + \Delta\phi$ surfaces
- $\mathcal{S}(\phi)$ is the area of the ϕ-hypersurface.

Take the continuous limit:

$$\sum_\phi \mathcal{V}(\phi, \phi + \Delta\phi) \times V(\phi)\Big|_{\Delta\phi\to 0} = \int \mathrm{d}^4x\sqrt{-g}\, V(\phi), \quad \sum_\phi \mathcal{S}(\phi) \times c_1(\phi)\Delta\phi\Big|_{\Delta\phi\to 0} = \int \mathrm{d}^3x\, \mathrm{d}\phi\, \sqrt{\gamma}\, c_1(\phi),$$

$$\int \mathrm{d}^3x\, \mathrm{d}\phi\, \sqrt{\gamma}\, c_1(\phi) = \int \mathrm{d}^4x\, \sqrt{-g}\, c_1(\phi)\frac{1}{N} = \int \mathrm{d}^4x\, \sqrt{-g}\, c_1(\phi)\sqrt{X}$$

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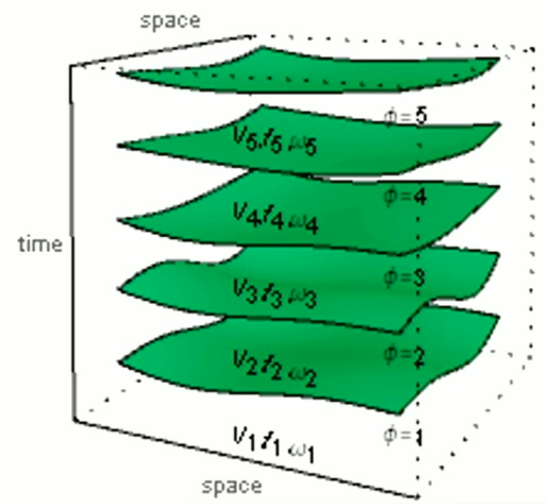

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Effective Cuscuton Theory (ECT)

No surface term

terms that live in the bulks

dimensions $\phi = \tilde{\phi}/\Lambda^3$

Action:

$$S_{CET} = \Lambda^4 \int \mathrm{d}^4x\sqrt{-g}\Big[V(\phi) + \frac{f(\phi)}{2\Lambda^2}R + \frac{\omega(\phi)}{2\Lambda^4}R_{GB} + \frac{c_1(\phi)}{\Lambda}\sqrt{X} - \varepsilon\frac{c_2(\phi)}{\Lambda^2}\sqrt{X}K + \frac{c_3(\phi)\sqrt{X}}{\Lambda^3}\mathcal{R} + \frac{c_4(\phi)}{\Lambda^4}\sqrt{X}\mathcal{K}_{GB}\Big]$$

terms that live on the boundaries

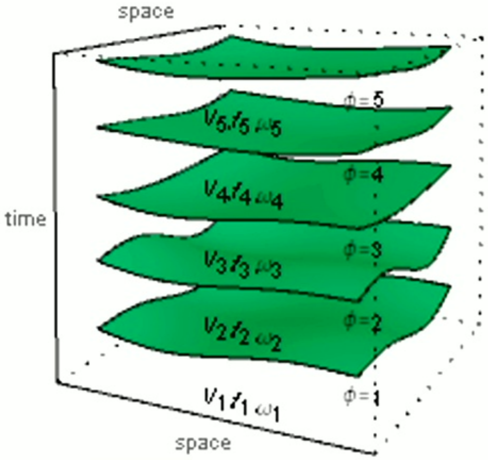
Brane thickness → 0

$$\mathcal{K}_{\rm GB} = -2\big(J - 2\varepsilon G^{ij}K_{ij}\big), \quad J_{ij} = \frac{1}{3}\big(2KK_{ic}K^c{}_j + K_{cd}K^{cd}K_{ij} - 2K_{ic}K^{cd}K_{dj} - K^2K_{ij}\big)$$

[Davis 2003]

In the continuous limit, the couplings are taken to be slow-varying functions of the scalar field ϕ.

Note: we can recover the original cuscuton action: $f = M_{Pl}^2, \quad c_1 = -\mu^2$ and set rest to zero.



Effective Cuscuton Theory (ECT)

Action:

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- Each bulk contains distinct values for the cosmological constant, gravitational constant and couplings to the Lovelock terms.
- Each brane contains distinct values for the coefficients of the gravitational surface terms.
- Whenever there is a discontinuity/jump in the bulk couplings, there will be a corresponding surface term associated with it.

Effective Cuscuton Theory (ECT)

Not all couplings are independent!

The effective couplings corresponding to the E.H. and G.B are complately determined by the geometry.

In the continuous limit…

$$c_2(\phi) = \lim_{\Delta\phi \to 0} \frac{f(\phi + \Delta\phi) - f(\phi)}{\Delta\phi} = \frac{\partial f(\phi)}{\partial \phi},$$

$$c_4(\phi) = \lim_{\Delta\phi \to 0} \frac{\omega(\phi + \Delta\phi) - \omega(\phi)}{\Delta\phi} = \frac{\partial \omega(\phi)}{\partial \phi}$$

… the surface couplings represent the rate of change of the bulk curvature couplings as we transition from one vacuum to another.

The boundary is bookkeeping that something physical is changing.

(Ensures the ECT propagates only two tensorial dof's)

Scalar-Tensor theory

Action:

$$S_{ECT} = \Lambda^4 \int \mathrm{d}^4x\, N\sqrt{\gamma}\Big[V(\phi) + c_1(\phi)\frac{\sqrt{X}}{\Lambda} + \frac{f(\phi)}{2\Lambda^2}\Big(K_{ab}K^{ab} + K^2 + \mathcal{R} - \frac{2D^aD_aN}{N} + 2\mathcal{L}_{\mathbf{n}}K\Big) - \varepsilon c_2(\phi)\frac{\sqrt{X}K}{\Lambda^2}\Big]$$

IBP:

$$S_{ECT} = \Lambda^4 \int \mathrm{d}^4x\, N\sqrt{\gamma}\Big[V(\phi) + c_1(\phi)\frac{\sqrt{X}}{\Lambda} + \frac{f(\phi)}{2\Lambda^2}\Big(K^2 - K_{ab}K^{ab} + \mathcal{R}\Big) - \frac{1}{\Lambda^2}f'(\phi)\sqrt{X}K - \varepsilon c_2(\phi)\frac{\sqrt{X}K}{\Lambda^2}\Big]$$

use: $\varepsilon = -1$ and $c_2(\phi) = f'(\phi)$

$$\Rightarrow \quad \boxed{S_{ECT} = \Lambda^4 \int \mathrm{d}^4x\, N\sqrt{\gamma}\Big[V(\phi) + c_1(\phi)\frac{\sqrt{X}}{\Lambda} + \frac{f(\phi)}{2\Lambda^2}\Big(K^2 - K_{ab}K^{ab} + \mathcal{R}\Big)\Big]}$$

Subset of MMG-II [Lin, Mukohyama (2017)]with $L = NF(K_{ij}, R_{ij}, \gamma^{ij}, t) + G(K_{ij}, R_{ij}, \gamma^{ij}, t)$ with no Einstein frame.

Some intuition from the geometric picture

The contributions from the surface terms, induce cancellations such that we get small corrections to the CMC condition

EOMs:

$$0 = V(\phi) - \frac{f(\phi)}{2}\left(K^2 - K_{cd}K^{cd} - \mathcal{R}\right),$$

$$0 = V'(\phi) - c_1(\phi) K \frac{\mathcal{L}_{\mathbf{n}}\phi}{\sqrt{X}} + \frac{f'(\phi)}{2}\Big(K^2 - K_{ab}K^{ab} + \mathcal{R}\Big),$$

Generalization of the CMC condition

$$K = \frac{1}{c_1(\phi)}\left[\frac{3}{2}\left(V'(\phi) + f'(\phi)\mathcal{R}\right)\right]$$

→

Background level

$$V'(\phi) - c_1(\phi)\sqrt{\frac{3\,V'(\phi)}{4\,f'(\phi)}} = 0$$

Need to add matter ρ_m to the mix.

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With GB

The action has the form:

$$S_{ECT} = \Lambda^4 \int \mathrm{d}^4x \, N\sqrt{\gamma} F(K_{ij}, R_{ij}, D_i, \gamma^{ij}, \mathcal{L}_{\mathbf{n}}, t)$$

which is of the form [Z.-B. Yao, M. Oliosi, X. Gao and S. Mukohyama, (2023)] apart of the Lie derivative…

(a Hamiltonian analysis may not be possible due to the mixing of time and spatial partial derivatives of the metric)

but with a suitable substitution…

$$K = \frac{1}{c_1(\phi)}\left[\frac{3}{2}V'(\phi) + f'(\phi)\mathcal{R} + \omega'(\phi)(B+C)\right]$$

$$B = 12K_a{}^c K^{ab} K_b{}^d K_{cd} + \frac{8}{3}K K_a{}^c K^{ab} K_{cd} + K_{ab}K^{ab}K_{cd}K^{cd} - 2K^2 K_{cd}K^{cd} + \frac{1}{3}K^4,$$

$$C = \mathcal{R}_{GB} + 8\, D_b D_a G^{ab} + 16\frac{D_c D_a N}{N}\left(K_a{}^c K^{ab} - K K^{bc}\right) + 8\frac{D_c D^c N}{N}\left(K^2 - K_{ab}K^{ab}\right),$$

Discussion

- Will the EFT propagate scalar dof?
- Is ECT ghost free?
- Perturbation theory?
- Phenomenology
- Relationship with VSL theories?
- Can ECT be extended further?

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