Title: Scattering amplitudes in high-energy limit of projectable Horava gravity

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Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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Abstract:

We study the high-energy limit of projectable Ho\v rava gravity using on-shell graviton scattering amplitudes. We compute the tree-level amplitudes using symbolic computer algebra and analyze their properties in the case of collisions with zero total momentum. The amplitudes grow with collision energy in the way consistent with tree-level unitarity. We discuss their angular dependence and derive the expression for the differential cross section that happens to depend only on the essential combinations of the couplings. One of our key results is that the amplitudes for arbitrary kinematics are finite when the coupling λ in the kinetic Lagrangian is taken to infinity -- the value corresponding to candidate asymptotically free ultraviolet fixed points of the theory. We formulate a modified action which reproduces the same amplitudes and is directly applicable at λ=∞, thereby establishing that the limit λ→∞ of projectable Ho\v rava gravity is regular. As an auxiliary result, we derive the generalized Ward identities for the amplitudes in non-relativistic gauge theories.

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Scattering Amplitudes in Horndeski Gravity

(JR, S. Sibiryakov PRD 108)

Jury Radkovski
50 years of Horndeski Gravity

Introduction and Motivation

Barvinsky '23, Herrero-Valea '23

- (projectable) Hořava Gravity (pHG) is a renormalisable Lorentz-symmetry-breaking QFT of gravity
- The action:

$$S = \frac{1}{2G}\int dt\Big(T[\partial_t \text{metric}; \lambda] - V[\text{metric}; \text{other couplings}]\Big)$$

$$V \sim V[\nabla^6\gamma] \implies \text{Propagators} \sim \frac{1}{\omega^2 - p^6}$$

Gumrukcuoglu, Mukohyama '11

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- The RG flow at $\lambda = \infty$ has asymptotically free UV fixed points
- But...

$$T \xrightarrow{\lambda \to \infty} \infty$$ Gumrukcuoglu, Mukohyama '11

- Consider an observable to see if this limit makes sense physically

Formulating the Theory

- Separating spacetime into space *and* time: the theory is postulated to be invariant at high energies and at tree level under anisotropic scaling Hořava '09

$$x \to b^{-1} x, \quad t \to b^{-d} t$$

- Theory is no longer diffeomorphism invariant, but symmetric under foliation preserving diffeomorphisms (FDiffs)

$$x \to \tilde{x} = x(\mathbf{x}, t), \quad t \to \tilde{t}(t), \quad \tilde{t}(t) - \text{monotonic function}$$

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Formulating the Theory

- The (*non-projectable*) action

$$S = \frac{1}{2G}\int d^3x dt N\sqrt{\gamma}\left(K_{ij}K_{ij} - \lambda K^2 - \mathcal{V}\right)$$

$K_{ij} \sim \dot{\gamma}_{ij}$

Passes observational tests
Gumrukcuoglu et al. '18

where

$$\mathcal{V} \sim 2\Lambda - \eta R + \mu R_{ij}^2 + \nu R_{ij}^3 + \nu'(\nabla R_{ij})^2 + \mathcal{V}_N\left[\partial_i \log N\right]$$

Extra spatial derivatives

- GR formally:

$$\mathcal{V}_N, \mu, \nu, \nu' \to 0, \quad \lambda, \eta \to 1$$

No stable Minkowski vacuum
Koyama et al. '09, Blas et al. '11

Running to $\lambda = 1$ Barvinsky et al. '23

Strong coupling Mukohyama '10, Izumi, Mukohyama '11

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$$N \to N(t) \implies \text{set } N \to 1,\ \mathcal{V}_N \to 0$$

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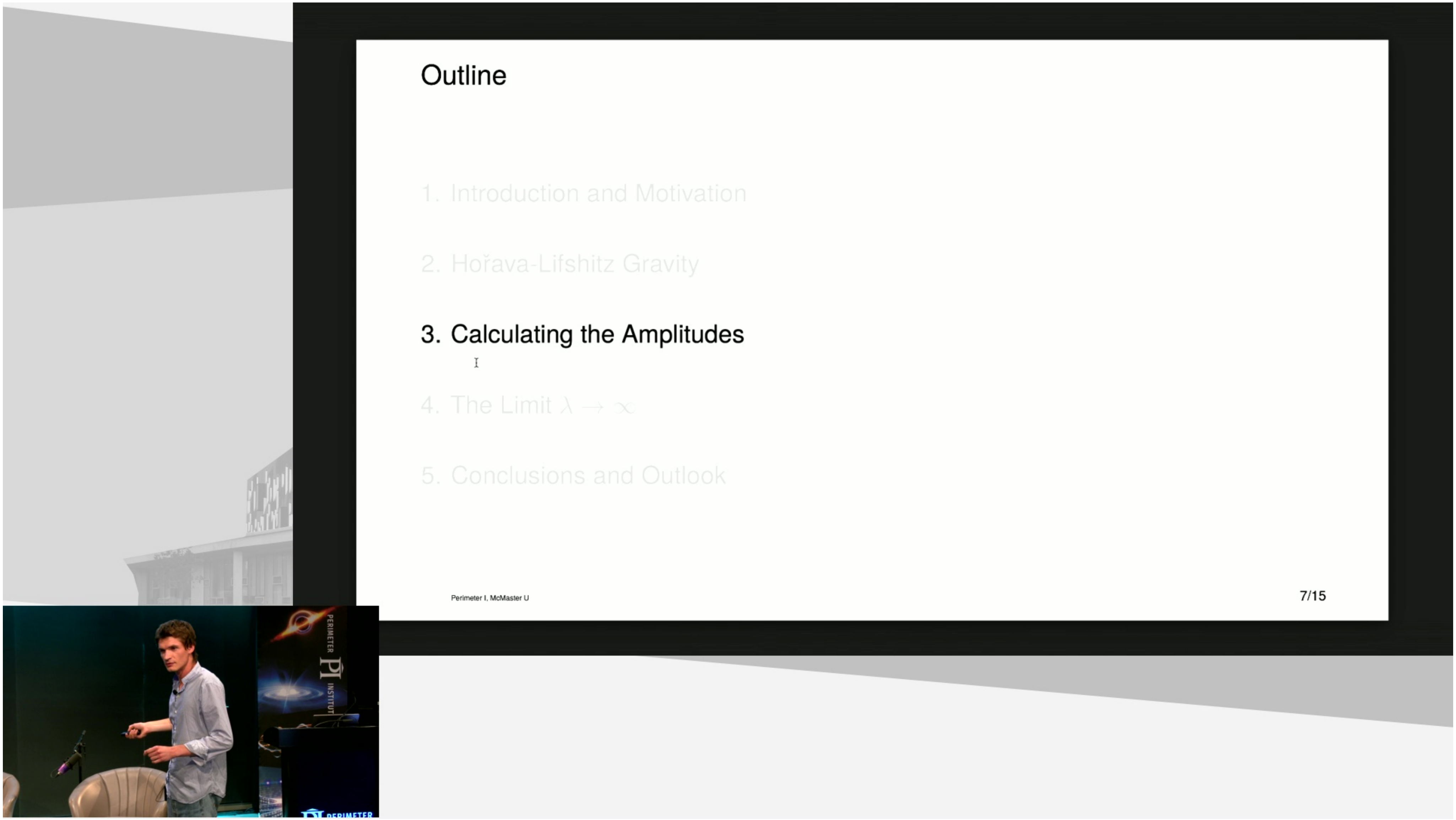
Outline
1. Introduction and Motivation
2. Hořava-Lifshitz Gravity
3. Calculating the Amplitudes
4. The Limit $\lambda \to \infty$
5. Conclusions and Outlook
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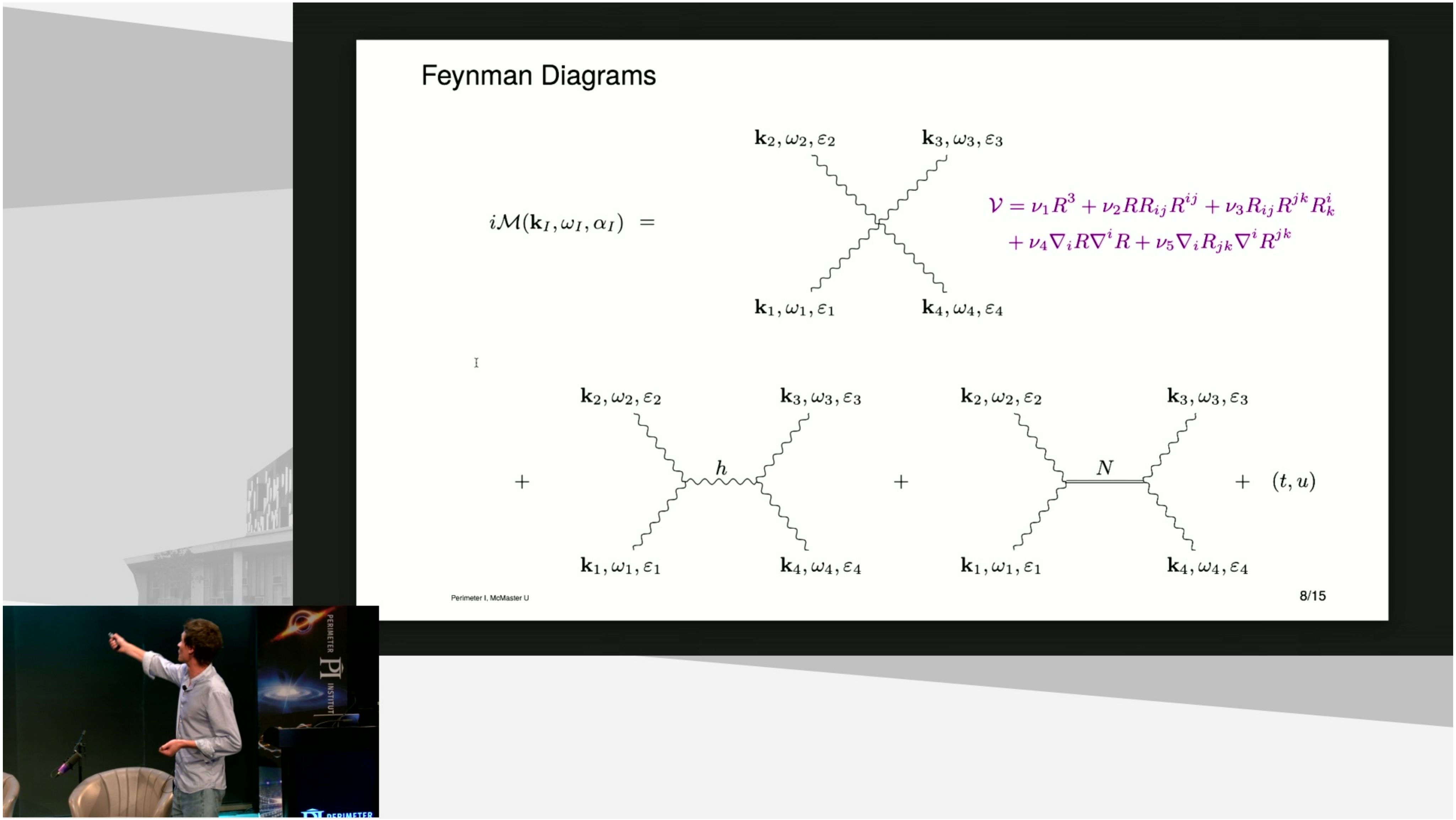
Feynman Diagrams
$i\mathcal{M}(\mathbf{k}_I, \omega_I, \alpha_I) =$
$\mathbf{k}_2, \omega_2, \varepsilon_2$
$\mathbf{k}_3, \omega_3, \varepsilon_3$
$\mathbf{k}_1, \omega_1, \varepsilon_1$
$\mathbf{k}_4, \omega_4, \varepsilon_4$
$\mathcal{V} = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R_{ij} R^{jk} R^i_k + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk}$
$+$
h
$+$
N
$+ \quad (t, u)$
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Overview

- Use standard Feynman rules (and Mathematica) to compute the amplitudes
- Similar to GR DeWitt ’67, Sannan ’86

$$\mathcal{M} = \sum \text{contractions of } \varepsilon_i \text{ and } \mathbf{k}_i$$

- In HG the structure is richer due to the presence of higher powers of momenta (higher spatial derivatives) in the vertices. Extra terms:

$$(\mathbf{k}_3\varepsilon_1\varepsilon_2\mathbf{k}_4)(\mathbf{k}_1\varepsilon_3\mathbf{k}_1)(\mathbf{k}_2\varepsilon_4\mathbf{k}_2)\ , \qquad (\mathbf{k}_2\varepsilon_1\mathbf{k}_2)(\mathbf{k}_1\varepsilon_2\mathbf{k}_1)(\mathbf{k}_4\varepsilon_3\mathbf{k}_4)(\mathbf{k}_3\varepsilon_4\mathbf{k}_3)$$

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- The amplitudes obey the generalised Ward identities and are independent of gauge parameters if on-shell

Head-On Amplitudes

- The helicity amplitudes have the form

$$\mathcal{M}_{\alpha_1\alpha_2,\alpha_3\alpha_4} = GE^2 f_{\alpha_1\alpha_2,\alpha_3\alpha_4}\left(\cos\theta; u_s, v_a, \lambda\right)$$

Blas et al. '10

$$u_s = \sqrt{\frac{\nu_s}{\nu_5}}, \quad v_a = \frac{\nu_a}{\nu_5}$$

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Head-On Amplitudes

GR: $f_{++,++} = \frac{1}{1-\cos^2\theta}$

HG: $f_{++,++} = \frac{P(\cos\theta; u_s, v_a, \lambda)}{(1-\cos^2\theta)^3}$

- The helicity amplitudes have the form

$$\mathcal{M}_{\alpha_1\alpha_2,\alpha_3\alpha_4} = GE^2 f_{\alpha_1\alpha_2,\alpha_3\alpha_4}(\cos\theta; u_s, v_a, \lambda)$$

$$u_s = \sqrt{\frac{\nu_s}{\nu_5}}, \quad v_a = \frac{\nu_a}{\nu_5}$$

- Scaling $\sim E^2$ is compatible with unitarity Blas et al. '10
- Cross section

$$\frac{d\sigma_{\alpha_1\alpha_2,\alpha_3\alpha_4}}{\sin\theta\, d\theta} = \frac{G^2}{72\pi\,\nu_5\, k^2}\left|f_{\alpha_1\alpha_2,\alpha_3\alpha_4}\right|^2 \propto \lambda^2_{\text{de Broglie}}.$$

diverges at small angles signaling the necessity of an infrared regulator

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Head-On Amplitudes

- This amplitude is absent in GR, but present in HG (and non-divergent!)

$$f_{++,--} = f_{--,++} =$$
$$= \frac{1}{512\hat{u}_s^2}\Big[3x^2\Big(-35 + 64v_2^2 - 16v_2(1-7v_3) + 501\hat{u}_s^2 + 3v_3^2(15-\hat{u}_s^2)$$
$$+ 2v_3(5-79\hat{u}_s^2)\Big) + 121 + 64v_2^2 - 1375\hat{u}_s^2 + 9v_3^2(1+\hat{u}_s^2) + 66v_3(1+13\hat{u}_s^2)$$
$$+ 16v_2(11+3v_3+32\hat{u}_s^2)\Big]$$

Collinear divergence can be compensated by the orbital wave functions from the angular momentum conservation

- Tensor-Scalar scattering

$$f_{\pm s,\pm s} = \frac{2(1-\lambda)}{(1-3\lambda)}\frac{P_{\pm s,\pm s}(x)}{64u_s^2(1-x)^3 g(x)}, \quad g(x) = \big((1-u_s)^2 - 8(1+x)^3\big)\big)\big((1-u_s)^2 - 8u_s^2(1+x)^3\big)$$

Poles at non-zero angles due to the decay

The Limit $\lambda \to \infty$

- Explicit cancellation of potentially divergent terms
- Manifest regularization of the theory by integrating in an auxiliary non-dynamical scalar field χ:

$$-\frac{\lambda}{2G}\sqrt{\gamma}\,K^2 \quad \longrightarrow \quad \frac{\sqrt{\gamma}}{G}\left[-\chi K + \frac{\chi^2}{2\lambda}\right]$$

Take the limit (with other couplings fixed) and get for the action of HG

$$S \xrightarrow[\lambda\to\infty]{} S' = \frac{1}{2G}\int d^3x dt \sqrt{\gamma}\left(K_{ij}K^{ij} - 2\chi K - \mathcal{V}\right)$$

- The field χ takes the role of a Lagrange multiplier constraining the extrinsic curvature to be traceless, $K = 0$

Conclusions and Outlook

Conclusions

- Generalised Ward identities for non-relativistic gauge theories
- Full set of tree level 2 to 2 scattering amplitudes
- The limit $\lambda \to \infty$ is a viable location for asymptotically free UV fixed points

Ground State of pHG, i.e. the fate of the instability

Outlook

- Spinor-Helicity formalism?
- Beyond tree level: running of the dispersion relations
- Amplitudes in non-projectable version

Renormalization of non-projectable theory

Bellorin et al. '23

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