Title: Formulating the complete initial boundary value problem in numerical relativity to model black hole echoes

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Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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Abstract:

Recently, there has been much interest in black hole echoes, based on the idea that there may be some mechanism (e.g., from quantum gravity) that waves/fields falling into a black hole could partially reflect off of an interface before reaching the horizon. There does not seem to be a good understanding of how to properly model a reflecting surface in numerical relativity, as the vast majority of the literature avoids the implementation of artificial boundaries, or applies transmitting boundary conditions. Here, we present a framework for reflecting a scalar field in a fully dynamical spherically symmetric spacetime, and implement it numerically. We study the evolution of a wave packet in this situation and its numerical convergence, including when the location of a reflecting boundary is very close to the horizon of a black hole. This opens the door to model exotic near-horizon physics within full numerical relativity.

Formulating the complete initial boundary value problem in numerical relativity to model black hole echoes

50 Years of Horndeski Gravity 2024

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July 16, 2024

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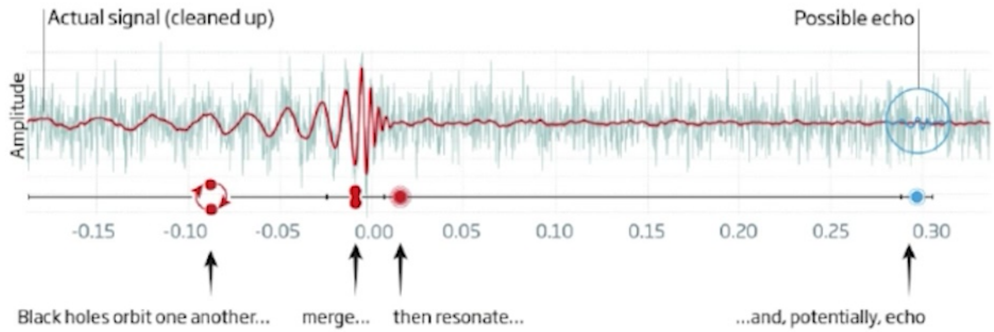
Black Hole Echoes

What are black hole echoes?

- Quantum gravity arguments suggest reflecting surface near the horizon of black holes
- May lead to reflections of gravitational waves/matter waves
- Would look like an echo of the original gravitational wave signal
- How do we simulate this in numerical relativity?



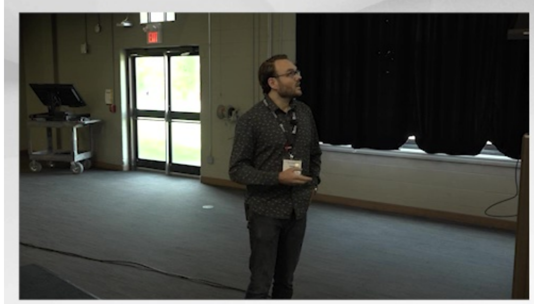
SOURCE: doi.org/bchw; NIAYESH AFSHORDI AND JAHED ABEDI

Numerical Methods

Simulate black hole echoes with boundary conditions on a numerical boundary near the horizon (This is difficult)

State of the art numerical methods for solving PDEs:

- Symmetric Hyperbolic formulations (Einstein-Christoffel, Generalized Harmonic)
- Summation by parts (SBP) derivative operators
- Boundary conditions implemented with Simultaneous Approximation Terms
- Constraint damping and constraint-preserving boundary conditions



Lessons from a scalar field in spherical symmetry

Scalar waves in GR: Reduction to a symmetric hyperbolic system

$$\nabla_\mu \nabla^\mu \phi = 0\,, \quad \Pi \equiv -\frac{1}{\alpha}(\partial_t \phi - \beta^r \partial_r \phi)\,, \quad \psi_r \equiv \partial_r \phi$$

Split into first order system in 1D

$$\begin{aligned} \partial_t \phi - \beta^r \partial_r \phi &= -\alpha \Pi \\ \partial_t \psi_r - \beta^r \partial_r \psi_r + \alpha \partial_r \Pi &= \cdots \\ \partial_t \Pi - \beta^r \partial_r \Pi + \frac{\alpha}{\gamma_{rr}} \partial_r \psi_r &= \cdots \end{aligned}$$

Identify the characteristic modes

$$U_\phi^\pm \equiv \Pi \pm \frac{\psi_r}{\sqrt{\gamma_{rr}}}\,, \quad c_\pm = -\beta^r \pm \frac{\alpha}{\sqrt{\gamma_{rr}}}$$

Lessons from a scalar field in spherical symmetry

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Local conservation of energy, BCs

$$\partial_t \int_a^b (\alpha\rho - \beta^r S_r)\sqrt{\gamma}\, dr = \frac{1}{4}\left[(c_- U_\phi^-)^2 - (c_+ U_\phi^+)^2\right] \gamma_{rr}\gamma_{\theta\theta}\Big|_a^b .$$

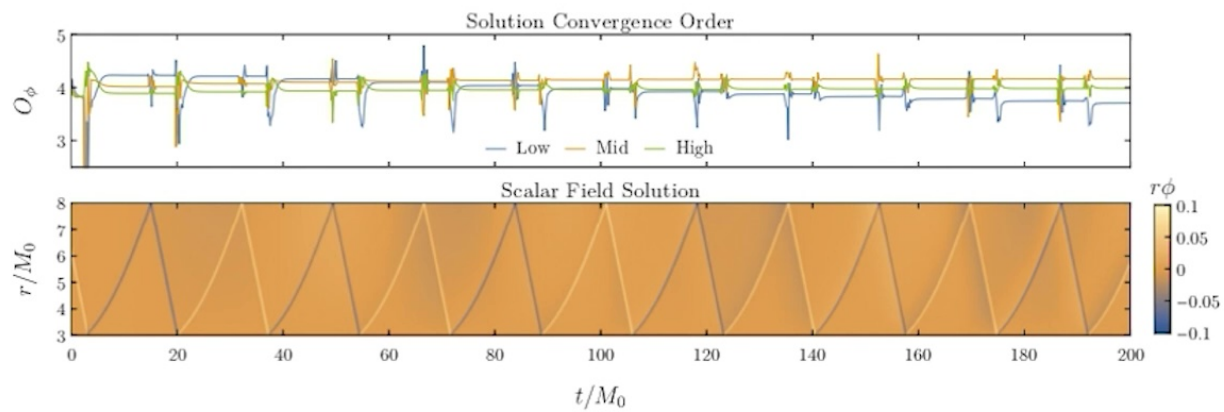
Implies BCs at $r = a$:

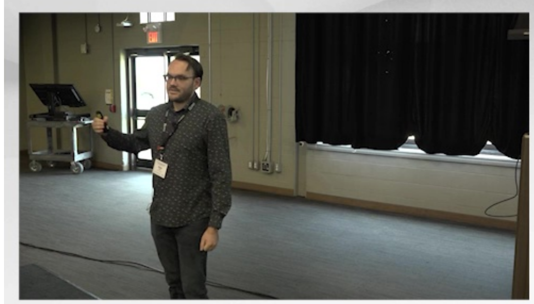
$$U_\phi^+(a) = -k_a \frac{c_-}{c_+} U_\phi^-$$

Reflecting Boundary Problems

Scalar wave around a fixed Schwarzschild black hole in Kerr-Schild coordinates

$$c_- = -1\,, \quad c_+ = \left(\frac{r-2M}{r+2M}\right)$$





Reflecting Boundary Conditions in GR

Based on Misner-Sharp Mass:

$$E_M = M(b) - M(a)\,, \qquad \partial_t E_M = \frac{\sqrt{\gamma}}{4\sqrt{\gamma_{\theta\theta}}}\left[c_+ U_\theta^- (U_\phi^+)^2 - c_- U_\theta^+ (U_\phi^-)^2\right]\Bigg|_a^b$$

Three boundary conditions at $r = a$:

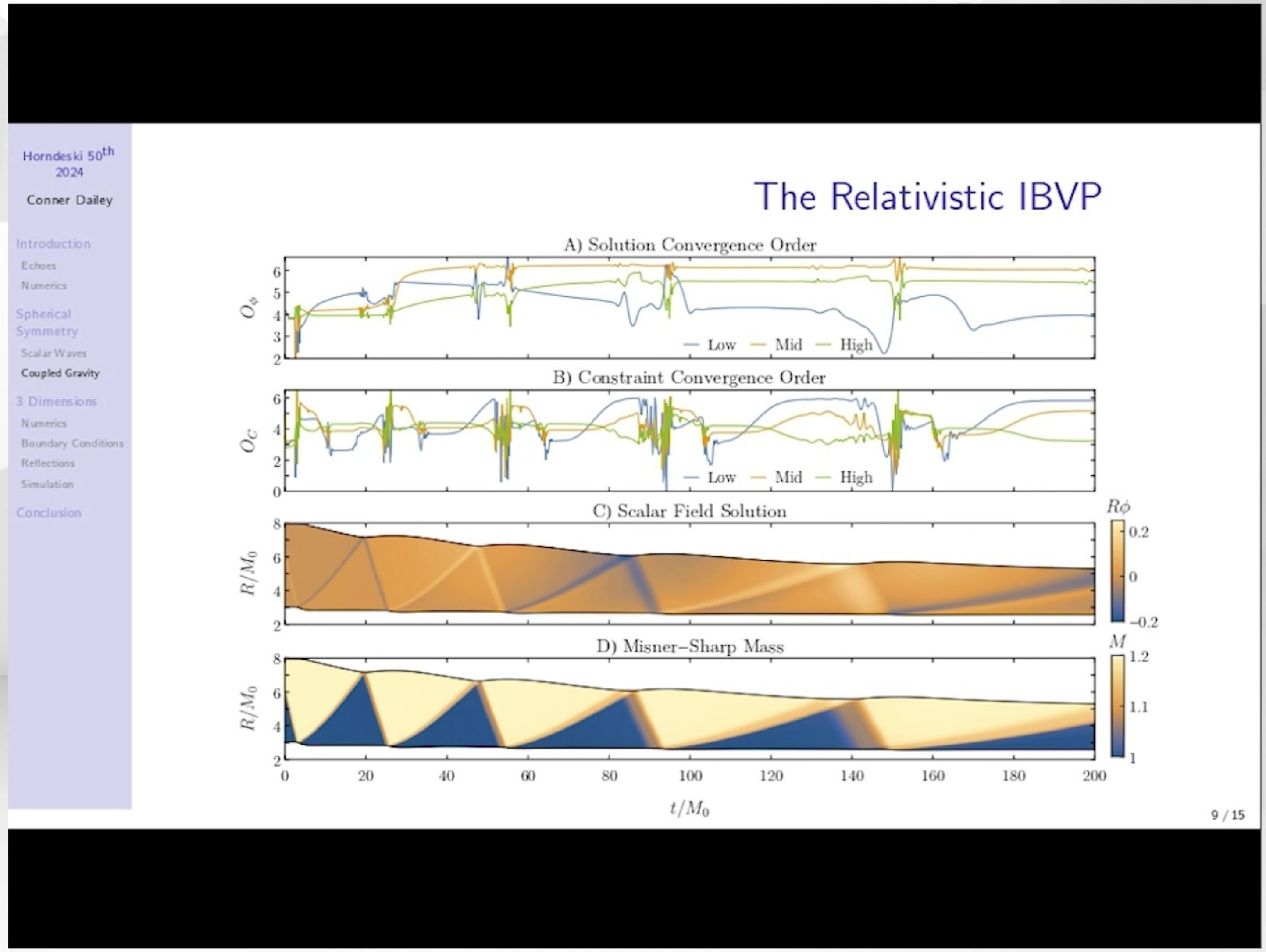
$$U_\phi^+(a) = -k_a \sqrt{\frac{c_- U_\theta^+}{c_+ U_\theta^-}}\, U_\phi^-$$

The k_a is a reflection coefficient, where $k_a = \pm 1$ corresponds to Neumann/Dirichlet style conditions

Angular mode comes from Misner-Sharp mass definition

$$U_\theta^+(a) = \frac{2M(a)\sqrt{\gamma_{\theta\theta}} - \gamma_{\theta\theta}}{U_\theta^-}$$

$U_r^+(a)$ modes arbitrary (controls boundary position)


Horndeski 50th 2024
Conner Dailey
Introduction
Echoes
Numerics
Spherical Symmetry
Scalar Waves
Coupled Gravity
3 Dimensions
Numerics
Boundary Conditions
Reflections
Simulation
Conclusion
The Relativistic IBVP
A) Solution Convergence Order
O_ϕ
Low
Mid
High
B) Constraint Convergence Order
O_C
Low
Mid
High
C) Scalar Field Solution
R/M_0
$R\phi$
0.2
0
−0.2
D) Misner−Sharp Mass
R/M_0
M
1.2
1.1
1
0
20
40
60
80
100
120
140
160
180
200
t/M_0
9 / 15

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3 Spatial Dimensions

Generalize this framework to 3D

1. Generalized Harmonic Symmetric Hyperbolic System
2. Embedded boundary numerical methods
3. Constraint preserving boundary conditions
4. Quasi-local conservation laws

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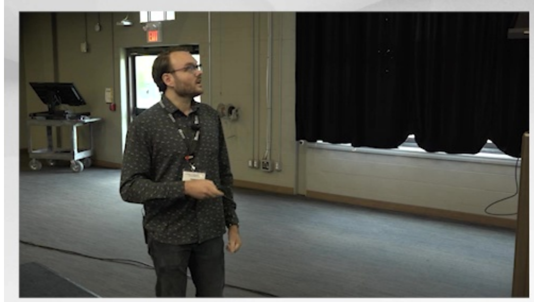
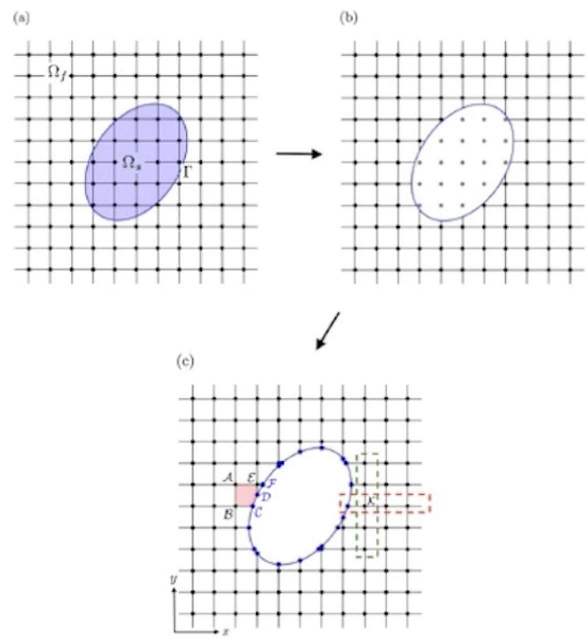


Embedded Boundary Methods[1]

- Store a rectangular grid array
- "Excise" a desired boundary region
- Defined embedded SBP differencing stencils
- Extrapolate the application of boundary conditions



[1] N. Sharan *et al.*, *Journal of Comp. Phys.* **464**, 111341 (2022).

Boundary Conditions

Choose incoming metric degrees of freedom $U^{-}_{\mu\nu} = \ell^{\alpha}\partial_{\alpha}g_{\mu\nu}$, with components split as:

$$\delta_{(\mu}{}^{(\alpha}\delta_{\nu)}{}^{\beta)} = \underbrace{C_{\mu\nu}{}^{\alpha\beta}}_{\text{4 D.O.F.}} + \underbrace{P_{\mu\nu}{}^{\alpha\beta}}_{\text{2 D.O.F.}} + \underbrace{G_{\mu\nu}{}^{\alpha\beta}}_{\text{4 D.O.F.}}$$

Dictates the satisfaction of the Einstein constraints	Dictates incoming gravitational waves	Dictates incoming gauge freedom

Ensures that the Einstein constraints are satisfied in the neighborhood of the boundary

Quasi-Local Conservation Laws

- Reflections based on Quasi-local Conservation laws

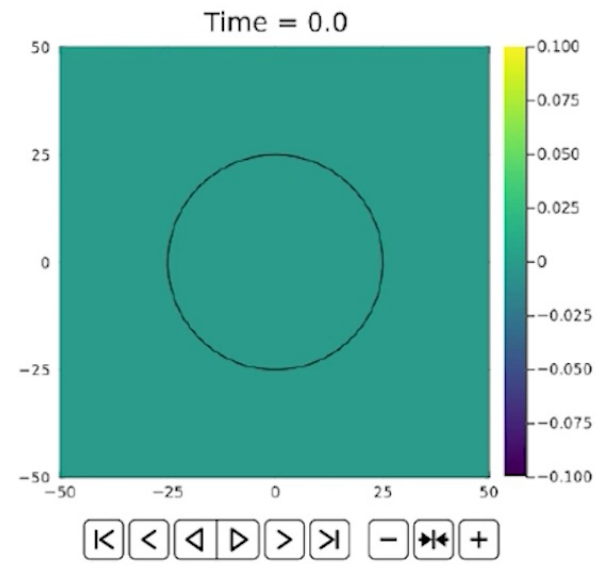
$$\partial_t \int \Pi_{ab} n^a n^b \, d\mathcal{S} = \int \left[T_{ab} s^a n^b - \Pi^{ab} K_{ab} \right] d\mathcal{S}$$

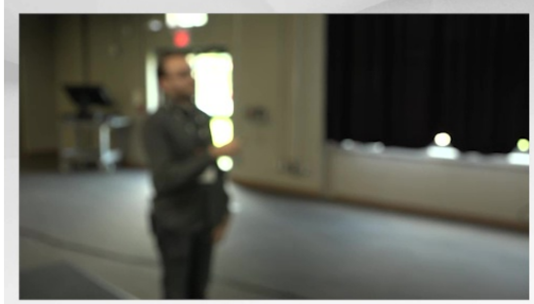
- Base reflections on the vanishing/control of the right hand side
- In principle, one can simulate gravitational plane wave scattering on a black hole surrounded by a "mirror"

Example with a spherically “excised” black hole

- Spherical boundary at $2.5M$
- Two incident gravitational wave pulses
- Time stable (checked to $t = 10,000M$)

- Working on evolving the boundary
- Working on applying reflecting conditions





Conclusion

- State of the art numerical methods can lead to stable relativistic IBVPs
- When a reflecting surface is near a horizon, this can model black hole echoes
- Stable IBVPs can help shrink domains when combined with Cauchy-characteristic matching
- Opens the door to model near horizon physics in a general fashion
- Ultimate goal: Pick your favorite quantum gravity/modified gravity near a horizon, use this framework to predict waveforms