Title: Quantum Energy Conditions and Bouncing Cosmologies

Speakers: Brayden Hull

Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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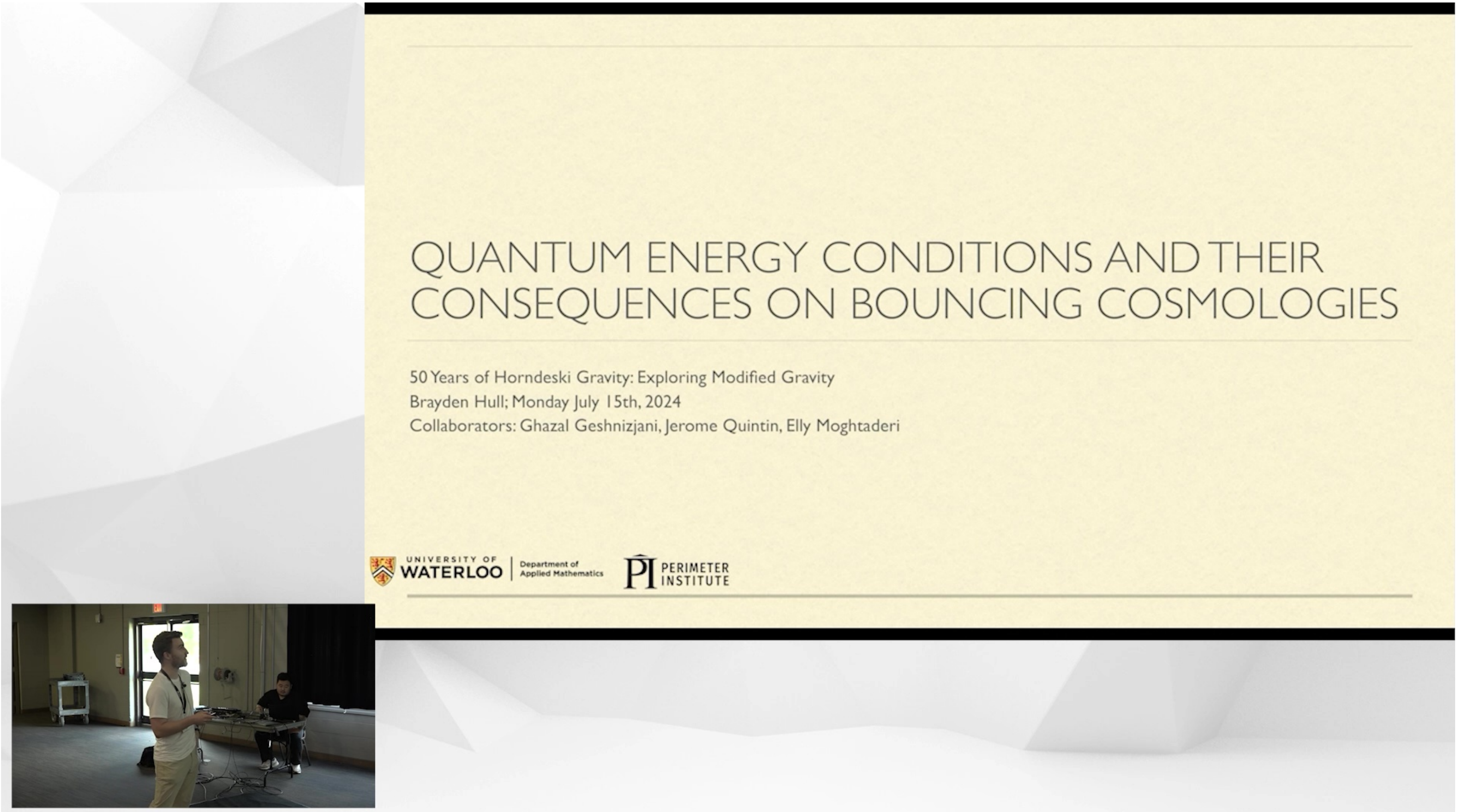
Abstract:

Classical energy conditions are used to provide restrictions on the matter fields present in the stress-energy tensor to avoid possible unphysical spacetimes. These classical energy conditions are imperative to the singularity theorems of Hawking and Penrose. However, we know that spacetime breaks down near said singularities and a quantum theory of gravity is needed. One insight into this area is semi-classical gravity where the spacetime is kept “classical” and the stress-energy tensor is quantized. In this regime one may ask what reasonable restrictions should be imposed on the quantum expectation of the stress-energy tensor? One such possibility is the smeared null energy conditions (SNEC). We will review motivation for the SNEC and explore its consequences in cosmological spacetimes that would otherwise violate the classical null energy condition, such as bouncing cosmologies.


QUANTUM ENERGY CONDITIONS AND THEIR CONSEQUENCES ON BOUNCING COSMOLOGIES
50 Years of Horndeski Gravity: Exploring Modified Gravity
Brayden Hull; Monday July 15th, 2024
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ENERGY CONDITIONS

The field equations of classical General Relativity can permit any solutions we want.

$$G_{\mu\nu} = T_{\mu\nu}$$

We want to consider only "realistic" matter fields so that our solutions are well behaved on physical grounds. The four "classical" energy conditions appear to be satisfied for ordinary matter.

$$\text{(DEC)} \quad \Rightarrow \quad \text{(WEC)} \quad \Rightarrow \quad \text{(NEC)}$$
$$\text{(SEC)} \quad \Rightarrow \quad \text{(NEC)}$$

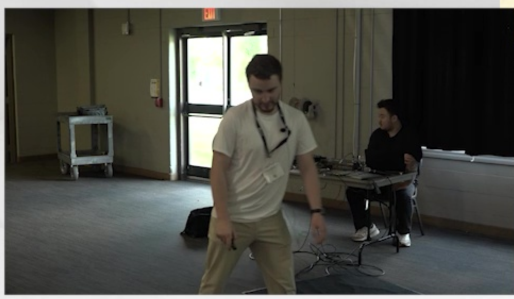
We know however that quantum fields can break the null energy condition.

Classical General Relativity breaks down at singularities which classical energy conditions are used to predict.

$$T_{\mu\nu}K^\mu K^\nu \geq 0 \quad \text{NEC} \quad \longrightarrow \quad R_{\mu\nu}K^\mu K^\nu \geq 0 \quad \text{(NCC)}$$

Penrose, 1965 PRL.

We expect Quantum Gravity to resolve this.



SEMI-CLASSICAL GENERAL RELATIVITY

Consider spacetime to remain "classical" while the matter fields and stress-energy tensor are quantized.

$$G_{\mu\nu}\left[g_{\alpha\beta}\right] = \langle T_{\mu\nu} \rangle$$

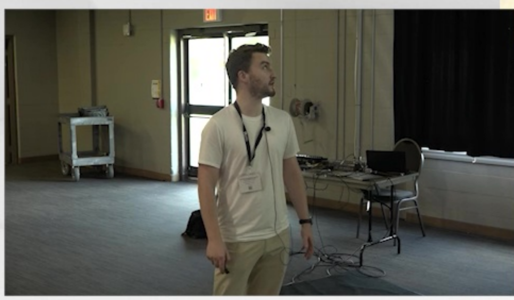
What conditions should exist on the expectation value of matter here?

These conditions are referred to as the quantum energy conditions.

Average Null Energy Condition Tipler, 1978 PRD.

$$\int_\gamma \langle T_{\mu\nu} \rangle K^\mu K^\nu \geq 0 \quad \Rightarrow \quad \int_{-\infty}^{\infty} \mathrm{d}\lambda \, \langle T_{\mu\nu}(\lambda) \rangle K^\mu K^\nu \geq 0$$

Global average, hard to apply in practice.



SMEARED NULLED ENERGY CONDITION(SNEC)

Builds up on the ANEC by considering a smearing along geodesic and a negative lower bound.

Key components: $\int_{-\infty}^{\infty} \mathrm{d}\lambda f(\lambda)^2 = 1$ Smearing function $f^2(\lambda) = \frac{1}{\sqrt{2\pi}\tau} \exp\left(-\frac{(\lambda-\lambda_0)^2}{2\tau^2}\right)$

$\langle F \rangle_\tau^{\mathrm{s}} \equiv \int_{-\infty}^{\infty} \mathrm{d}\lambda f(\lambda)^2 F(\lambda)$ Smearing definition

$\frac{1}{\tau^2} \equiv 4 \int_{-\infty}^{\infty} \mathrm{d}\lambda \left(\frac{\mathrm{d}f}{\mathrm{d}\lambda}\right)^2$ Length scale

Krommydas, Freivogel 2018 JHEP.

SNEC $\langle\langle T_{\mu\nu}\rangle K^\mu K^\nu\rangle_\tau^{\mathrm{s}} \geq -\frac{B}{G_{\mathrm{N}}\tau^2}$ → $\langle R_{\mu\nu} K^\mu K^\nu\rangle_\tau^{\mathrm{s}} \geq -\frac{8\pi B}{\tau^2}$

Note: c and $\hbar$ are both set to unity

B is $\mathcal{O}(1)$ number

How does this apply to cosmology?

How does this restrict bouncing models?

COSMOLOGIES AND SNEC

We model our cosmology with a FLRW metric

$$\mathrm{d}s^2 \equiv g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a(t)^2 \left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \right)$$

$K^\mu = a^{-1}(1,1,0,0)$ with $\mathrm{d}\lambda = a \ \mathrm{d}t$ Null vector and affine parameter

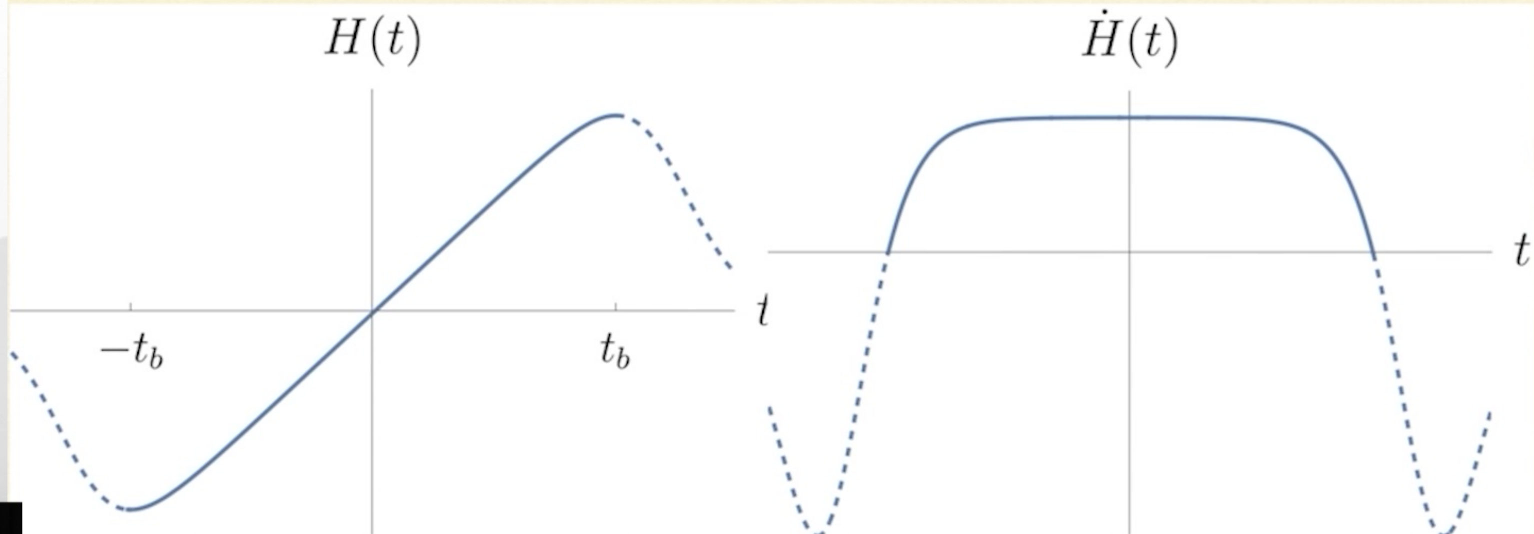
$$\int_{-\infty}^{\infty} \mathrm{d}t f(\lambda(t))^2 \frac{\dot{H}(t)}{a(t)} \leq \frac{4\pi B}{\tau^2}$$

We set $B = \dfrac{1}{32\pi}$

We now wish to apply this to bouncing cosmologies.

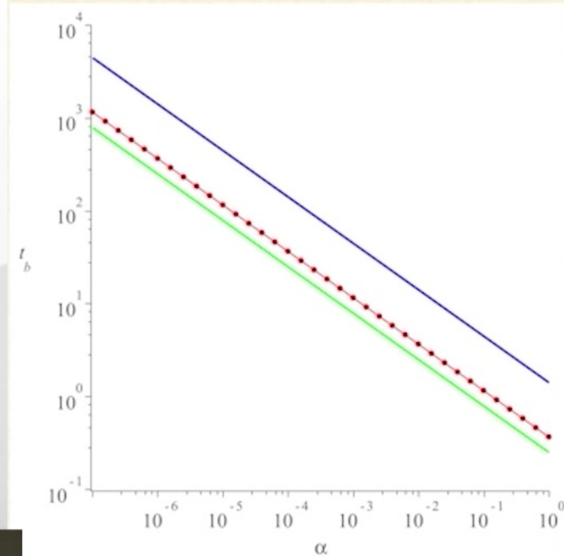
BOUNCING COSMOLOGIES

Generic features of bouncing cosmology.



BOUNCING COSMOLOGIES - SINGLE COMPONENT

Simplest parameterization $H(t) = \alpha t$ $\tau = \lambda(t_b)$



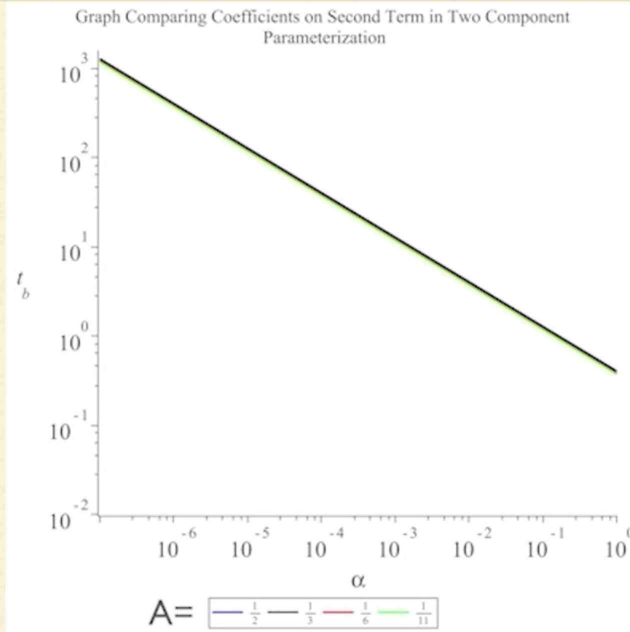
Everything below the dotted line satisfies the SNEC while above violates.

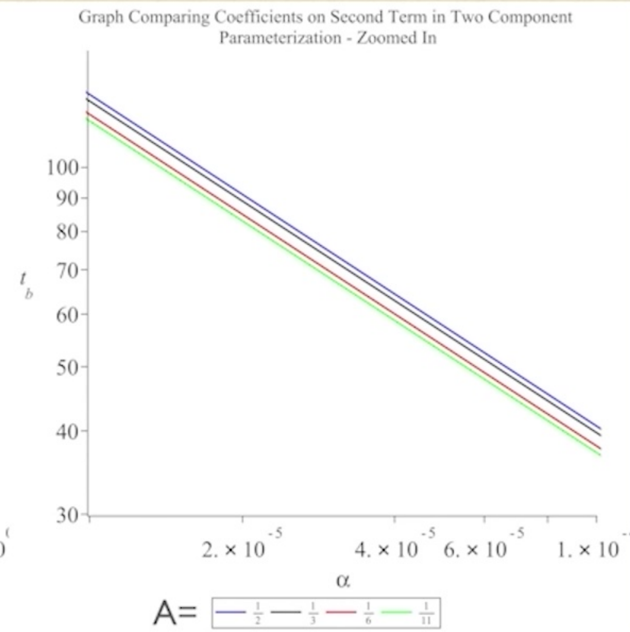
$$t_b = \sqrt{\frac{2}{\alpha}}$$ Upper bound from $a(t)$

$$t_b = \sqrt{\frac{1}{16\alpha}}$$ Lower bound from SNEC

BOUNCING COSMOLOGIES - DOUBLE COMPONENT

$$H(t) = \alpha t \left(1 - \alpha A t^2\right)$$





CONCLUSION

- Any model can be inserted into the SNEC and get constraints on your theory.
- We are currently investigation the Cuscuton bounce in this regard.
- Elly will present a poster later today with how the SNEC is used to constrain the equation of state parameter for a single component universe.
- We hope to extent the SNEC into black hole spacetimes in the near future.

Thank you!