Title: Generalized disformal Horndeski theories: consistency of matter coupling and cosmological perturbations

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Abstract:

Invertible disformal transformations are a useful tool to investigate ghost-free scalar-tensor theories. By performing a higher-derivative generalization of the invertible disformal transformation on Horndeski theories, we construct a novel class of ghost-free scalar-tensor theories, which we dub generalized disformal Horndeski theories. In the talk, I will clarify the basic idea for constructing the invertible disformal transformation with higher derivatives. I will also discuss some aspects of the generalized disformal Horndeski theories, including the consistency of matter coupling and cosmological perturbations.

Generalized disformal Horndeski theories: consistency of matter coupling and cosmological perturbations

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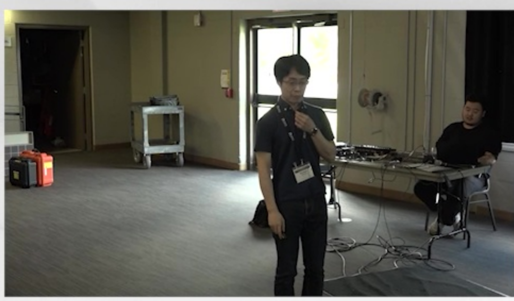


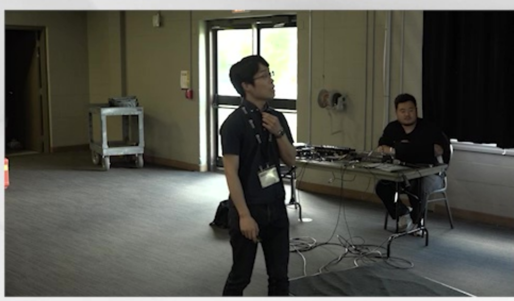
Collaborators: M. Minamitsuji (Osaka Butsuryo), H. Motohashi (Kogakuin)

- KT, M. Minamitsuji, H. Motohashi, *PTEP* **2023**, 013E01 (2023) [arXiv: 2209.02176]
- KT, M. Minamitsuji, H. Motohashi, *JCAP* **07**, 009 (2023) [arXiv: 2304.08624]

➢ Scalar-tensor theories

$$S[g,\phi] = \int \mathrm{d}^4x \sqrt{-g}\, \mathcal{L}(g, \partial g, \partial^2 g, \cdots; \phi, \partial\phi, \partial^2\phi, \cdots)$$

- Simple extension of GR, useful framework for cosmology/BHs
- Have a relatively long history
  - Jordan (1955), Brans-Dicke (1961), scalar-Gauss-Bonnet, ...
  - Horndeski (1974) [rediscovered as generalized Galileons in 2011]
    = The most general class of scalar-tensor theories with
    2nd-order Euler-Lagrange equations in 4D
    (No Ostrogradsky ghost associated with higher derivatives)
  - DHOST [Degenerate Higher-Order Scalar-Tensor] (2015–2016)
    - EL eqs. involve higher derivatives, but they can be removed by an appropriate linear combination of the eqs. due to the degeneracy.
    - ∃ subclass that can be generated from Horndeski via disformal trnsf.



➢ DHOST and disformal transformations

- Disformal transformation: field redefinition in scalar-tensor theories

$$(g_{\mu\nu}, \phi) \mapsto (\bar{g}_{\mu\nu}, \phi)$$

The most general trnsf. up to $\partial\phi$ (and without ∂g) is given by

$$\bar{g}_{\mu\nu}[g,\phi] = F_0(\phi, X) g_{\mu\nu} + F_1(\phi, X) \nabla_\mu \phi \nabla_\nu \phi \quad (X := g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi)$$

✓Invertible in general (i.e., uniquely solvable for $g_{\mu\nu}$)

- Disformal transformations map a scalar-tensor theory to another.

$$S[g_{\mu\nu}, \phi] \mapsto S[\bar{g}_{\mu\nu}, \phi] =: \tilde{S}[g_{\mu\nu}, \phi]$$

  - Preserves the ghost-freeness [Domènech+ (2015)] [KT+ (2017)]
  - ∃ subclass of DHOST that can be generated from Horndeski = "disformal Horndeski" (DH)
  - DHOST lying outside the DH class do not admit viable cosmology, and hence are phenomenologically disfavored. [Langlois+ (2017)], [KT, Kobayashi (2017)], [Langlois+ (2018)]

➢ Beyond DHOST?

DHOST: general but *not* the most general ghost-free scalar-tensor theory

Going beyond DHOST...

- would enlarge our knowledge on ghost-free scalar-tensor theories
- would lead to novel phenomenology
  cf. DHOST exhibits peculiar phenomena compared to Horndeski,
  e.g., Vainshtein breaking inside astrophysical bodies
  [Kobayashi+ (2014)], [Koyama+ (2015)], [Saito+ (2015)], ...

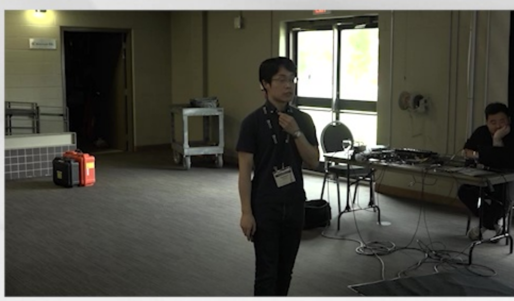
How can we go beyond DHOST?

- One could enlarge the underlying theory space & impose degeneracy.

$$\mathcal{L} = F_2 R + F_3 G^{\mu\nu}\phi_{\mu\nu} + C_2(\partial^2\phi)^2 + C_3(\partial^2\phi)^3$$
$$+\#(\partial^2\phi)^4 + \#\partial^3\phi + \cdots$$

NB Within DHOST, only "disformal Horndeski" admits viable cosmology.

- More efficient if we can generalize the disformal transformation



Disformal trnsf. with higher derivatives

- Conventional disformal transformations

$$\bar{g}_{\mu\nu} = F_0(\phi, X) g_{\mu\nu} + F_1(\phi, X) \nabla_\mu \phi \nabla_\nu \phi$$

form the most general class of trnsf. up to $\partial\phi$ (and without ∂g)

✓Invertible (necessary to generate ghost-free theories)

- Group structure of disformal trnsf. is useful to construct its inverse.
  - If $\bar{g}' \circ \bar{g} = \bar{g}''$, then the inverse of $\bar{g}$ is obtained by putting $\bar{g}'' = \mathrm{id}$.
- Closedness under the functional composition is nontrivial when generalized to disformal trnsf. with higher derivatives.
  - For trnsf. with $\nabla\nabla\phi$,

$$\bar{g}, \bar{g}' \ni \nabla\nabla\phi \supset \Gamma \supset \partial g$$

which generically yields unwanted higher derivatives in

$$\bar{g}' \circ \bar{g} \ni \partial\bar{g} \supset \partial^3\phi \notin \bar{g}, \bar{g}'$$

→ Need to tune the trnsf. law so that the operation is closed

➢ Generalized disformal transformation

■ Such a tuning is available for trnsf. of the form

$$\bar{g}_{\mu\nu} = F_0(\phi, X, Y, Z) g_{\mu\nu} + F_1(\phi, X, Y, Z) \nabla_\mu \phi \nabla_\nu \phi$$
$$+ 2 F_2(\phi, X, Y, Z) \nabla_{(\mu} \phi \nabla_{\nu)} X + F_3(\phi, X, Y, Z) \nabla_\mu X \nabla_\nu X$$

which involves

$$Y \coloneqq g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta X, \qquad Z \coloneqq g^{\alpha\beta} \nabla_\alpha X \nabla_\beta X$$

$X \coloneqq \nabla_\alpha \phi \nabla^\alpha \phi$

- Most general trnsf. constructed out of $\phi, \partial\phi, \partial X$ $(\ni \partial^2 \phi)$
- Includes the conventional disformal trnsf. as a special case

$$\bar{g}_{\mu\nu} = F_0(\phi, X) g_{\mu\nu} + F_1(\phi, X) \nabla_\mu \phi \nabla_\nu \phi$$

- Can be extended to include arbitrary higher derivatives [KT (2023)]

✓ The trnsf. is invertible if $F_\#$'s satisfy some conditions. [KT+ (2021)]

$$F_0 \neq 0, \quad \mathcal{F} \neq 0, \quad \bar{X}_Y = \bar{X}_Z = 0, \quad \bar{X}_X \neq 0, \quad \left| \frac{\partial(\bar{Y}, \bar{Z})}{\partial(Y, Z)} \right| \neq 0$$

➤ Generalized disformal Horndeski (1)

$G_\# = G_\#(\phi, X)$
$\phi_{\mu\nu} \coloneqq \nabla_\mu \nabla_\nu \phi$

- Horndeski theories described by the action

$$S_{\rm H} = \int {\rm d}^4 x \sqrt{-g}\, \Big[G_2 + G_3 \Box\phi + G_4 R - 4 G_{4X} g^{\alpha[\mu|} g^{\beta|\nu]} \phi_{\alpha\mu}\phi_{\beta\nu}$$
$$+ G_5 G^{\mu\nu}\phi_{\mu\nu} + 2G_{5X} g^{\alpha[\mu|} g^{\beta|\nu|} g^{\gamma|\lambda]} \phi_{\alpha\mu}\phi_{\beta\nu}\phi_{\gamma\lambda} \Big]$$

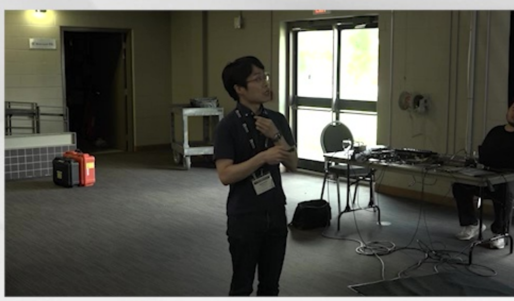
are mapped to new ghost-free theories.

= "Generalized Disformal Horndeski" (GDH) theories

- Transformation law for each building block
  - $\sqrt{-\bar{g}} = \sqrt{-g}\, \mathcal{J}$
  - $\bar{R}_{\mu\nu} = R_{\mu\nu} + 2\nabla_{[\alpha} C^{\alpha}{}_{\nu]\mu} + 2C^{\alpha}{}_{\beta[\alpha} C^{\beta}{}_{\nu]\mu}$
  - $\bar{\nabla}_\mu \bar{\nabla}_\nu \phi = \phi_{\mu\nu} - C^{\lambda}{}_{\mu\nu}\phi_\lambda$

$$\mathcal{J} \coloneqq F_0 [F_0^2 + F_0(XF_1 + 2YF_2 + ZF_3) + (Y^2 - XZ)(F_2^2 - F_1F_3)]^{1/2}$$
$$C^{\lambda}{}_{\mu\nu} \coloneqq \bar{\Gamma}^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\mu\nu} = \bar{g}^{\lambda\alpha}\left(\nabla_{(\mu}\bar{g}_{\nu)\alpha} - \frac{1}{2}\nabla_\alpha \bar{g}_{\mu\nu} \right) \ni \partial^3\phi$$

2024/7/15 GENERALIZED DISFORMAL HORNDESKI THEORIES Kazufumi Takahashi



➢ Generalized disformal Horndeski (2)

■ The action of GDH theories is written as

$$S_{\rm GDH}[g,\phi] \coloneqq S_{\rm H}[\bar{g},\phi] = \int {\rm d}^4x\sqrt{-g}\,\mathcal{J}\sum_{I=2}^{5}\tilde{\mathcal{L}}_I[g,\phi]$$

$$G_I = G_I\big(\phi,\bar{X}(\phi,X)\big)$$
$$\bar{g}^{\mu\nu} = \#g^{\mu\nu} + \#\phi^\mu\phi^\nu + \#\phi^{(\mu}X^{\nu)} + \#X^\mu X^\nu$$

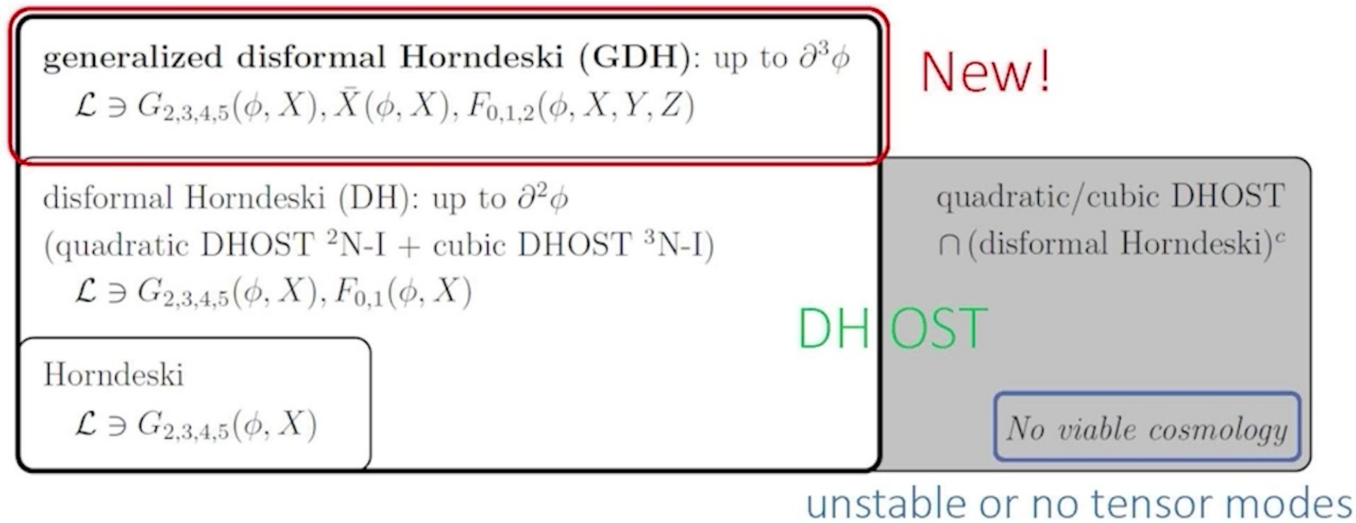
$$\tilde{\mathcal{L}}_2 \coloneqq G_2$$
$$\tilde{\mathcal{L}}_3 \coloneqq G_3\bar{g}^{\mu\nu}\big(\phi_{\mu\nu} - C^\rho{}_{\mu\nu}\phi_\rho\big)$$
$$\tilde{\mathcal{L}}_4 \coloneqq G_4\bar{g}^{\mu\nu}\left(R_{\mu\nu} - 2C^\alpha{}_{\beta[\alpha}C^\beta{}_{\nu]\mu}\right) - 2\big(G_{4\phi}\phi_\alpha + G_{4X}\bar{X}_\alpha\big)\bar{g}^{\mu[\nu}C^{\alpha]}{}_{\mu\nu}$$
$$-4G_{4X}\bar{g}^{\alpha[\mu|}\bar{g}^{\beta|\nu]}\big(\phi_{\alpha\mu} - C^\sigma{}_{\alpha\mu}\phi_\sigma\big)\big(\phi_{\beta\nu} - C^\rho{}_{\beta\nu}\phi_\rho\big)$$
$$\tilde{\mathcal{L}}_5 \coloneqq G_5\left(\bar{g}^{\mu\lambda}\bar{g}^{\nu\sigma} - \frac{1}{2}\bar{g}^{\mu\nu}\bar{g}^{\lambda\sigma}\right)\big(\phi_{\mu\nu} - C^\rho{}_{\mu\nu}\phi_\rho\big)\left(R_{\lambda\sigma} + 2\nabla_{[\alpha}C^\alpha{}_{\sigma]\lambda} + 2C^\alpha{}_{\beta[\alpha}C^\beta{}_{\sigma]\lambda}\right)$$
$$+2G_{5X}\bar{g}^{\alpha[\mu|}\bar{g}^{\beta|\nu|}\bar{g}^{\gamma|\lambda]}\big(\phi_{\alpha\mu} - C^\sigma{}_{\alpha\mu}\phi_\sigma\big)\big(\phi_{\beta\nu} - C^\rho{}_{\beta\nu}\phi_\rho\big)\big(\phi_{\gamma\lambda} - C^\eta{}_{\gamma\lambda}\phi_\eta\big)$$

- Contains $\partial^3\phi$ through $C^\lambda{}_{\mu\nu}$
- Free functions: $G_{2,3,4,5}(\phi,X)$, $\bar{X}(\phi,X)$, $F_{0,1,2}(\phi,X,Y,Z)$.

Horndeski　　generalized disformal trnsf.

Theory space

■ Inclusion relation among ghost-free scalar-tensor theories



- Within known DHOST, only the DH class can admit viable cosmology.
- GDH ⊃ DH ⊃ Horndeski because

$$\{\text{generalized disformal}\} \supset \{\text{disformal}\} \ni \text{id}$$

Consistency of matter coupling

Matter coupling in GDH theory can introduce a ghost.

NB No such problem in DHOST [Deffayet, Garcia-Saenz (2020)]

- Consider GDH gravity + matter field(s) Ψ

$$S[g,\phi,\Psi] = S_{\rm GDH}[g,\phi] + S_{\rm m}[g,\Psi], \quad S_{\rm GDH}[\bar{g},\phi] \coloneqq S_{\rm H}[g,\phi]$$

By construction of GDH theories, ∃ "Horndeski" frame:

$$S[\bar{g},\phi,\Psi] = S_{\rm H}[g,\phi] + S_{\rm m}[\bar{g},\Psi]$$

- Under the unitary gauge $\phi = \phi(t)$, the dynamics of the system is governed by the metric variables (N, N_i, γ_{ij}) and matter (Ψ).
- $\dot{N} \in S_{\rm m}[\bar{g},\Psi]$ makes N dynamical in general (= Ostrogradsky mode).

NB $\bar{g} \ni \nabla\nabla\phi \supset \Gamma \supset \dot{N}$ in general

✓∃ subclass of GDH where matter fields can be consistently coupled [KT, Minamitsuji, Motohashi (2022)], [Naruko+ (2022)], [KT, Kimura, Motohashi (2022)] [Ikeda, KT, Kobayashi (2023)]

➢ EFT of cosmological perturbations

✓Provide a model-indep. description of cosmological perturbations

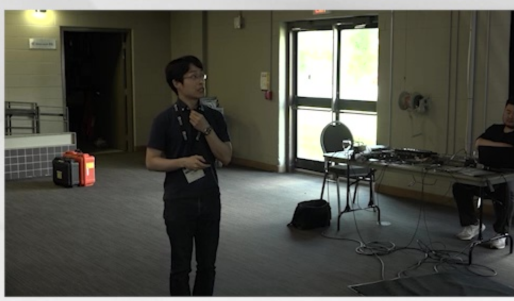
- ϕ has a timelike profile on a cosmological background.

  Under the unitary gauge $\phi = \phi(t)$, the action for perturbations is written in terms of not only 4D covariant objects but also spatially covariant objects, such as K_{ij} and ${}^{(3)}R$.

- Quadratic action at the leading order in derivatives

$$\delta_2 S = \int \mathrm{d}t\mathrm{d}^3x\, N a^3 \frac{M^2}{2}\left[\delta K^i_j \delta K^j_i - \left(1+\frac{2}{3}\alpha_{\mathrm{L}}\right)\delta K^2 + (1+\alpha_{\mathrm{T}})\delta_2\left({}^{(3)}R\frac{\sqrt{\gamma}}{a^3}\right) + H^2\alpha_{\mathrm{K}}\frac{\delta N^2}{N^2} + 4H\alpha_{\mathrm{B}}\frac{\delta N}{N}\delta K + (1+\alpha_{\mathrm{H}})\frac{\delta N}{N}\delta\,{}^{(3)}R\right]$$

  - Each parameter has some physical meaning. (e.g., $\alpha_{\mathrm{T}} = c^2_{\mathrm{GW}} - 1$)
  - Horndeski theory amounts to a particular choice of M^2 and α's.



➢ EFT action for GDH theories

- Performing the generalized disformal trnsf. yields

$$\delta_2 S = \int \mathrm{d}t\mathrm{d}^3x\, N a^3 \frac{M^2}{2}\left\{\delta K^i_j \delta K^j_i - \left(1+\frac{2}{3}\alpha_{\mathrm{L}}\right)\delta K^2 + (1+\alpha_{\mathrm{T}})\delta_2\left({}^{(3)}\!R\frac{\sqrt{\gamma}}{a^3}\right)\right.$$

$$+H^2\alpha_{\mathrm{K}}\frac{\delta N^2}{N^2} + 4H\alpha_{\mathrm{B}}\frac{\delta N}{N}\delta K + (1+\alpha_{\mathrm{H}})\frac{\delta N}{N}\delta\,{}^{(3)}\!R$$

$$(\text{New in DHOST} \to)\quad +\frac{4\beta_1}{N}\left(\frac{\delta N}{N}\right)^{\cdot}\delta K + \frac{\beta_2}{N^2}\left[\left(\frac{\delta N}{N}\right)^{\cdot}\right]^2 + \beta_3\frac{(\partial_i \delta N)^2}{N^2 a^2}$$

$$\left.(\text{New in GDH} \to)\quad +\frac{\gamma_1}{M_{\mathrm{D}}}\delta K^i_j \frac{\partial_i\partial^j \delta N}{N a^2} + \frac{\gamma_2}{M_{\mathrm{D}}}\delta K\frac{\partial_i^2 \delta N}{N a^2} + \frac{\gamma_3}{M_{\mathrm{D}}^2}\frac{\left(\partial_i^2 \delta N\right)^2}{N^2 a^4}\right\}$$

- $\beta_\#$'s are those newly appear in DHOST. Effects on CMB have been studied in [Hiramatsu+ (2020)] [Hiramatsu (2022)].

- $\gamma_\#$'s are proportional to (some combination of) the new disformal factors, with M_{D} being an associated mass scale. All these new operators contain higher spatial derivative of δN.

➢ LIGO/Virgo bound

- The simultaneous detection of GW170817 & GRB 170817A implies $c_{\mathrm{GW}}^2 = 1$ in the late-time Universe, which corresponds to $\alpha_{\mathrm{T}} = 0$.

- In terms of the GDH functions, we have

$$\alpha_{\mathrm{T}} = \frac{X G_4(\phi, \bar{X})}{\bar{X} F_0 [G_4(\phi, \bar{X}) - 2\bar{X} G_{4\bar{X}}(\phi, \bar{X})]} - 1.$$

F_0: conformal factor
$\bar{X} = \bar{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi$

<u>NB</u> G_5 also contributes to α_{T}, but it has been omitted for simplicity.

- Therefore, $\alpha_{\mathrm{T}} = 0$ requires ($G_5 = 0$ and)

$$\frac{\bar{X} F_0}{X} = \frac{G_4(\phi, \bar{X})}{G_4(\phi, \bar{X}) - 2\bar{X} G_{4\bar{X}}(\phi, \bar{X})}.$$

✓ A nontrivial subclass of GDH theories still survives.

➢ Summary & future works

- Constructed "generalized disformal Horndeski" theories by invertible generalized disformal trnsf. of Horndeski theories
  - ✓ Matter fields can be consistently coupled.
  - ✓ Extended the EFT of inflation/DE to accommodate GDH

- Many possible future works:
  - BH solutions
  - neutron stars
  - screening mechanism

  etc.