Title: Photon Rings and Shadow Size for General Axi-Symmetric and Stationary Integrable spacetimes

Speakers: Kiana Salehi

Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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Abstract:

There are now multiple direct probes of the region near black hole horizons, including direct imaging with the Event Horizon Telescope (EHT). As a result, it is now of considerable interest to identify what aspects of the underlying spacetime are constrained by these observations. For this purpose, we present a new formulation of an existing broad class of integrable, axisymmetric, stationary spinning black hole spacetimes, specified by four free radial functions, that makes manifest which functions are responsible for setting the location and morphology of the event horizon and ergosphere. We explore the size of the black hole shadow and high-order photon rings for polar observers, approximately appropriate for the EHT observations of M87*, finding analogous expressions to those for general spherical spacetimes. Of particular interest, we find that these are independent of the properties of the ergosphere, but does directly probe on the free function that defines the event horizon. Based on these, we extend the nonperturbative, nonparametric characterization of the gravitational implications of various near-horizon measurements to spinning spacetimes. Finally, we demonstrate this characterization for a handful of explicit alternative spacetimes.


Photon Rings and Shadow Size for a General Class of Integrable Space Times
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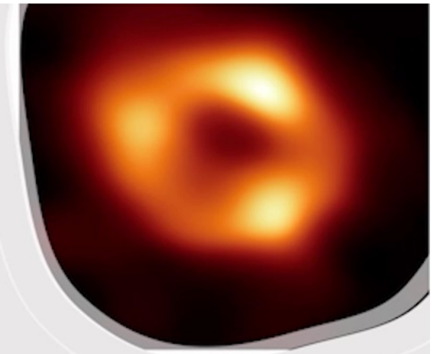
No Hair Theorem

How to quantify these deviations?

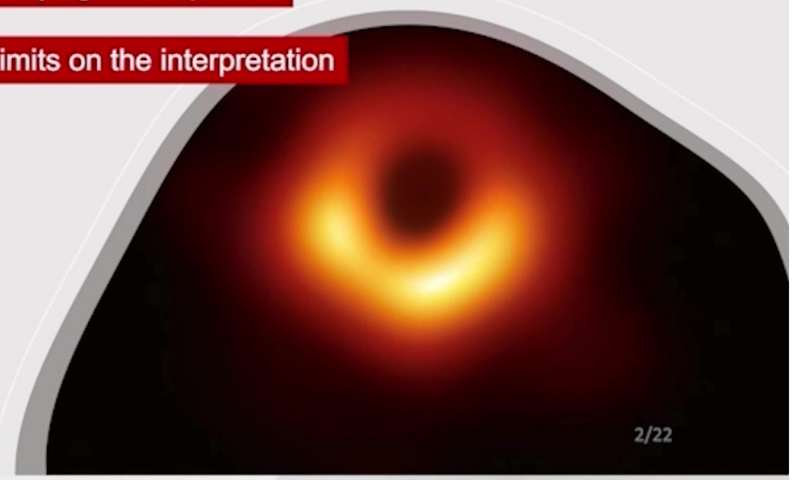
Alternatives?

parametrized



strong underlying assumptions

impose strong limits on the interpretation



- EHT data for M87 and Sgr A*
- Constraints → the possible deviations.

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We need

a Non-perturbative and non-parametric framework to describe/compare near horizon tests

Simple Case

Spherically Symmetric and Static Spacetime

Symmetries :

$$\partial_t \rightarrow E = g_{tt} \frac{dt}{d\lambda}$$

$$\partial_\varphi \rightarrow L_z = g_{\varphi\varphi} \frac{d\varphi}{d\lambda}$$

A general spherically symmetric static :

$$ds^2 = -N(r)^2 dt^2 + \frac{B(r)^2}{N(r)^2} dr^2 + r^2 d\Omega$$

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Photon Circular Orbit

$$ds^2 = -N(r)^2 dt^2 + \frac{B(r)^2}{N(r)^2} dr^2 + r^2 d\Omega$$

rearrange :

$$\dot{r}^2 = -\frac{g^{tt} + b^2 g^{\varphi\varphi}}{g_{rr}} = 0$$

Where,

$$\dot{r} = \frac{dr}{d\lambda}$$

$$b = \frac{L_z}{E}$$

Taking a derivative

$$\ddot{r} = \frac{1}{2} \frac{N(r_\gamma)^2}{{r_\gamma}^2 B(r_\gamma)^2} \left(\frac{r^2}{N(r)^2} \right)' \Bigg|_{r_\gamma} = 0$$

Solving simultaneously :

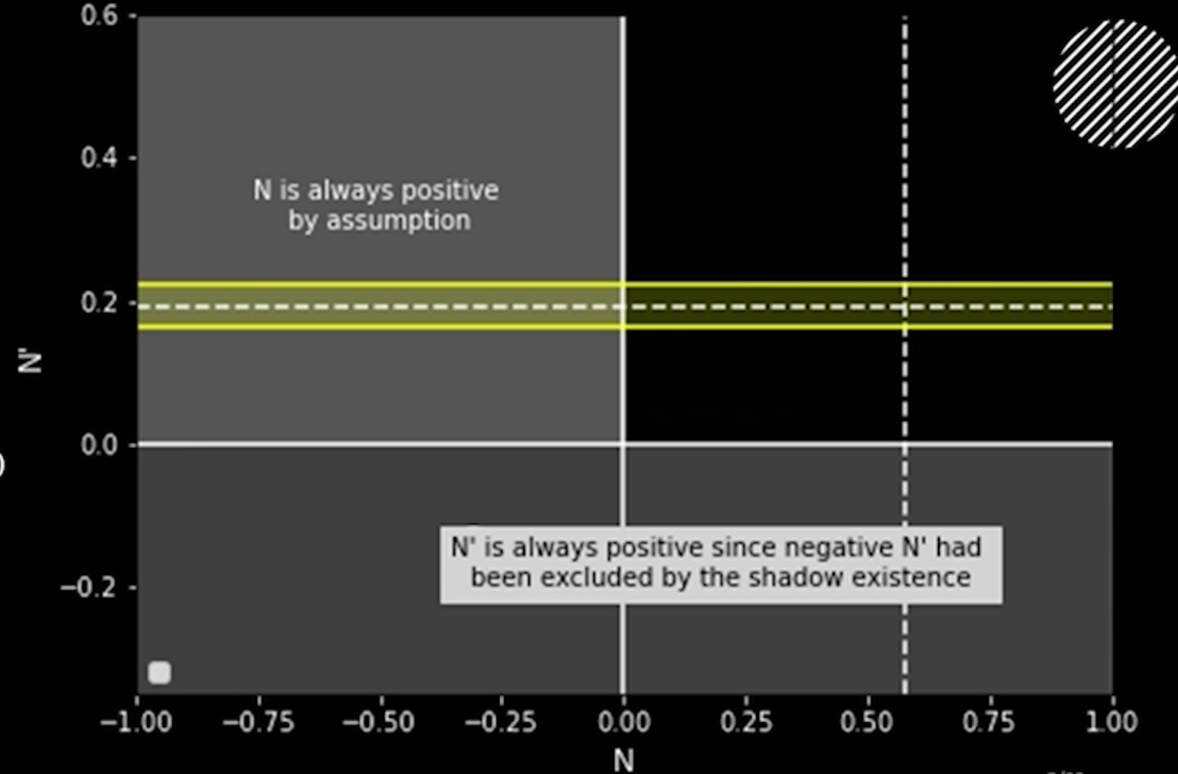
$$r_\gamma = \frac{N(r_\gamma)}{N'(r_\gamma)}$$

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$

he shadow size is :

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$

Shadow exist $\rightarrow N' > 0$



Reminder $g_{tt}(r) = -N^2(r)$

$N > 0$

$N < 0$

Choose $N > 0$

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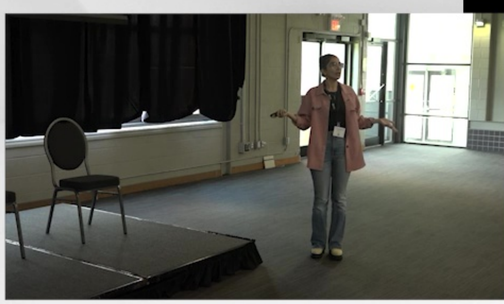


Multiple Photon Ring

$b = b_\gamma + \delta b$

$\ddot{r} = \ddot{r}|_{r_\gamma} + \ddot{r}'|_{r_\gamma}\, \delta r + \mathcal{O}(\delta r^2)$

$\ddot{r}|_{r_\gamma}=0$

$\delta r=\delta r_0 e^{\omega\tau}$

which

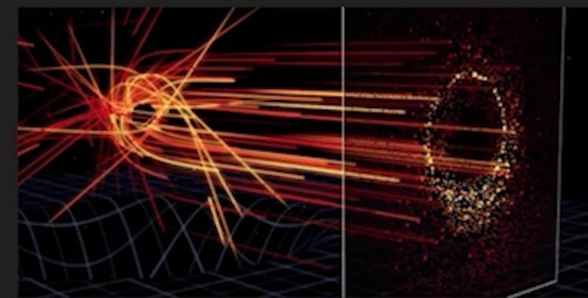
$\omega^2 = \ddot{r}'|_{r_\gamma}$

Lyapunov exponent

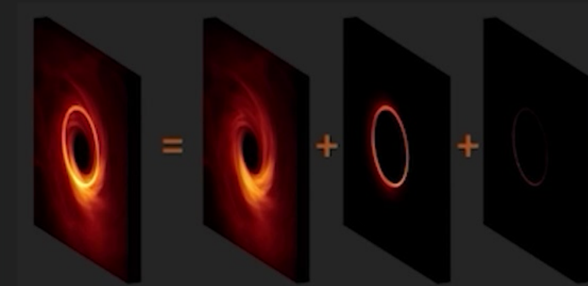
$$\pi \frac{d\delta r}{d\varphi} = \gamma\, \delta r$$

$$\delta r = \delta r_0 \exp\left(\frac{\gamma}{\pi}\varphi\right)$$

$$\gamma = \pi \frac{N^{3/2}}{N'} \sqrt{\frac{-N''}{B^2}}$$



How strongly lensed light creates a photon ring. Credit: Center for Astrophysics, Harvard & Smithsonian



Different photon paths create layers of light. Credit: George Wong (UIUC) and Michael Johnson (CfA)

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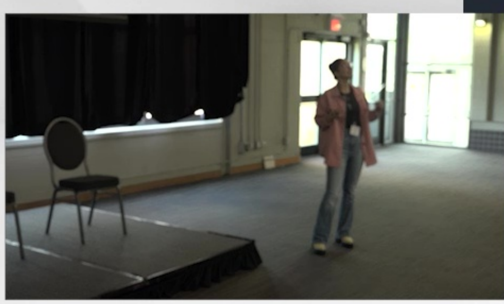
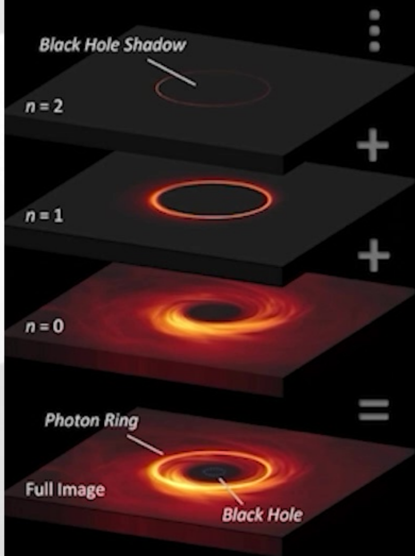
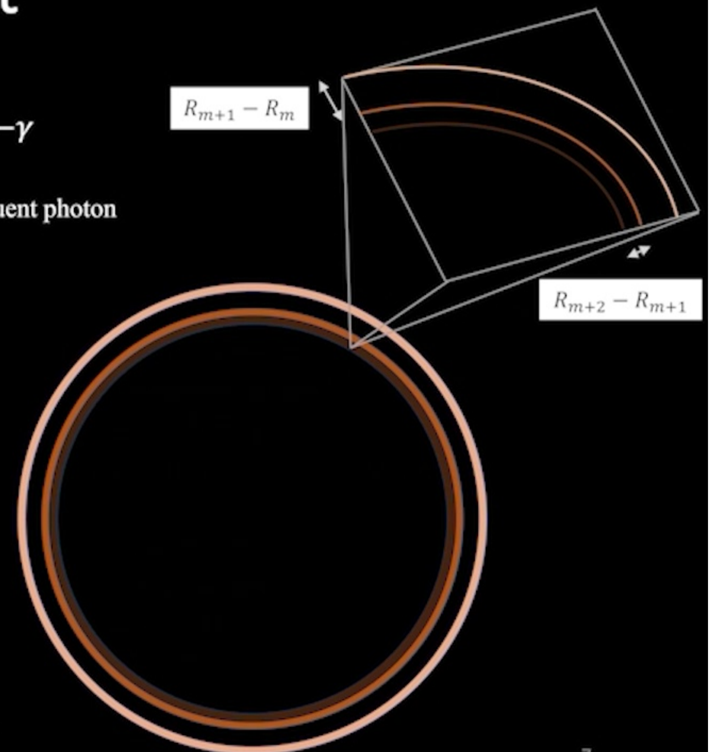


Observing Lyapunov Exponent



$$\frac{R_{m+2}-R_{m+1}}{R_{m+1}-R_m} = e^{-\gamma}$$

R_m: radius of the m/2 subsequent photon



Reference: *Universal Interferometric Signatures of a Black Hole's Photon Ring*
dit: Michael D. Johnson (CfA), Simulation: George Wong (UIUC)

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$N - N'$ diagram

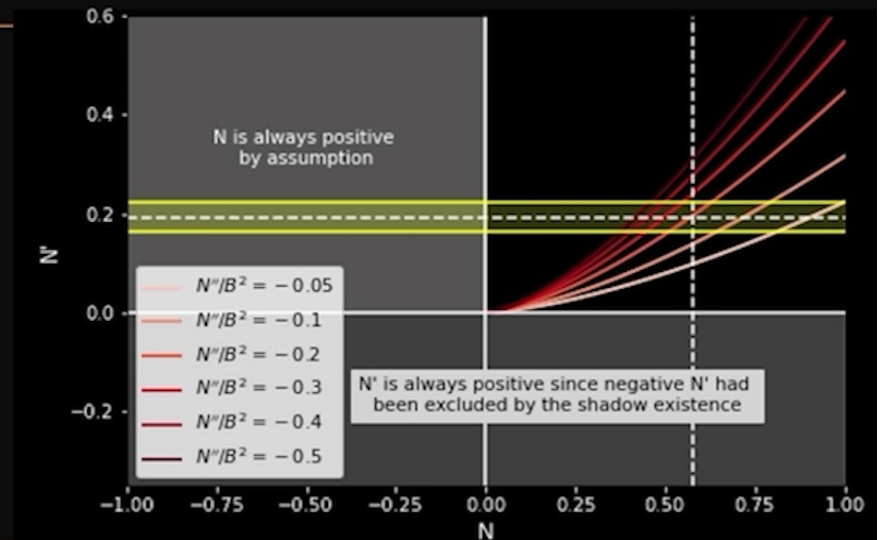
- **Reminder**

Shadow size:

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$

Lyapunov exponent:

$$\gamma = \pi \frac{N^{3/2}}{N'} \sqrt{\frac{-N''}{B^2}}$$



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$N - N''/B^2$ diagram

- **Reminder**

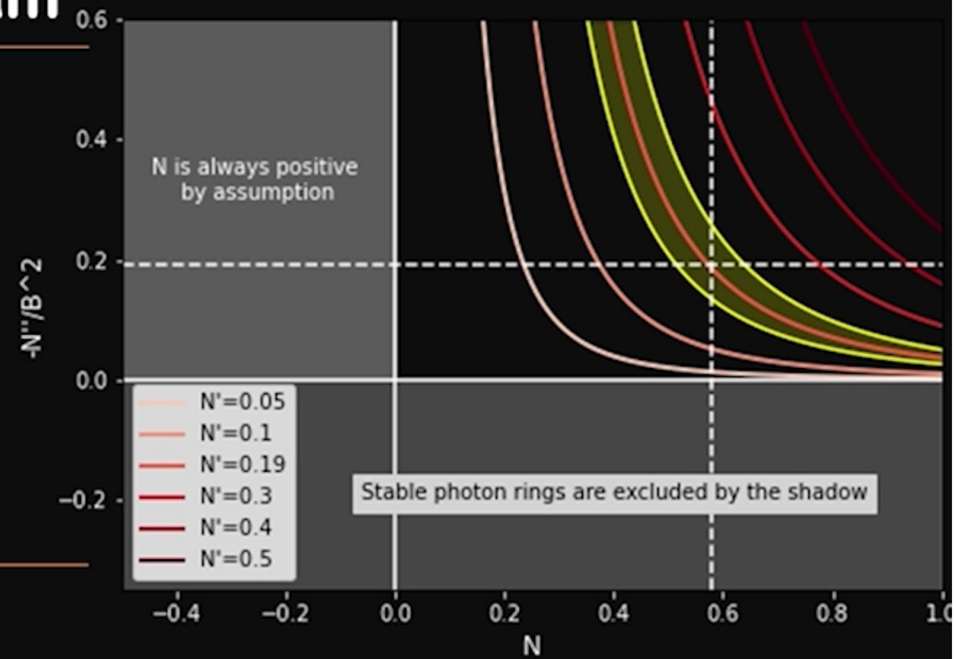
Shadow size:

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$

Lyapunov exponent:

$$\gamma = \pi \frac{N^{3/2}}{N'} \sqrt{\frac{-N''}{B^2}}$$

$$\text{thus} \rightarrow \frac{-N''}{B^2} > 0$$



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General Spherically Symmetric and Static

↓

Non-perturbative and non-parametric

↓

$N - N'$ diagram $N - N''/B^2$ diagram

↓

different spherically symmetric spacetimes

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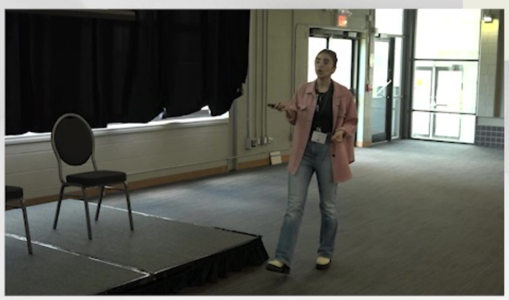
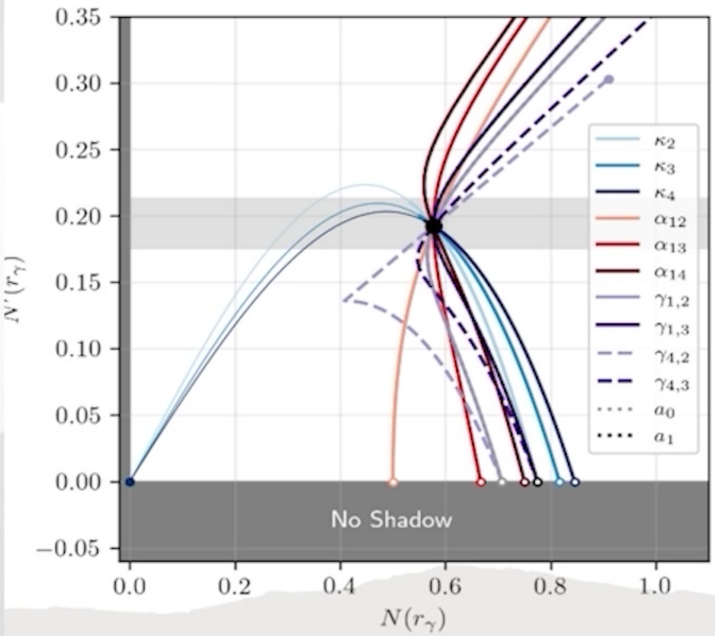
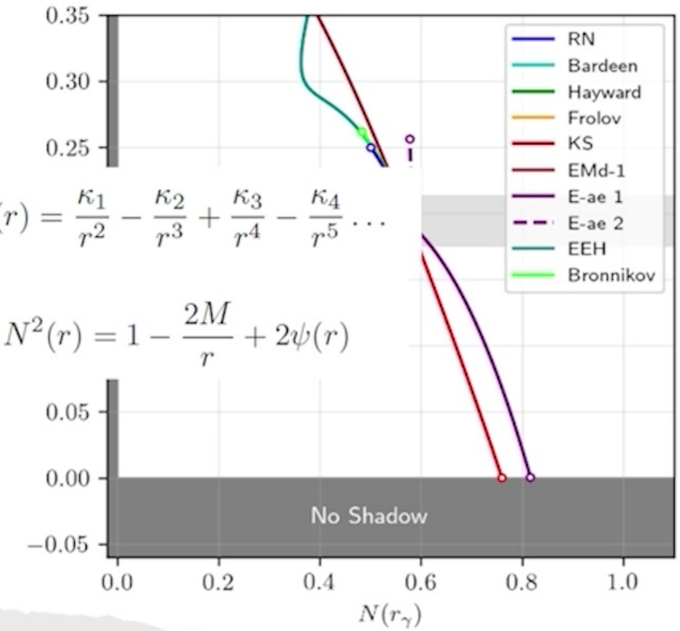


Metric Expansions



Alternative Spacetimes



$$\psi(r) = \frac{\kappa_1}{r^2} - \frac{\kappa_2}{r^3} + \frac{\kappa_3}{r^4} - \frac{\kappa_4}{r^5} \dots$$

$$N^2(r) = 1 - \frac{2M}{r} + 2\psi(r)$$

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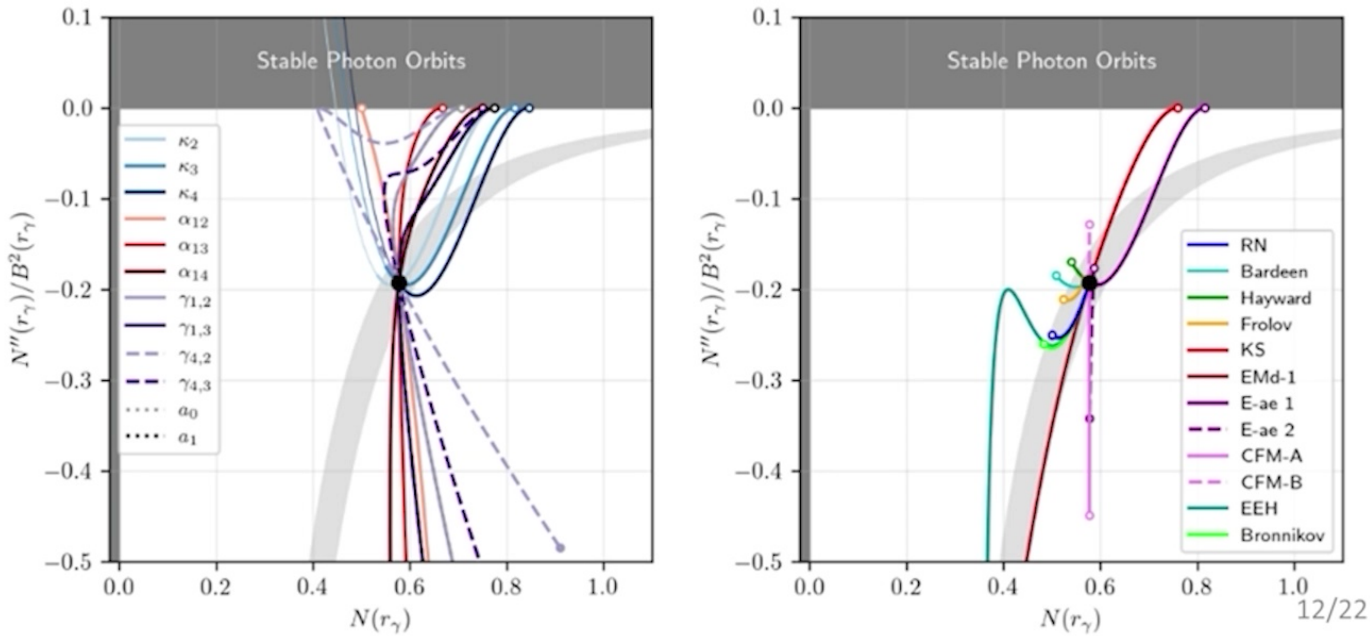
$N - N''/B^2$ diagram



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Summary

arXiv:2307.15120
arXiv:2311.01495

- **a non-perturbative and non-parametric** framework to describe/compare near horizon tests.
- Shadow size measurements:

$$b_\gamma = \frac{1}{N'(r_\gamma)}$$

- Relative radii of the subsequent photon rings:

$$\frac{R_{m+2} - R_{m+1}}{R_{m+1} - R_m} = e^{-\gamma} \quad , \quad \bar{\gamma} = -\pi \frac{N^{1.5}}{N'} \sqrt{\frac{-N''}{B^2}}\, 2K\left(\left(\frac{a}{b_\gamma}\right)^2\right)$$

For spherically symmetric and a general class of axisymmetric spacetimes.

- Compare them to other horizon scale tests of GR such as GW.

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Relaxing our conditions

General Spherically Symmetric and Static Spacetime → General Axisymmetric and Stationary Spacetime

Integrability

4 DoF

General Spherically Symmetric and Static Spacetime:
1. μ
2. E
3. L_z
4. L

General Axisymmetric and Stationary Spacetime:
1. μ
2. E
3. L_z
4. ?

Carter Constant?

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General integrable Axi-Symmetric and Stationary Spacetime

$$g_{tt} = -\tilde{\Sigma}\frac{N^2 - a^2F^2\sin^2\theta}{[r - a^2F\sin^2\theta]^2}$$

$$g_{rr} = \tilde{\Sigma}\frac{B^2}{N^2}$$

$$g_{\theta\theta} = \tilde{\Sigma}$$

$$g_{\phi\phi} = \tilde{\Sigma}\sin^2\theta\frac{[r^2 - a^2N^2\sin^2\theta]}{[r - a^2F \ldots}$$

$$g_{t\phi} = \tilde{\Sigma}a\sin^2\theta\frac{[N \ldots}{[r - a^2F\sin^2\theta]^2}.$$

Where,

$$\tilde{\Sigma} = r^2 + a^2\cos^2\theta + f(r),$$

COMPLICATED!

Regular black hole metric with three constants of motion

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[2] Canadian Institute for Theoreti...
... Waterloo, ...
...r 16, 2015)

Baines-Visser Metric

Isi and Farr

According to the no-hair theorem, astrophysical black holes are uniquely characterized by their masses and spins and are described by the Kerr metric. Several parametric spacetimes which deviate from the Kerr metric have been proposed in order to test this theorem with observations of black holes in both the electromagnetic and gravitational-wave spectra. Such metrics often contain naked singularities or closed timelike curves in the vicinity of the compact objects that can limit the applicability of the metrics to compact objects that do not spin rapidly, and generally admit only two constants of motion. The existence of a third constant, however, can facilitate the calculation

- ...err-like metrics
- Separable Action → Carter constant.
- General way to have a modified Carter Constant

$$Q \equiv C - (L_z - aE)^2,$$

Where,

$$C = \frac{1}{\sin^2\theta}\left[A_3(\theta)L_z - aA_4(\theta)E\sin^2\theta\right]^2 + \mu^2\left[a^2\cos^2\theta + g(\theta)\right] + A_6(\theta)\left(\frac{\partial S_\theta}{\partial\theta}\right)^2$$

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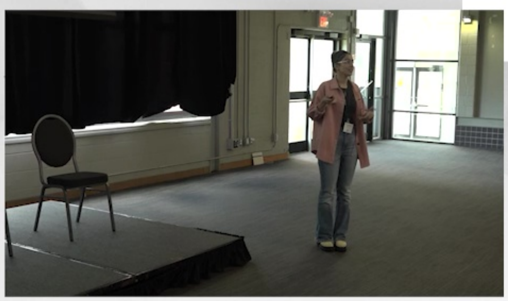


$$\dot{r} = 0 \quad \& \quad \ddot{r} = 0$$

$$\left(\frac{1}{\mathrm{N}^2(\mathrm{r})} - \frac{\beta_\gamma^2}{\mathrm{r}^2}\right) = 0$$

$$\left(\frac{1}{\mathrm{N}^2(\mathrm{r})} - \frac{\beta_\gamma^2}{\mathrm{r}^2}\right)' = 0$$

Very similar form to the general spherically symmetric space times!

Shadow Size

$$\beta_\gamma = \sqrt{q + a^2} = \frac{1}{N'(r_\gamma)}$$

Photon Circle Radius

$$r_\gamma = \frac{N(r_\gamma)}{N'(r_\gamma)}$$

$$\ddot{r} = \ddot{r}(r_\gamma) + \ddot{r}'(r_\gamma)\delta r \rightarrow \delta\ddot{r} = \ddot{r}'(r_\gamma)\delta r$$

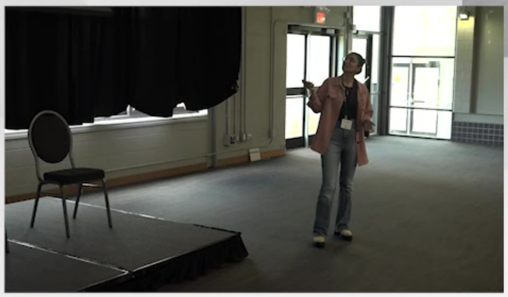
$$\delta r = e^{\omega\tau}\delta r_0$$

$$\omega^2 = \frac{E^2}{N(r_\gamma)}\left(\frac{-N''(r_\gamma)}{B^2(r_\gamma)}\right)\frac{r_\gamma^2}{\Sigma}\frac{\Delta(r_\gamma)}{1 - \frac{2M}{r_\gamma}}$$

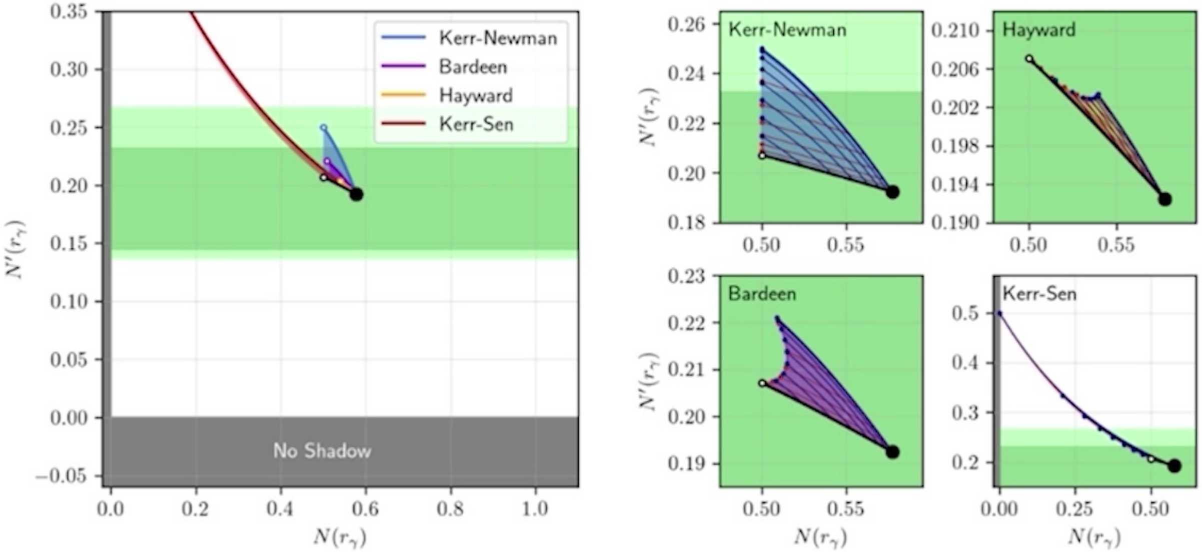
$$\pi \frac{\quad}{d\theta} = \gamma\,\delta r$$

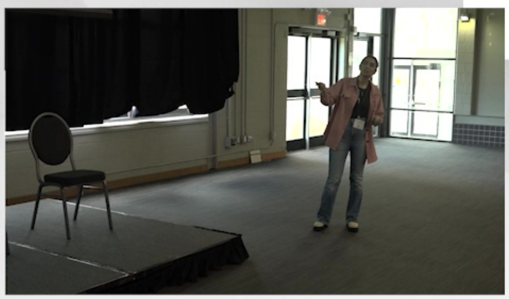
$$\bar{\gamma} = -\pi\frac{N^{3/2}}{N'}\sqrt{\frac{-N''}{B^2}}\,2K\left(\left(\frac{a}{\beta_\gamma}\right)^2\right)$$

where K(k) is the complete elliptic integral of the first kind.



$N - N'$ diagram





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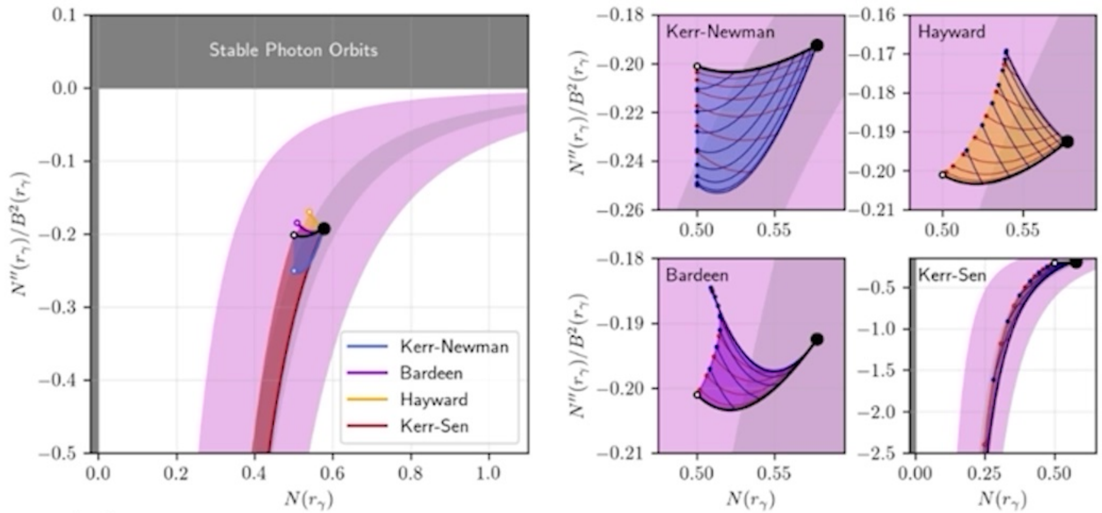
$N - N''/B^2$ diagram

Gravitational Waves

$$\gamma = 2\pi \lim_{l\to\infty} l \frac{\omega_{I,l}}{\omega_{R,l}}$$

$$\lim_{l\to\infty} \omega_{I,l} = \frac{N'(r_\gamma)\gamma}{2\pi} \text{ and } \lim_{l\to\infty} \frac{\omega_{R,l}}{l} = N'(r_\gamma)$$



elating Black Hole Shadow to Quasinormal Modes for Rotating Black Holes

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Observation of Gravitational Waves from a Binary Black Hole Merger

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(LIGO Scientific Collaboration and Virgo Collaboration)

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