Title: Collinear singularities from a double cover of twistor space

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Abstract:

Based on an idea of Kevin Costello, I will show how to construct a double cover of the twistor space of $\mathbb{R}^4$, $X = \pi^*(\mathcal{O}(1)\oplus\mathcal{O}(1))\to\Sigma$ where Σ is an (hyper)elliptic curve. I then discuss how holomorphic theories such as BF and Chern-Simons theory on X descend to theories on ordinary twistor space. Once on twistor space, compactifying along the $\mathbb{CP}^1$ direction of twistor space produces a corresponding 4d theory where we can study the algebra of collinear singularities. I will present my calculations which show that this algebra lives on the elliptic curve defining the double cover of twistor space.

Building celestial chiral algebras living on elliptic curves

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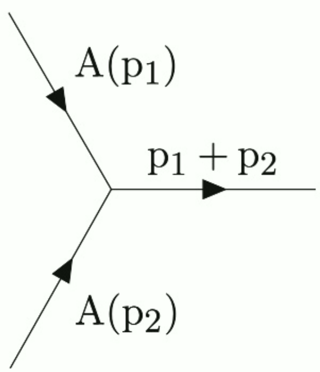
Twistor space setup

Recall that $\mathbb{PT} = (\mathcal{O}(1) \oplus \mathcal{O}(1) \to \mathbb{CP}^1) \cong \mathbb{CP}^1 \times \mathbb{R}^4$ and

$$\underbrace{\int_{\mathbb{PT}} \mathrm{tr}(bF^{0,2}(a))}_{\text{BF Theory}} \xleftrightarrow{\text{Penrose transform}} \underbrace{\int_{\mathbb{R}^4} \mathrm{tr}(B \wedge F(A)_+)}_{\text{SDYM}}.$$

Scattering plane waves in 4d SDYM gives collinear singularities that live on $\mathbb{CP}^1$. Eg.



$$\sim \frac{1}{\langle 12 \rangle} e^{ix\cdot(p_1+p_2)} = \frac{1}{z_1 - z_2} e^{ix\cdot(p_1+p_2)}.$$

Elliptic curve setup

Let $w^2 = \Pi_i^n (z - z_i) = H(z)$ define an (hyper)elliptic curve $\Sigma \xrightarrow[2:1]{\pi} \mathbb{CP}^1$.
We can construct

$$X = (\pi^* \mathcal{O}(1) \oplus \pi^* \mathcal{O}(1) \to \Sigma) \cong \Sigma \times \mathbb{R}^4 \xrightarrow[2:1]{} \mathbb{PT}$$

and consider

$$\int_X \beta F^{0,2}(\alpha) \xrightarrow{\pi_*} \int_{\mathbb{PT}} b\bar{\partial}a + \tilde{b}\bar{\partial}\tilde{a} + baa + a\tilde{a}\tilde{b} + b\tilde{a}\tilde{a}H(z).$$

Example: Take $\deg H = 4$ (so $\Sigma = \mathbb{T}^2$). The 4d action is

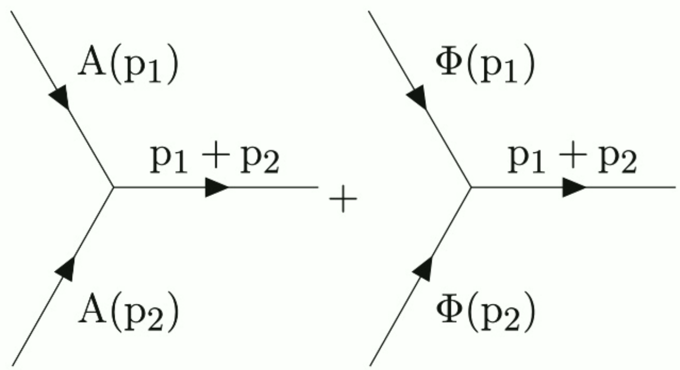
$$\int_{\mathbb{R}^4} BF(A) + \tilde{\Phi}\Delta\Phi + A^{\alpha}_{\dot{\alpha}} D^{\dot{\alpha}}_{\alpha} \Phi \tilde{\Phi} + B_{\alpha\beta} D_{\dot{\alpha}\delta} \Phi D^{\dot{\alpha}}_{\gamma} \Phi H^{\alpha\beta\delta\gamma} + \ldots$$

a deformation of SDYM with two scalars.

Celestial chiral algebra lives on the elliptic curve

$$\int_{\mathbb{R}^4} BF(A) + \tilde{\Phi}\Delta\Phi + A^{\alpha}_{\dot{\alpha}} D^{\dot{\alpha}}_{\alpha} \Phi\tilde{\Phi} + B_{\alpha\beta} D_{\dot{\alpha}\delta} \Phi D^{\dot{\alpha}}_{\gamma} \Phi H^{\alpha\beta\delta\gamma} + \ldots$$

Scattering plane waves on the double cover gives diagrams like



$$\sim \frac{\omega(p_1)\omega(p_2) + \tilde{\omega}(p_1)\tilde{\omega}(p_2)}{z_1 - z_2}.$$

Singular only when $z_1 \to z_2$ on the same branch of the double cover.

Where $\omega, \tilde{\omega}$ are the even and odd components of the plane wave on X.