Title: A Celestial Dual for MHV Amplitudes

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Abstract:

We show that a 2D CFT consisting of a central charge c Liouville theory, a chiral level one, rank N Kac-Moody algebra and a weight −3/2 free fermion holographically generates 4D MHV leaf amplitudes associated to a single hyperbolic slice of flat space. Celestial amplitudes arise in a large-N and semiclassical large-c limit, according to the holographic dictionary, as a translationally-invariant combination of leaf amplitudes. A step in the demonstration is showing that the semiclassical limit of Liouville correlators are given by contact AdS3 Witten diagrams.

A Celestial Dual for MHV Amplitudes

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Celestial Holography Summer School

Celestial Holography and Translation Invariance

- Bulk translation invariance is not a symmetry of normal 2D CFTs.
- Translation invariance forces low-point functions to be distributional.
- We construct a theory that computes an intermediate object called a ‘leaf amplitude.’
- Leaf amplitudes integrate an interaction vertex over a single hyperbolic slice of spacetime $x^2 = -\tau^2$.

MHV Leaf Amplitudes

$$\mathcal{A}_{\mathrm{MHV},n} = \frac{z_{12}^3}{z_{23}\cdots z_{n1}} \int \frac{\mathrm{d}^4 x}{(2\pi)^4} \prod_{i=1}^{n} \frac{\Gamma(2\bar{h}_i)}{(\epsilon - \mathrm{i}\hat{q}_i \cdot x)^{2\bar{h}_i}} ,$$

$$= \frac{z_{12}^3}{z_{23}\cdots z_{n1}} \int \frac{d\tau}{(2\pi)^4} \tau^{-1-\beta} \Bigg(\int_{\hat{x}^2=-1} d^3\hat{x} \prod_{i=1}^{n} \frac{\Gamma(2\bar{h}_i)}{(\epsilon - i\hat{q}_i \cdot \hat{x})^{2\bar{h}_i}} + \int_{\hat{x}^2=1} d^3\hat{x} \prod_{i=1}^{n} \frac{\Gamma(2\bar{h}_i)}{(\epsilon - i\hat{q}_i \cdot \hat{x})^{2\bar{h}_i}} \Bigg)$$

$$= \frac{\delta(\beta)}{(2\pi)^3} \left(\mathcal{L}_{\mathrm{MHV},n}(z_j, \bar{z}_j) + \mathcal{L}_{\mathrm{MHV},n}(z_j, -\bar{z}_j) \right)$$

$$x^\mu = \tau \hat{x}^\mu, \; d^4x = \tau^3 d\tau d^3\hat{x}$$

$$\hat{q}_i = \varepsilon_i (1 + z_i\bar{z}_i, z_i + \bar{z}_i, z_i - \bar{z}_i, 1 - z_i\bar{z}_i)$$

The Three-Point Leaf Amplitude

The three-point MHV leaf amplitude can be found explicitly by integrating over an $\mathrm{AdS}_3/\mathbb{Z}$ slice of Klein space:

$$\mathcal{L}_{\mathrm{MHV},3} = \frac{\mathrm{i}\pi\mathcal{N}}{2}\frac{z_{12}^3}{z_{23}z_{31}} \times \left(\frac{1}{\prod_{i\neq j\neq k}(\varepsilon_i\varepsilon_j z_{ij}\bar{z}_{ij}+\mathrm{i}\epsilon)^{\bar{h}_i+\bar{h}_j-\bar{h}_k}} - \frac{1}{\prod_{i\neq j\neq k}(\varepsilon_i\varepsilon_j z_{ij}\bar{z}_{ij}-\mathrm{i}\epsilon)^{\bar{h}_i+\bar{h}_j-\bar{h}_k}}\right)$$

$$\mathcal{N} = \Gamma\left(1+\frac{\beta}{2}\right)\Gamma(\bar{h}_1+\bar{h}_2-\bar{h}_3)\Gamma(\bar{h}_1-\bar{h}_2+\bar{h}_3)\Gamma(-\bar{h}_1+\bar{h}_2+\bar{h}_3)$$

Extracting the Three-Point MHV Gluon Amplitude

$$\begin{aligned}
\mathcal{A}_{\mathrm{MHV},3} &= \frac{\delta(\beta)}{8\pi^3}\left(\mathcal{L}(z_i,\bar{z}_i)+\mathcal{L}(z_i,-\bar{z}_i)\right)\\
&= \frac{\mathcal{N}}{8\pi^2}\,\delta(\beta)\sin\left(\frac{\pi\beta}{4}\right)\frac{z_{12}^3}{z_{23}z_{31}}\\
&\times\left\{\frac{\mathrm{e}^{\mathrm{i}\pi\beta/4}}{\prod_{i\neq j\neq k}(\varepsilon_i\varepsilon_j z_{ij}\bar{z}_{ij}+\mathrm{i}\epsilon)^{\bar{h}_i+\bar{h}_j-\bar{h}_k}}+\frac{\mathrm{e}^{-\mathrm{i}\pi\beta/4}}{\prod_{i\neq j\neq k}(\varepsilon_i\varepsilon_j z_{ij}\bar{z}_{ij}-\mathrm{i}\epsilon)^{\bar{h}_i+\bar{h}_j-\bar{h}_k}}\right\}\\
&= \frac{\pi\delta(\beta)}{2}\frac{\mathrm{sign}(z_{12}z_{23}z_{31})\delta(\bar{z}_{13})\,\delta(\bar{z}_{23})}{|z_{12}|^{h_1+h_2-h_3}|z_{23}|^{h_2+h_3-h_1}|z_{31}|^{h_1+h_3-h_2}}\Theta\left(\frac{\varepsilon_3 z_{23}}{\varepsilon_1 z_{12}}\right)\Theta\left(\frac{\varepsilon_3 z_{31}}{\varepsilon_2 z_{12}}\right)
\end{aligned}$$

A Celestial Dual for MHV Leaf Amplitudes

- The dual leaf CFT for MHV amplitudes has

$$S = \underbrace{\frac{1}{8\pi}\int d^2z\psi^i\bar{\partial}\psi^i}_{\text{N (1/2,0) fermions}} + \underbrace{\frac{1}{4\pi}\int d^2z\rho\bar{\partial}\eta}_{\text{(-3/2,0) fermion }\eta} + \underbrace{\frac{1}{4\pi}\int d^2z[\partial\phi\bar{\partial}\phi + 4\pi\mu e^{2b\phi}]}_{\text{Liouville Theory}}$$

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- This theory contains the 'gluon' operators

$$\mathcal{O}_{\Delta}^{+a} \propto T^a_{ij} : \psi^i\psi^j : e^{(\Delta-1)b\phi}(z,\bar{z})$$
$$\mathcal{O}_{\Delta}^{-a} \propto \eta\partial\eta\, T^a_{ij} : \psi^i\psi^j : e^{(\Delta+1)b\phi}(z,\bar{z})$$

- In the semiclassical limit $b \to 0$

$$\langle \mathcal{O}_{\Delta_1}^{-a_1}\mathcal{O}_{\Delta_2}^{-a_2}\cdots\mathcal{O}_{\Delta_n}^{+a_n}\rangle = \frac{\mathrm{Tr}[T^{a_1}\cdots T^{a_n}]z_{12}^4}{z_{12}\cdots z_{n1}}\int_{\hat{x}^2=-1} d^3\hat{x}\prod_{j=1}^{n}\frac{\Gamma(2\bar{h}_j)}{(\epsilon - \mathrm{i}\hat{q}_j\hat{x})^{2\bar{h}_j}}$$
$$= \mathcal{L}_{\mathrm{MHV},n}$$