

**Title:** Theoretical status of Horava gravity

**Speakers:** Sergey Sibiryakov

**Collection/Series:** 50 Years of Horndeski Gravity: Exploring Modified Gravity

**Date:** July 18, 2024 - 3:15 PM

**URL:** <https://pirsa.org/24070075>

**Abstract:**

I'll review the models of quantum gravity postulating invariance with respect to anisotropic (Lifshitz) scaling in the deep ultraviolet domain. At low energies they reduce to scalar-tensor gravity, with a timelike gradient of the scalar field breaking local Lorentz invariance. The models come in two versions differing by the dynamics in the scalar sector. The first, projectable, model has been shown to be perturbatively renormalizable and the full renormalization group (RG) flow of its marginal operators has been computed. The flow possesses a number of asymptotically free fixed points with one of them being connected by RG trajectories to the region of the parameter space where the kinetic term of the theory acquires the general relativistic form. The gravitational coupling exhibits non-monotonic behavior along the flow, vanishing both in the ultraviolet and the infrared. I'll mention the challenges facing the model in the infrared domain. The second, non-projectable, model is known to reproduce the phenomenology of general relativity in a certain region of parameters. Full proof of its renormalizability is still missing due to its complicated structure. I'll review recent progress towards constructing such proof.

# Theoretical Status of Horava Gravity

**Sergey Sibiryakov**



*50 Years of Horndeski Gravity, 18 July 24*

# ~~Theoretical~~ Status of Horava Gravity


**Sergey Sibiryakov**



*50 Years of Horndeski Gravity, 18 July 24*

## Quest for quantum gravity

**General Relativity** is a **unitary relativistic local** field theory

- + Describes classical phenomena from  $10^{-2}\text{cm}$  to  $10^{28}\text{cm}$
- + Can be quantized in a **controllable** way as an effective field theory at energies  $< M_P \simeq 2 \times 10^{19}\text{GeV}$
- Fails at higher energies:
  - interaction strength grows with  $E$  (power counting)
  - non-renormalizable  infinite # of parameters
  - loss of predictive power

Needs a **UV completion** which would be valid at all energies  
*hopefully, it'll have testable signatures*

## Quest for quantum gravity

$R_{\mu\nu}^2$ -gravity

General Relativity is a ~~unitary relativistic local~~ field theory


- + Describes classical phenomena from  $10^{-2}$ cm to  $10^{28}$ cm
- + Can be quantized in a **controllable** way as an effective field theory at energies  $< M_P \simeq 2 \times 10^{19}$ GeV
- Fails at higher energies:
  - interaction strength grows with  $E$  (power counting)
  - non-renormalizable  $\longleftrightarrow$  infinite # of parameters
  - loss of predictive power

Needs a **UV completion** which would be valid at all energies  
*hopefully, it'll have testable signatures*

## Quest for quantum gravity

string theory,  
LQG, CDT

General Relativity is a **unitary relativistic** ~~local~~ field theory


- + Describes classical phenomena from  $10^{-2}$ cm to  $10^{28}$ cm
- + Can be quantized in a **controllable** way as an effective field theory at energies  $< M_P \simeq 2 \times 10^{19}$ GeV
- Fails at higher energies:
  - interaction strength grows with  $E$  (power counting)
  - non-renormalizable  infinite # of parameters
  - loss of predictive power

Needs a **UV completion** which would be valid at all energies  
*hopefully, it'll have testable signatures*

## Quest for quantum gravity

### Horava gravity

General Relativity is a **unitary** ~~relativistic~~ **local** field theory

- + Describes classical phenomena from  $10^{-2}\text{cm}$  to  $10^{28}\text{cm}$
- + Can be quantized in a **controllable** way as an effective field theory at energies  $< M_P \simeq 2 \times 10^{19}\text{GeV}$
- Fails at higher energies:
  - interaction strength grows with  $E$  (power counting)
  - non-renormalizable  infinite # of parameters
  - loss of predictive power

Needs a **UV completion** which would be valid at all energies  
*hopefully, it'll have testable signatures*

## Geometric framework

Imagine that spacetime is endowed with a preferred spacelike foliation

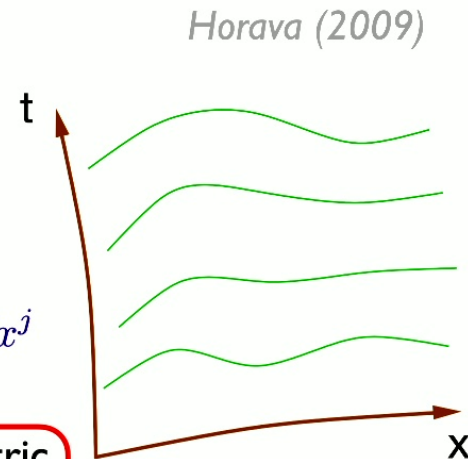
Metric in the ADM decomposition:

$$ds^2 = (-N^2 + N_i N^i) dt^2 + 2N_i dx^i dt + \gamma_{ij} dx^i dx^j$$

↑  
lapse

↑  
shift

↑  
spatial metric



We want to preserve as many symmetries, as possible

$$\text{foliation-preserving diffeos (FDiff)} \begin{cases} x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \\ t \mapsto \tilde{t}(t) \end{cases}$$



## Anisotropic scaling / new power counting

Write Lagrangians with **more than 2 space derivatives** (but still 2 time derivatives). Use different scaling of time and space (*Lifshitz scaling*)

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}, \quad t \mapsto b^{-z}t$$

Most general Lagrangian with only marginal and relevant operators:

$$\mathcal{L} = \frac{1}{2G} \sqrt{\gamma} N (K_{ij} K^{ij} - \lambda K^2 - \mathcal{V}[\gamma_{ij}, N])$$

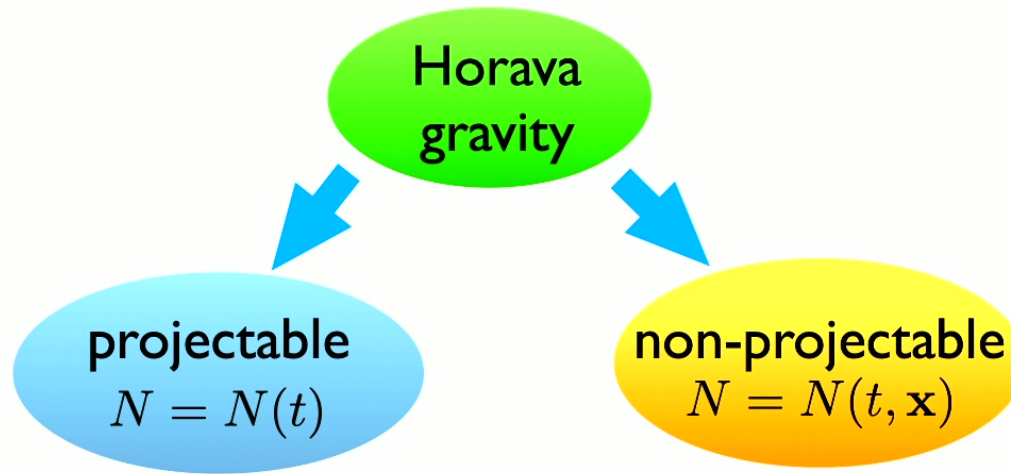
contains terms with up to **2d spatial** derivatives

Reduces to a scalar-tensor gravity at low energies

*Blas, Pujolas, S.S. (2009, 2010)*


$$\mathcal{L} = M_P^2 \sqrt{g} R + \mathcal{L}_\chi[g_{\mu\nu}, \chi]$$

## Two flavours of HG



theory

AAA

A

pheno

BB

A

## Projectable HG: UV complete quantum theory

Renormalizable beyond power counting:

*Barvinsky et al. (2015)*

- ☑ It is possible to fix the gauge without spoiling convergence of loop integrals

*Barvinsky et al. (2017)*

- ☑ The gauge (BRST) structure is preserved by renormalization

Full beta-functions of marginal couplings are available in (2+1)d and (3+1)d

*Barvinsky et al. (2017)*

*Barvinsky, Herrero-Valea, S.S. (2019)*

*Barvinsky, Kurov, S.S. (2021)*

- ☑ Possesses asymptotically free fixed points

- ☑ In (3+1) an attractor RG trajectory from  $\lambda = \infty$  in UV to  $\lambda \rightarrow 1$  in IR

## RG in (3+1)d projectable Horava gravity

6 essential marginal couplings

- ✓ 3 asymptotically free UV fixed points at  $\lambda \rightarrow \infty$

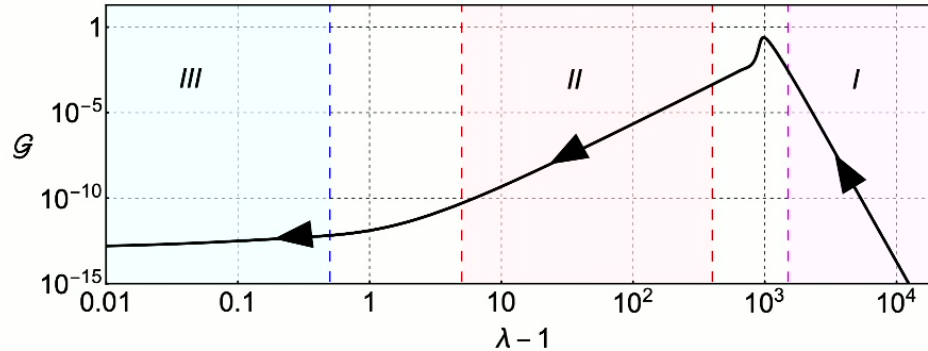
regular limit

*Gümrukçüoglu, Mukohyama (2011)*

*Radkovski, S.S. (2023)*

- ✓ unique attractor trajectory that flows towards  $\lambda \sim 1$

*Barvinsky, Kurov, S.S. (2023)*



gravitational  
coupling changes  
non-monotonically

## Pheno status of projectable HG

- The extra scalar destabilizes Minkowski space at low energies.

Instability suppressed if  $\lambda \approx 1$

- + The extra scalar behaves as dark matter

*Mukohyama (2009)*

- But loss of perturbative control...

Suggestions for way out in *Mukohyama (2010)*, *Izumi, Mukohyama (2011)*,  
*Gümrukçüoglu, Mukohyama, Wang (2011)*

## What about Non-projectable HG?

### *Is it renormalizable?*



We don't know: divergences may be non-local



Non-localities cancel at one loop!

*Bellorin, Borquez, Droguett (2022)*



Can work at higher loops if canonical structure associated to 2nd class constraints is preserved

### *Is it phenomenologically viable?*

**Yes ...** with fine-tuning to recover Lorentz invariance

*Blas, Pujolas, S.S., (2009, 2010, 2011)*

*Gümrukçüoğlu, Saravani, Sotiriou (2017)*

## Low energy limit of NP HG

khrono-metric Lagrangian:

$$\mathcal{L} = -\frac{1}{2}M_P^2\sqrt{-g}(R + \alpha a_\mu a^\mu + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu + \lambda' \vartheta^2)$$

$$u_\mu = \frac{\partial_\mu \chi}{\sqrt{(\partial\chi)^2}}$$

$$a_\mu = u^\nu \nabla_\nu u_\mu$$

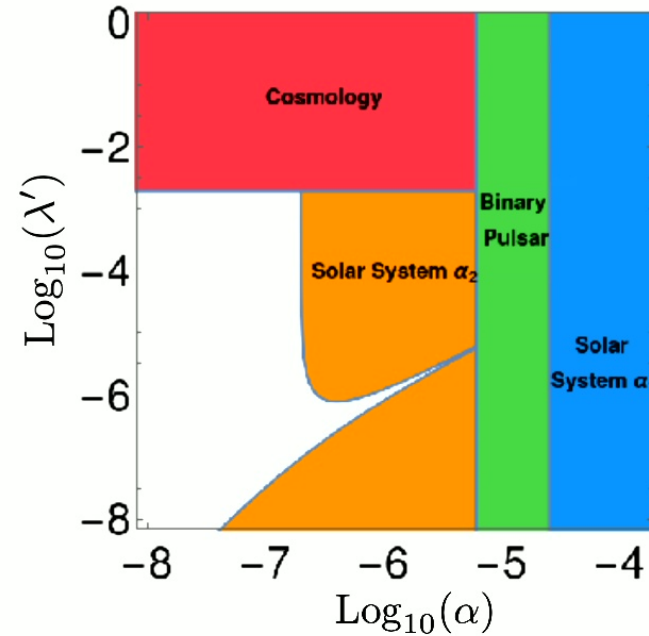
$$\vartheta = \nabla_\nu u^\nu$$

$$\lambda' = \lambda - 1$$

similar to Einstein-aether

*Jacobson, Mattingly (2000)*

$$\text{GW170817: } |\beta| \lesssim 10^{-15}$$



## Low energy limit of NP HG

khrono-metric Lagrangian:

$$\mathcal{L} = -\frac{1}{2}M_P^2\sqrt{-g}(R + \alpha a_\mu a^\mu + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu + \lambda' \vartheta^2)$$

$$u_\mu = \frac{\partial_\mu \chi}{\sqrt{(\partial\chi)^2}}$$

$$a_\mu = u^\nu \nabla_\nu u_\mu$$

$$\vartheta = \nabla_\nu u^\nu$$

$$\lambda' = \lambda - 1$$

similar to Einstein-aether

*Jacobson, Mattingly (2000)*

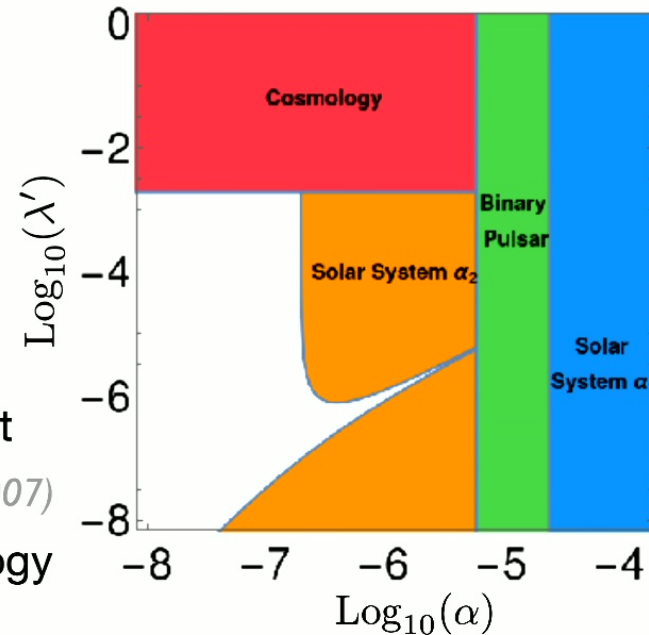
GW170817:  $|\beta| \lesssim 10^{-15}$

**NB.** Reduces to cuscuton in the limit

$$\alpha = \beta = 0$$

*Afshordi et al. (2007)*

➡ constraints only from cosmology





## Summary

- ✓ Horava gravity provides a class of renormalizable quantum gravity theories
- ✓ Full RG flow available in the **projectable** version (2+1) and (3+1)d. [Asymptotically free](#)
- ✓ Progress towards renormalizability of the **non-projectable** version
- ✓ Viability phenomenology constrained by the data