Title: Bouncing cosmology; a solution to the singularity problem and more.

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Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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Abstract:

Finding a complete explanation for cosmological evolution in its very early stages (about 13 billion years ago) can significantly advance our understanding of physics. Over the past few decades, several models have been proposed, with the majority falling into a category called inflationary universes, where the universe experiences rapid exponential expansion. Despite numerous achievements of inflationary models in explaining the origin of the universe, it has been shown that inflationary models generically suffer from being geodesically past incomplete, which is a representation of singularity. Motivated by addressing the singularity problem, we review a recent model of the early universe, called Cuscuton bounce. This model utilizes a theory of modified gravity by the same name, i.e., Cuscuton, which was originally proposed as a dark-energy candidate, to produce a bouncing cosmology. It has been shown that within the Cuscuton model, we can have a regular bounce without violation of the null energy condition in the matter sector, which is a common problem in most bouncing-cosmology models. In addition, the perturbations do not show any instabilities, and with the help of a spectator field, can generate a scale-invariant scalar power spectrum. We will then set out to investigate if this model has a strong coupling problem or any distinguishing and detectable signatures for non-Gaussianities. We expand the action to the third order and obtain all the interaction terms that can generate non-Gaussianities or potentially lead to a strong coupling problem (breakdown of the perturbation theory). While we do not expect the breakdown of the theory, any distinct and detectable sign of non-Gaussianities would provide an exciting opportunity to test the model with upcoming cosmological observations over the next decade.

Bouncing Cosmology via Cuscuton gravity

Amir Dehghani

50 years of Horndeski gravity

University of Waterloo

18 July 2024

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Cuscuton Gravity

- k-essence gravity:
  (special case of Horndeski gravity where $G_3 = G_4 = G_5 = 0$)

$$S = \int \sqrt{-g}[R + P(\varphi, X)], \;\; X = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$$

- Cuscuton gravity: special case of k-essence.

  In FLRW, the equation of motion for Cuscuton reads: $(P_{,X} + 2XP_{XX})\ddot{\varphi} + \cdots = 0$

- In order to not add a new degree of freedom: $P_{,X} + 2XP_{,XX} = 0$

- This leads to unique definition of: $P(\varphi, X) + V(\varphi) \propto \sqrt{X}$

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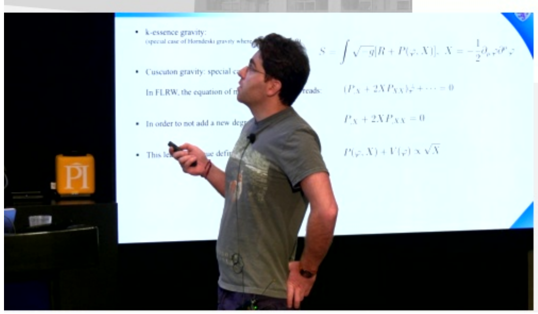

Cuscuton Gravity

- Incompressible limit of k-essence
  (N.Afshordi, D.J.H. Chung, G.Geshnizjani, 2007)
- Low energy limit of Horava-Lifshitz gravity
  (N.Afshordi, 2009)
- Symmetric Superfluid
  (E,Pajer, D,Stefanyszyn, 2019)
  (T.Grall, S.Jazayeri, E.Pajer, 2019)
- Zero temperature effective fluid
  (V.Faraoni, A.Giusti, S.Jose, S.Giardino, 2022)
- A Geometrical picture: Fundamentally discrete ‘field’ labelling transitions in spacetime. Cuscuton is then the continuous limit (many transitions).
  (M. Mylova, N. Afshordi, 2023)

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Motivation

- The main motivation is to have a resolution of the singularity problem.
- Inflation is the leading paradigm for early universe, but it generally does not offer a resolution of the singularity problem.
- We are proposing an alternative to inflation and addressing the singularity problem.


Jerome Quintin


Ghazal Geshnizjani


Supranta Sarma Boruah


Jungjoon Leo Kim


Hyung Jin Kim (Tony)

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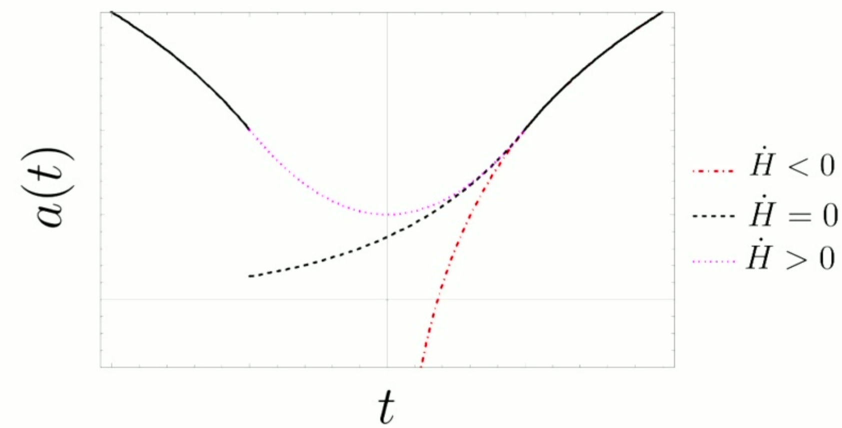
Bouncing Models

Assuming Einstein gravity

$\dot{H} > 0$

$\rho + p < 0$ → ~~NEC~~



$$\dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}$$

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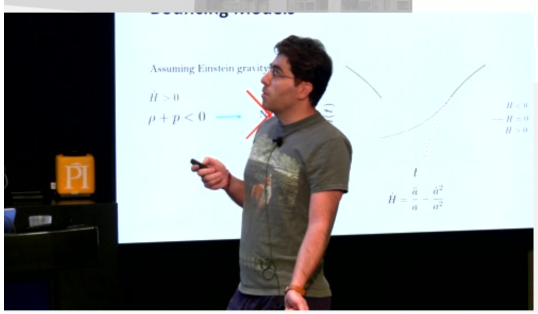

Cuscuton

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R + \mathcal{L}_\varphi + \mathcal{L}_\psi + \mathcal{L}_\chi\right)$$

- Matter Field $\quad \mathcal{L}_\psi = -\frac{1}{2}\partial_\mu\psi\partial^\mu\psi$
- Cuscuton $\quad \mathcal{L}_\varphi = -\mu^2 M_{\mathrm{Pl}}^2\sqrt{-\partial_\mu\varphi\partial^\mu\varphi} - U(\varphi)$
- Spectator Field $\quad \mathcal{L}_\chi = -\frac{1}{2}M_{\mathrm{Pl}}^2 F(\psi, \partial_\mu\psi)\partial_\mu\chi\partial^\mu\chi$

$$g_{\mu\nu}dx^\mu dx^\nu = -(1+2\alpha)dt^2 + 2a\partial_i\beta dt dx^i + a^2(1+2\zeta)\delta_{ij}dx^i dx^j\,.$$

$$\psi(t,\boldsymbol{x}) = \bar{\psi}(t)$$

$$\delta\psi(t,\boldsymbol{x}) \equiv \psi(t,\boldsymbol{x}) - \bar{\psi}(t) = 0$$

$$\varphi(t,\boldsymbol{x}) = \bar{\varphi}(t) + \delta\varphi(t,\boldsymbol{x})$$

$$\chi(t,\boldsymbol{x}) = \bar{\chi}(t) + \delta\chi(t,\boldsymbol{x})$$

$$\dot{\bar{\chi}} = 0$$

$$\delta\chi(t,\boldsymbol{x}) = \chi(t,\boldsymbol{x})$$

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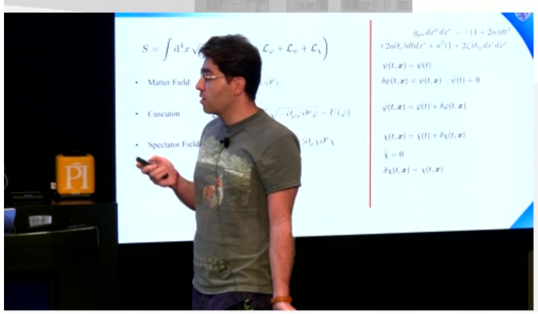
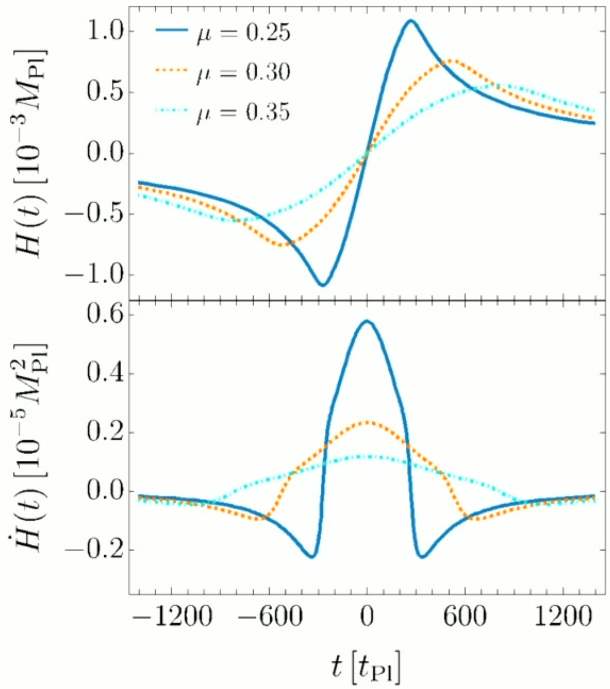


Cuscuton



Background EoMs :

$$3M_{\rm Pl}^2H^2 = \frac{1}{2}\dot{\psi}^2 + U(\bar{\varphi})\,,$$

$$\boxed{-\,2\dot{H} = M_{\rm Pl}^{-2}\dot{\psi}^2 - \mu^2|\dot{\bar{\varphi}}|\,.}$$

Equation of motion for Cuscuton
Non-dynamical, a constraint.

$$\boxed{3\mu^2 H = M_{\rm Pl}^{-2}U_{,\varphi}(\bar{\varphi})\,.}$$

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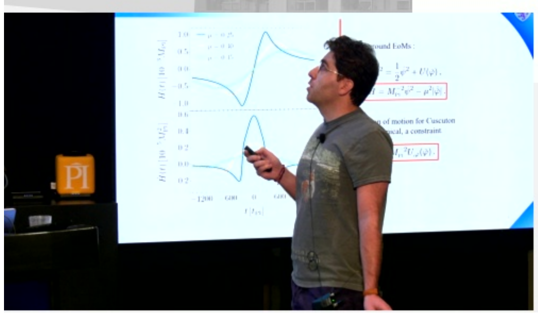
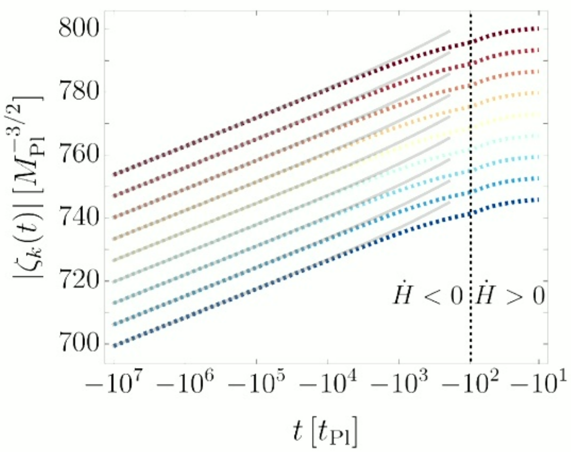
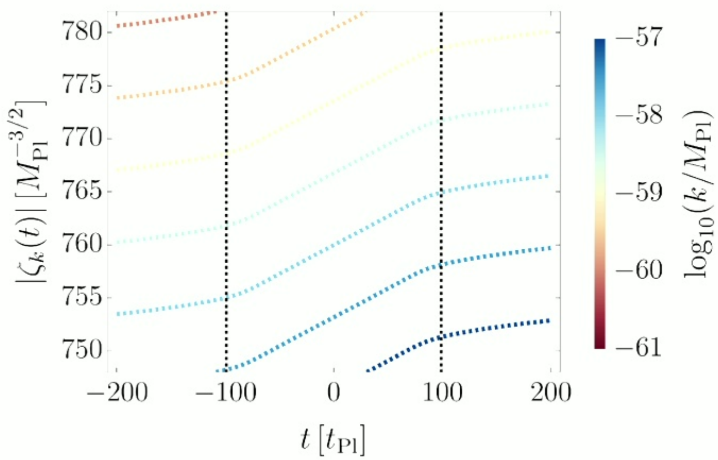


Numerical solutions





$$S_\zeta^{(2)} = \frac{M_{\rm Pl}^2}{2}\int_{\boldsymbol{k}}\int \mathrm{d}\tau\, z^2\left(|\zeta_{\boldsymbol{k}}'|^2 - c_{\rm s}^2 k^2 |\zeta_{\boldsymbol{k}}|^2\right)$$

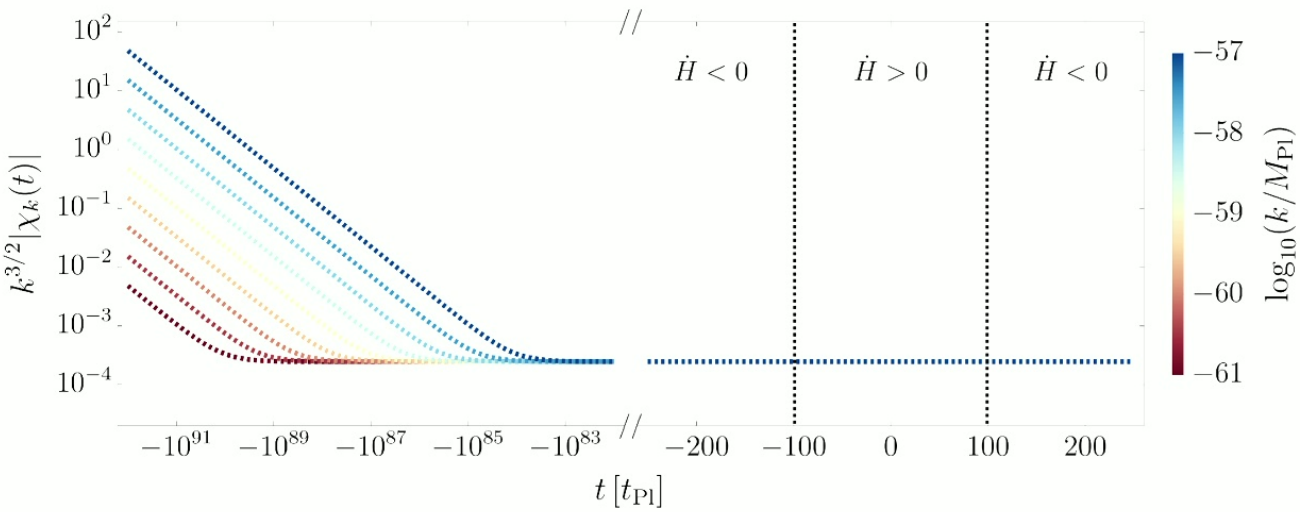
$$\boxed{\begin{aligned} z^2 &> 0 \\ c_S^2 &> 0 \end{aligned}}$$

$$\mathcal{P}_\zeta(k) \propto k^3$$

- Blue Spectrum, unobservable.

Numerical solutions

Entropy perturbations



$$S_\chi^{(2)} = \frac{M_{\rm Pl}^2}{2}\int_{\boldsymbol{k}}\int \mathrm{d}\tau\, z_\chi^2 \left(|\chi_{\boldsymbol{k}}'|^2 - k^2|\chi_{\boldsymbol{k}}|^2\right)$$

$$\boxed{z_\chi^2 > 0}$$

$\mathcal{P}_\chi(k) \propto k^0$

- Near scale-invariant, it matches the observations once is converted to curvature perturbations.

So far

- Cuscuton is a non-dynamical field (both in background and perturbations) which can generate a regular bounce.
- With choosing a proper potential, we can have the desired behavior in the background.
- The theory is ghost-free and instability free (no gradient or tachyonic instability).
  It is true for both scalar and tensor modes.
- Perturbations are stable before, after, and through the bounce.
- With adding a second field with a proper coupling constant, the power spectrum would be scale invariant.

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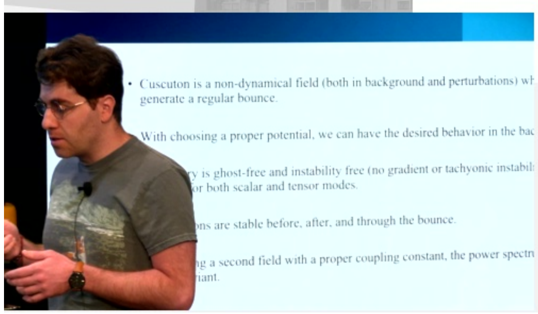

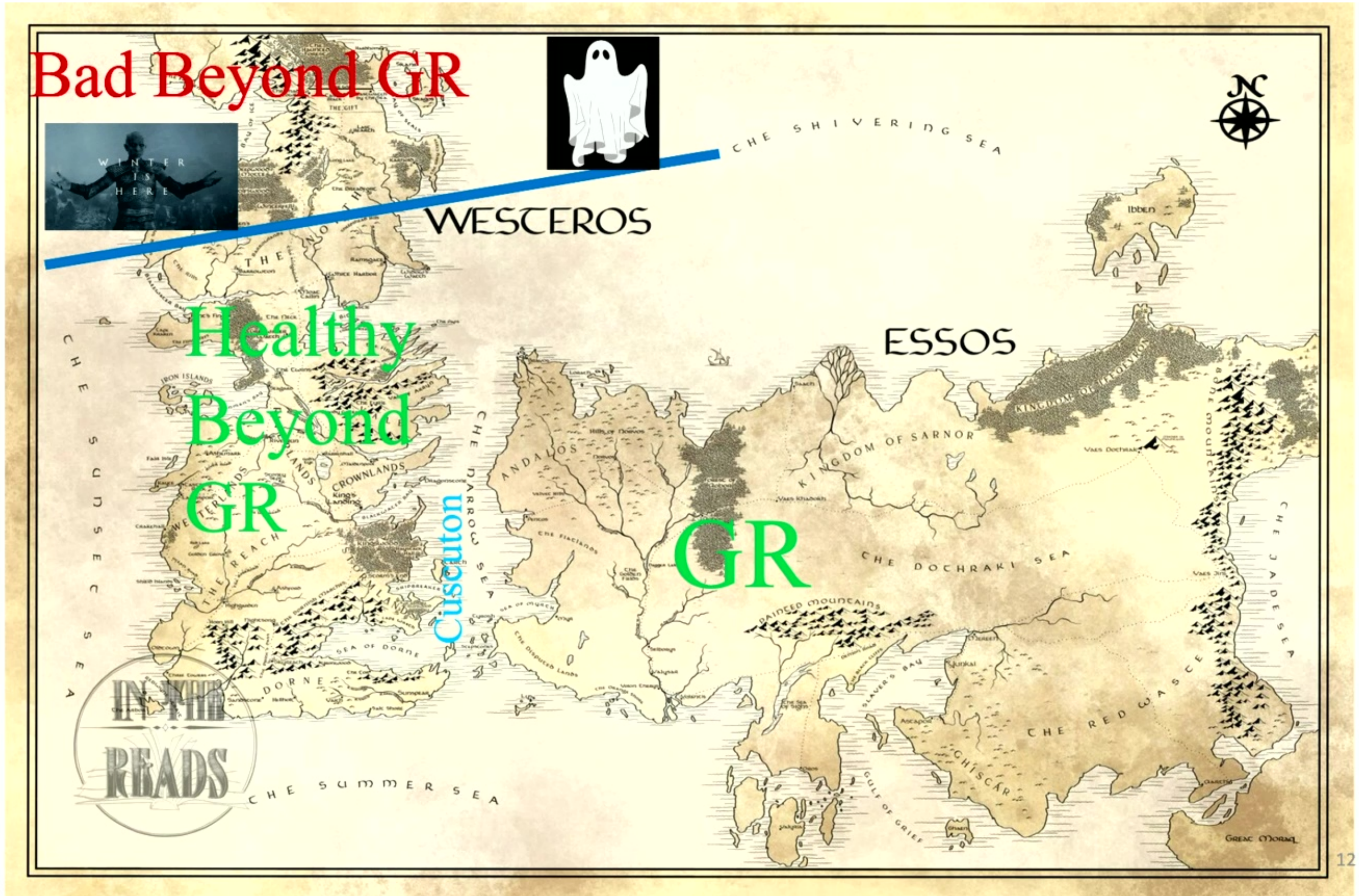
Bad Beyond GR
WINTER IS HERE
WESTEROS
ESSOS
THE SHIVERING SEA
Healthy
Beyond
GR
Cuscuton
GR
THE SUNSET SEA
THE NARROW SEA
THE DOTHRAKI SEA
THE SUMMER SEA
THE JADE SEA
THE RED WASTE
KINGDOM OF SARNOR
IN THE READS
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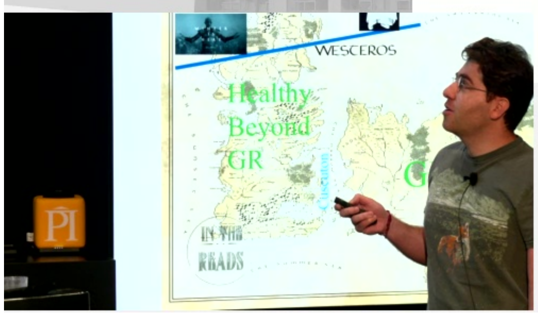
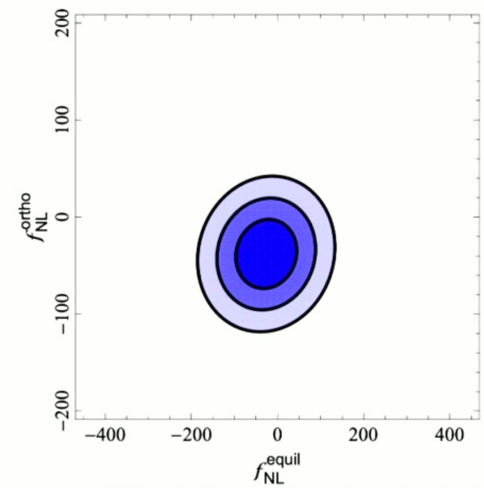


Non-Gaussianities

- Cubic terms in the action act as interactions, and alter the gaussian feature of the field.
- High interactions (strong coupling) should be avoided, otherwise the perturbation theory breaks down.
- "It is challenging to produce scale-invariant fluctuations that are weakly coupled over the range of wavelengths accessible to cosmological observations."
  [D.Baumann, L.Senatore, M.Zaldarriaga, 2011]
- We need to investigate if theory is viable beyond quadratic order.
- Very small non-Gaussianity can be a distinguisher between different models.



Planck 2018 results. IX. Constraints on primordial non-Gaussianity
Planck Collaboration, 2018

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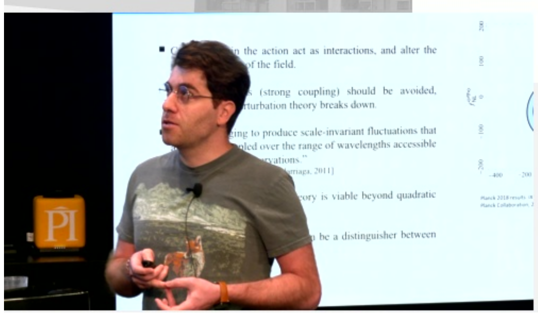

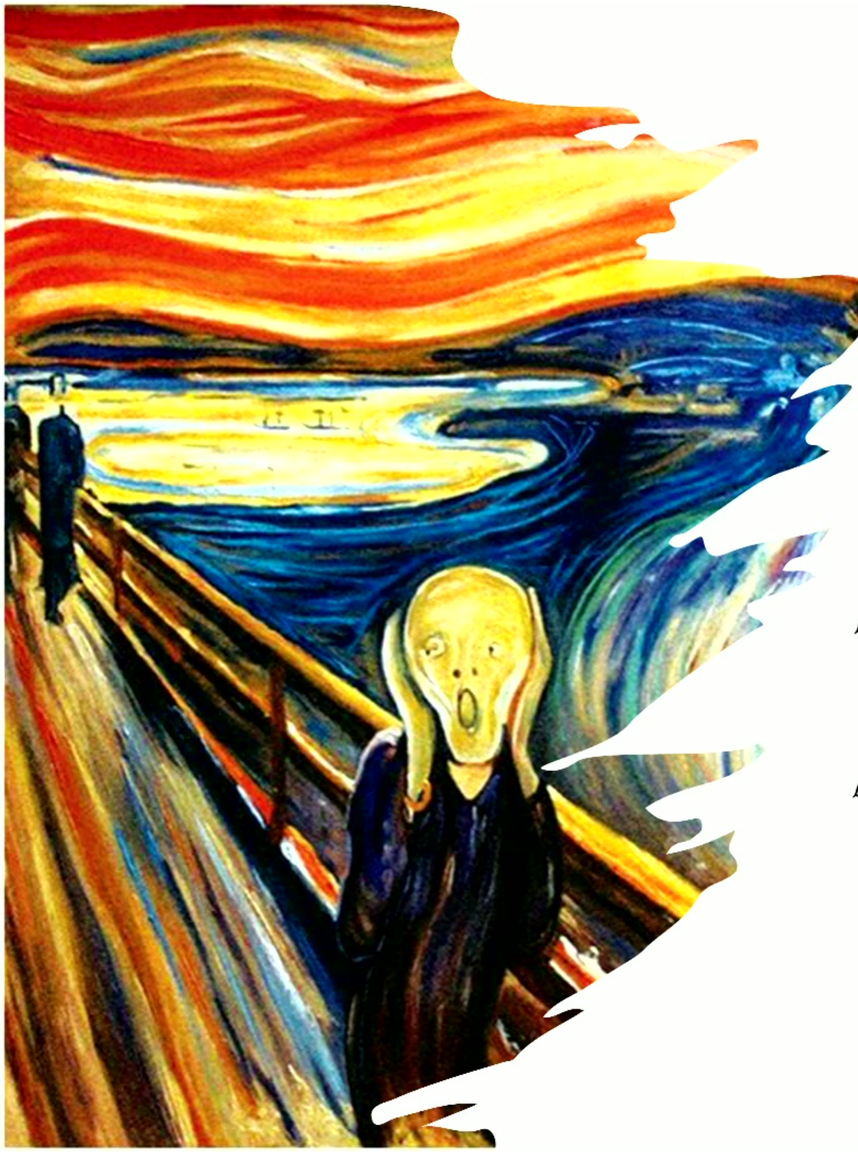

Non-Gaussianities

The devil is in the details.

$$S^{(3)} = M_{\rm Pl}^2 \int \mathrm{d}^3\boldsymbol{x}\,\mathrm{d}t\,a^3 \left(\mathcal{L}^{(3)}_{(\chi)} + \mathcal{L}^{(3)}_{(\delta\varphi)} + \mathcal{L}^{(3)}_{(\rm rest)}\right) .$$

$$\mathcal{L}^{(3)}_{(\chi)} = -\frac{1}{2}F(\psi,X)\left((\alpha-3\zeta)\dot\chi^2 + (\alpha+\zeta)\frac{(\partial_i\chi)^2}{a^2} + 2\frac{\partial^i\beta\partial_i\chi}{a^2}\dot\chi\right) - XF_{,X}\alpha\left(\dot\chi^2 - \frac{(\partial_i\chi)^2}{a^2}\right)$$

$$\mathcal{L}^{(3)}_{(\delta\varphi)} = 9\mu^2(\dot\zeta - H\alpha)\zeta\delta\varphi + \frac{3\mu^4\epsilon(\alpha+3\zeta)}{4(\epsilon_\psi-\epsilon)}\delta\varphi^2 + \frac{\mu^6\epsilon\epsilon_\psi(2\epsilon-\eta-6)}{8H(\epsilon-\epsilon_\psi)^3}\delta\varphi^3 + \mu^2\zeta\frac{\partial^i\beta\partial_i\delta\varphi}{a^2}$$
$$-\frac{\mu^4(2\alpha+\zeta)(\partial_i\delta\varphi)^2}{4(\epsilon-\epsilon_\psi)a^2H^2} - \frac{\mu^6\dot{\delta\varphi}(\partial_i\delta\varphi)^2}{8(\epsilon-\epsilon_\psi)^2a^2H^4},$$

$$\mathcal{L}^{(3)}_{(\rm rest)} = (3-\epsilon_\psi)H^2\alpha^3 + 3(\epsilon_\psi-3)H^2\alpha^2\zeta - 6H\alpha^2\dot\zeta + 18H\alpha\zeta\dot\zeta + 3\alpha\dot\zeta^2 - 9\zeta\dot\zeta^2 + 2H\frac{\partial^i\alpha\partial_i\beta}{a^2}\zeta$$
$$+\frac{(\partial_i\zeta)^2}{a^2}(\zeta-\alpha) - 2\frac{\partial^i\beta\partial_i\dot\zeta + \alpha\partial^2\zeta}{a^2}\zeta + 2\frac{\partial^2\beta}{a^2}(H\alpha-\dot\zeta)\alpha + \frac{(\partial^2\beta)^2-(\partial_i\partial_j\beta)^2}{2a^4}\alpha$$
$$+\frac{\partial^2\beta\partial^i\beta\partial_i\zeta}{a^4} + \frac{3\partial^i\beta\partial^j\beta\partial_i\partial_j\zeta}{2a^4}.$$

$$\alpha = \frac{1}{H}\left(\dot\zeta - \frac{\mu^2}{2}\delta\varphi\right), \qquad \beta = -\frac{\zeta}{H} + a^2\epsilon_\psi\xi, \qquad \delta\varphi = \frac{2}{\mu^2}H(\epsilon-\epsilon_\psi)\mathcal{D}^{-2}\left(\epsilon_\psi H\dot\zeta - \frac{\partial^2}{a^2}\zeta\right),$$

$$\epsilon \equiv -\frac{\dot H}{H^2}, \qquad \epsilon_\psi \equiv \frac{\dot\psi^2}{2M_{\rm Pl}^2H^2}, \qquad \eta \equiv \frac{\dot\epsilon}{H\epsilon}, \qquad \eta_\psi \equiv \frac{\dot\epsilon_\psi}{H\epsilon_\psi}.$$

Non-Gaussianities

- First task is to simplify the expressions and find the dominant terms. $\langle \zeta^3 \rangle$ is small compared to $\langle \zeta \chi^2 \rangle$. There is no $\langle \chi^3 \rangle$.

- Bi-spectrum and the three-point correlation function.

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, t_{\mathrm{e}}) \delta^{(3)} \Big(\sum_{j=1}^{3} \boldsymbol{k}_j \Big) = \langle \hat{\zeta}_{\boldsymbol{k}_1}(t_{\mathrm{e}}) \hat{\chi}_{\boldsymbol{k}_2}(t_{\mathrm{e}}) \hat{\chi}_{\boldsymbol{k}_3}(t_{\mathrm{e}}) \rangle \,,$$

$$\tilde{B}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, t_{\mathrm{e}}) = \frac{(k_1 k_2 k_3)^3}{k_1^3 + k_2^3 + k_3^3} B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, t_{\mathrm{e}}) \,.$$

- A quantitative definition of non-Gaussianities for a real random field.

$$\zeta(t, \boldsymbol{x}) = \zeta^{\mathrm{G}}(t, \boldsymbol{x}) - f_{\mathrm{NL}}^{\zeta} [\zeta^{\mathrm{G}}(t, \boldsymbol{x})]^2 - C_{\mathrm{NL}}^{\zeta} \zeta^{\mathrm{G}}(t, \boldsymbol{x}) \chi^{\mathrm{G}}(t, \boldsymbol{x}) \,,$$

$$\chi(t, \boldsymbol{x}) = \chi^{\mathrm{G}}(t, \boldsymbol{x}) - f_{\mathrm{NL}}^{\chi} [\chi^{\mathrm{G}}(t, \boldsymbol{x})]^2 - C_{\mathrm{NL}}^{\chi} \zeta^{\mathrm{G}}(t, \boldsymbol{x}) \chi^{\mathrm{G}}(t, \boldsymbol{x}) \,.$$

Non-Gaussianities

- First task is to simplify the expressions and find the dominant terms. $\langle\zeta^3\rangle$ is small compared to $\langle\zeta\chi^2\rangle$. There is no $\langle\chi^3\rangle$.

- Bi-spectrum and the three-point correlation function.

$$B(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,t_\mathrm{e})\delta^{(3)}\Big(\sum_{j=1}^{3}\boldsymbol{k}_j\Big)=\langle\hat{\zeta}_{\boldsymbol{k}_1}(t_\mathrm{e})\hat{\chi}_{\boldsymbol{k}_2}(t_\mathrm{e})\hat{\chi}_{\boldsymbol{k}_3}(t_\mathrm{e})\rangle\,,$$

$$\tilde{B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,t_\mathrm{e})=\frac{(k_1k_2k_3)^3}{k_1^3+k_2^3+k_3^3}B(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,t_\mathrm{e})\,.$$

- A quantitative definition of non-Gaussianities for a real random field.

$$\zeta(t,\boldsymbol{x})=\zeta^\mathrm{G}(t,\boldsymbol{x})-f^{\zeta}_{\mathrm{NL}}[\zeta^\mathrm{G}(t,\boldsymbol{x})]^2-C^{\zeta}_{\mathrm{NL}}\zeta^\mathrm{G}(t,\boldsymbol{x})\chi^\mathrm{G}(t,\boldsymbol{x})\,,$$

$$\chi(t,\boldsymbol{x})=\chi^\mathrm{G}(t,\boldsymbol{x})-f^{\chi}_{\mathrm{NL}}[\chi^\mathrm{G}(t,\boldsymbol{x})]^2-C^{\chi}_{\mathrm{NL}}\zeta^\mathrm{G}(t,\boldsymbol{x})\chi^\mathrm{G}(t,\boldsymbol{x})\,.$$

What matters for $\langle\zeta\chi^2\rangle$

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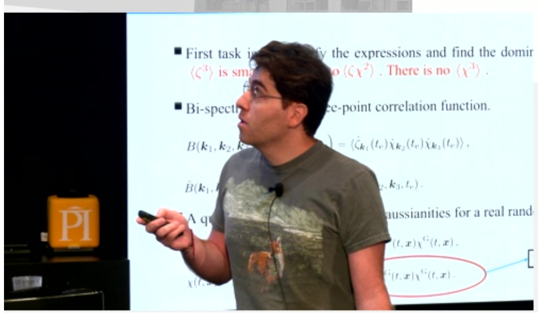
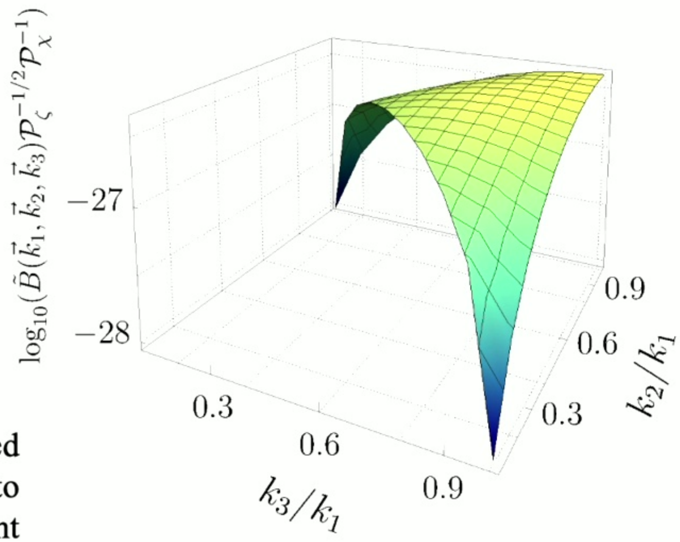


Non-Gaussianities

- $C_{\mathrm{NL}}^{\chi}\zeta^{\mathrm{G}}(t,\boldsymbol{x})$ Which represents the level of non-Gaussianities is order of 10^{-26}
- $f_{\mathrm{NL}}^{\zeta}\zeta^{\mathrm{G}}(t,\boldsymbol{x})$ is even more suppressed.
- The theory stays perfectly Gaussian.
- Some $O(1)$ non-Gaussianities may be generated during the conversion of entropy perturbations to adiabatic perturbations. This will be the dominant signal.



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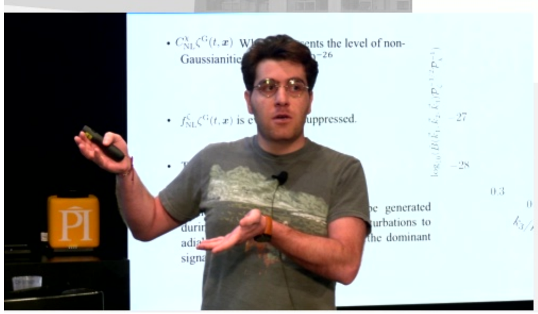
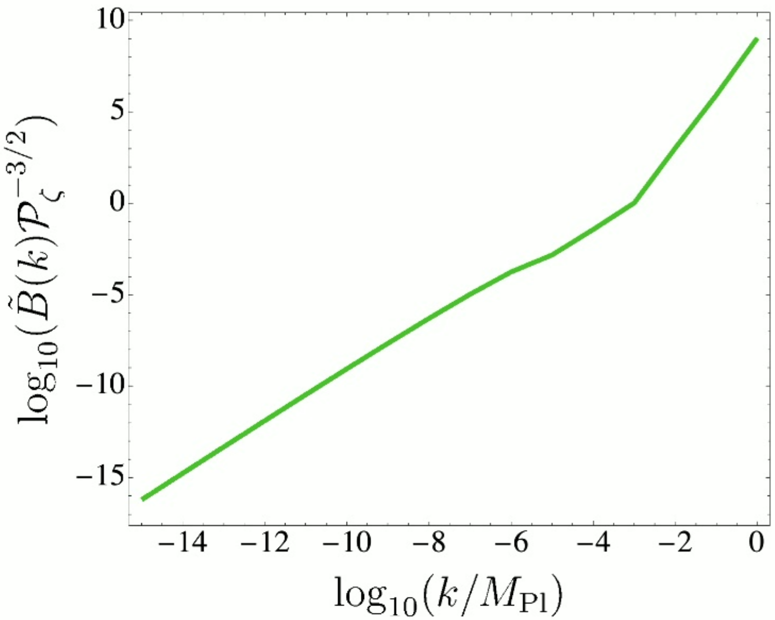
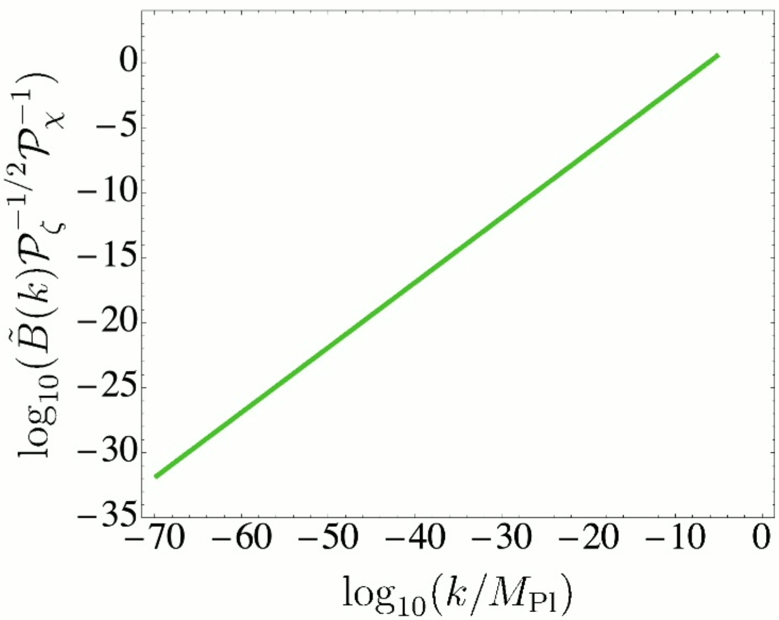


Strong coupling scale

- Curvature perturbations



- Entropy perturbations



Conclusion

- ✓ Avoiding cosmological singularities seems to be quite challenging.
- ✓ One way to go around the cosmological singularity is the bouncing cosmology, however, this does not come free. One has to deal with several theoretical challenges including the violation of null energy condition.
- ✓ Cuscuton bounce is a special modified gravity theory which generates a bounce precisely such that the theory does not propagate any new local degree of freedom (DoF) compared to GR
- ✓ The theory is ghost-free, instability-free, and perturbations are stable before, after, and through the bounce.
- ✓ We showed consistent numerical solutions accompanying our analytical solutions for super-Hubble modes
- ✓ The non-Gaussianities remain small and the model does not suffer from any strong coupling issue.