Title: Constructing rotating black holes in Horndeski gravity

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Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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Abstract:

Exploring the structure of compact objects in modified theories of gravity is mandatory to parametrize the possible deviations w.r.t general relativity and confront these theories to the current and future observations. While important efforts have been devoted to understand the phenomenology of stars and black holes, it is still a challenging task to provide new exact analytical solutions describing rotating black hole in such theories. In this talk, I propose to recent efforts to construct such solutions. Concretely, I will review how one can mix the disformal field redefinitions affect the Petrov type of a given gravitational field and how this can be used to constrain the derivation of rotating black hole. Then, I will review the main properties of a new solution of a subset of Horndeski theories called the disformal Kerr black hole and comment on the most promising directions to derive exact rotating black hole solutions in scalar-tensor theories.

This talk will be based on the two articles: https://inspirehep.net/literature/1800972, https://inspirehep.net/literature/1877661

Constructing rotating compact objects in Horndeski gravity

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50 years of Horndeski gravity: Exploring modified gravity
Waterloo
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Based on
J. BA, H. Roussille [to appear]
J. BA, A. De Felice, M. A. Gorji, S. Mukohyama, M. Pookkillath [JCAP '21]
J. BA, H. Liu, H. Motohashi, S. Mukohyama, K. Noui [JCAP '20]

Context and Motivations

Observational motivations

- Current and future missions $\rightarrow$ increasing accurate data on the strong field regime of gravity
- GRAVITY: orbits of stars around Sagitarius A* (2020)
- EHT: imaging black hole environment (2019)
- BLACK HOLE EXPLORER: imaging black hole photon rings structure [Lupsasca, Cardenas-Avendano, Palumbo, Johnson, Gralla '24]
- LISA : BBH merger and ringdown $\rightarrow$ black hole spectroscopy

Goal

- Community of observers need tractable model of compact objects to test w.r.t GR

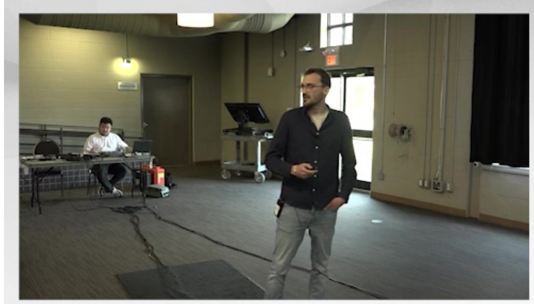
Startegy

- Step 1: construct suitable ad hoc model of rotating compacts objects [Johannsen-Psaltis '20]
- Step 2: derive exact solutions for rotating compact objects in modified gravity
  key challenge $\rightarrow$ loss integrability of the axi-symmetric phase space of GR
  need to adapt old or identify new solution-generating techniques
  (Newman-Penrose formalism, Ernst formalism, Janis-Newman trick)
- Step 3: understand their properties / preservation or loss of Kerr symmetries

$\rightarrow$ review efforts in this direction in the context of Horndeski gravity

Outlines

- Disformal solution-generating method
- Two families of exact solutions and their properties
- Disformal transformation of the Petrov type



Disformal structure of DHOST gravity

- Degenerate higher order scalar tensor theories

$$S[g,\varphi] = \int \mathrm{d}^4x\sqrt{|g|}\,[f(\varphi,X)\mathcal{R} + P(\varphi,X) + A_I(\varphi,X)\mathcal{L}_I] \qquad (1)$$

$$\Psi(A_I, \partial_\varphi A_I, \partial_X A_I) = 0 \qquad (2)$$

- Class Ia corresponds to all disformally related theories to Horndeski gravity

$$L_1 = (\Box\varphi)^2 \qquad L_2 = \varphi_{\mu\nu}\varphi^{\mu\nu} \qquad A_1 = -A_2 \qquad (3)$$

- Healthy (degenerate) higher order scalar-tensor theories are organized into equivalence class under disformal field transformations (DFT)

$$(\tilde{g}_{\mu\nu}, \varphi) \to (\tilde{g}_{\mu\nu} = A(\varphi,X)g_{\mu\nu} + B(\varphi,X)\varphi_\mu\varphi_\nu, \varphi) \qquad (4)$$

  Degeneracy conditions are preserved under DFT

$$\Psi(A_I, \partial_\varphi A_I, \partial_X A_I) = 0 \qquad \to \qquad \Psi(\tilde{A}_I, \partial_\varphi \tilde{A}_I, \partial_{\tilde{X}} \tilde{A}_I) = 0 \qquad (5)$$

  [**BA**, Noui, Langlois '16]

- **Solution generating map** provided we know a solution of one of the DHOST theory [**BA**, Liu, Mukohyama '19]
- Mix this transformation with known methods to guide us in exploring the solution space (disformal NP formalism)

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Disformal map

- Consider the simplest disformal transformation (DT) of the Einstein-Scalar system

$$S[g_{\mu\nu}, \phi] = \int \mathrm{d}^4 x \sqrt{|g|} \left[\mathcal{R} - \frac{X}{2} \right] \tag{6}$$

- Consider the simplest disformal field redefinition

$$(g_{\mu\nu}, \varphi) \to (\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_0 \varphi_\mu \varphi_\nu, \varphi) \tag{7}$$

- The new action is given by the following Horndeski theory

$$\tilde{S}[\tilde{g}_{\mu\nu}, \varphi] = \int \mathrm{d}^4 x \sqrt{|\tilde{g}|} \left[G_2(\tilde{X}) + G_4(\tilde{X}) \mathcal{R} - 2 G_{4\tilde{X}}(\tilde{X}) \left((\Box\varphi)^2 - \varphi_{\mu\nu}\varphi^{\mu\nu} \right) \right] \tag{8}$$

with the two functions (G_2, G_4):

$$G_2(\tilde{X}) = \frac{\tilde{X}}{2} \qquad G_4(\tilde{X}) = \frac{1}{\sqrt{1 - B_0 \tilde{X}}} \tag{9}$$

Key points:

- DT are a pure field redefinition
- New physics show up by (implicitly) assuming that test fields couple to $\tilde{g}_{\mu\nu}$
  - → different causal structure
  - → different Petrov type [**BA**, De Felice, Gorji, Mukohyama, Pookilath '22]
  - → different principal null directions, spin coefficients, GW content [**BA**, Gorji, Roussille '24]
  - → different geodesics (photon rings, QNM spectrum)
- Disformal transformation of the seed solution reveals the effects of the higher order terms (controlled by B_0) in the presence of the very same scalar source

From naked singularities to black holes

Naked singularities

GR solutions with scalar field

- Classification of exact axi-symmetric solutions of the Einstein-Scalar system [Janis, Newman, Winicour '68] .... [Bogush, Gal'tsov ' 20]
- Example: quadrupolar non-rotating exact solution sourced by a standard scalar field in GR

$$\mathrm{d}s^2 = -e^{\gamma f(r)}\mathrm{d}t^2 + e^{\beta f(r)}F^{\nu}(r,\theta)\left[e^{-f(r)}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2\right] + r^2 f^{1-\gamma}(r)\sin^2\theta\,\mathrm{d}\phi^2 \qquad (10)$$

$$\phi(r) = \frac{1-\gamma^2-\nu}{\ell_P}f(r) \qquad (11)$$

$$f(r) = \log\left(1-\frac{m}{r}\right) \qquad F(r,\theta) = 1-\frac{2m}{r}+\frac{m^2\sin^2(\theta)}{r^2} \qquad (12)$$

$$\nu = 1-\gamma-\beta \qquad \nu \leqslant 1-\gamma^2 \qquad \beta \geqslant \gamma^2-\gamma \qquad (13)$$

- Extension to rotation obtained very recently [Mirza, Kangazi, Sadeghi '23]

Full four-parameter family of rotating compact objects for Horndeski

- Effect of the higher order terms for the scalar profile are encoded in the parameter

$$g_{rr} = e^{\beta f(r)}F^{\nu}(r,\theta)e^{-f} \qquad \rightarrow \qquad \tilde{g}_{rr} = e^{\beta f(r)}F^{\nu}(r,\theta)\left[e^{-f} + B_0\beta^2(f')^2\right] \qquad (14)$$

- Change the radial behavior of the geometry: photon ring, ISCO, shadow, geodesics ...
- All geometries are Petrov type I and represent naked singularities
- No-go: can't generate any black holes through this path (under some assumptions) spherical symmetry [BA, Liu, Mukohyama '19], work in progress to extend to axi-symmetry [BA, Roussille '24]

Way out to get black holes ? Identify no hair theorem for rotating black holes

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No-hair theorems as guides

- No hair theorem: from GR to scalar-tensor theories
  [Bekenstein '72] [Hui, Nicolis '13] [Graham, Jha '14] [Lebehel, Babichev, Charmousis '17]
- Recent generalization to rotating black holes for DHOST theories
  Provided:
  1) the metric is circular: $(t \to -t, \phi \to -\phi)$
  2) asymptotically flat
  3) the lagrangian and the field equations are shift symmetric: $\varphi \to \varphi + c$
  4) the scalar profile shares the same symmetries as the metric: $\mathcal{L}_\xi \varphi = 0$
  5) Conserver Noether current of shift symmetry reduces asymptotically to $J_\mu = -\partial_\mu \varphi$
  6) $J^\mu J_\mu$ is regular on the horizon

then any black hole cannot carries a non-trivial scalar charge: a contribution in $1/r$ at large radii
[Capuano, Santoni, Barausse '23]

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Evading the no-hair theorems: disformal Kerr black hole

- **Stealth Kerr**: configuration such that the scalar field does not gravitate : $\mathcal{T}_{\mu\nu} = 0$

$$\mathrm{d}s^2 = \mathrm{d}s^2_{\text{Kerr}} \qquad \varphi(t,r) = Et \pm \int \mathrm{d}r \frac{\mathcal{R}}{\Delta} \tag{15}$$

  with $\mathcal{R}(r) = \sqrt{2Mr(r^2+a^2)}$ and $\Delta(r) = r^2 - 2Mr + a^2$
- Scalar field regular on the past and future horizons : follow the geodesic curves
- Conditions for a stealth Kerr black hole [Charmousis, Crisostomi, Gregory, Stergioulas '19]

- **Disformed Kerr black hole**

$$\mathrm{d}s^2 = \mathrm{d}s^2_{\text{Kerr}} + B_0 \left(\mathrm{d}t + \frac{\mathcal{R}}{\Delta}\mathrm{d}r\right)^2 \qquad \varphi(t,r) = Et \pm \int \mathrm{d}r \frac{\mathcal{R}}{\Delta} \tag{16}$$

  [**BA**, Liu, Mukohyama, Noui '20] [Anson, Babichev, Charmousis, Cisterna, Hassaine '20]
- Same ergoregion, non-circular geometry [Ghosh, Chakravarti '24]
- Affect EMRI signals [Babichev, Charmousis, Doneva, Gyulchev, Yazadjiev '24]
- Challenge the definition of multipoles: no ACMC coordinates available [Mayerson '23]
- Disformed Kerr geometry no longer algebraically special: Petrov type I
  [**BA**, De Felice, Gorji, Mukohyama, Pookilath '22]
- Loss of symmetry : Killing-Yano tensor only for type D and N [Collinson '76]
  → explain why it miss the elegant symmetry of Kerr ...
  → need to find algebraically special solutions !

How do we keep control on the change of Petrov type under disformal transformation ?

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Disformal transformation on the tetrad field

- Usually, DT are written at the level of the metric

$$(g_{\mu\nu}, \phi) \to (\tilde{g}_{\mu\nu} = A g_{\mu\nu} + B\phi_\mu \phi_\nu, \phi) \tag{17}$$

with $A := A(\phi, X)$, $B := B(\phi, X)$ and $X = g^{\mu\nu}\phi_\mu\phi_\nu$.

- To understand the change in the Newman-Penrose quantities, implement DT on the tetrad
- Introduce the J-map

$$J^A{}_B = \sqrt{A}\left(\delta^A{}_B + \frac{\beta}{1-\beta X}\phi^A\phi_B\right) \qquad \beta = \frac{1}{X}\left[1 - \frac{\sqrt{A}}{\sqrt{A+BX}}\right] \tag{18}$$

with $\phi_A = E^\mu_A \phi_\mu$ and $X = \phi_A\phi^A = \phi_\mu\phi^\mu$

- Allows one to implement DT in a local rest frame:

$$\boxed{\tilde{E}^A_\mu = J^A{}_B E^B_\mu \qquad \tilde{g}_{\mu\nu} = (J^A{}_C E^C_\mu)(J^B{}_D E^D_\nu)\eta_{CD} = A g_{\mu\nu} + B\phi_\mu\phi_\nu} \tag{19}$$

- Close formula for the disformed null directions:

$$\ell^\mu = E^\mu_A \ell^A \qquad \to \qquad \tilde{\ell}^\mu = (J^B{}_A E^\mu_B)(J^A{}_C \ell^C) = \frac{1}{\sqrt{A}}\left[\ell^\mu + \beta \ell^\alpha \phi_\alpha \phi^\mu\right] \tag{20}$$

- Close formula for the disformed optical scalars: expansion Θ, twist ω, shear σ

$$\tilde{\Theta} = \frac{1}{\sqrt{A}}\left[\Theta - \ell^\mu \nabla_\mu(\sqrt{A}/J) - \frac{\sqrt{A}}{J}\nabla_\mu\left(\frac{\beta J}{\sqrt{A}}\ell^\alpha\phi_\alpha\phi^\mu\right)\right] \tag{21}$$

- Formula for the change of Weyl scalars and thus of Petrov type :
  $\to$ guide for new exact solutions [BA, De Felice, Gorji, Mukohyama, Pookilath '22]

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Conclusion

- Obtaining exact black hole solutions in Horndeski gravity is challenging : no-hair theorem [Capuano, Santoni, Barausse '23]
- Disformal solution generating map provides one efficient way to construct such solution [**BA**, Liu, Mukohyama '19] [**BA**, Roussille '24]
- First family: naked singularity with mass, spin, quadrupole and scalar charges
- Second family: disformed Kerr black hole solution → test non-circularity [Ghosh, Chakravarti '24]
- None of them preserve the Petrov type D → loss of Kerr symmetry
- Challenge: look for algebraically special solutions in modified gravity

Strategy

- Preserve the Petrov type under a disformal transformation
- Provide concrete formula to do so : depend on the profile of the scalar field [**BA**, De Felice, Gorji, Mukohyama, Pookilath '22]
- Pave the way for deriving Petrov type D solutions with separable wave equation: Work in progress
- Develop Newman-Penrose formalism / Ernst-like equations for scalar-tensor theories [**BA**, Roussille '24]
- Need systematic method to explore the solution space

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