**Title:** Against Horndeski

**Speakers:** Cliff Burgess

**Collection/Series:** 50 Years of Horndeski Gravity: Exploring Modified Gravity

**Subject:** Cosmology, Strong Gravity, Mathematical physics

**Date:** July 18, 2024 - 11:45 AM

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**Abstract:**

The Horndeski program is motivated by arguing that scalar-tensor modifications to gravity should have two properties: effective interactions that are at most second-order in time derivatives and only a single scalar. I will argue against both of these criteria. First I argue why the low-energy limit of known well-behaved theories can have more than two-derivative field equations. Second I argue why the scalar-tensor interactions most likely to be found competing with gravity at very low energies typically are those with two derivatives, at least when semiclassical methods are justified, and this suggests exploring multiple-scalar models.

# Against Horndeski


CPB @ McMaster University

*50 Years of Horndeski*

PI & UW, July 18, 2024

# UV Info at Very Low Energies


CPB @ McMaster University

*50 Years of Horndeski*

PI & UW, July 18, 2024


UV Info at Very Low Energies
(the Highland Program)
CPB @ McMaster University
PERIMETER INSTITUTE
50 Years of Horndeski
PI & UW, July 18, 2024

P. Brax

A. Davis

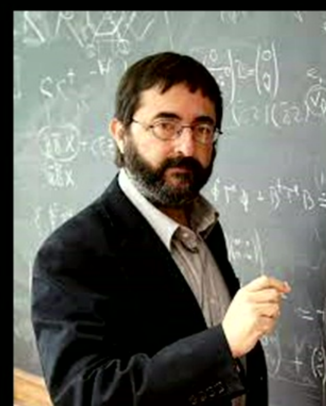
D. Dineen

*Axiodilaton tests*
2212.14870
*Screening*
2310.02092
*CMB*
2407.xxxxx

*Yoga models*
2111.07286
*dS & inflation*
2202.05344

M. Mylova

A. Smith



F. Quevedo

C. van de Bruck

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Based on earlier work on ubiquity of accidental symmetries in EFTs for string vacua
S. Krippendorf M. Cicoli
M. Ciupke
M. Mylova
A. Smith
F. Quevedo
C. van de Bruck

# The Horndeski Assumptions

**Single scalar field**

*Simplicity; $\varepsilon^2$ physics*

**Second-order field equations**

*Fear of ghosts; well-posedness*

# The Horndeski Assumptions

Single scalar field

*Simplicity; $\varepsilon^2$ physics*

*Light scalars correlate with small DE*

*More is different*

Second-order field equations

*Fear of ghosts; well-posedness*

*UV implications for the IR?*

*Ghosts begone!*

*Classical approximation & derivative counting*

# Outline

EFTs, ghosts and well-posedness

*Ostrogradski garlic*

UV implications for the IR

*Which interactions like to compete with gravity?*

*More is different*

Axio-dilatons

*Motivations, preliminary results*

*(Having a wonderful time, wish you were here!)*


# EFTs, ghosts & well-posedness

*Who you gonna call?*

GHOSTBUSTERS

# Ostrogradsky Instability

Higher-derivative toy model

$$L = \frac{v^2}{2}\left(\dot{\vartheta}^2 + \frac{1}{M^2}\ddot{\vartheta}^2\right)$$

Field equation

$$-\ddot{\vartheta} + \frac{1}{M^2}\ddddot{\vartheta} = 0$$

General solution

$$\vartheta(t) = C_1 + C_2 t + \boxed{C_+ e^{Mt} + C_- e^{-Mt}}$$

Must specify: $\vartheta(0),\ \dot{\vartheta}(0),\ \ddot{\vartheta}(0),\ \dddot{\vartheta}(0)$

# Ostrogradsky Instability

*Ostrogradsky 1850*

*1506.0221*

Very general result

$$L = L(\vartheta, \dot{\vartheta}, \ddot{\vartheta})$$

Canonical variables

$$x_1 = \vartheta \qquad p_1 = \frac{\partial L}{\partial \dot{\vartheta}} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \ddot{\vartheta}}\right)$$

$$x_2 = \dot{\vartheta} \qquad p_2 = \frac{\partial L}{\partial \ddot{\vartheta}}$$

$$\ddot{\vartheta} = A(x_1, x_2, p_2)$$

Hamiltonian

$$H(x_1, x_2, p_1, p_2) = p_1 x_2 + p_2 A(x_1, x_2, p_2) - L[x_1, x_2, A(x_1, x_2, p_2)]$$

# EFTs and Higher Derivatives

*1404.2236*

*1709.09695*

Do higher derivatives actually arise in EFTs obtained by integrating out heavy fields in sensible UV completions? *(Yes, as it turns out.)*

$$L = (\partial\phi)^*(\partial\phi) - \frac{\lambda}{4}(\phi^*\phi - v^2)^2$$

Fields

$$\phi = \rho(x)\, e^{i\vartheta(x)} \qquad M^2 = \lambda v^2$$

EFT for the massless field

$$-\frac{\mathcal{L}_{\rm eff}}{v^2} = \frac{1}{2}(\partial\vartheta)^2 - \frac{1}{2M^2}(\partial\vartheta)^4 + \frac{2}{M^4}\partial_{\mu\nu}\vartheta\partial^{\mu\rho}\vartheta\partial^\nu\vartheta\partial_\rho\vartheta +$$

$$= \frac{1}{2}(\partial\vartheta)^2 + \text{Horndeski} + \frac{c}{M^6}(\partial_{\mu\nu}\vartheta\partial^{\mu\nu}\vartheta)^2 + \cdots$$

1404.2236

# EFTs and Higher Derivatives

How can the EFT have instabilities when the full theory does not?

1: *EFT is only local at a fixed order in 1/M*

$$-\frac{\mathcal{L}_{\rm eff}}{v^2} = \frac{1}{2}(\partial\vartheta)^2 - \frac{1}{2M^2}(\partial\vartheta)^4 + \cdots$$

2. *Should only trust EFT solutions at fixed order in 1/M*

$$\vartheta(t) = C_1 + C_2 t + \underbrace{C_+ e^{Mt} + C_- e^{-Mt}}$$

*These are exponentially small in 1/M*

*In practice well-posedness remains an issue:*
eg when numerical evolution can generate spurious runaway behaviour.


# UV implications for the IR

*A non-swampy overview including some faults*

# A Light-Scalar Surprise

Particle physicists usually argue that light scalar fields are NOT generic at low energies

A technically natural Dark Energy density makes them *more* likely rather than less likely

BUT we are likely looking for them in the wrong way (by doing so using eg Horndeski models).

# Light Gravitating Scalars

What should the low-energy dynamics of gravitating scalars look like?

$$\mathcal{L}_W = -\sqrt{-g}\left[v^4 U(\phi) + \tfrac{1}{2}M_p^2 R + \tfrac{1}{2}f^2 \mathcal{G}_{ab}(\phi)\partial_\mu\phi^a\partial^\mu\phi^b \boxed{+ c_2 R^2 + \cdots}\right]$$

Four derivative terms and so on

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It is also technically natural for $v$ to be large, but typically $v^2 = H\, M_p$ and $H \ll M_p$ if the derivative expansion is to be valid (*the cc problem*)

$$M_p^2\, R_{\mu\nu} + M_p^2\, \mathcal{G}_{ab}(\phi)\partial_\mu\phi^a\partial_\nu\phi^b + v^4 U(\phi) g_{\mu\nu} + \cdots = 0$$

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If $v$ is small and if $U$ and $G_{ab}$ are order unity then the scalar mass is generically: $\mu \sim \frac{v^2}{f} \sim \frac{v^2}{M_p}$

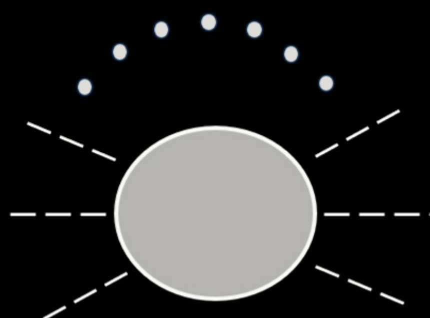
*In a world where it is understood why the cc problem is solved* **any** *gravitationally coupled scalar has a Hubble-scale mass!*

*astro-ph/0107573*

# Light Gravitating Scalars

Will now argue why the derivative expansion is *compulsory*
if one works semiclassically (as everyone does)

$$\mathcal{L}_W = -\sqrt{-g}\Big[v^4 U(\phi) + \tfrac{1}{2}M_p^2 R + \tfrac{1}{2}M_p^2\,\mathcal{G}_{ab}(\phi)\partial_\mu\phi^a\partial^\mu\phi^b + c_2 R^2 + \frac{c_3}{m^2}R^3 + \cdots\Big]$$



Evaluate a correlation function with *E* external lines, *L* loops and $V_n$ vertices involving $d_n$ derivatives with curvature *H* and external momenta *k/a=H*

0902.4465

1708.07443

$$\mathcal{B}_E(H) \simeq M_p\left(\frac{H^2}{M_p}\right)^{E-1}\left(\frac{H}{4\pi\,M_p}\right)^{2L} \times \prod_{d_n\geq 4}\left[\left(\frac{H}{M_p}\right)^2\left(\frac{H}{m}\right)^{d_n-4}\right]^{V_n}\prod_{d_n=0}\left(\frac{v^4}{H^2M_p^2}\right)^{V_n}$$

# Light Gravitating Scalars

This shows what controls semiclassical perturbation theory

$$\mathcal{B}_E(H) \simeq M_p \left(\frac{H^2}{M_p}\right)^{E-1} \left(\frac{H}{4\pi\, M_p}\right)^{2L} \times \prod_{d_n \geq 4} \left[\left(\frac{H}{M_p}\right)^2 \left(\frac{H}{m}\right)^{d_n-4}\right]^{V_n} \prod_{d_n=0} \left(\frac{v^4}{H^2 M_p^2}\right)^{V_n}$$

Each loop costs: $\left(\frac{H}{4\pi\, M_p}\right)^2 = \frac{GH^2}{2\pi}$

The semiclassical approximation *relies* on the derivative expansion

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Each higher-derivative interaction costs an *additional*: $\left(\frac{H}{M_p}\right)^2 \left(\frac{H}{m}\right)^{d_n - 4}$

4- and higher-derivative interactions are ***always*** suppressed at low energies when the semiclassical approximation is under control

*Keeping powers of $(\partial\phi)^2$ while dropping $\partial\partial\phi$ is usually inconsistent with semiclassically methods (except for DBI models) 1910.05277*

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Each zero-derivative interaction **_amplifies_** by an *additional*: $\frac{v^4}{H^2 M_p^2}$

This generically undermines the derivative expansion (and semiclassical control)

This need not be a problem ***if*** $v^2 = HM_p$ or smaller

# Light Gravitating Scalars

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There is *no penalty* for fields being large

This is why trans-Planckian field excursions need not be a problem

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There is *no penalty* for 2-derivative terms

This is why GR nonlinearities cannot be neglected at low energies

# Light Gravitating Scalars

We should expect two-derivative scalar self-interactions to compete with GR for any scalars light enough to be relevant to observations

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# Light Gravitating Scalars

We should expect two-derivative scalar self-interactions to compete with GR for any scalars light enough to be relevant to observations

*These two-derivative interactions can be removed using a field redefinition if the metric $G_{ab}$ is flat*

$$\mathcal{A}(\vartheta^a + \vartheta^b \leftrightarrow \vartheta^c + \vartheta^d) \propto \mathcal{R}^{acbd}$$

Alvarez-Gaume & Freedman 1981

2111.03045

# Light Gravitating Scalars

We should expect two-derivative scalar self-interactions to compete with GR for any scalars light enough to be relevant to observations

*These two-derivative interactions can be removed using a field redefinition if the metric $G_{ab}$ is flat*

**BAD NEWS**

Most tests of scalar-tensor theories for simplicity specialize to a single scalar

For all single-field models the metric $G_{ab}$ is flat

# Light Gravitating Scalars

We should expect two-derivative scalar self-interactions to compete with GR for any scalars light enough to be relevant to observations

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BAD NEWS

Most tests of scalar-tensor theories for simplicity specialize to a single scalar

For all single-field models the metric $G_{ab}$ is flat

This is why higher-derivative interactions are studied so much

This is also why it is hard to get single-scalar models to compete with gravity at low energies in a controlled way

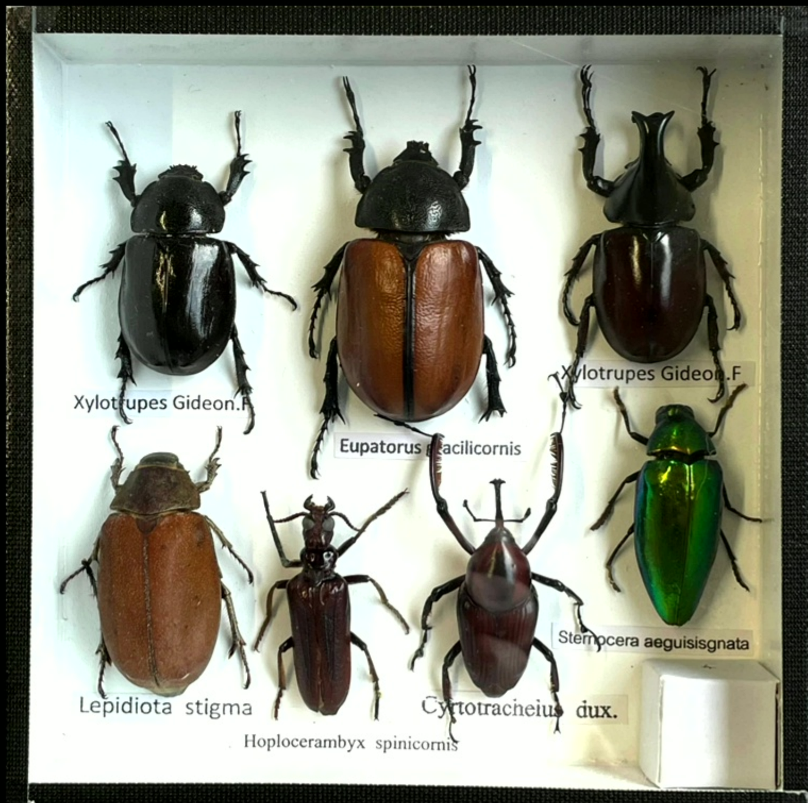
# The Dark Sector Opportunity

The success of cosmology *requires* Nature to have features (eg small scalar potential) NOT generic at low energies

There are many scalar-tensor models and cosmological observations alone cannot distinguish amongst most of them.

Demanding scalar tensor models be consistent with the rest of physics (eg EFT power counting) is an important clue .

Multiple-scalar models are potentially the most interesting (but relatively poorly explored)




etsy.com

# Axio-dilatons

*Beginnings of multiple-scalar taxonomy*

# Minimal Multiple-Scalar Models

Two-field models are the simplest ones that allow non-trivial two-derivative interactions

$$\mathcal{L} = \sqrt{-g}\Big[V(\vartheta) + M_p^2\mathcal{R} + \mathcal{G}_{ab}\,\partial_\mu\vartheta^a\partial^\mu\vartheta^b + \cdots\Big]$$

$$\mathcal{G}_{ab} = Z^2(\vartheta^1,\vartheta^2)\,\delta_{ab}$$

Nontrivial $G_{ab}$ can be consistent with shift symmetries designed to suppress V

$$\delta\vartheta^a = \xi^a(\vartheta)$$

# Minimal Multiple-Scalar Models

*Axisymmetric target spaces:*

suppose $G_{ab}$ is invariant under shifts of one of the fields

$$\mathcal{G}_{ab}\, \mathrm{d}\vartheta^a \mathrm{d}\vartheta^b = Z^2(\tau)\Big[\mathrm{d}\tau^2 + \mathrm{d}a^2\Big]$$

$$= \mathrm{d}\chi^2 + W^2(\chi)\mathrm{d}a^2$$

where $\mathrm{d}\chi = Z(\tau)\,\mathrm{d}\tau$ and $W(\chi) = Z[\tau(\chi)]$

*Specified by a single function $W(\chi)$ and by the scalar couplings to matter*

eg $\mathcal{L}_m = \mathcal{L}_m(\tilde{g}_{\mu\nu}, \psi)$ with $\tilde{g}_{\mu\nu} = A^2(\chi) g_{\mu\nu}$

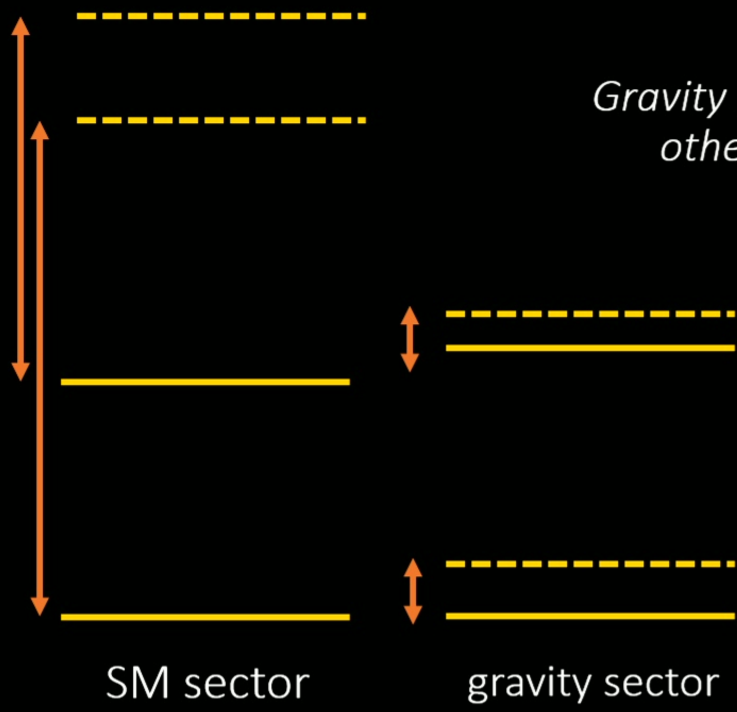
# Minimal Multiple-Scalar Models

*Pairing of 'axion' field a with non-axion field $\chi$ is very common in supersymmetric models, where each spin-half fermion is paired with a complex scalar*

$$\Phi = \frac{1}{2}\Big(\tau + ia\Big)$$

# Supersymmetry of the gravity sector

How can supersymmetry play a role at low energies when LHC finds no evidence for supersymmetry?




*Gravity multiplet typically split by less than others because gravity is weakest force*

$$\Delta m^2 = m_B^2 - m_F^2 \sim gF$$

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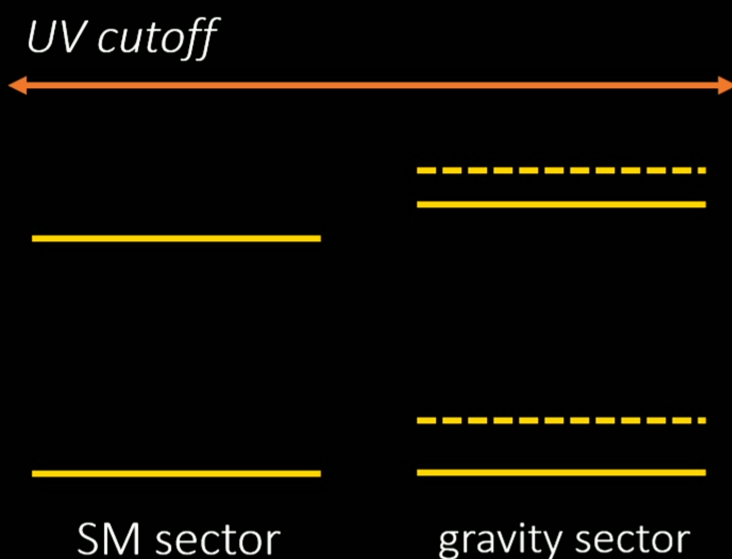



*Should expect gravity sector to be more supersymmetric at low energies than particle physics sector*

*We now know how to couple supergravity to matter that is not supersymmetric*

Komargodsky & Seiberg 09
Bergshoeff et al 15
Dallagata & Farakos 15
Schillo et al 15
Antoniadis et al 21
Dudas et al 21

# Minimal Multiple-Scalar Models

*Pairing of 'axion' field a with non-axion field $\chi$ is very common in supersymmetric models, where each spin-half fermion is paired with two scalars*

$$\Phi = \tfrac{1}{2}\Big(\tau + ia\Big)$$

shift symmetry if

$$K(\Phi, \Phi^*) = K(\Phi + \Phi^*) \qquad\qquad W(\Phi) = W_0$$

also

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} \ni e^{-K/3}\,\tilde{\mathcal{R}}$$

so

$$\tilde{g}_{\mu\nu} = e^{K/3}\,g_{\mu\nu}$$

implying

$$\boxed{A^2(\tau) = e^{K/3}}$$

*Both W and A are fixed by the single function K*

# Minimal Multiple-Scalar Models

The complex scalar is an *axio-dilaton* if the field $\chi$ is a dilaton

$$e^{-K/3} = \tau^{\lambda} \quad \text{so} \quad K = -3\lambda \log \tau$$

then

$$A^2(\tau) = e^{K/3} = \tau^{-\lambda}$$

and

$$Z^2(\tau) = \tfrac{1}{2} K'' = \frac{3\lambda}{2\tau^2}$$

implying

$$\tau = \tau_0 \exp\left(\sqrt{\frac{2}{3\lambda}}\,\chi\right)$$

*so $\chi$ couples to matter like a Brans-Dicke scalar, A = exp(g $\chi$), with* $g = -\sqrt{\frac{\lambda}{6}}$

# Minimal Multiple-Scalar Models

The case $\lambda = 1$ is particularly interesting because the scalar potential is

$$V = e^K \left( K^{ij*} K_i K_{j*} - 3 \right) |W_0|^2$$
$$= 3(\lambda - 1) \frac{|W_0|^2}{\tau^{3\lambda}}$$

This includes the class of 'Yoga' models, for which a relaxation mechanism suppresses the scalar potential in a way that gives an attractive framework for understanding why the Dark Energy can be so small

2111.07286

## Dangers and Opportunities?

Dilaton couples to matter like a Brans-Dicke scalar with gravitational strength

Main (possible) way out:

Particle coupling strength need not equal coupling strength to macroscopic objects (screening)

To be consistent with UV physics screening should arise due to two-derivative interactions (unlike existing mechanisms: eg chameleon or symmetron)

# Multi-field Screening

Best example so far:

$$\mathcal{L} = -\frac{1}{2} M_p^2 \sqrt{-g}\left[(\partial\phi)^2 + W^2(\phi)(\partial a)^2\right]$$

If axion experiences different minimum inside/outside of matter

*Hook & Huang 17*

then axion necessarily has gradient near object's surface, whose interaction with the dilaton reduces the object's dilaton charge

$$\phi'(R_+) \simeq \phi'(R_-) + \left(\frac{WW'}{2\ell}\right)_{r=R} (a_+ - a_-)^2$$

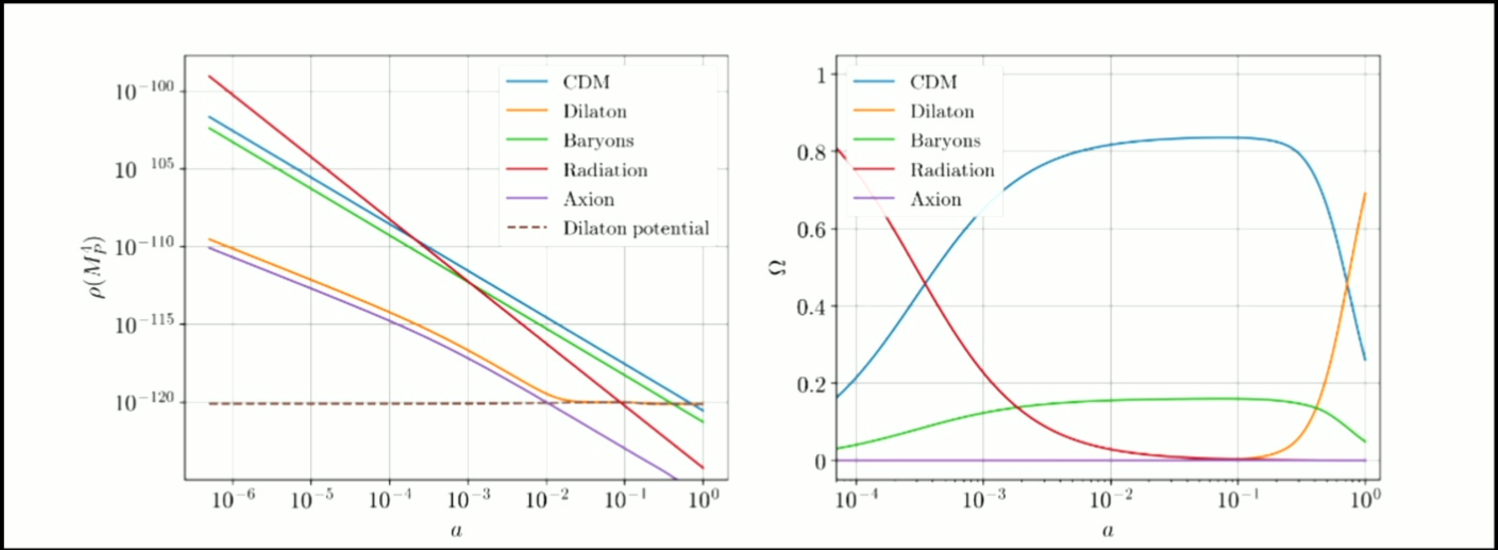
(narrow width approximation)

*2310.02092*

# Viable cosmological evolution?

Coupled dilaton-axion evolution seems possible even for large Brans-Dicke couplings to matter

$$g_B = -\sqrt{\frac{1}{6}} \qquad g_C \simeq -k g_B$$




Density evolution for cosmic fluid components

2407.xxxxx

# Conclusions

Knowing that cosmology is the low-energy limit of something more fundamental is an important clue

*For scalar-tensor theories it suggests that two-derivative interactions are likely to be the ones that compete with gravity in any controlled approximation.*

Such interactions require at least two scalars

*Axio-dilaton models provide a broad minimal but well-motivated class to explore*

Remarkably rich physics possible at very low energies

*EFT arguments are restrictive but not prohibitive for predicting things to be tested in GW (and other gravity) tests*

# *Thanks for your time & attention!*