Title: Conference Talk

Speakers:

Collection: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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Galileon Duality

$$S = \int \sum_n c_n \Pi \, (dx)^{4-n} (d\partial\Pi)^n$$

$$(dx)^3 (d\,\partial\Pi) = \varepsilon_{abcd}\, dx^a \wedge dx^b \wedge dx^c \wedge d\,\partial^d\Pi$$

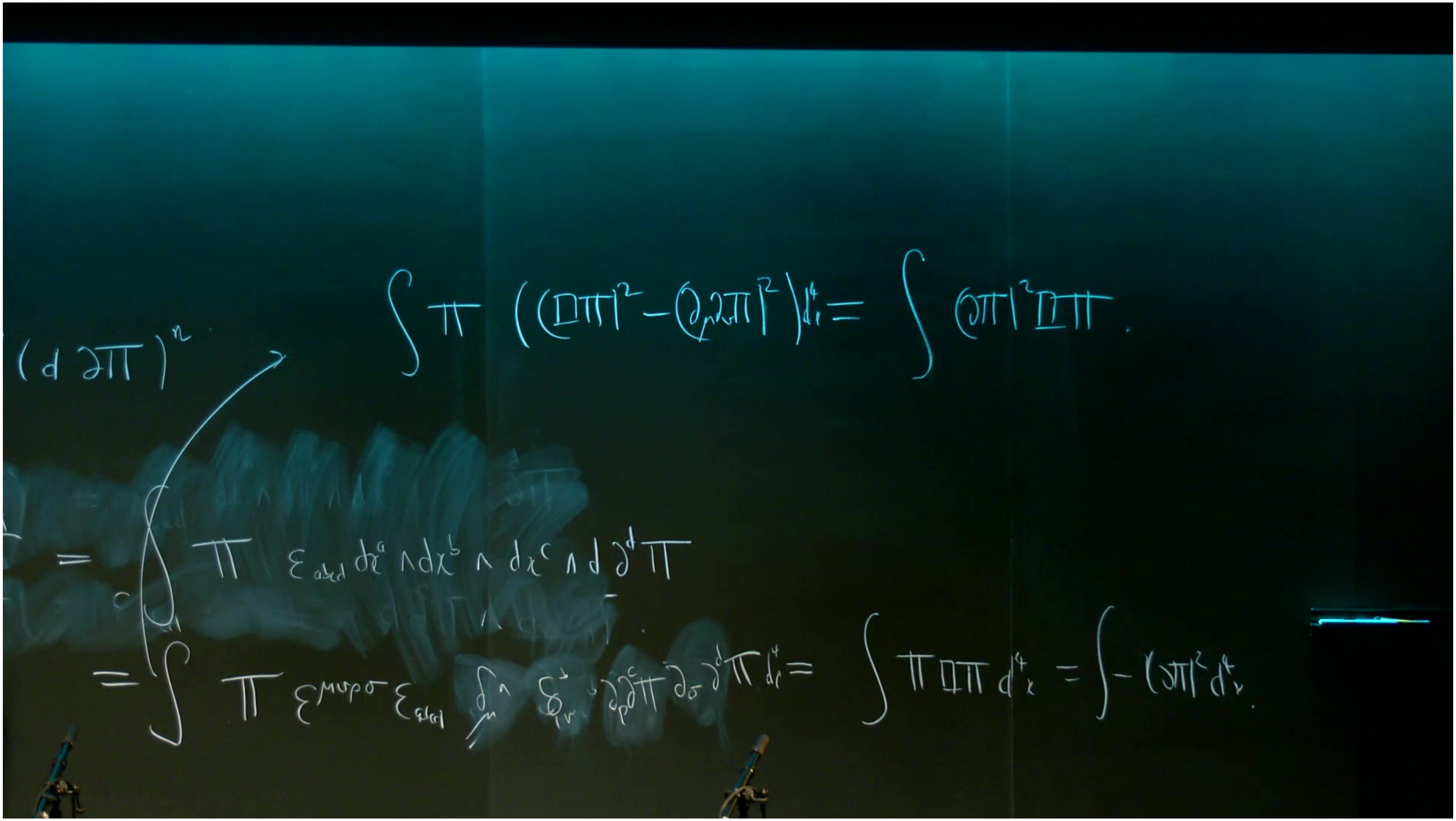
$$(dx)^2 (d\,\partial\Pi) = \varepsilon_{abcd}\, dx^a \wedge dx^b \wedge d\partial^c\Pi \wedge d\partial^d\Pi .$$

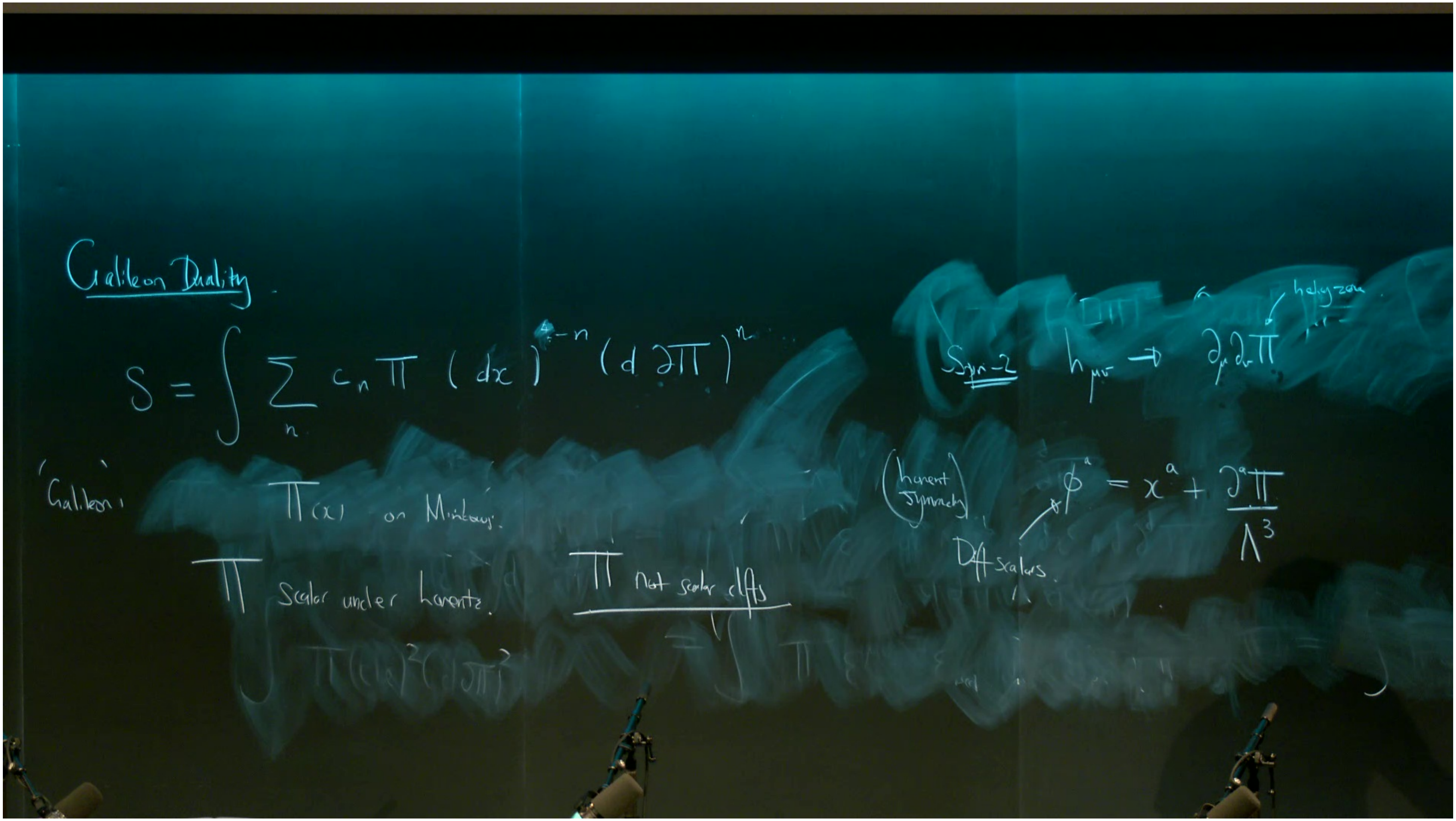
Galileon Duality

$$S = \int \sum_n c_n \pi \, (dx)^{4-n} (d\partial\pi)^n$$

$$\int \pi (dx)^3 (d\partial\pi)^1 = \int \pi \, \varepsilon_{abcd} \, dx^a \wedge dx^b \wedge dx^c \wedge d\partial^d \pi$$

$$= \int \pi \, \varepsilon^{\mu\nu\rho\sigma} \varepsilon_{abcd} \, \partial_\mu x^a \partial_\nu x^b \partial_\rho x^c \partial_\sigma \partial^d \pi$$


$(d\,\partial\pi)^n$
$\int \pi \left((\Box\pi)^2 - (\partial_\mu\partial\pi)^2\right) d^4x = \int (\partial\pi)^2 \Box\pi$.
$= \int \pi\, \varepsilon_{abcd}\, dx^a \wedge dx^b \wedge dx^c \wedge d\,\partial^d\pi$
$= \int \pi\, \varepsilon^{\mu\nu\rho\sigma} \varepsilon_{abcd}\, \delta^a_\mu\, \delta^b_\nu\, \partial_\rho\partial^c\pi\, \partial_\sigma\partial^d\pi\, d^4x = \int \pi\Box\pi\, d^4x = \int -(\partial\pi)^2 d^4x$.


Galileon Duality
S = ∫ Σn cn π (dx)^4-n (d∂π)^n
'Galileon'
π(x) on Minkowski
π scalar under Lorentz
π not scalar diffs
Spin-2 hμν → ∂μ∂ν π
helicity zero
(Inherent Symmetry)
φ^a = x^a + ∂^a π / Λ^3
Diff scalars

Field dependent diffeomorph.

$$\tilde{x}^\mu = x^\mu + s\,\partial^\mu \Pi(x)$$

$$\frac{\partial \tilde{x}^\mu}{\partial x^\nu} = \eta^\mu_\nu + s\,\partial_\nu \partial^\mu \Pi$$

$$\tilde{\Pi}(\tilde{x}) = \Pi(x) + \frac{s}{2}\partial_\mu \Pi\,\partial^\mu \Pi.$$

$$\tilde{\Pi}(x^\mu + s\,\partial^\mu \Pi) = \Pi(x) + \frac{s}{2}\partial_\mu \Pi\,\partial^\mu \Pi$$

Field dependent diffeomorph.

$$\boxed{\begin{aligned}\tilde{x}^\mu &= x^\mu + s\,\partial^\mu \Pi(x)\\ \tilde{\Pi}(\tilde{x}) &= \Pi(x) + \frac{s}{2}\,\partial_\mu \Pi\, \partial^\mu \Pi\end{aligned}}$$

$$\tilde{\Pi}(x^\mu + s\,\partial^\mu \Pi) = \Pi(x) + \frac{s}{2}\,\partial_\mu \Pi\, \partial^\mu \Pi$$

$$\frac{\partial \tilde{x}^\mu}{\partial x^\nu} = \eta^\mu_\nu + s\,\partial_\nu \partial^\mu \Pi$$

$$\boxed{\tilde{\partial}_\mu \tilde{\Pi}(\tilde{x}) = \partial_\mu \Pi(x)}$$

Galileon Duality

$$\boxed{X = -\frac{1}{2}(\partial\pi)^2}$$

$$S = \int \sum_n c_n \tilde{\pi}\, (d\tilde{x})^{4-n} (d\tilde{\partial}\tilde{\pi})^n$$

$$= \int \sum_n c_n (\pi - sX)\,(dx + s\, d\partial\pi)^{4-n} (d\partial\pi)^n$$

$$= \int \sum_n c_n (\pi - sX)\, \frac{(4-n)!}{r!(4-n-r)!}\, s^r\, (dx)^{4-n-r} (d\partial\pi)^{n+r}$$

Field dependent diffe

$$\tilde{x}^\mu = x^\mu +$$

$$\tilde{\pi}(\tilde{x}) = \pi$$

$$\tilde{\pi}(x^\mu + s\partial^\mu\pi)$$

$X\,(dx)^{4-n}(d\partial\pi)^{n} \Rightarrow$ Galileon ter

Galileon Duality

$$\boxed{X = -\frac{1}{2}(\partial\pi)^2}$$

$$S = \int \sum_n c_n \tilde{\pi}\,(d\tilde{x})^{4-n}(d\tilde{\partial}\tilde{\pi})^{n}$$

$$= \int \sum_n c_n (\pi - sX)\,(dx + s\,d\partial\pi)^{4-n}(d\partial\pi)^{n}$$

$$= \int \sum_n c_n (\pi - sX)\,\frac{(4-n)!}{r!(4-n-r)!}\,s^{r}(dx)^{4-n-r}(d\partial\pi)^{n+r}$$

$$= \sum_n b_n (dx)^{4-n}(d\partial\pi)^{n}$$

Field dependent diffeo

$$\tilde{x}^{\mu} = x^{\mu} +$$

$$\tilde{\pi}(\tilde{x}) = \pi$$

$$\tilde{\pi}(x^{\mu} + s\,\partial^{\mu}\pi)$$

Galileon Duality

$$\times\ (dx)^{4-n}(d\partial\pi)^n \Rightarrow \text{Galileon term}$$

$$S = \int d^4\tilde{x}\ -\frac{1}{2}(\tilde{\partial}\tilde{\pi})^2$$

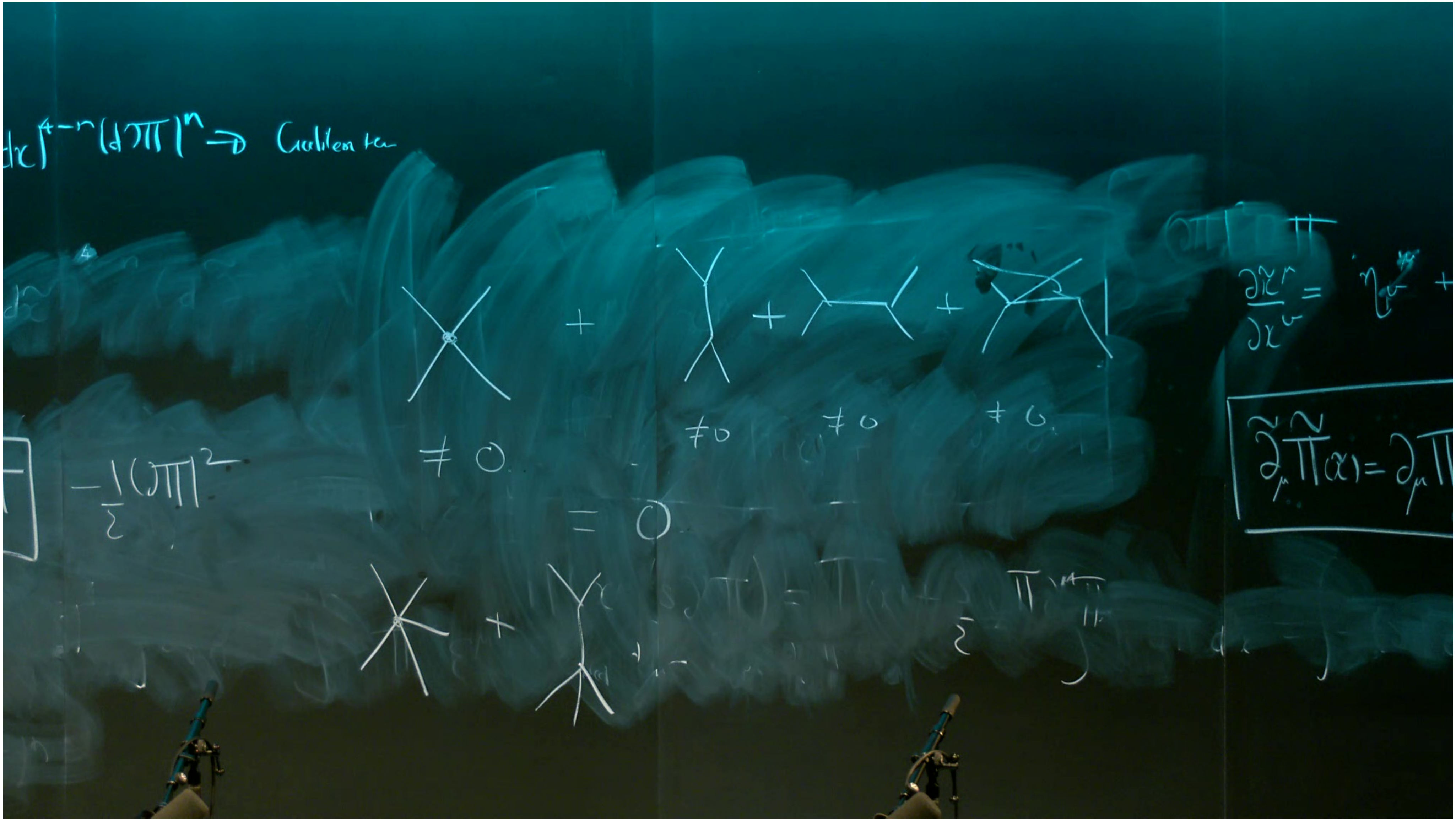
Quintic Galileon

$$= \int d^4x\ \det\left[\eta_\mu^{\ \nu} + s\,\partial_\mu\partial^\nu\pi\right] - \frac{1}{2}(\partial\pi)^2$$

Field dependent diffe

$$\boxed{\begin{aligned}\tilde{x}^\mu &= x^\mu + \\ \tilde{\pi}(\tilde{x}) &= \pi\end{aligned}}$$

$$\tilde{\pi}(x^\mu + s\,\partial^\mu\pi)$$


≠ 0
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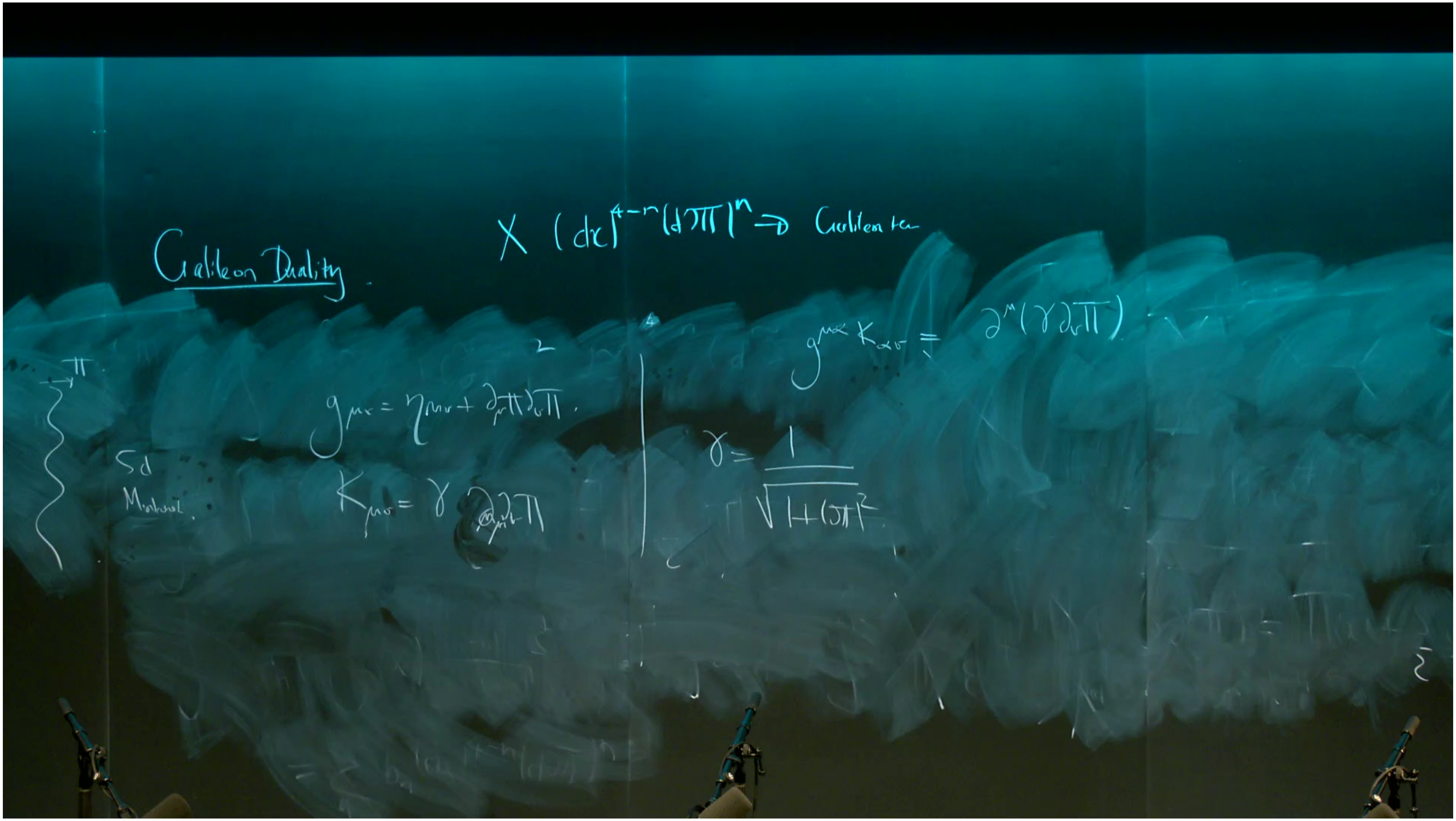
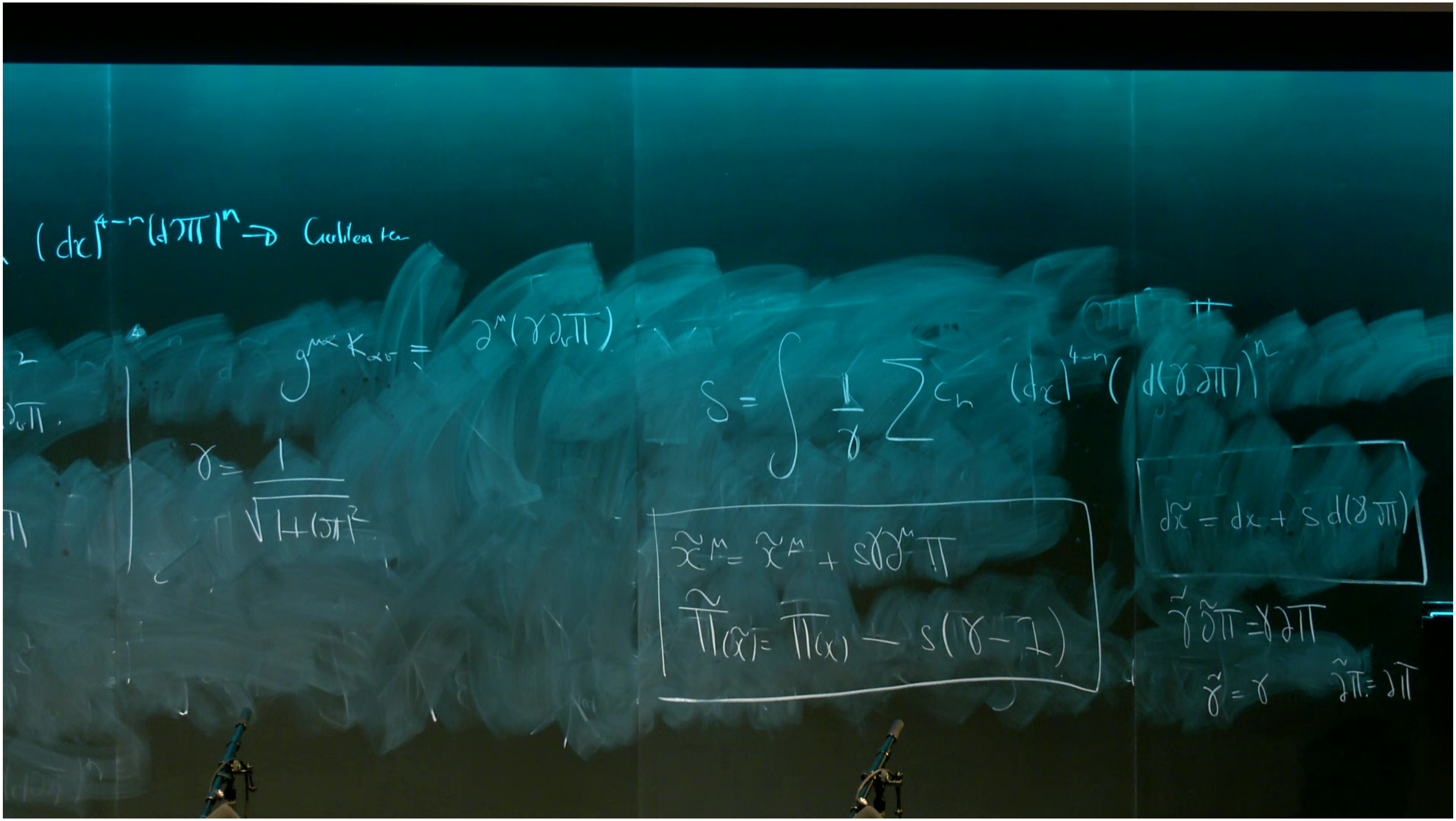
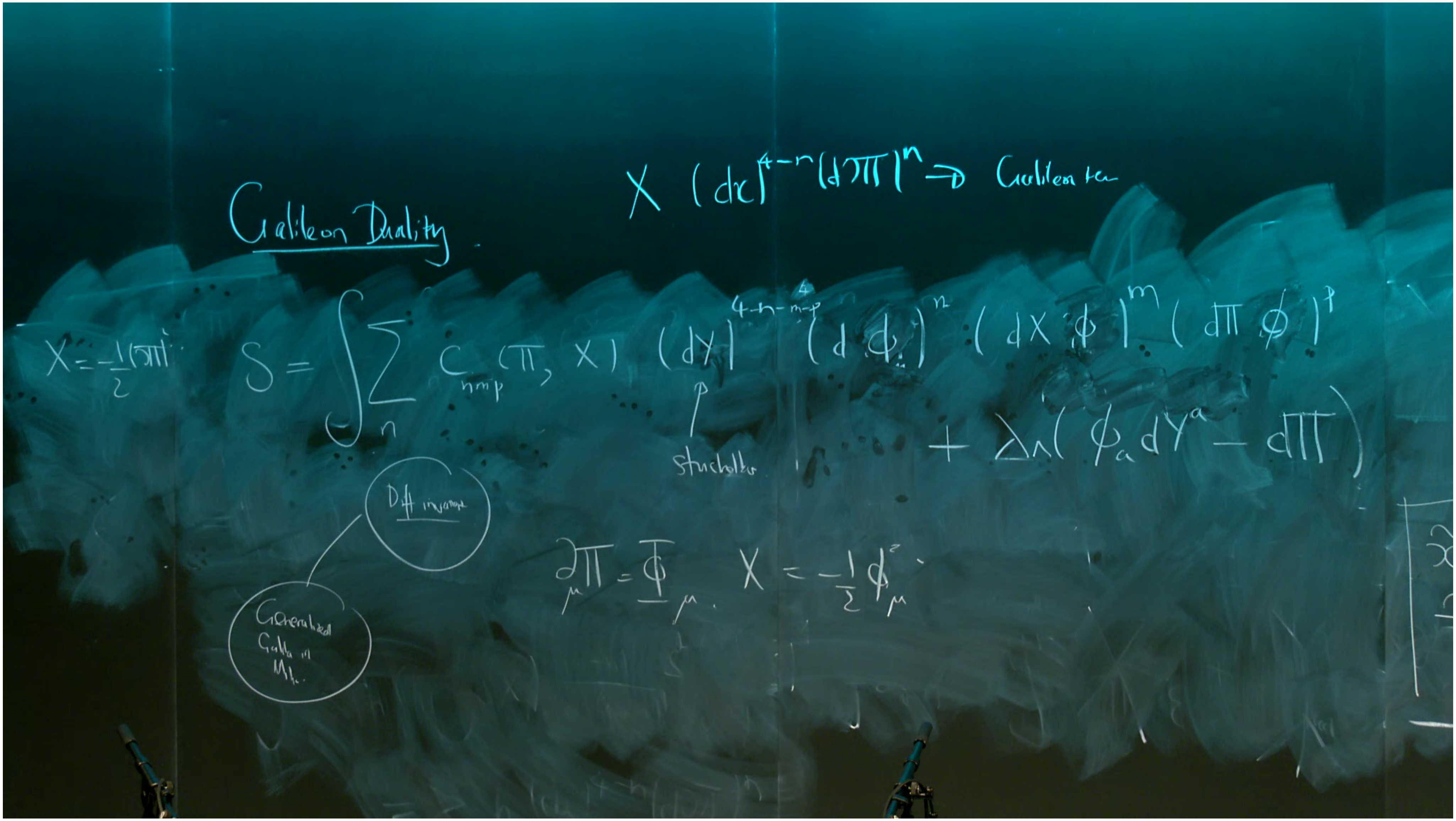
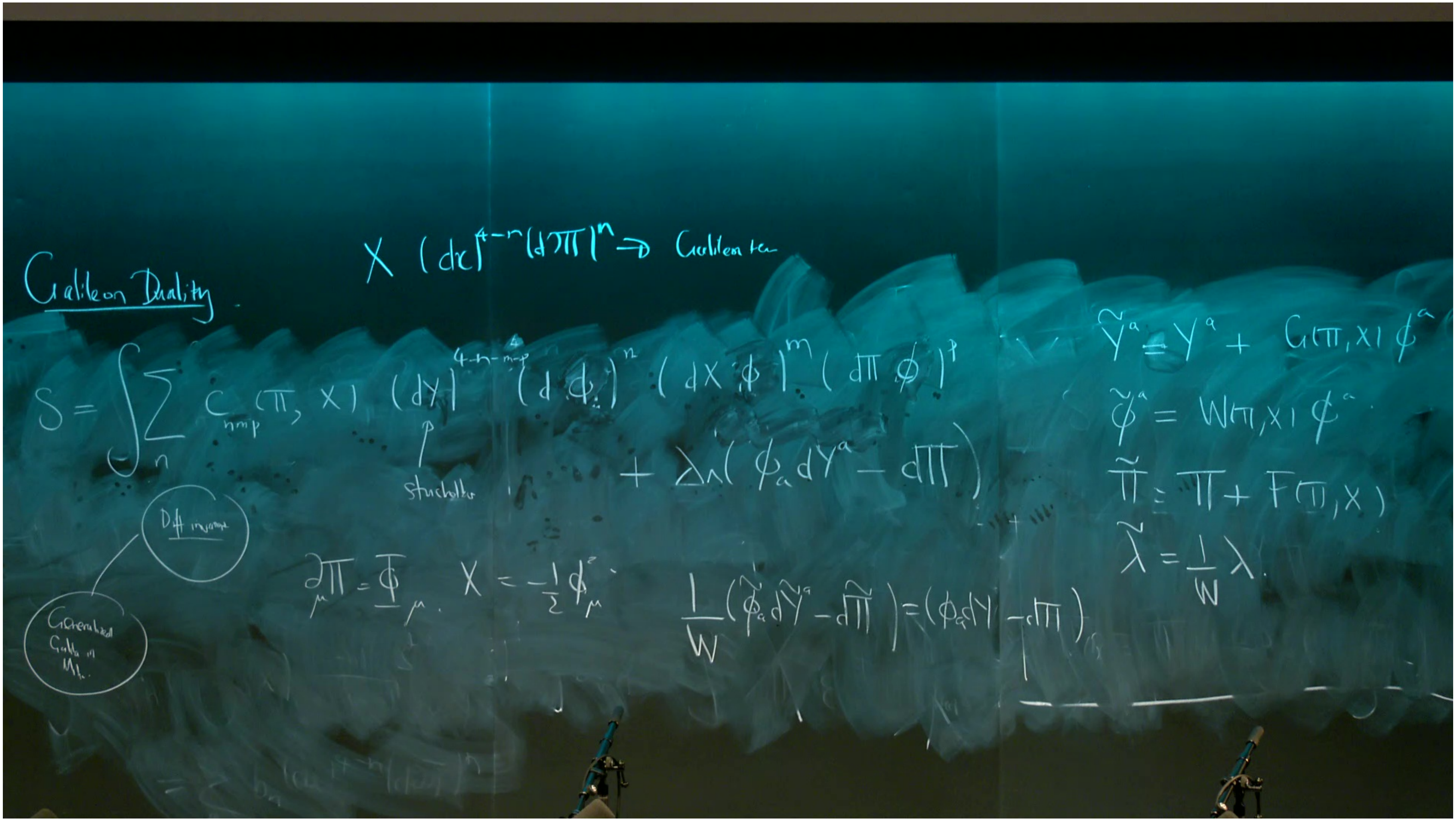
≠ 0
≠ 0
= 0



Galileon Duality
X (dx)^4−n (dπ)^n → Galileon ter
g_μν = η_μν + ∂_μπ ∂_νπ
K_μν = γ ∂_μ∂_νπ
5d
Minkowski
g^μα K_αν = ∂^μ(γ ∂_νπ)
γ = 1 / √(1+(∂π)²)


$(dx)^{4-n}(d\partial\pi)^n \Rightarrow$ Galileon
$g^{\mu\alpha} K_{\alpha\sigma} = \partial^\mu(\gamma\partial_\sigma\pi)$
$\gamma = \frac{1}{\sqrt{1+(\partial\pi)^2}}$
$S = \int \frac{1}{\gamma} \sum c_n (dx)^{4-n} (d(\gamma\partial\pi))^n$
$\tilde{x}^\mu = x^\mu + s\gamma\partial^\mu\pi$
$\tilde{\pi}(\tilde{x}) = \pi(x) - s(\gamma - 1)$
$d\tilde{x} = dx + s\,d(\gamma\partial\pi)$
$\tilde{\gamma}\tilde{\partial}\pi = \gamma\partial\pi$
$\tilde{\gamma} = \gamma \quad \tilde{\partial}\pi = \partial\pi$


Galileon Duality
$X\,(dx)^{4-n}(d\pi)^n \to$
$X = -\frac{1}{2}(\partial\pi)^2$
$S = \int \sum_n C_{nmp}(\pi, X)\,(dY)^{4-n-m-p}\,(d\phi_a)^n\,(dX\phi)^m\,(d\pi\phi)^p$
$+ \Lambda(\phi_a\, dY^a - d\pi)$
Stuckelberg
$\partial_\mu \pi = \Phi_\mu$
$X = -\frac{1}{2}\phi_\mu^2$
Generalized


Galileon Duality