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Speakers:

Collection: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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50 Years of Horndeski Gravity: Exploring Modified Gravity
07/18/2024

# Post-Newtonian limit of Lorentz-violating scalar-tensor theories


**Tsutomu Kobayashi**

Rikkyo University, Tokyo, Japan



Extreme Universe
A New Paradigm for Spacetime and Matter from Quantum Information
Grant-in-Aid for Transformative Research Areas (A)


Based on:
TK and Takashi Hiramatsu
Phys.Rev.D 109 (2024) 6, 064091[2310.11041]
Jin Saito, Zhibang Yao, and TK
JCAP06(2024)040 [2402.10459]

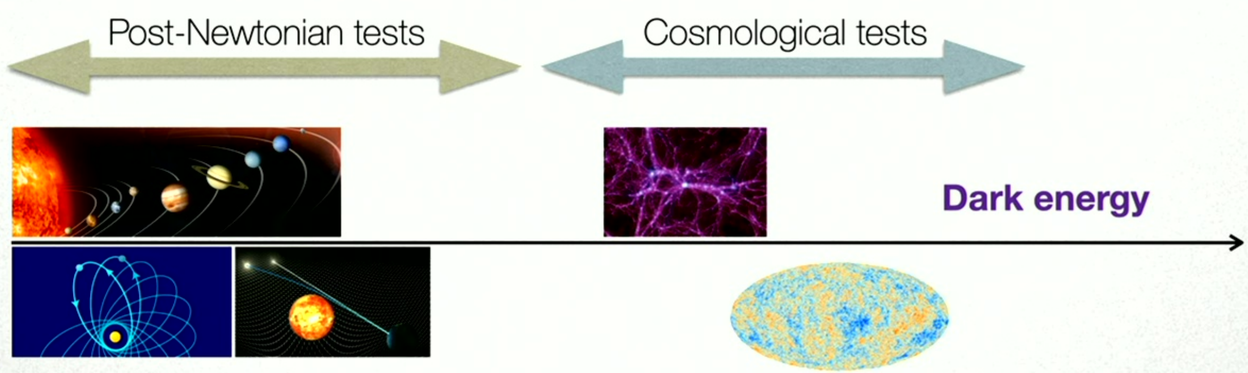

T. Hiramatsu


J. Saito


Z. Yao

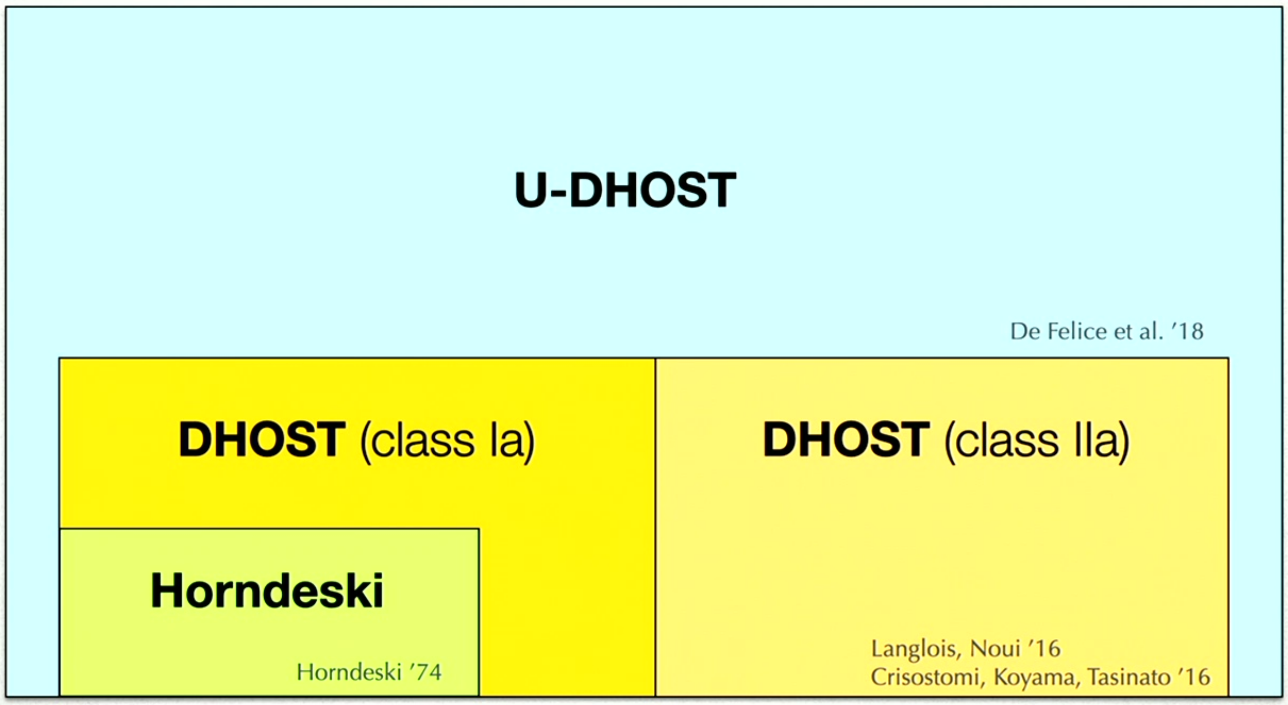
# Confronting modified gravity with experiments

- Gravity modification persists in Newtonian and post-Newtonian (PN) regimes, which has been tightly constrained by experiments.
- Efficient screening of “fifth force” or constraints on theory parameters.
- Complementary to cosmological tests.



# Scalar–tensor theory space

**2** tensor + **1** scalar DOFs, ghost-free




[talk by D. Langlois]

# Horndeski theory

Horndeski '74

Deffayet, Gao, Steer, Zahariade '11
TK, Yamaguchi, Yokoyama '11

- Ostrogradsky's theorem: Higher-order EOMs lead to ghost instability (if Lagrangian is non-degenerate).
- The most general scalar-tensor theory with second-order field equations (⇨ ghost-free):

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]$$
$$+ G_5 G^{\mu\nu}\phi_{\mu\nu} - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 + \dots\right].$$

Notations

$$\phi_\mu := \nabla_\mu \phi$$
$$\phi_{\mu\nu} := \nabla_\mu \nabla_\nu \phi$$
$$X := -\phi^\mu \phi_\mu / 2$$

# DHOST theories

[talk by D. Langlois]

- **D**egenerate **H**igher-**O**rder **S**calar-**T**ensor theories:
  Field equations are of higher-order, but their combination results in second-order equations because the system is degenerate (<u>for any field configurations</u>).

⇨ 2 tensor + 1 scalar DOFs, ghost-free

Langlois, Noui '16
Crisostomi, Koyama, Tasinato '16

- <u>Quadratic</u> DHOST Lagrangian:

(Extension to cubic order: Ben Achour et al. '16)

$$\mathcal{L} = G_2 - G_3 \Box\phi + f(\phi, X) R + \sum_{I=1}^{5} A_I(\phi, X) L_I,$$

A set of degeneracy condtions must be satisfied.
(class Ia/class IIa)

$$\underbrace{(A_1 + A_2)}_{=0}\underbrace{(\ldots\ldots)}_{=0} = 0$$

$=0$ → class I or $=0$ → class II

where (all possible terms of the form $\{g^{\mu_3\nu_3}, \phi^{\mu_4}\phi^{\nu_4}\} \times \phi_{\mu_1\nu_1}\phi_{\mu_2\nu_2}$ )

$$L_1 = \phi_{\mu\nu}\phi^{\mu\nu}, \quad L_2 = (\Box\phi)^2, \quad L_3 = (\Box\phi)\phi^\mu\phi^\nu\phi_{\mu\nu}, \quad L_4 = \phi^\mu\phi_{\mu\lambda}\phi^\nu\phi_\nu^\lambda, \quad L_5 = (\phi^\mu\phi^\nu\phi_{\mu\nu})^2.$$

# U-DHOST theories

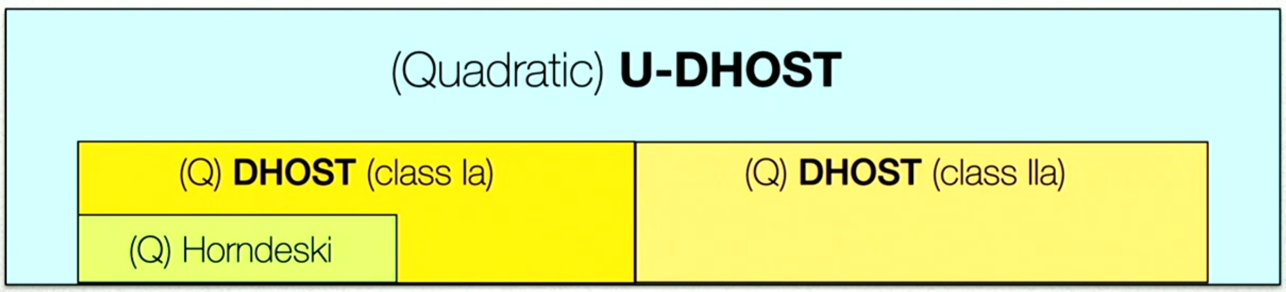
De Felice, et al. '18
De Felice, et al. '21

- **U**(nitary)-degenerate theories:
  Degeneracy conditions are satisfied when restricted to the unitary gauge, $\phi = \phi(t)$.

- The apparent extra mode obeys an elliptic equation if $\phi_\mu$ is timelike, and hence does not propagate.

⇨ 2 tensor + 1 scalar DOFs, ghost-free

  - Degeneracy condtions are less restrictive than in fully degenerate theories.



# Effective field theory of dark energy (EFTofDE)

Gubitosi, Piazza, Vernizzi '13
Gleyzes et al. '13
Bellini, Sawicki '14

- General description of perturbations of scalar-tensor theories around flat FLRW in the unitary gauge.
- Efficient way of testing dark energy/modified gravity models with cosmological observations.
- Time-dependent EFT coefficients (in the $\alpha$-basis):

$$S = \int \mathrm{d}t\mathrm{d}^3x\sqrt{\gamma}\frac{M^2}{2}\Big\{(1+\delta N)\left[\delta K_i^j\delta K_j^i - \left(1+\frac{2}{3}\alpha_L\right)\delta K^2\right] + (1+\alpha_T)R^{(3)} + H^2\alpha_K\delta N^2 + 4H\alpha_B\delta N\delta K + (1+\alpha_H)\delta N R^{(3)} + 4\beta_1\delta K\delta V + \beta_2\delta V^2 + \beta_3\mathbf{a}_i\mathbf{a}^i - \alpha_{V1}\delta N\delta K_i^j\delta K_j^i + \alpha_{V2}\delta N\delta K^2 + \dots\Big\},$$

where $\delta K_i^j = K_i^j - H\delta_i^j, \quad \delta N = N-1, \quad \delta V = \frac{\dot{N}}{N} - \frac{N^i}{N}\partial_i N, \quad \mathbf{a}_i = \frac{\partial_i N}{N}.$

ADM variables

# Physical significance of (some of) EFTofDE coefficients

$$S = \int \mathrm{d}t\mathrm{d}^3x\sqrt{\gamma}\frac{M^2}{2}\left\{(1+\delta N)\left[\delta K^j_i\delta K^i_j - \left(1+\frac{2}{3}\alpha_L\right)\delta K^2\right] + (1+\alpha_T)R^{(3)} + H^2\alpha_K\delta N^2 + 4H\alpha_B\delta N\delta K + (1+\alpha_H)\delta N R^{(3)} + 4\beta_1\delta K\delta V + \beta_2\delta V^2 + \beta_3\mathbf{a}_i\mathbf{a}^i - \alpha_{V1}\delta N\delta K^j_i\delta K^i_j + \alpha_{V2}\delta N\delta K^2 + \ldots\right\}$$

- Gravitational waves: $c_{\rm GW}^{-2}\ddot{h}_{ij} - \nabla^2 h_{ij} \sim 16\pi G_{\rm GW}T_{ij}$

  with $c^2_{\rm GW} = 1+\alpha_T$, $8\pi G_{\rm GW} = 1/c^2_{\rm GW}M^2$.

  **Speed of GWs** **Effective gravitational coupling for GWs**

- $\alpha_L$ : detuning of $K_{ij}K^{ij}$ and $K^2$. $\alpha_L \neq 0$ in khronometric theory and the low-energy limit of Horava-Lifshitz gravity.

  $\alpha_L \neq 0$ : **“Lorentz-violating theories”**

# Mapping to Horndeski/ (U-)DHOST functions

- EFT coefficients are given in terms of functions in the action of scalar-tensor theories (explicit expressions are not important):

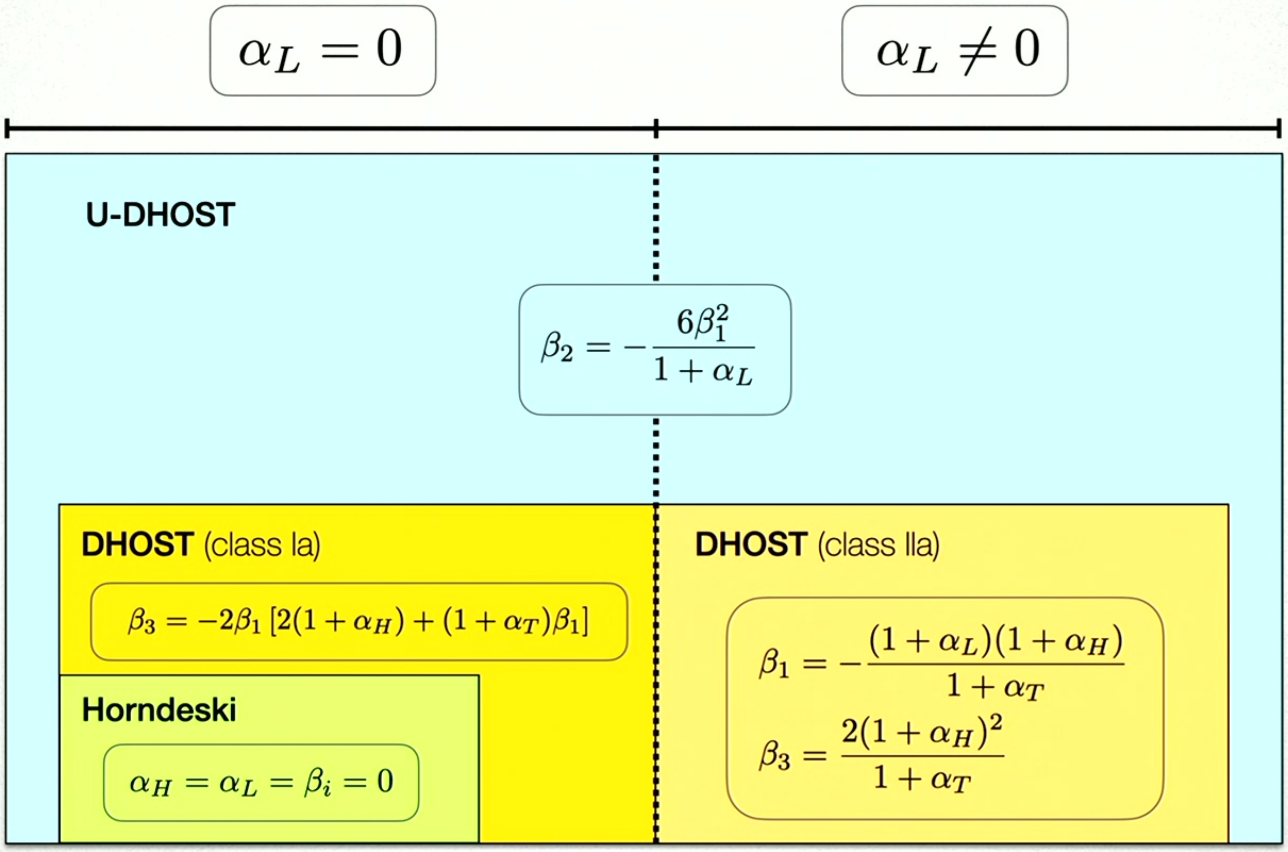
$$M^2 = 2(f + 2XA_1), \quad M^2(1+\alpha_T) = 2f, \quad M^2(1+\alpha_H) = 2(f - 2Xf_{,X}),$$
$$M^2\alpha_L = -6X(A_1 + A_2), \quad M^2\beta_1 = 2\left[Xf_{,X} - X(A_2 - XA_3)\right],$$
$$M^2\beta_2 = 4X\left[A_1 + A_2 - 2X(A_3 + A_4) + 4X^2 A_5\right], \quad M^2\beta_3 = -8\left[Xf_{,X} + X(A_1 - XA_4)\right],$$
$$M^2\alpha_{V1} = 4X\left[f_{,X} + 2\left(A_1 + XA_{1,X}\right)\right], \quad M^2\alpha_{V2} = 4X\left[f_{,X} - 2\left(A_2 + XA_{2,X}\right)\right], \quad \ldots$$

$$S = \int \mathrm{d}t\mathrm{d}^3x\sqrt{\gamma}\frac{M^2}{2}\Big\{(1+\delta N)\left[\delta K_i^j\delta K_j^i - \left(1+\frac{2}{3}\alpha_L\right)\delta K^2\right] + (1+\alpha_T)R^{(3)} + H^2\alpha_K\delta N^2$$
$$+ 4H\alpha_B\delta N\delta K + (1+\alpha_H)\delta N R^{(3)} + 4\beta_1\delta K\delta V + \beta_2\delta V^2 + \beta_3\mathbf{a}_i\mathbf{a}^i$$
$$- \alpha_{V1}\delta N\delta K_i^j\delta K_j^i + \alpha_{V2}\delta N\delta K^2 + \ldots\Big\}$$

$$\mathcal{L} = G_2 - G_3\Box\phi + f(\phi, X)R + \sum_{I=1}^{5} A_I(\phi, X)L_I$$

# EFTofDE and classification of scalar-tensor theories

Langlois et al. '17

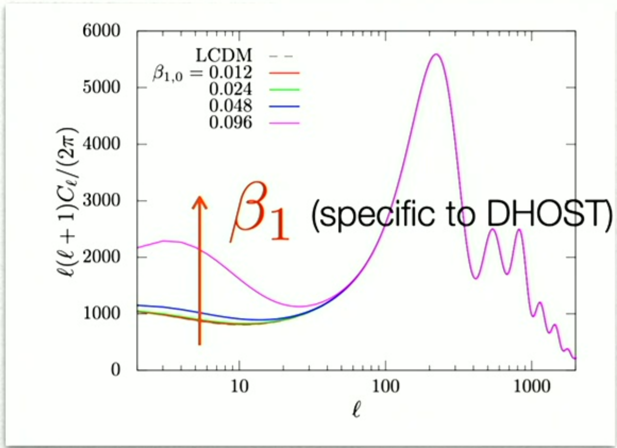


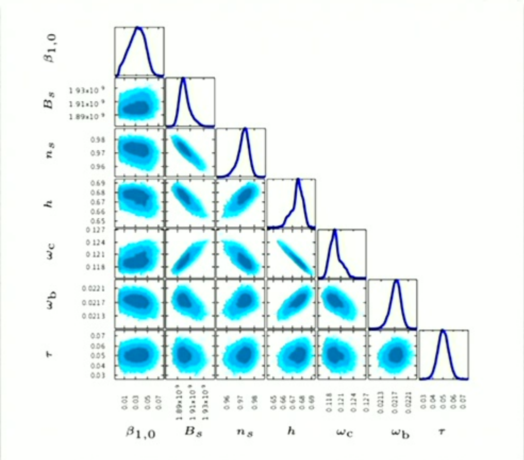
# Cosmological constraints on EFTofDE coefficients

- Numerical codes for Horndeski/DHOST/EFTofDE

Huang '12, Hu et al. '14, Zumalacárregui et al. '17, Hiramatsu, Yamauchi '20

Hiramatsu, Yamauchi '20, Hiramatsu '22




See also: Raveri et al. '14, Bellini et al. '16, Kreisch, Komatsu '18, Brando et al. '19, Noller, Nicola '20, Ye, Silvestri '24 …

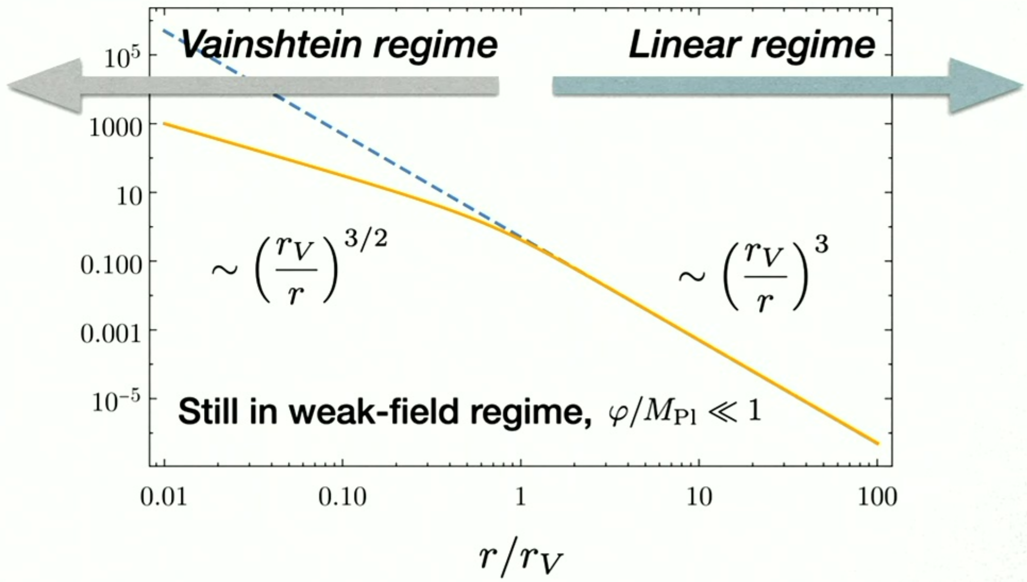
# Vainshtein mechanism

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2\Lambda^3}(\partial\varphi)^2\Box\varphi - \frac{\beta}{M_{\rm Pl}}\varphi\rho.$$

Terms with the higherst number of derivatives per field are improtant.

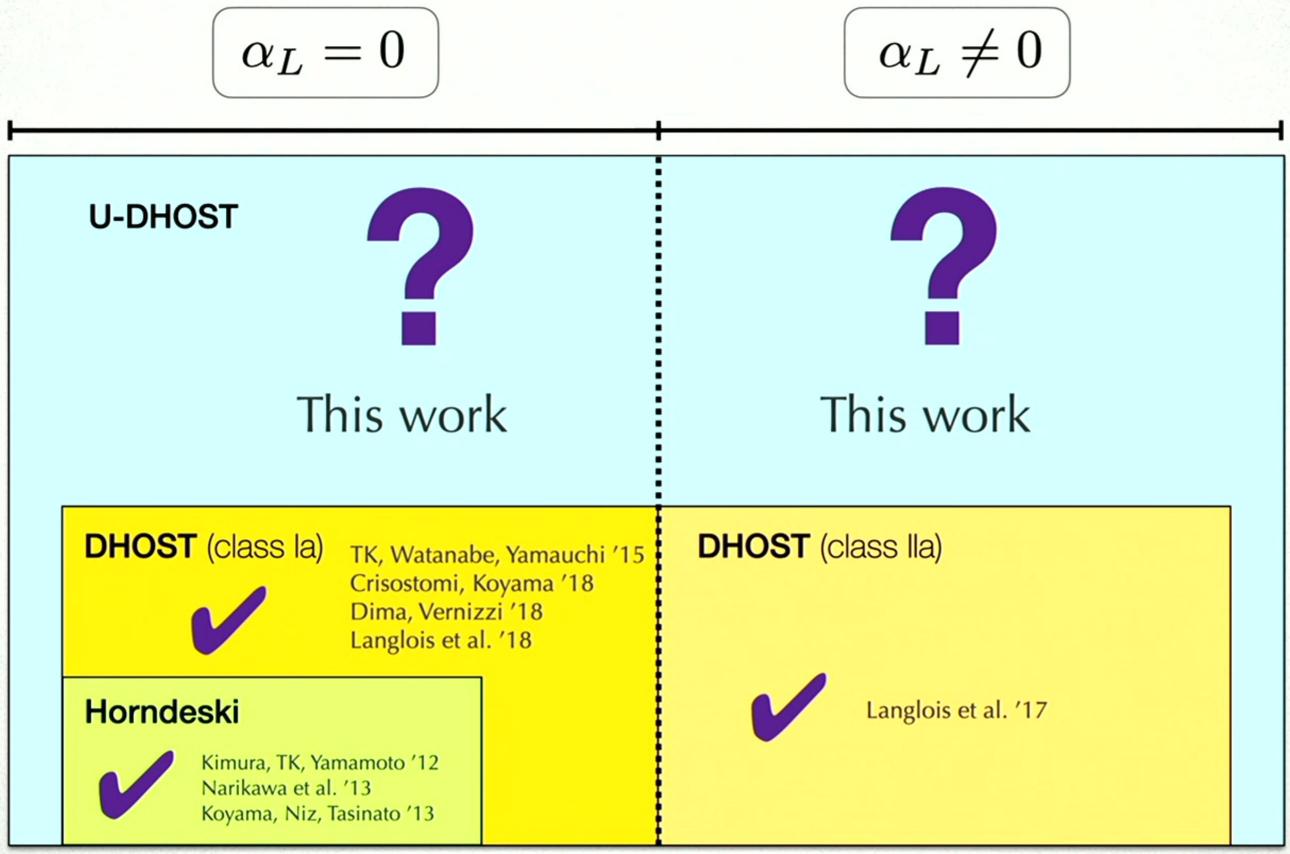
$$\frac{\partial_r\varphi}{\Lambda^3 r} \quad \left(\sim \frac{\Box\varphi}{\Lambda^3}\right)$$

$$\Lambda \sim (M_{\rm Pl}H_0^2)^{1/3}$$




$$r_V = \left(\frac{M_{\rm Pl} r_{\rm Sch}}{\Lambda^3}\right)^{1/3}$$ : **Vainshtein radius** (typically, ~ 100 pc for the Sun)

# Weak gravitational fields



# Class Ia DHOST theories

- Metric and scalar field:

$$\mathrm{d}s^2 = -\left[1 + 2\Phi(t,\vec{x})\right]\mathrm{d}t^2 + a^2(t)\left[1 - 2\Psi(t,\vec{x})\right]\mathrm{d}\vec{x}^2,$$
$$\phi = \phi_0(t) + \pi(t,\vec{x}), \quad \Phi, \Psi, \pi/M_{\mathrm{Pl}} = \mathcal{O}(\varepsilon).$$

- Quasi-static approximation:

$$\dot{\Phi} \sim H_0\Phi \ll \nabla\Phi, \quad \cdots .$$

- Orders of magnitude:

$$\dot{\phi}_0 = \mathcal{O}(M_{\mathrm{Pl}}H_0), \quad M = \mathcal{O}(M_{\mathrm{Pl}}), \quad \alpha_i, \beta_i \lesssim \mathcal{O}(1).$$

- **Linear regime**:

$$\Phi' \sim \Psi' \sim \frac{\pi'}{M_{\mathrm{Pl}}} = \mathcal{O}(r_{\mathrm{Sch}}/r^2),$$
$$\Phi - \Psi \sim \pi.$$

# Class Ia DHOST theories

corrections to standard gravity

- **Vainshtein regime** ($r \ll r_V$):

$$\Phi' = \frac{G_N \mathcal{M}}{r^2} - \frac{(\alpha_H + \beta_1)^2}{2(\alpha_H + 2\beta_1)} G_N \mathcal{M}''$$

$$\Psi' = \frac{G_N \mathcal{M}}{r^2} + \alpha_H \frac{G_N \mathcal{M}'}{r} - \frac{\beta_1(\alpha_H + \beta_1)}{2(\alpha_H + 2\beta_1)} G_N \mathcal{M}''$$

Crisostomi, Koyama '18
Dima, Vernizzi '18
Langlois et al. '18

where $8\pi G_N = \frac{1}{M^2(1 - \alpha_H - 3\beta_1)} = \frac{8\pi G_{\rm GW}}{1 - \alpha_H - 3\beta_1}$

$$\mathcal{M}(r) = 4\pi \int_0^r \rho(x) x^2 \mathrm{d}x.$$

( $\alpha_T \ (= c_{\rm GW}^2 - 1) = 0$ for simplicity)

- Horndeski ($\alpha_H = \beta_1 = 0$): efficient screening in the Vainshtein regime.

Kimura, TK, Yamamoto '12

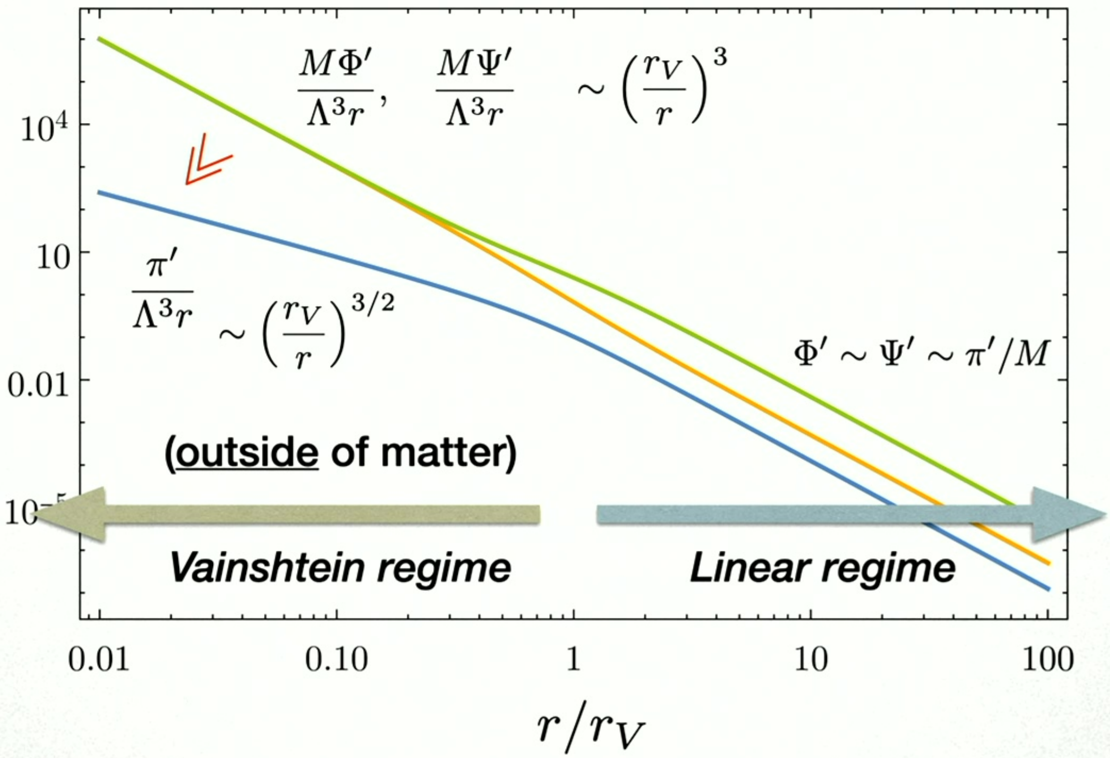
- Class Ia DHOST: efficient screening outside matter/breaking of screening inside matter. ⇨ Many tests have been proposed.

TK, Watanabe, Yamauchi '15

Koyama, Sakstein '15
Sakstein '15 Jain et al. '16 ……

(Separate treatment needed for class IIa)

# Class Ia DHOST theories



# Special case

Hirano, TK, Yamauchi '19
Crisostomi, Lewandowski, Vernizzi '19

$$\Phi' = \frac{G_N \mathcal{M}}{r^2} - \frac{(\alpha_H + \beta_1)^2}{2(\alpha_H + 2\beta_1)} G_N \mathcal{M}''$$

$$\Psi' = \frac{G_N \mathcal{M}}{r^2} + \alpha_H \frac{G_N \mathcal{M}'}{r} - \frac{\beta_1(\alpha_H + \beta_1)}{2(\alpha_H + 2\beta_1)} G_N \mathcal{M}''$$

Vainshtein mechanism operates differently for $\alpha_H + 2\beta_1 = 0$

- Different $G_N$ outside and inside
  - Outside of matter (screening occurs provided that $\alpha_B + \beta_1 + \cdots = 0$.)

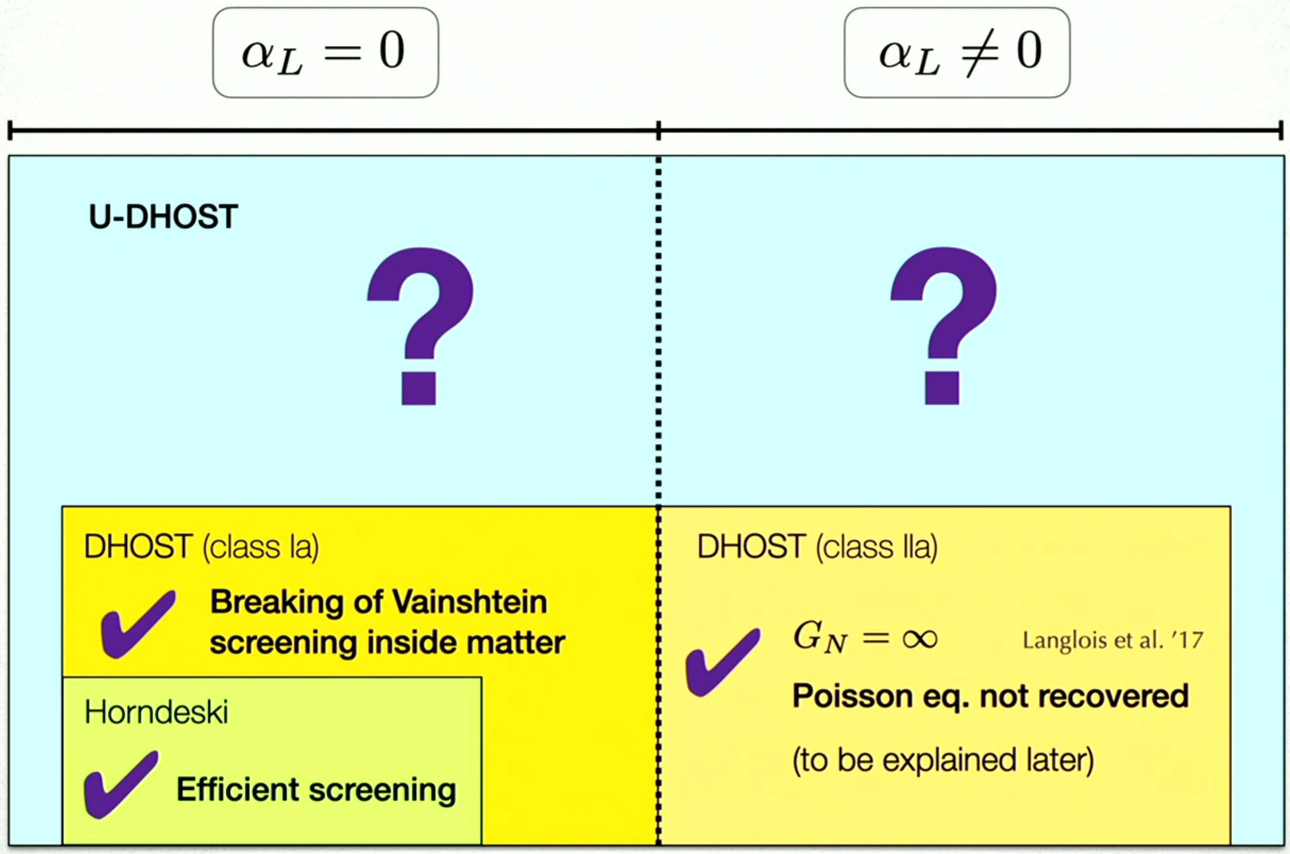
$$\Phi' = \Psi' = \frac{G_{N,\mathrm{out}} \mathcal{M}}{r^2}, \quad 8\pi G_{N,\mathrm{out}} = \frac{1}{M^2(1-\beta_1)} = \frac{8\pi G_{\mathrm{GW}}}{1-\beta_1}.$$

  - Inside of matter

$$\Phi' = \frac{G_{N,\mathrm{in}} \mathcal{M}}{r^2}, \quad \Psi = (1-2\beta_1)\Phi, \quad G_{N,\mathrm{in}} = \frac{G_{N,\mathrm{out}}}{1-\beta_1}.$$

- Physically most interesting case — no graviton decay into dark energy if $\alpha_T = \alpha_H + 2\beta_1 = 0$. Creminelli et al. '18

# Summary of previous results



TK, Hiramatsu Phys.Rev.D 109 (2024) 6, 064091[2310.11041]

# Weak fields in U-DHOST theories

$$\alpha_L \neq 0$$

- Now we have a new term in the perturbative expansion of the Lagrangian:

$$\mathcal{L} = \frac{M^2}{2}\left[(\ldots)\Phi\nabla^2\pi + (\ldots)\Psi\nabla^2\pi + \ldots - \frac{2\alpha_L}{3\dot{\phi}_0^2}(\nabla^2\pi)^2 + (\ldots)(\nabla\pi)^2\nabla^2\pi + \ldots\right] - \Phi\rho.$$

Nonlinear derivative interactions

Highest number of derivatives per field.

TK, Hiramatsu Phys.Rev.D 109 (2024) 6, 064091[2310.11041]

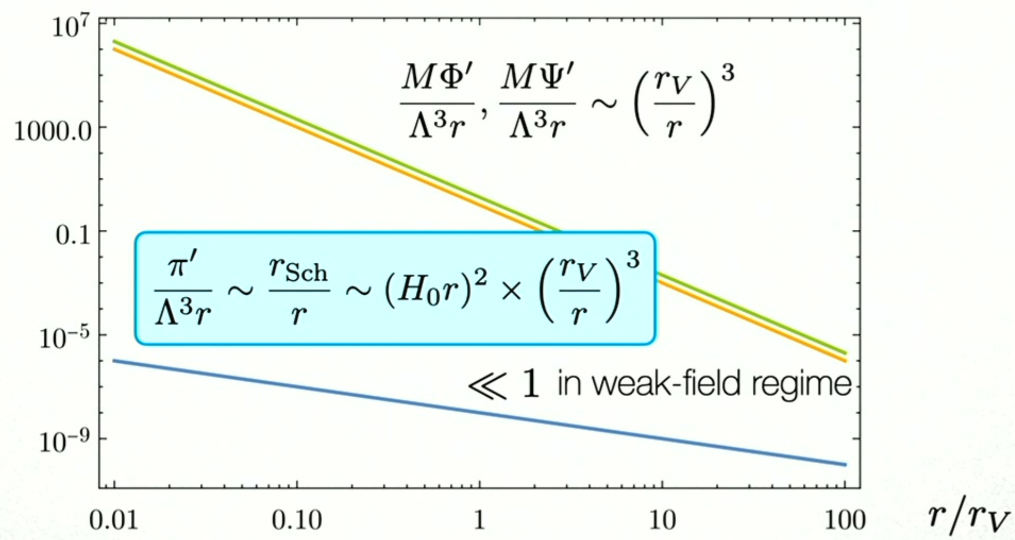
# Linear regime

$$\gamma^{\rm PPN} = \frac{1+\alpha_H}{1+\alpha_T}$$

$$\nabla^2\Phi = 4\pi G_N\rho, \quad \Psi = \left(\frac{1+\alpha_H}{1+\alpha_T}\right)\Phi, \quad \frac{M}{\dot{\phi}_0^2}\cdot\alpha_L\nabla^4\pi = \mathcal{O}(G_N\rho)$$

$$8\pi G_N = \left[\frac{2(1+\alpha_T)}{2(1+\alpha_H)^2-(1+\alpha_T)\beta_3}\right]\frac{1}{M^2} = 8\pi G_{\rm GW}\cdot\left[\frac{2(1+\alpha_T)^2}{2(1+\alpha_H)^2-(1+\alpha_T)\beta_3}\right].$$

- No Vainshtein regime. Linear analysis is sufficient.



# The case of class IIa DHOST

$$\nabla^2\Phi = 4\pi G_N\rho, \quad \Psi = \left(\frac{1+\alpha_H}{1+\alpha_T}\right)\Phi, \quad \frac{M}{\dot{\phi}_0^2}\cdot\alpha_L\nabla^4\pi = \mathcal{O}(G_N\rho)$$

$$8\pi G_N = \left[\frac{2(1+\alpha_T)}{2(1+\alpha_H)^2-(1+\alpha_T)\beta_3}\right]\frac{1}{M^2} = 8\pi G_{\rm GW}\cdot\left[\frac{2(1+\alpha_T)^2}{2(1+\alpha_H)^2-(1+\alpha_T)\beta_3}\right].$$

- Class IIa degeneracy condition (which is not imposed in U-DHOST theories):

$$\beta_3 = \frac{2(1+\alpha_H)^2}{1+\alpha_T} \quad \Rightarrow \quad G_N = \infty.$$

Poisson equation is not recovered. Langlois et al. '17

TK, Hiramatsu Phys.Rev.D 109 (2024) 6, 064091[2310.11041]

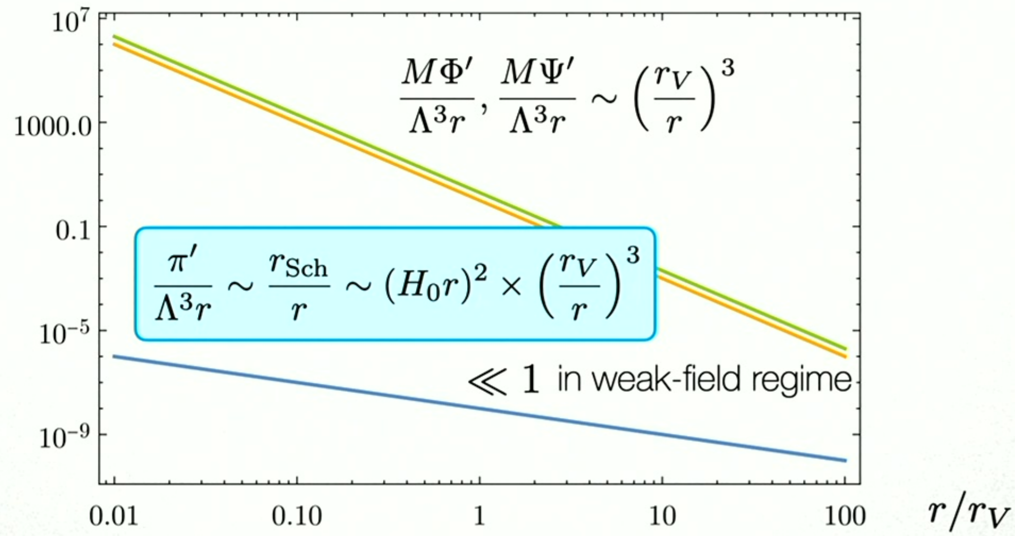
# Linear regime

$$\gamma^{\rm PPN} = \frac{1+\alpha_H}{1+\alpha_T}$$

$$\nabla^2\Phi = 4\pi G_N\rho, \quad \Psi = \left(\frac{1+\alpha_H}{1+\alpha_T}\right)\Phi, \quad \frac{M}{\dot\phi_0^2}\cdot\alpha_L\nabla^4\pi = \mathcal{O}(G_N\rho)$$

$$8\pi G_N = \left[\frac{2(1+\alpha_T)}{2(1+\alpha_H)^2-(1+\alpha_T)\beta_3}\right]\frac{1}{M^2} = 8\pi G_{\rm GW}\cdot\left[\frac{2(1+\alpha_T)^2}{2(1+\alpha_H)^2-(1+\alpha_T)\beta_3}\right].$$

- No Vainshtein regime. Linear analysis is sufficient.



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# Weak fields in U-DHOST theories

$$\alpha_L = 0$$

- $\nabla^2\pi/\Lambda^3$ can be as large as 1. Nonlinear derivative interactions are important. Technically much more involved.
- EOMs are of third order for $\Phi$ and $\Psi$, and of fourth order for $\pi$.
- Screening is not efficient even in the matter exterior:

$$\Phi' = \frac{G_N\mathcal{M}}{r^2}\left[1+\Delta_\Phi(r)\right], \quad \Psi' = \frac{G_N\mathcal{M}}{r^2}\left[1+\Delta_\Psi(r)\right], \qquad (r \ll r_V)$$

where $\Delta_\Phi = C_1 r^{n_+} + C_2 r^{n_-}, \quad \Delta_\Psi = C_3 r^{n_+} + C_4 r^{n_-}.$

$n_\pm = n_\pm(\alpha_H, \beta_1, \beta_3)$

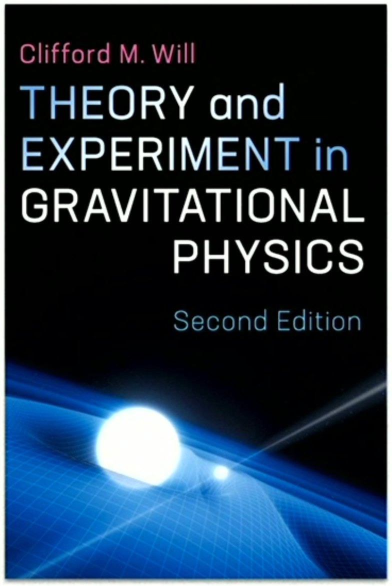
$C_1, C_2, C_3, C_4$ ↑ determined from boundary conditions

# Summary so far

| $\alpha_L = 0$ | $\alpha_L \neq 0$ |
|---|---|
| **U-DHOST**<br>**Screening is not efficient even outside matter.**<br>(complicated…)<br>TK, Hiramatsu '24 | TK, Hiramatsu '24<br>**No Vainshtein screening**<br>$\gamma^{\mathrm{PPN}} = \dfrac{1+\alpha_H}{1+\alpha_T}$<br>? ***Other PPN parameters?***<br>Saito, Yao, TK '24 |
| DHOST (class Ia)<br>Breaking of Vainshtein screening inside matter | DHOST (class IIa)<br>$G_N = \infty$<br>Poisson eq. not recovered |
| Horndeski<br>Efficient screening | |

# PPN

- **Parametrized post-Newtonian (PPN) parameters** characterize the metric in the PN limit and its deviations from GR.
- Stringent experimental constraints in terms of PPN parameters.



# PPN and EFTofDE

- **Parametrized post-Newtonian (PPN) parameters** characterize the metric in the PN limit and its deviations from GR.
- Stringent experimental constraints in terms of PPN parameters.
- **Connecting PPN parameters and EFTofDE coefficients would be useful for constraining modified gravity!**

PPN parameters

$$\gamma^{\rm PPN}, \beta^{\rm PPN}, \xi^{\rm PPN}, \ldots$$

↑ Tests of PN gravity

↔

EFTofDE coefficients

$$\alpha_L, \alpha_H, \alpha_T, \beta_1, \ldots$$

↑ Cosmological tests

Similar spirit with Lombriser '18 and
Renevey, Kennedy, Lombriser '20

# PN expansion

- Field equations (with shift symmetry)

$$\mathcal{E}_{\mu\nu} := f R_{\mu\nu} + \cdots - f_{,X} R \phi_\mu \phi_\nu + \cdots - A_1 g_{\mu\nu} \phi_{\rho\lambda} \phi^{\rho\lambda} + \dots$$
$$= T_{\mu\nu}$$

Perfect fluid: $T^{\mu\nu} = (\rho + \rho\Pi + p)\, u^\mu u^\nu + p g^{\mu\nu}$

pressure

rest-mass density

energy density/rest-mass density

- PN bookkeeping
  - Small quantity in PN expansion: $v = \mathcal{O}(\epsilon), \quad \partial_t = \mathcal{O}(\epsilon \partial_i)$
  - Newtonian gravitational potential:

$$U(t, \vec{x}) = G_N \int \mathrm{d}^3 y \frac{\rho^*(t, \vec{y})}{|\vec{x} - \vec{y}|} \quad \Leftrightarrow \quad \Delta U = -4\pi G_N \rho^*. \qquad (\rho^* := \sqrt{-g}\rho u^0)$$

Then, $U \sim v^2 \sim p/\rho \sim \Pi = \mathcal{O}(\epsilon^2)$.

# Potentials

$$\mathcal{X} = G_N \int \mathrm{d}^3 y \rho^*(t,\vec{y})|\vec{x}-\vec{y}|, \quad = \mathcal{O}(\epsilon^2)$$

$$V_i = G_N \int \mathrm{d}^3 y \frac{\rho^*(t,\vec{y}) v_i(t,\vec{y})}{|\vec{x}-\vec{y}|}, \quad = \mathcal{O}(\epsilon^3)$$

$$\Phi_1 = G_N \int \mathrm{d}^3 y \frac{\rho^*(t,\vec{y}) v^2(t,\vec{y})}{|\vec{x}-\vec{y}|},$$

$$\Phi_2 = G_N \int \mathrm{d}^3 y \frac{\rho^*(t,\vec{y}) U(t,\vec{y})}{|\vec{x}-\vec{y}|},$$

$$\Phi_3 = G_N \int \mathrm{d}^3 y \frac{\rho^*(t,\vec{y}) \Pi(t,\vec{y})}{|\vec{x}-\vec{y}|},$$

$$\Phi_4 = G_N \int \mathrm{d}^3 y \frac{p(t,\vec{y})}{|\vec{x}-\vec{y}|},$$

$$\Phi_6 = G_N \int \mathrm{d}^3 y \frac{\rho^*(t,\vec{y}) \left[\vec{v}(t,\vec{y}) \cdot (\vec{x}-\vec{y})\right]^2}{|\vec{x}-\vec{y}|^3},$$

$$\Phi_W = G_N \int \mathrm{d}^3 y \mathrm{d}^3 z \rho^*(t,\vec{y}) \rho^*(t,\vec{z}) \frac{(\vec{x}-\vec{y})}{|\vec{x}-\vec{y}|^3} \cdot \left[\frac{(\vec{y}-\vec{z})}{|\vec{x}-\vec{z}|} - \frac{(\vec{x}-\vec{z})}{|\vec{y}-\vec{z}|}\right].$$

$$= \mathcal{O}(\epsilon^4)$$

# PN metric and PPN parameters

- Metric in the standard PPN gauge:

$$g_{00} = -1 + 2U + 2\left(\psi - \beta^{\rm PPN} U^2\right) + \mathcal{O}(\epsilon^6),$$
$$\quad = \mathcal{O}(\epsilon^2) \qquad = \mathcal{O}(\epsilon^4)$$

$$g_{0i} = -\left[2\left(1 + \gamma^{\rm PPN}\right) + \frac{\alpha_1^{\rm PPN}}{2}\right] V_i - \frac{1}{2}\left(1 + \alpha_2^{\rm PPN} - \zeta_1^{\rm PPN} + 2\xi^{\rm PPN}\right)\partial_i \dot{\mathcal{X}} + \mathcal{O}(\epsilon^5),$$
$$= \mathcal{O}(\epsilon^3)$$

$$g_{ij} = \left(1 + 2\gamma^{\rm PPN} U + 2C\right)\delta_{ij} + D_{ij} + \mathcal{O}(\epsilon^6),$$
$$\quad = \mathcal{O}(\epsilon^2) \qquad = \mathcal{O}(\epsilon^4)$$

PPN parameters

where

$$\psi := \frac{1}{2}\left(2\gamma^{\rm PPN} + 1 + \alpha_3^{\rm PPN} + \zeta_1^{\rm PPN} - 2\xi^{\rm PPN}\right)\Phi_1 + \left(1 - \beta^{\rm PPN} + \zeta_2^{\rm PPN} + \xi^{\rm PPN}\right)\Phi_2$$
$$+ \left(1 + \zeta_3^{\rm PPN}\right)\Phi_3 + \left(3\gamma^{\rm PPN} + 3\zeta_4^{\rm PPN} - 2\xi^{\rm PPN}\right)\Phi_4 - \frac{1}{2}\left(\zeta_1^{\rm PPN} - 2\xi^{\rm PPN}\right)\Phi_6 - \xi^{\rm PPN}\Phi_W,$$
$$C := d_{UU}U^2 + d_W\Phi_W + d_1\Phi_1 + d_2\Phi_2 + d_3\Phi_3 + d_4\Phi_4 + d_6\Phi_6 + d_{\mathcal{X}}\ddot{\mathcal{X}}.$$

# Scalar field

$$\phi = q\left(t + \gamma_{\mathcal{X}}^{\rm PPN}\dot{\mathcal{X}}\right) + \mathcal{O}(\epsilon^5).$$

$$= \mathcal{O}(\epsilon^3)$$

Constant determined by $\mathcal{O}(\epsilon^0)$ part of field equations.

$$\mathcal{X} = G_N \int \mathrm{d}^3 y \rho^*(t, \vec{y})|\vec{x} - \vec{y}|$$

- No $\mathcal{O}(\epsilon^2)$ piece: $\pi = \phi - qt \ll MU = \mathcal{O}(\epsilon^2)$
- $\gamma_{\mathcal{X}}^{\rm PPN}$ cannot be measured by experiments.
- One can instead work in the unitary gauge, but then the metric no longer takes the standard PPN form. In either gauge, one arrives at the same results.

# Calculations of PPN parameters

To determine the metric in the PN limit, solve at each order

$$
\begin{aligned}
\mathcal{E}_{00} &= \mathcal{E}_{00}^{(0)} + \mathcal{E}_{00}^{(2)}\epsilon^2 + \mathcal{E}_{00}^{(4)}\epsilon^4 + \dots \\
\mathcal{E}_{0i} &= \mathcal{E}_{0i}^{(3)}\epsilon^3 + \dots \\
\mathcal{E}_{ij} &= \mathcal{E}_{ij}^{(0)} + \mathcal{E}_{ij}^{(2)}\epsilon^2 + \mathcal{E}_{ij}^{(4)}\epsilon^4 + \dots
\end{aligned}
\quad = T_{\mu\nu}
$$

$\mathcal{E}^{(0)} \rightarrow q$

$\mathcal{E}^{(2)} \rightarrow \gamma^{\mathrm{PPN}}, G_N$

$\mathcal{E}_{0i}^{(3)} \rightarrow \alpha_1^{\mathrm{PPN}}$

$\mathcal{E}^{(4)} \rightarrow \beta^{\mathrm{PPN}}, \alpha_2^{\mathrm{PPN}}, \dots$

Saito, Yao, TK, JCAP06(2024)040 [2402.10459]

# PPN in terms of EFTofDE

| Parameter | GR | Higher-order scalar-tensor theories | Constraints |
|---|---|---|---|
| $G_{\rm N}$ | $G_{\rm GW}$ | $\left[\frac{2(1+\alpha_T)^2}{2(1+\alpha_H)^2-(1+\alpha_T)\beta_3}\right] G_{\rm GW}$ | $0.995 \lesssim G_{\rm GW}/G_{\rm N} c_{\rm GW} \lesssim 1.00$ [57][a] |
| $c_{\rm GW}^2$ | 1 | $1+\alpha_T$ | $-3\times10^{-15} < c_{\rm GW}-1 < 7\times10^{-16}$ [58, 59] |
| $\gamma^{\rm PPN}$ | 1 | $\frac{1+\alpha_H}{1+\alpha_T}$ | $\gamma^{\rm PPN}-1=(2.1\pm2.3)\times10^{-5}$ [60] |
| $\beta^{\rm PPN}$ | 1 | $\frac{4\gamma^{\rm PPN}\left[c_{\rm GW}^2\gamma^{\rm PPN}\left(1+\gamma^{\rm PPN}\right)+2\delta_1\right]-\beta_3\left(3+\gamma^{\rm PPN}\right)-2\delta_2}{4\left[2c_{\rm GW}^2(\gamma^{\rm PPN})^2-\beta_3\right]}$ | $\beta^{\rm PPN}-1=(0.2\pm2.5)\times10^{-5}$ [61] |
| $\xi^{\rm PPN}$ | 0 | 0 | ✓ |
| $\alpha_1^{\rm PPN}$ | 0 | $4\left[2c_{\rm GW}^2(\gamma^{\rm PPN})^2-\gamma^{\rm PPN}-1-\beta_3\right]$ | $\alpha_1^{\rm PPN}=-0.4^{+3.7}_{-3.1}\times10^{-5}$ [62] |
| $\alpha_2^{\rm PPN}$ | 0 | Eq. (70) | $\lvert\alpha_2^{\rm PPN}\rvert<1.6\times10^{-9}$ [63] |
| $\alpha_3^{\rm PPN}$ | 0 | 0 | ✓ |
| $\zeta_{1,2,3,4}^{\rm PPN}$ | 0 | 0 | ✓ |
| $\gamma_{\mathcal{X}}^{\rm PPN}$ | N/A | Eq. (72) | N/A |

[a] The scalar gravitational radiation is ignored.

complicated…

Saito, Yao, TK, JCAP06(2024)040 [2402.10459]

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| $\alpha_3^{\rm PPN}$ | 0 | | ✓ |
| $\zeta_{1,2,3,4}^{\rm PPN}$ | 0 | | ✓ |
| $\gamma_X^{\rm PPN}$ | N/A | Eq. (72) | N/A |

[a] The scalar gravitational radiation is ignored.

*How much nonlinearity in superposition of gravity?*

Need new parameters in addition to the existing ones characterizing linear cosmology:

$$M^2\delta_1 := 2X(f-2Xf_{,X})_{,X},$$
$$M^2\delta_2 := -8X\left[X(f_{,X}+A_1-XA_4)\right]_{,X}.$$

Saito, Yao, TK, JCAP06(2024)040 [2402.10459]

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| $\xi^{\rm PPN}$ | 0 | 0 | ✓ |
| $\alpha_1^{\rm PPN}$ | | | |
| $\alpha_2^{\rm PPN}$ | | | |
| $\alpha_3^{\rm PPN}$ | | | |
| $\zeta_{1,2,3,4}^{\rm PPN}$ | 0 | 0 | ✓ |
| $\gamma_{\mathcal{X}}^{\rm PPN}$ | N/A | Eq. (72) | N/A |

[a] The scalar gravitational radiation is ignored.

$$\alpha_2^{\rm PPN} = \frac{3\left[2\left(\gamma^{\rm PPN}+\beta_1\right)-2c_{\rm GW}^2(\gamma^{\rm PPN})^2-\beta_3\right]^2}{2\left[2c_{\rm GW}^2(\gamma^{\rm PPN})^2-\beta_3\right]}\left(\frac{1}{\alpha_L}+1\right)-1+c_{\rm GW}^2(\gamma^{\rm PPN})^2+6\beta_1-\frac{\beta_3}{2} + \frac{\beta_2-6\beta_1^2-12\beta_1\gamma^{\rm PPN}}{2c_{\rm GW}^2(\gamma^{\rm PPN})^2-\beta_3}.$$

# Example: khronometric theory

$$S = \frac{M_*^2}{2}\int d^4x\sqrt{-g}\left[\mathcal{R} + c_1\nabla_\mu u_\nu \nabla^\mu u^\nu + c_2(\nabla_\mu u^\mu)^2 + c_3\nabla_\mu u_\nu\nabla^\nu u^\mu + c_4 u^\mu u^\nu \nabla_\mu u_\lambda \nabla_\nu u^\lambda\right],$$

where $u_\mu = -\phi_\mu/\sqrt{2X}$.

Blas, Pujolas, Sibiryakov '09

- EFTofDE coefficients

$$M^2 = (1+c_3)M_*^2, \quad \alpha_T = \alpha_H = -\frac{c_3}{1+c_3}, \quad \alpha_L = -\frac{3}{2}\frac{c_2+c_3}{1+c_3}, \quad \beta_1 = \beta_2 = 0, \quad \beta_3 = \frac{c_4}{1+c_3}, \quad \delta_1 = \delta_2 = 0.$$

- PPN parameters Blas, Sanctuary '11

$$8\pi G_N = \frac{1}{(1-c_4/2)M_*^2}, \quad \gamma^{\mathrm{PPN}} = 1, \quad \beta = 1, \quad \alpha_1^{\mathrm{PPN}} = -\frac{4(2c_3+c_4)}{1+c_3},$$
$$\alpha_2^{\mathrm{PPN}} = -2 + \frac{4}{1+c_3} - \frac{2c_2}{c_2+c_3} - \frac{3(2-3c_2-c_3)}{3c_2+c_3-c_4} + \frac{(1-2c_2-c_3)c_4}{(1+c_3)(c_2+c_3)},$$
$$\alpha_3^{\mathrm{PPN}} = \zeta_1^{\mathrm{PPN}} = \zeta_2^{\mathrm{PPN}} = \zeta_3^{\mathrm{PPN}} = \zeta_4^{\mathrm{PPN}} = \xi^{\mathrm{PPN}} = 0.$$

Previous results in the literature can be reproduced.

# Application 1: Experimental constraints

on theories with $c_{\rm GW} = 1$

$$\alpha_T \ (= c_{\rm GW}^2 - 1) = 0$$

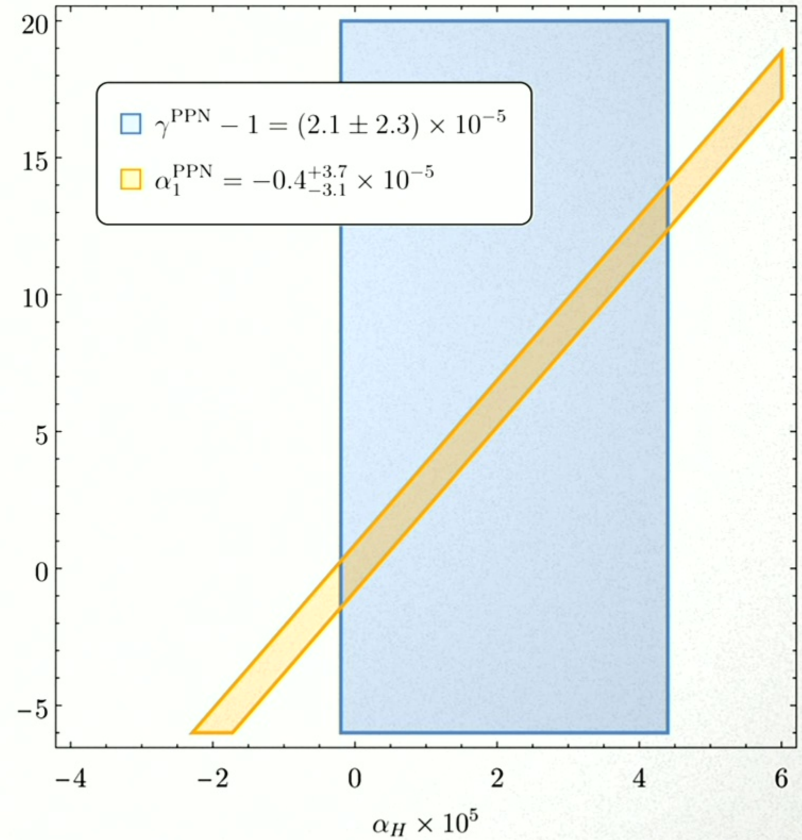
⇩

$$\gamma^{\rm PPN} = 1 + \alpha_H ,$$
$$\alpha_1^{\rm PPN} = 4\left[2(\gamma^{\rm PPN})^2 - \gamma^{\rm PPN} - 1 - \beta_3\right].$$

⇩

Constraints on EFTofDE

$(\alpha_H, \beta_3)$



# Application 2: A model with no deviation from GR in the PN limit

- All PPN parameters = GR values, $c_{\rm GW} = 1,\ G_N = G_{\rm GW}$

  ⇨ $\alpha_T = \alpha_H = \beta_1 = \beta_2 = \beta_3 = 0,$

  but $\alpha_L$ is completely arbitrary (as long as $\alpha_L \neq 0$).

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\left[R - \frac{\alpha(X)}{3X}\left(\Box\phi + \frac{\phi^\mu\phi_{\mu\nu}\phi^\nu}{2X}\right)^2\right] + P(X)$$

⇨ $\alpha_L(t) = \alpha|_{\rm FLRW}$

$$\sim K_{ij}K^{ij} - \left[1 + \frac{2}{3}\alpha(N)\right]K^2 + R^{(3)}$$

cf. Blas, Pujolas, Sibiryakov '11

- Cosmological tests?

Ongoing work with T. Hiramatsu and Z. Yao

# Summary

- No Vainshtein screening in Lorentz-violating ($\alpha_L \neq 0$) U-DHOST theories.
- All PPN parameters are obtained in terms of EFTofDE coefficients (+ some new parameters).
  - Experimental constraints on PPN parameters can be translated to constraints on EFTofDE coefficients.
- A model with no deviation from GR in the PN limit.
  - *Cosmological tests?*
  - *Gravitational radiation?*

Thank you!

# Summary of our new results

$\alpha_L = 0$

$\alpha_L \neq 0$

**U-DHOST**

✔ **Screening is not efficient even outside matter.**
(complicated…)

TK, Hiramatsu '24

TK, Hiramatsu '24

✔ **No Vainshtein screening**

✔ **All PPN parameters are determined.**

Saito, Yao, TK '24

DHOST (class Ia)

✔ Breaking of Vainshtein screening inside matter

Horndeski

✔ Efficient screening

DHOST (class IIa)

✔ Poisson eq. not recovered