Title: Non-linear dark energy simulations

Speakers: Martin Kunz

Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

Subject: Cosmology, Strong Gravity, Mathematical physics

Date: July 17, 2024 - 11:00 AM

URL: https://pirsa.org/24070042

Abstract:

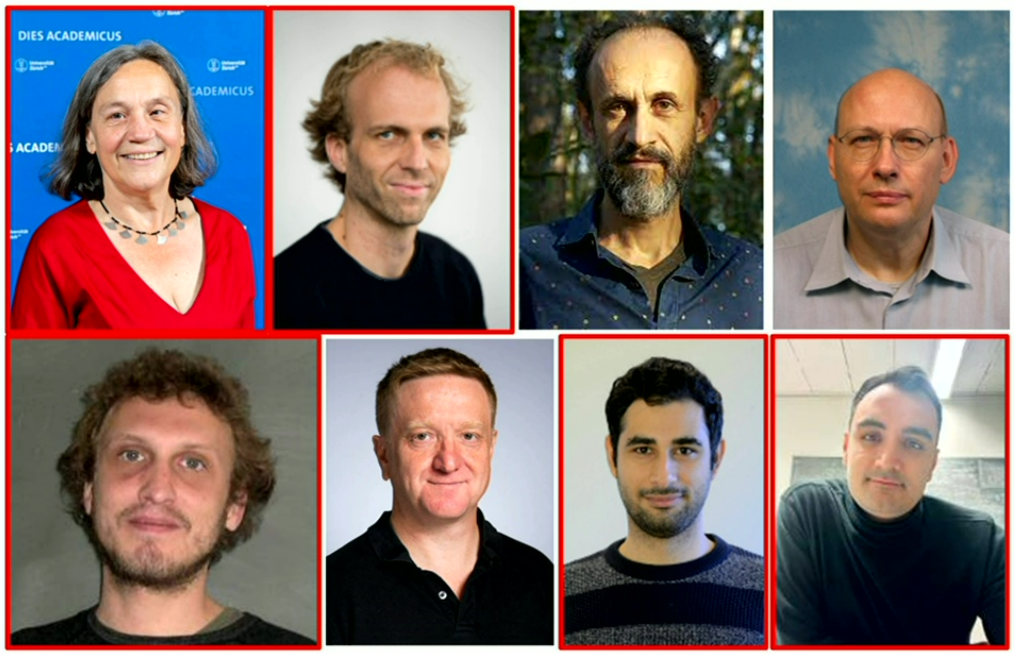
The coming years will see an amazing increase in data on the large-scale structure of the Universe, ushering in a new phase for "precision cosmology". One of the major questions in fundamental physics concerns the nature of the dark energy, and the new data may help to shed light on this issue. But in order to unlock the full power of the future data to test alternative models like Horndeski Gravity, we need theoretical predictions that are as accurate as the new observations on all scales, including non-linear scales. In my presentation I will introduce our relativistic N-body code for cosmological simulations, gevolution, and how we are using it to look at non-linear effects in the Universe. In particular I will discuss our k-essence simulations, how to use them for cosmology, and what can happen when dark energy clustering becomes non-linear in models with low speed of sound.

Non-linear DE simulations

Martin Kunz

University of Geneva

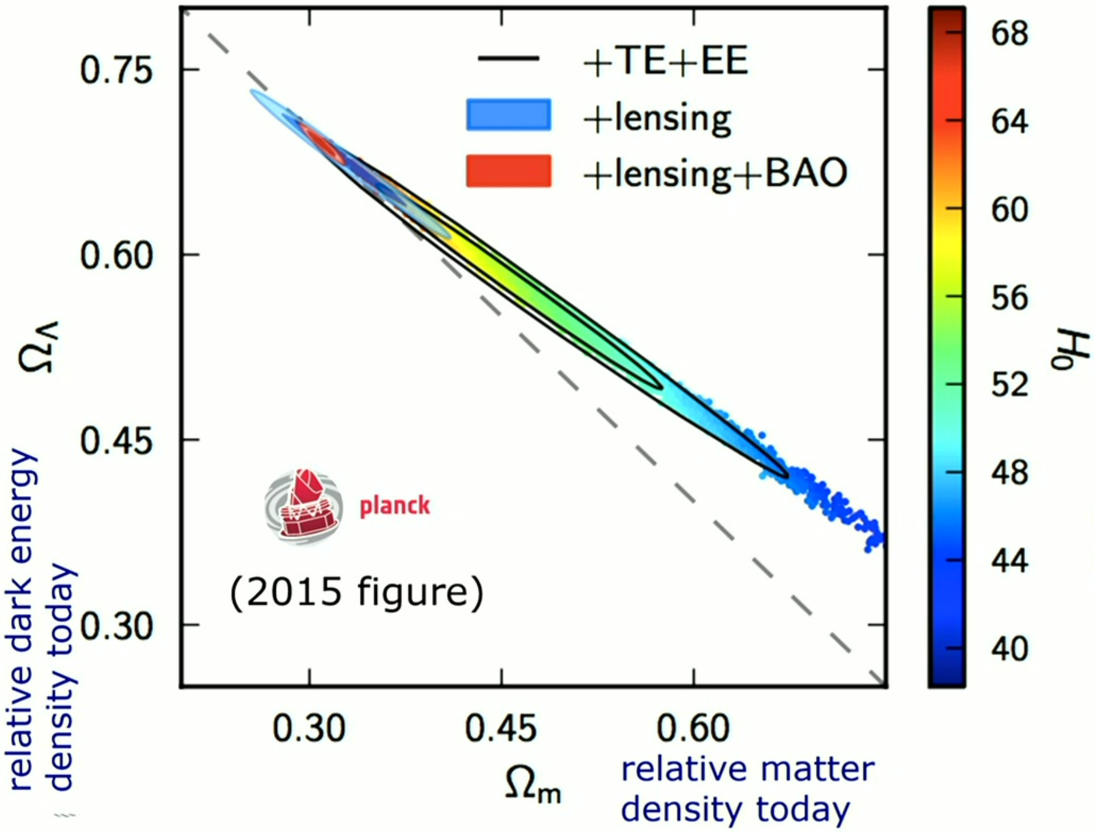
and many friends



What is the Universe made of?

Planck 2018 paper VI

spatial curvature: $\Omega_k=0.001\pm0.004$ (95%)



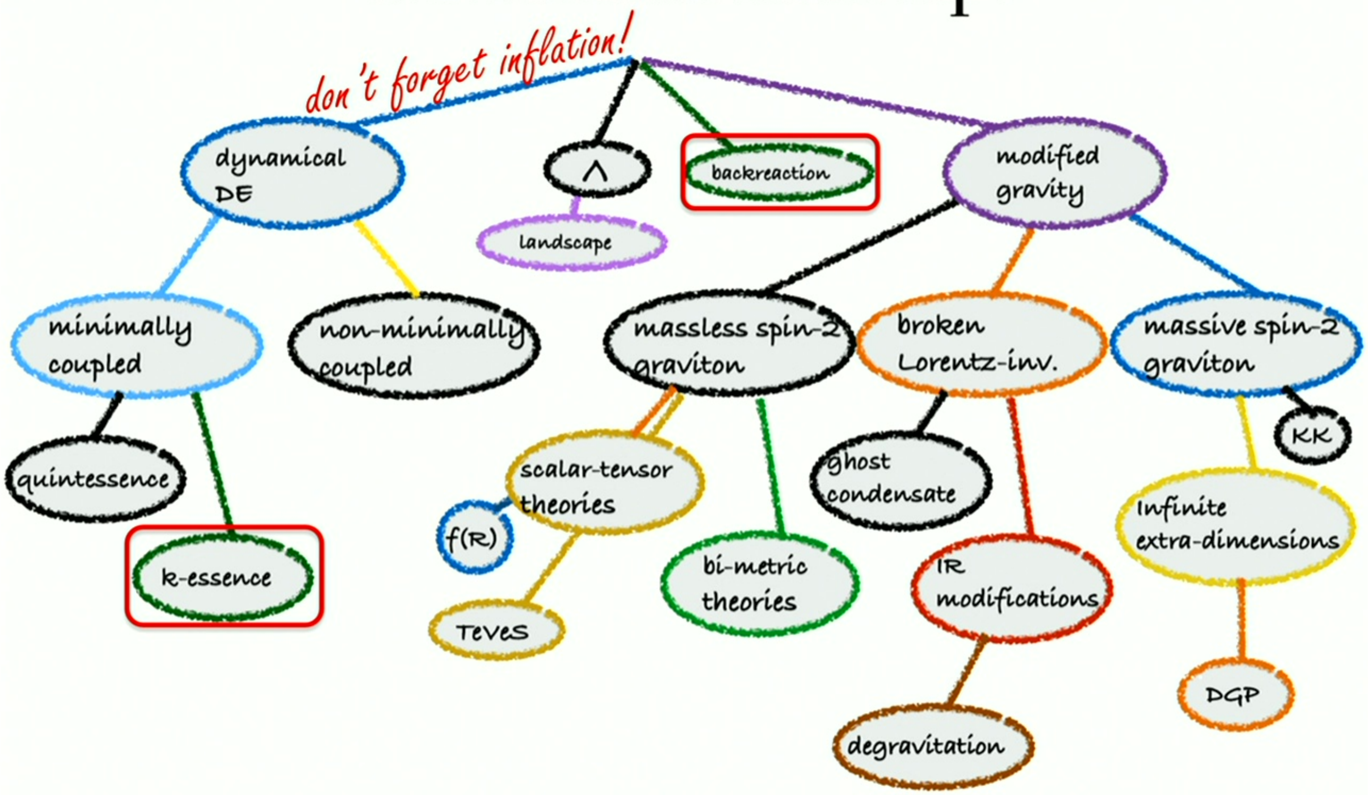
flat universe:
(1σ errors)

age [Gyr]:
13.79 ± 0.02

expansion rate:
H_0 = 67.7 ± 0.4
[km/s/Mpc]

initial spectrum:
n_s = 0.967 ± 0.004

Gravitational landscape



(Thank you, Alessandra!)

Outline

I. Introduction

II. How to simulate the Universe
- Everyone uses Newtonian gravity
- But General Relativity would be more appropriate!

III. The non-linear evolution of the Universe
- General Relativity is a non-linear theory
- Can this affect the average evolution of the Universe…
- … and make dark energy obsolete?

IV. Non-linear effects in dark energy
- If we need dark energy, can it have non-linear effects?
- If yes, are they important?

https://github.com/gevolution-code

II. How to simulate the Universe

- Everyone uses Newtonian gravity (yes, really!)
- Full numerical General Relativity is difficult (no global coordinate system, hard pde's, stability issues, …)
- In standard cosmology we are close to FLRW …

$$ds^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\mathbf{x}^2\right]$$

- … and the potentials should remain small on *all* scales!

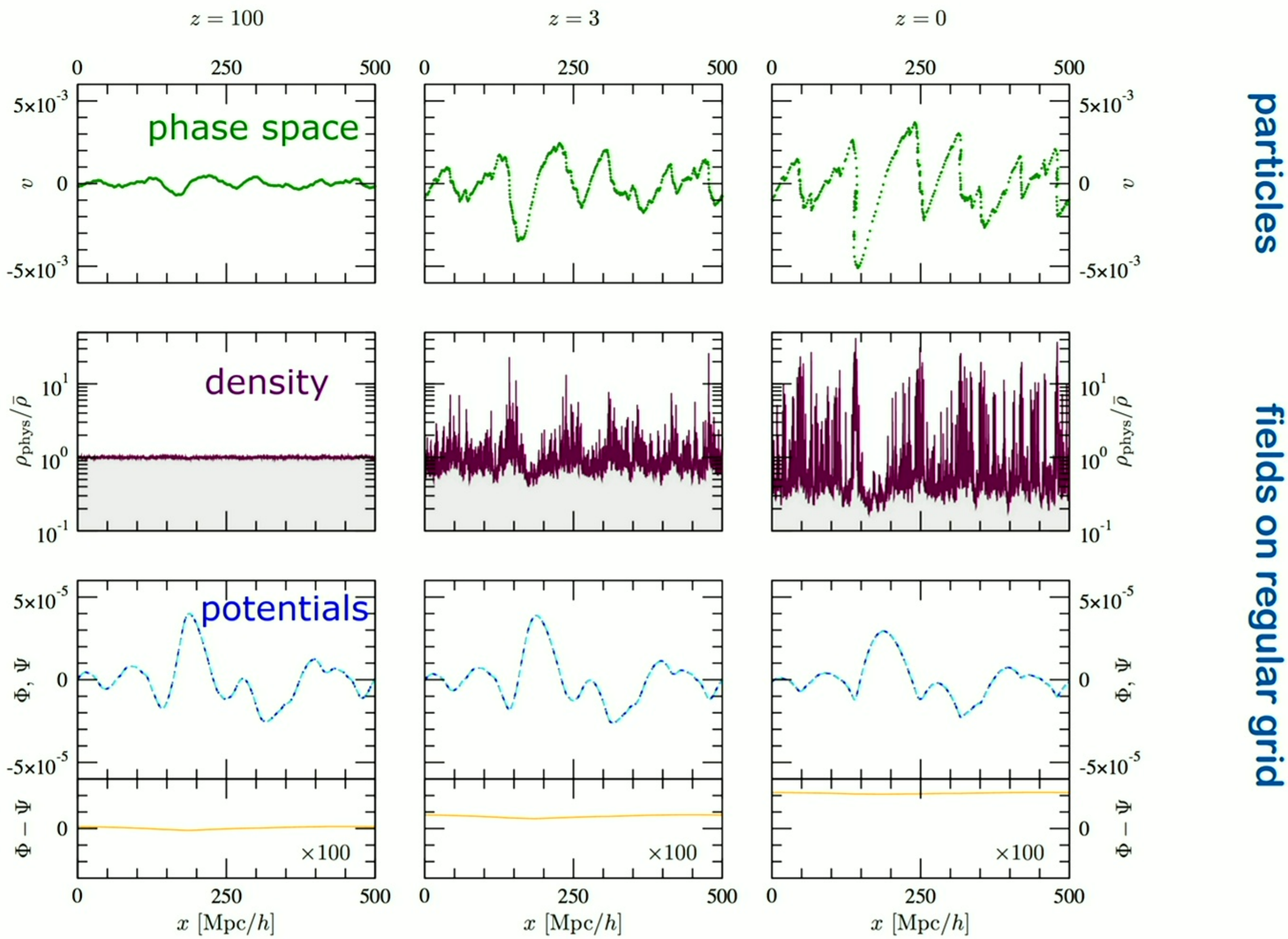
$$\Delta\Phi = 4\pi G a^2 \bar{\rho}\ \delta$$

($\Delta \sim -k^2$ → small scales: k large, δ large, Φ stays small)

- → Use weak field approximation
  - metric perturbations stay small: all okay
  - metric perturbations become large: uh oh

The '1D' universe

Adamek, Daverio, Durrer, MK, arXiv:1308.6524



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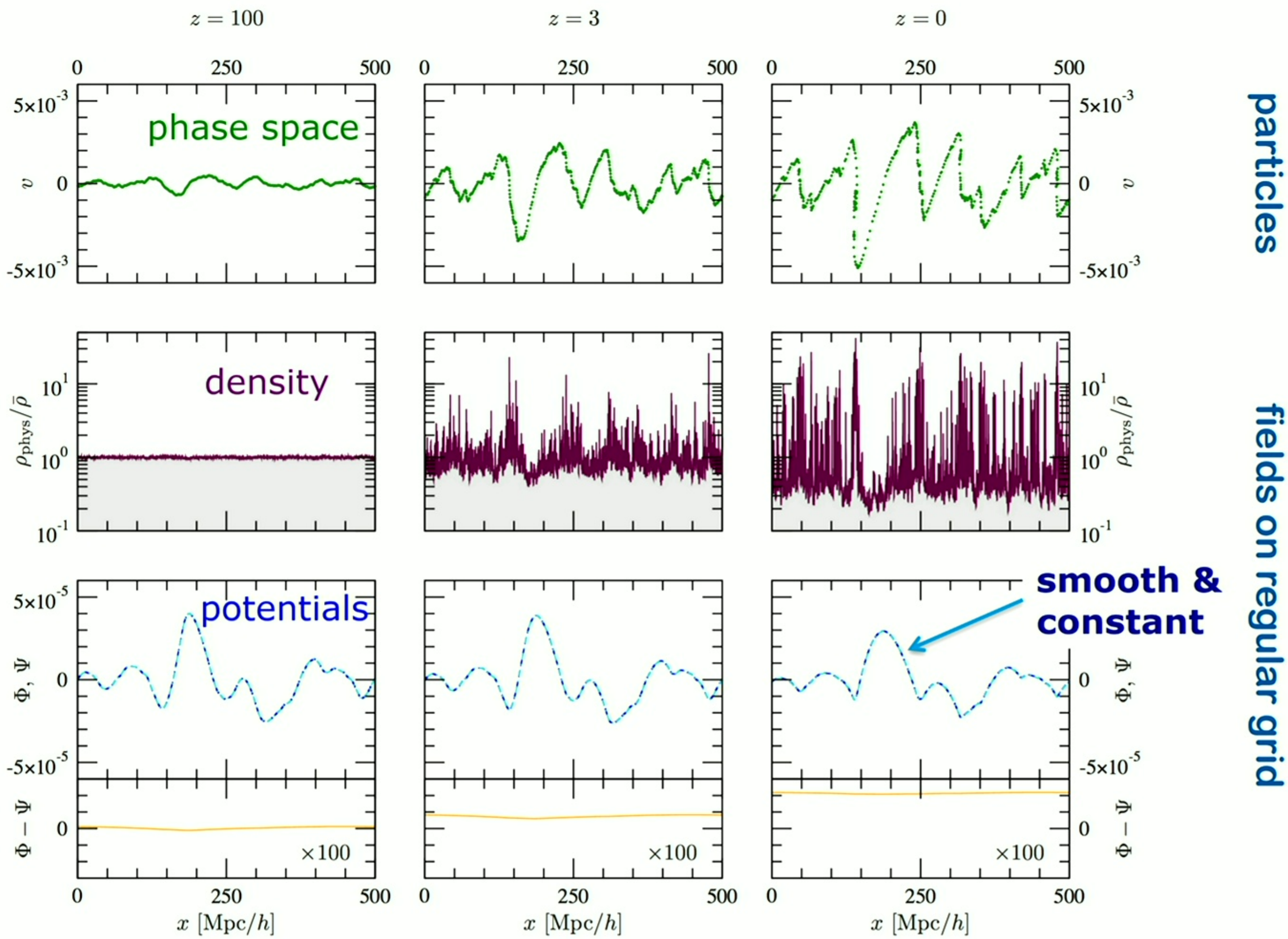

Formalism I : relativistic Poisson eq.

Now just 'crank the handle': compute Einstein and geodesic equations

example: 0-0 Einstein equation for LCDM (→ Poisson eq.):

$$(1+4\Phi)\,\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi + \frac{3}{2}\delta^{ij}\Phi_{,i}\Phi_{,j}$$
$$= 4\pi G a^2 \bar{\rho}\left[\delta + 3\Phi\,(1+\delta) + \frac{1}{2}\,(1+\delta)\,\langle v^2\rangle\right]$$

→ diffusion-type equation for Φ, estimate of diffusion to dynamical (free-fall) time scale for structure of size r:

$$\frac{t_{\rm diff}}{t_{\rm dyn}} \simeq \frac{r^2}{r_H^2}\sqrt{1+\delta}$$ <<1 for r << r_H

→ expect to be driven towards 'equilibrium' solution, which is given by solution of Poisson eq.

Formalism II : 'beyond-Newtonian' quantities

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a^2(\tau)\left[-e^{2\psi}d\tau^2 - 2B_i dx^i d\tau + \left(e^{-2\phi}\delta_{ij} + h_{ij}\right)dx^i dx^j\right]$$

Other Einstein field equations:

$$\Delta^2 B_i = 16\pi G a^2 P_i^j T_j^0 \,,$$

$$\Delta^2\chi - \left(3\delta^{ik}\delta^{jl}\frac{\partial^2}{\partial x^k \partial x^l} - \delta^{ij}\Delta\right)\phi_{,i}\phi_{,j} = 4\pi G a^2\left(3\delta^{ik}\frac{\partial^2}{\partial x^j \partial x^k} - \delta^i_j\Delta\right)T^j_i \,,$$

$$\Delta^2\left(h''_{ij} + 2\mathcal{H}h'_{ij} - \Delta h_{ij}\right) - 4\left(P_i^k P_j^l - \frac{1}{2}P_{ij}P^{kl}\right)\phi_{,k}\phi_{,l} = 16\pi G a^2\left(P_{ik}P_j^l - \frac{1}{2}P_{ij}P_k^l\right)T_l^k$$

- χ=Φ-Ψ is our second scalar variable
- we solve these equations in Fourier space, but we also have a multi-grid solver
- also particle motion is changed (we actually use fully relativistic version with momenta – also for Newtonian!)

$$q'_i = -\frac{2q^2 + m^2a^2}{\sqrt{q^2 + m^2a^2}}\phi_{,i} + \sqrt{q^2 + m^2a^2}\chi_{,i} - q^j B_{j,i} + \frac{1}{2}\frac{q^j q^k h_{jk,i}}{\sqrt{q^2 + m^2a^2}}$$

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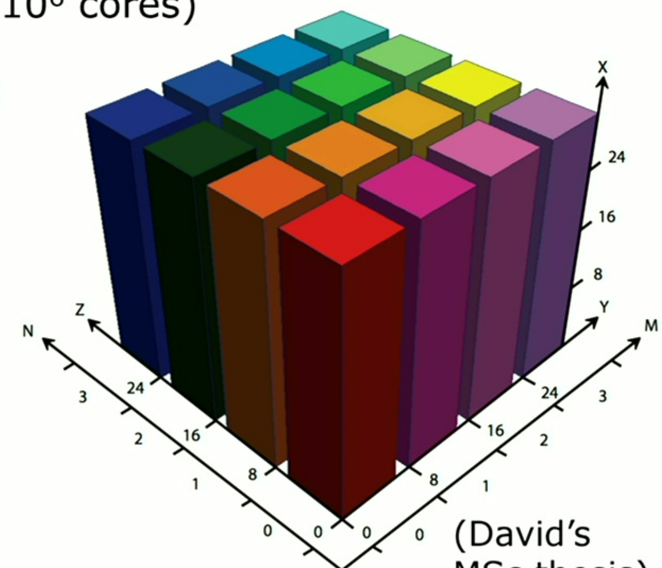
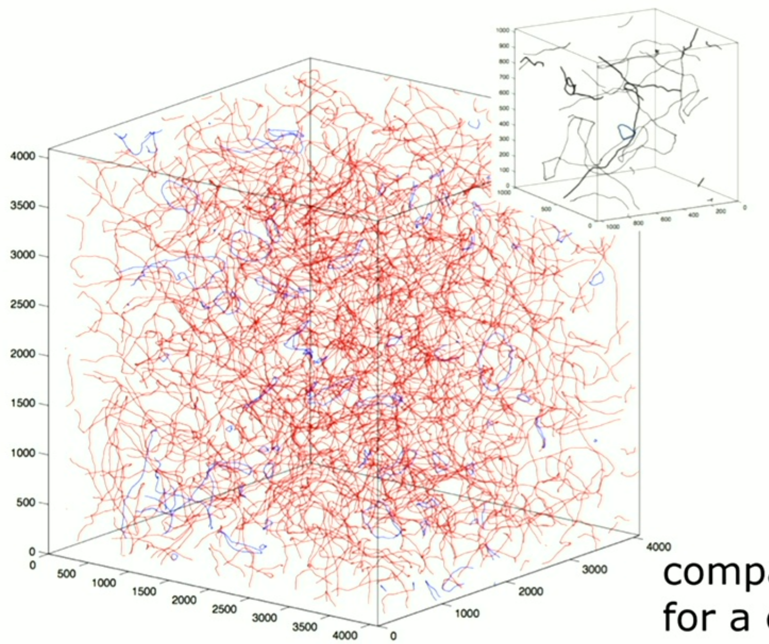
3D simulation framework: LATfield2

A C++ framework for parallel field simulations. Hides all the parallelization. No need to think about it from 4 cores to …. (tested up to 72,000, designed to scale to > 10^6 cores)

focus: easy to use & efficient

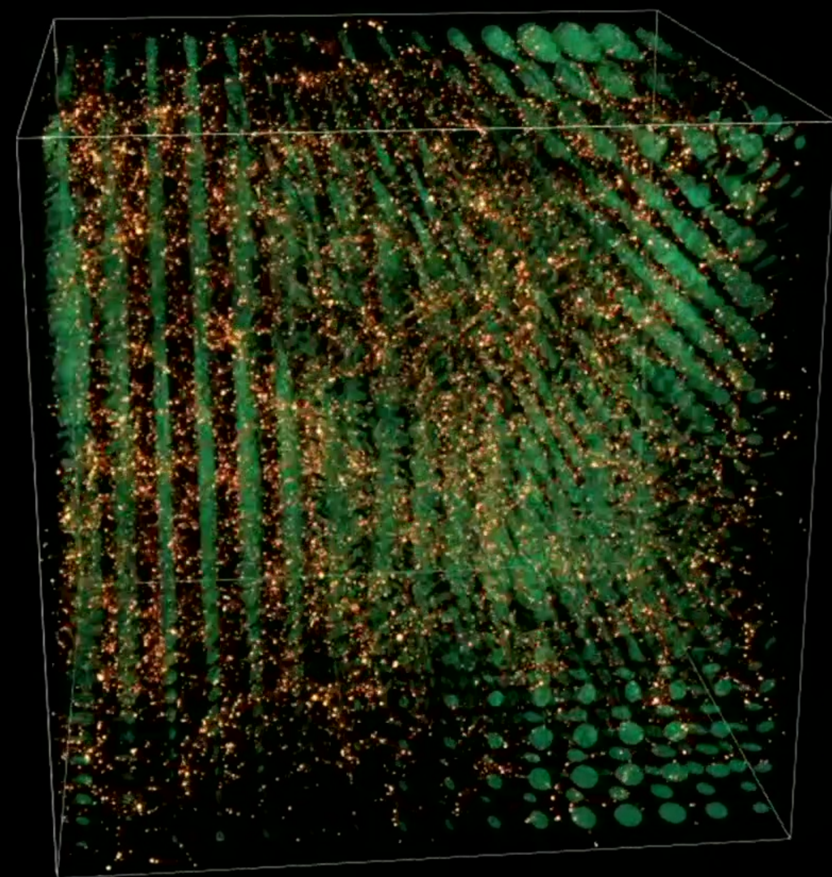


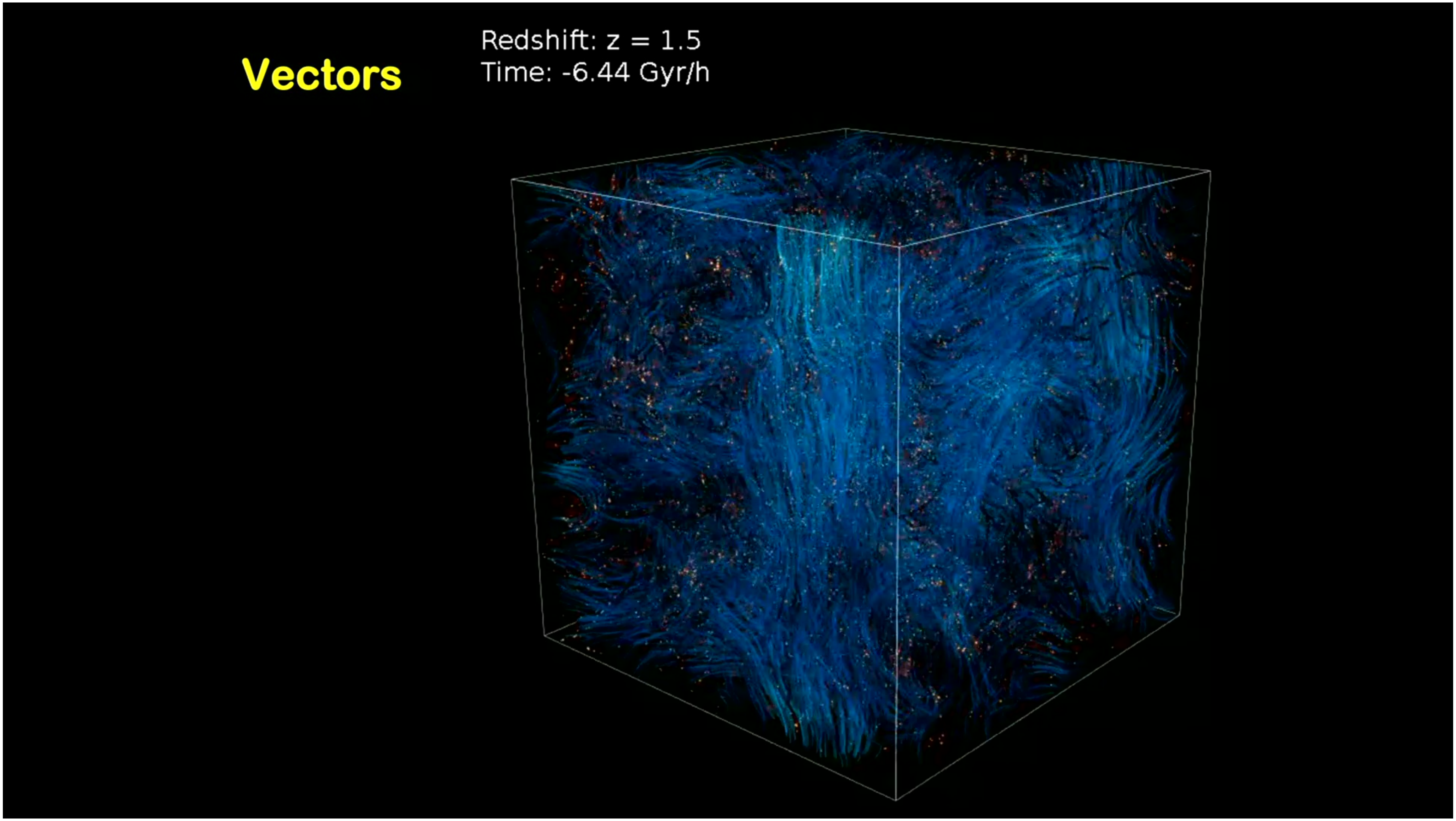
(David's MSc thesis)



comparison $(4096)^3$ to $(1024)^3$ grid for a cosmic string simulation

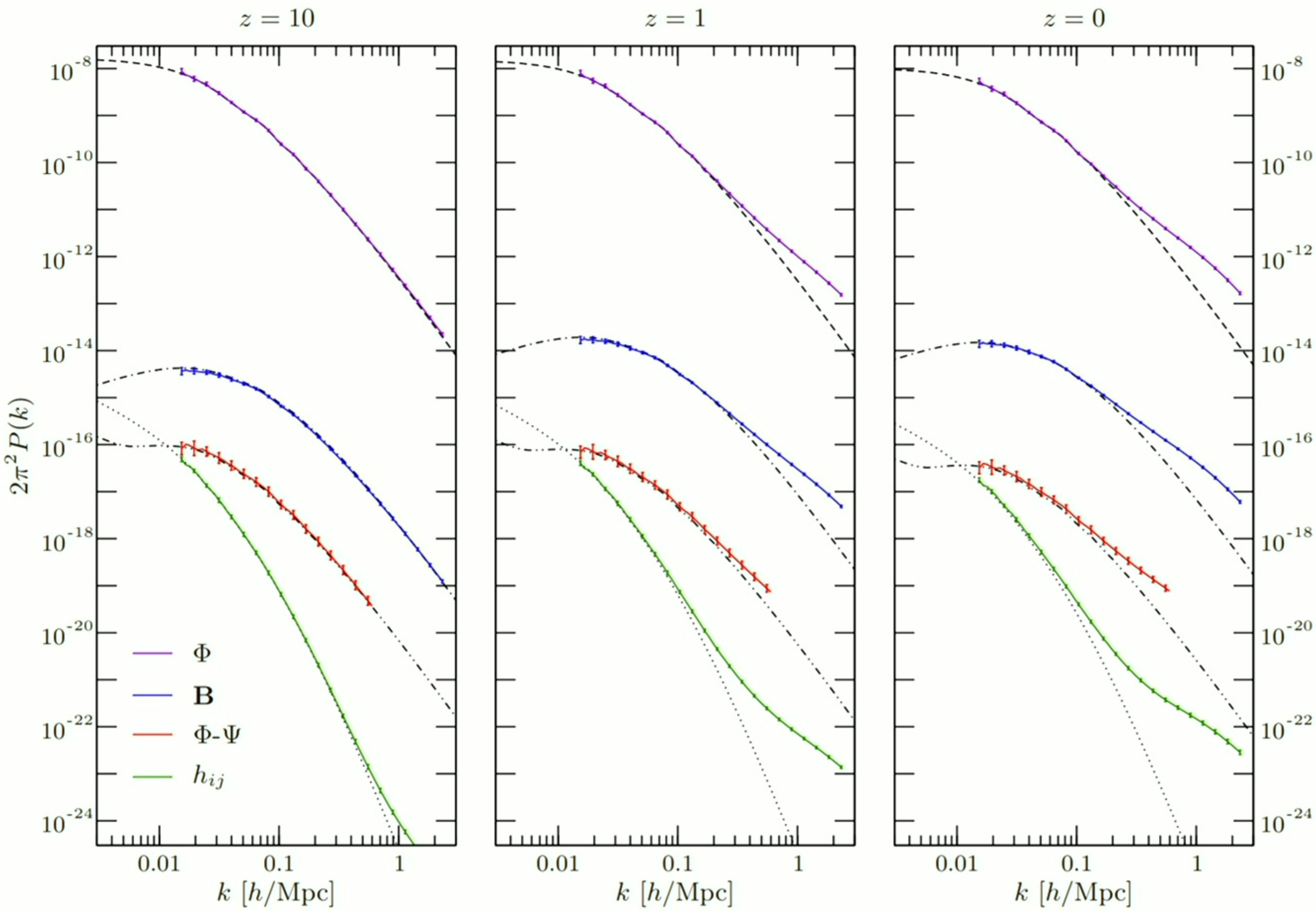
Tensors




Vectors
Redshift: z = 1.5
Time: -6.44 Gyr/h

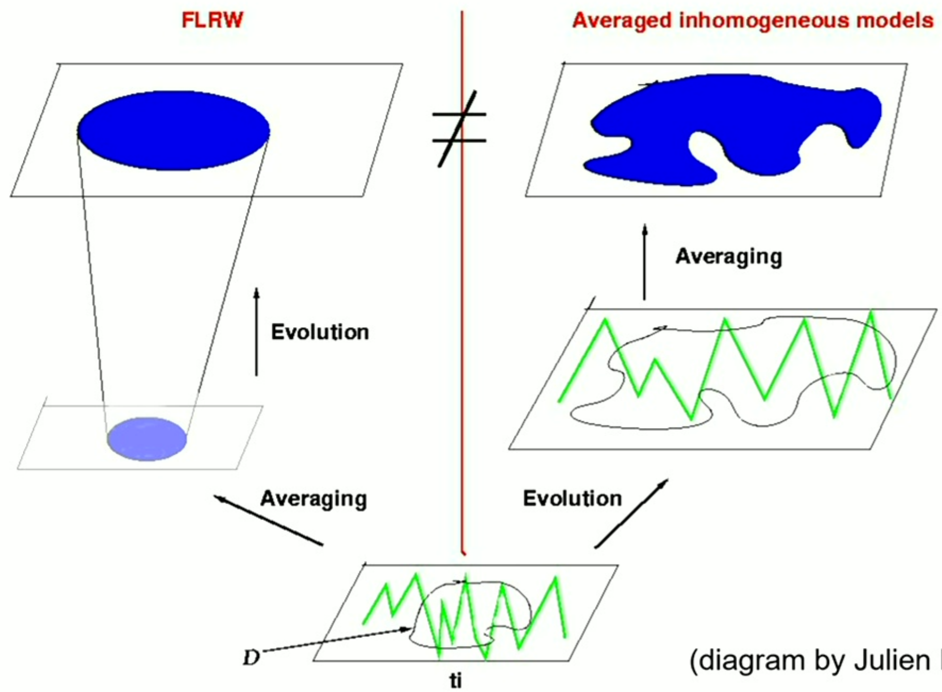
Adamek, Daverio, Durrer, MK, arXiv:1604.06065, 1509.01699 & 1408.3352

Spectra



III. The non-linear universe: average and evolution

Under non-linear evolution, the average of the evolved universe is in general not the evolution of the averaged universe!



(diagram by Julien Larena)

effect would become important around structure formation, same as dark energy

Buchert equations

- Einstein eqs, irrotational dust, 3+1 split (as defined by freely-falling observers)
- averaging over spatial domain D
- $a_D \sim V_D^{1/3}$ [<-> enforce isotropic & homogen. coord. sys.]
- set of effective, averaged, local eqs.:

$$\frac{\dot{a}_D^2}{a_D^2} = \frac{8\pi G}{3}\langle\rho\rangle_D - \frac{1}{6}\left(\mathscr{Q} + \langle\mathscr{R}\rangle_D\right) \qquad 3\frac{\ddot{a}_D}{a_D} = -4\pi G\langle\rho\rangle_D + \mathscr{Q}$$

$$\mathscr{Q} = \frac{2}{3}\left\langle(\theta - \langle\theta\rangle_D)^2\right\rangle_D - \langle\sigma_{ij}\sigma^{ij}\rangle_D$$

if this is positive then
it looks like dark energy!

(θ expansion rate, σ shear, from expansion tensor Θ)

- $<\rho> \sim a^{-3}$
- looks like Friedmann eqs., but with extra contribution!

T. Buchert, 2008 Gen. Rel. Grav. 40 467

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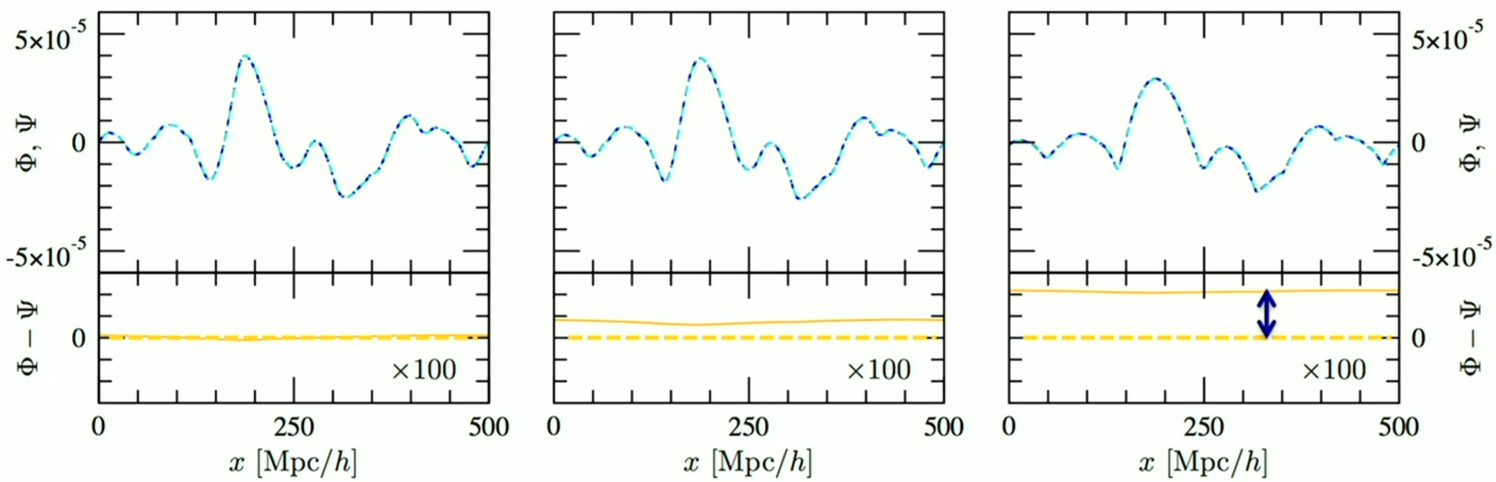
Deviation from FLRW background

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)dx^2$$

- absorb Ψ zero mode (constant in x) into time redefinition
- interpret Φ zero mode as correction to chosen background evolution a(t)
- can check if background evolves differently than in FLRW
  → not possible in Newtonian simulations (boundary term) nor in perturbation theory (divergent result)!

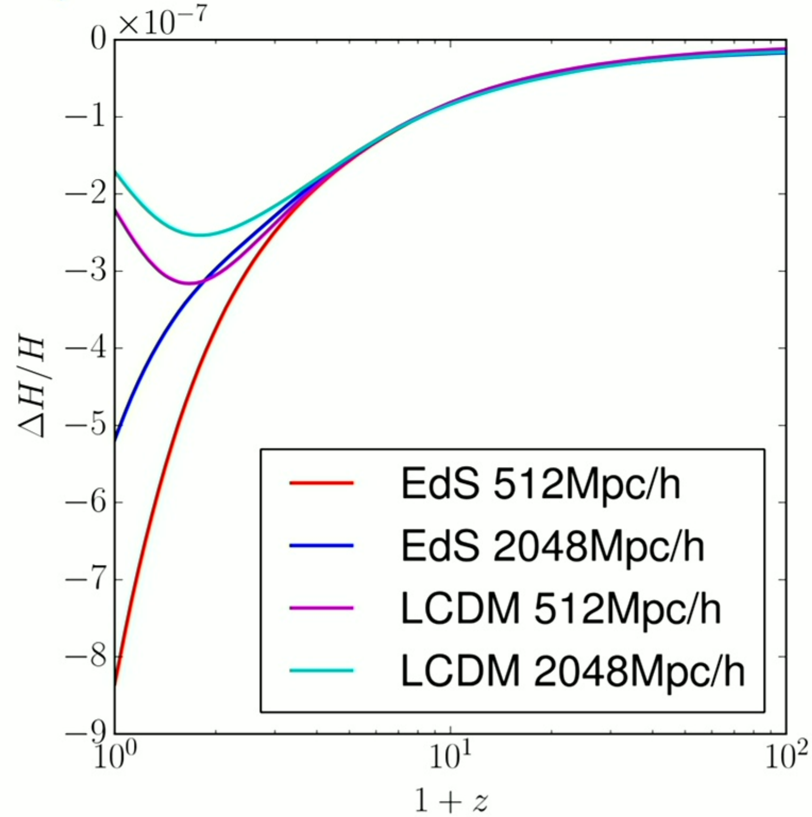


Backreaction on expansion rate:

Full box backreaction is very small, $\Delta H/H \sim 10^{-7}$ (slowing expansion down)

Onset of accelerated exp-ansion slows down back-reaction…

… and non-linear structure formation does not seem to make it stronger.



Adamek, Clarkson, Daverio, Durrer, MK, arXiv:1706.09309
Adamek, Clarkson, Coates, Durrer, MK, arXiv:1812.04336

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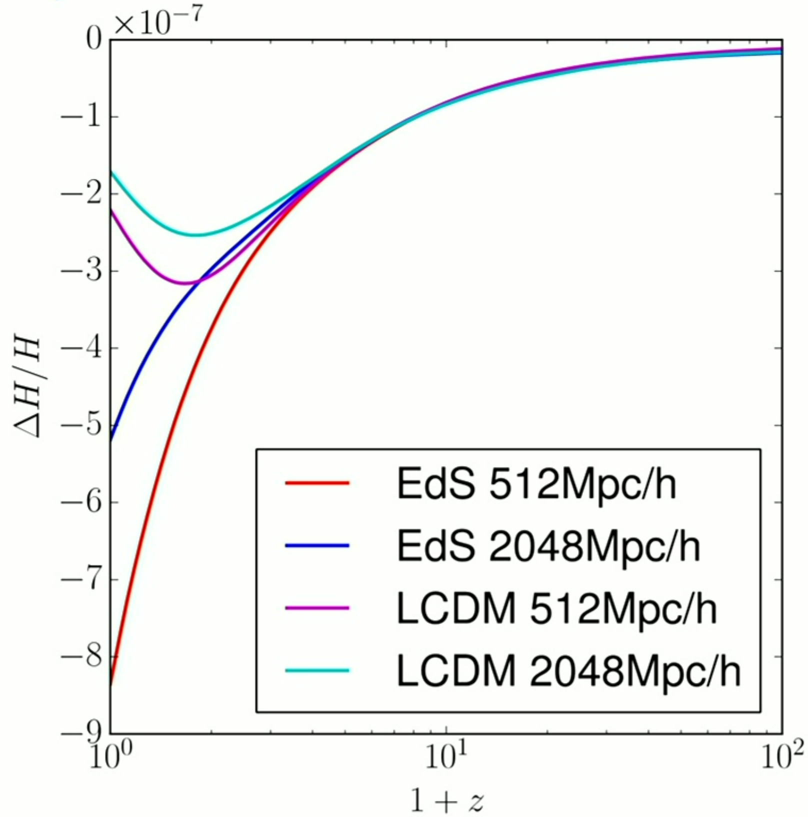


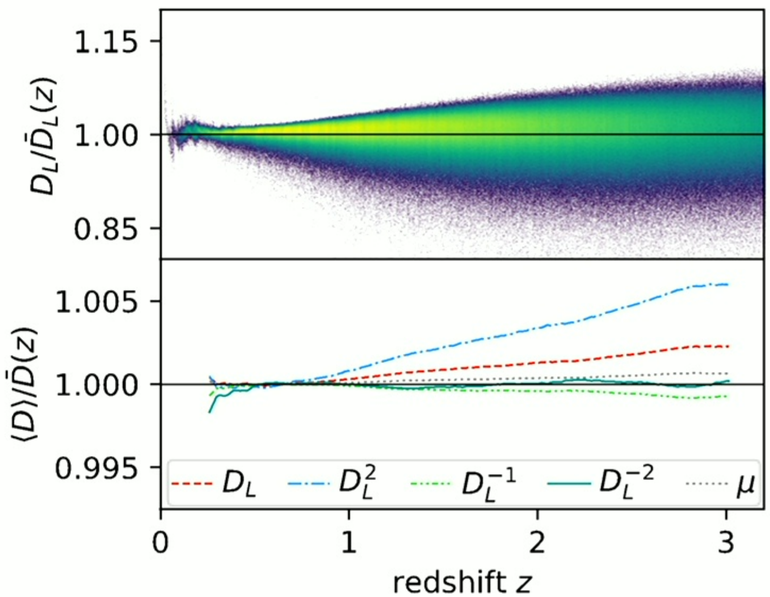
BUT:

- This is gauge / coordinate dependent; in comoving coordinates the effect is of order unity! (volume collapsing with structures)
- Only **observations** are relevant for us! → need ray-tracing

gevolution light-cone simulations

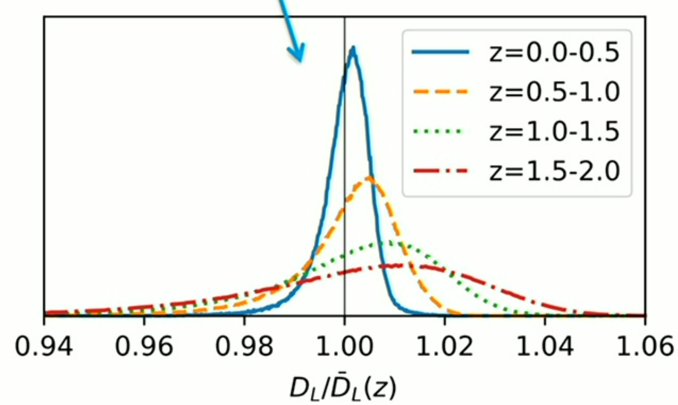
- relativistic N-body simulation from gevolution
- 4.5×10^{11} 'particles' in volume of $(2.4\ \mathrm{Gpc/h})^3$, $2.6 \times 10^9\ M_\odot/h$ per particle
- metric sampled on Cartesian 7680^3 grid [resolution 312.5 kpc/h]
- light-cone saved for circular 450 deg^2 beam to distance 4.5 Gpc/h
- ray tracing backwards from observer to halos (shooting method)

→ ***observables (raytracing available for any gevolution sims!)***



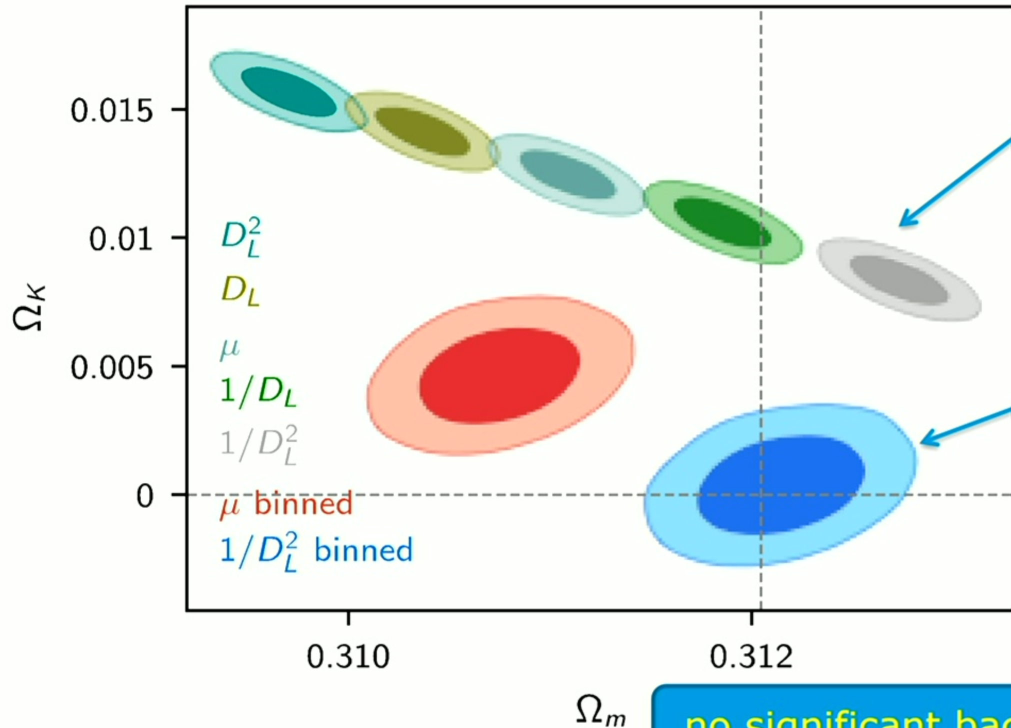
distances for 11M halos

pdf (mostly lensing)



How important is 'generalized backreaction' for supernovae?

Analysing a 'super-supernova' sample with ~500k `perfect' standard candles, assuming a standard Gaussian likelihood:



small shifts, mostly due to pdf not being Gaussian

shifts can be reduced/ removed by
- binning in redshift (gaussianizing pdf)
- using $1/D_L^2$

(of course better to use full info!)

no significant backreaction on distances

IV. Non-linear dark energy

- Upcoming data (e.g. **Euclid**) need sub-percent accurate model predictions also on **non-linear scales**.
- MG models are covariant, GR is easier than Newtonian! ☺
- Basic idea: expand the 'EFT of dark energy' action to higher order
  - non-linear behaviour in a general framework of theories
  - can be implemented in **gevolution** in a straightforward way (and then have access to all gevolution observables)
- Simplest case: k-essence → **k-evolution**

Modeling levels:

1. Boltzmann codes: everything (matter & DE) is linear.
2. 'gevolution': Matter fully non-linear, linear realization of DE.
3. Linear k-evolution: linear k-essence equations but sourced by non-linear matter.
4. Nonlinear k-evolution: second-order k-essence equations.

k-essence equations of motion

Hassani, Adamek, MK, Vernizzi, arXiv:1910.01104

π : k-essence field
ζ : ~ π' (time derivative)

$$\zeta \equiv \pi' + \mathcal{H}\pi - \Psi$$

Non-linear terms

$$\zeta' - 3w\mathcal{H}\zeta + 3c_s^2\left(\mathcal{H}^2\pi - \mathcal{H}\Psi - \mathcal{H}'\pi - \Phi'\right) - c_s^2\nabla^2\pi$$
$$+ \vec{\nabla}\left[2(c_s^2-1)\zeta + c_s^2\Phi - \Psi\right]\cdot\vec{\nabla}\pi + \left[(c_s^2-1)\zeta + c_s^2\Phi - c_s^2\Psi\right]\nabla^2\pi$$
$$+ \frac{\mathcal{H}}{2}\left[(2+3w+c_s^2)(\vec{\nabla}\pi)^2 + 6c_s^2(1+w)\pi\nabla^2\pi\right] - \frac{c_s^2-1}{2}\nabla_i\left(\nabla_i\pi(\vec{\nabla}\pi)^2\right) = 0$$

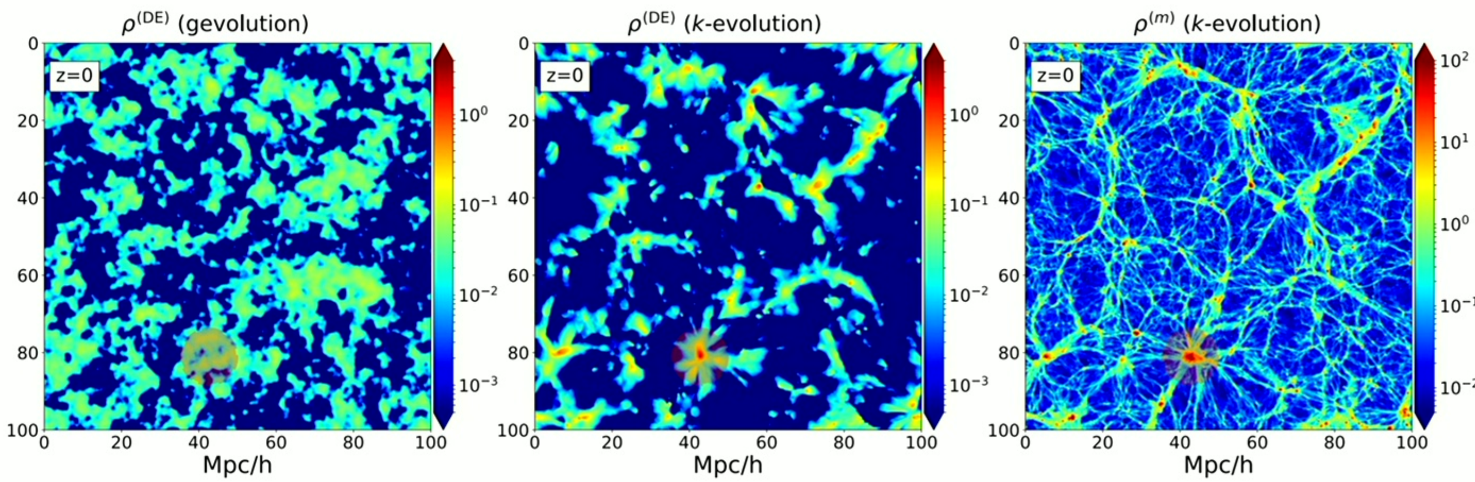
$$T^0_0 = -\rho + \frac{\rho+p}{c_s^2}\left[3c_s^2\mathcal{H}\pi - \zeta - \frac{2c_s^2-1}{2}(\vec{\nabla}\pi)^2\right] + T^0_0(\text{particles})$$

$$T^i_j = p\delta^i_j - (\rho+p)\left[3w\mathcal{H}\pi - \zeta + \frac{1}{2}(\vec{\nabla}\pi)^2\right]\delta^i_j + (\rho+p)\delta^{ik}\partial_k\pi\partial_j\pi + T^0_i(\text{particles})$$

$$T^0_i = -(\rho+p)\left[1 - \frac{1}{c_s^2}\left(3c_s^2(1+w)\mathcal{H}\pi - \zeta + c_s^2\Psi\right) + \frac{c_s^2-1}{2c_s^2}(\vec{\nabla}\pi)^2\right]\partial_i\pi \; + T^j_i(\text{particles})$$

Linear k-essence evolution

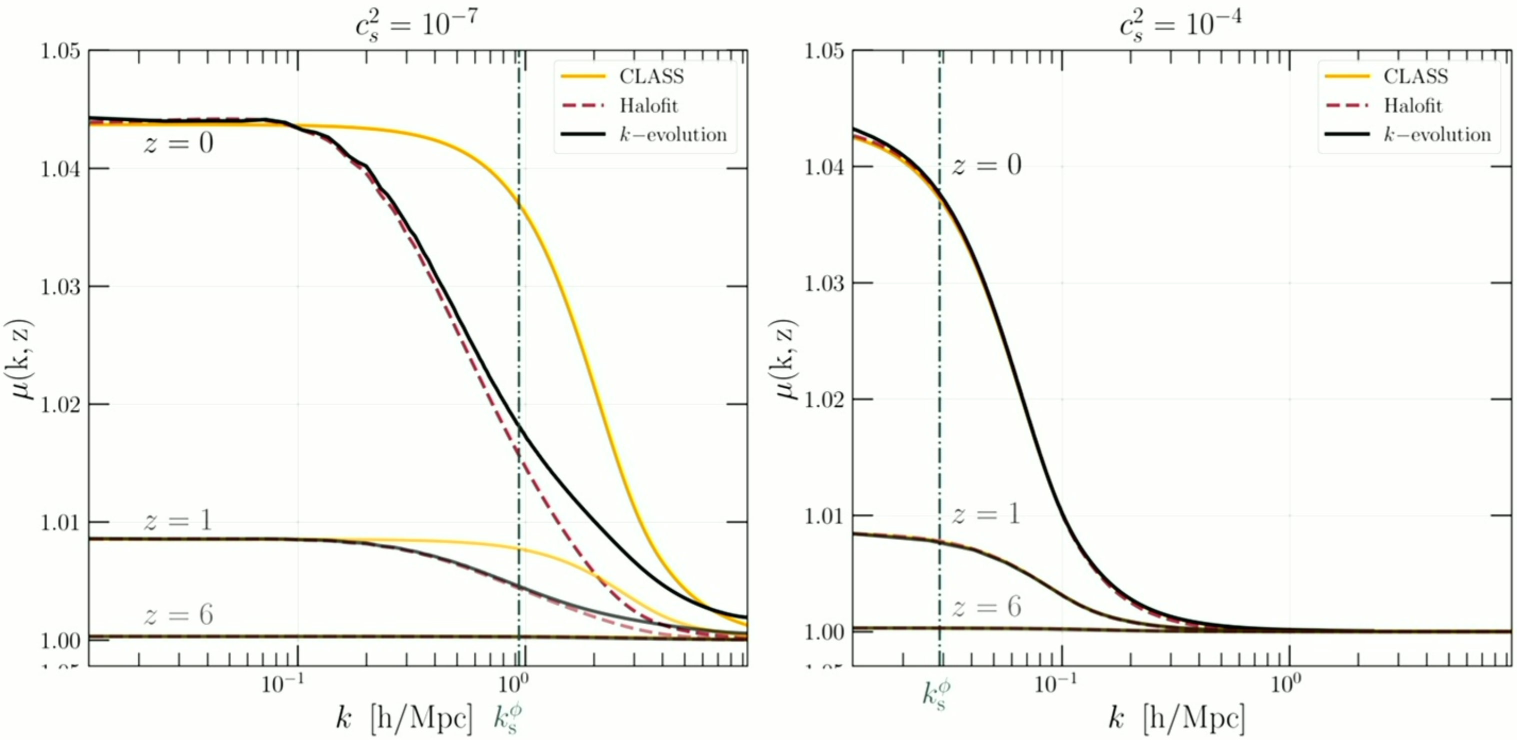
- Dark matter density becomes highly non-linear.
- Dark energy density contrast suppressed inside sound horizon.
- For ‘high’ sound speed, $c_s^2 > 10^{-4}$, DE non-linear clustering is small.
- For smaller sound speed, DE is dragged into DM halos even if we turn off non-linear DE contributions (here $c_s^2 = 10^{-7}$).
- DE clustering changes particularly the gravitational potential.



Quantifying DE clustering

Contribution of DE clustering to ϕ : $-k^2\Phi = 4\pi G a^2 \mu(k,z) \sum_{X\backslash \mathrm{DE}} \bar{\rho}_X \Delta_X$

(For k-essence ϕ=ψ, and the higher order contributions are negligible)

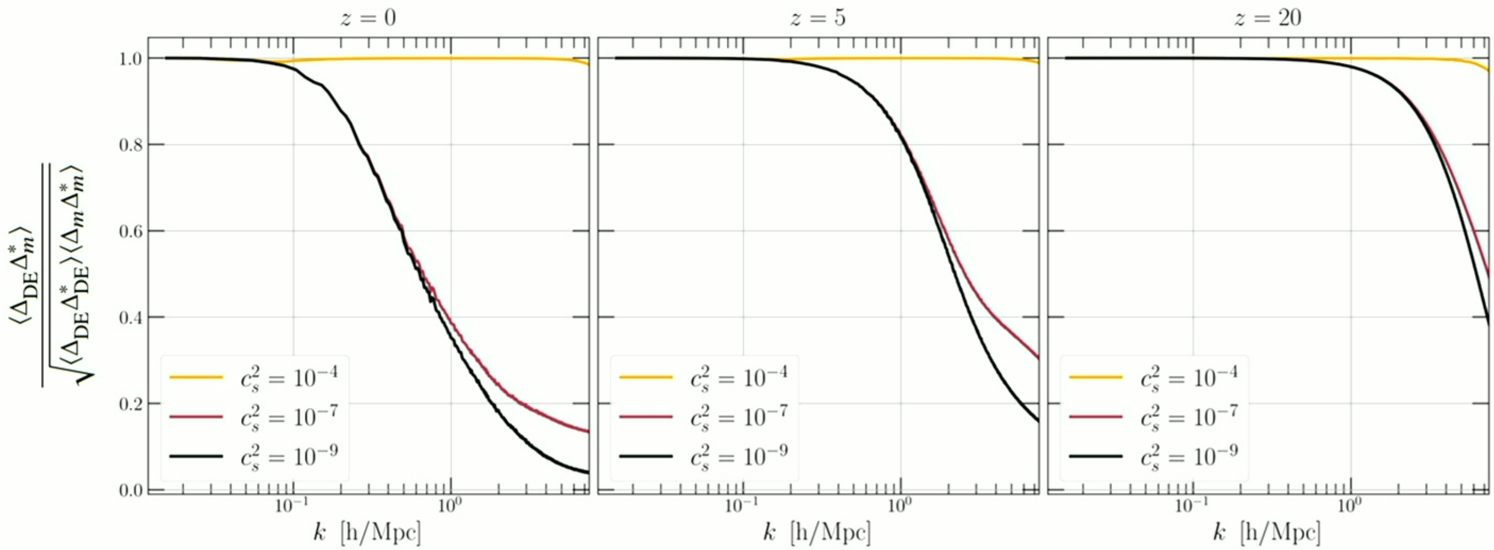


(Hassani, L'Huillier, Shafieloo, MK, Adamek, arXiv:1910.01105 ; Nouri-Zonoz, Hassani, MK, arXiv:2405.10424)

An aside on 2pt functions

For <ϕϕ> we need <μμ> which involves DE-m cross terms:

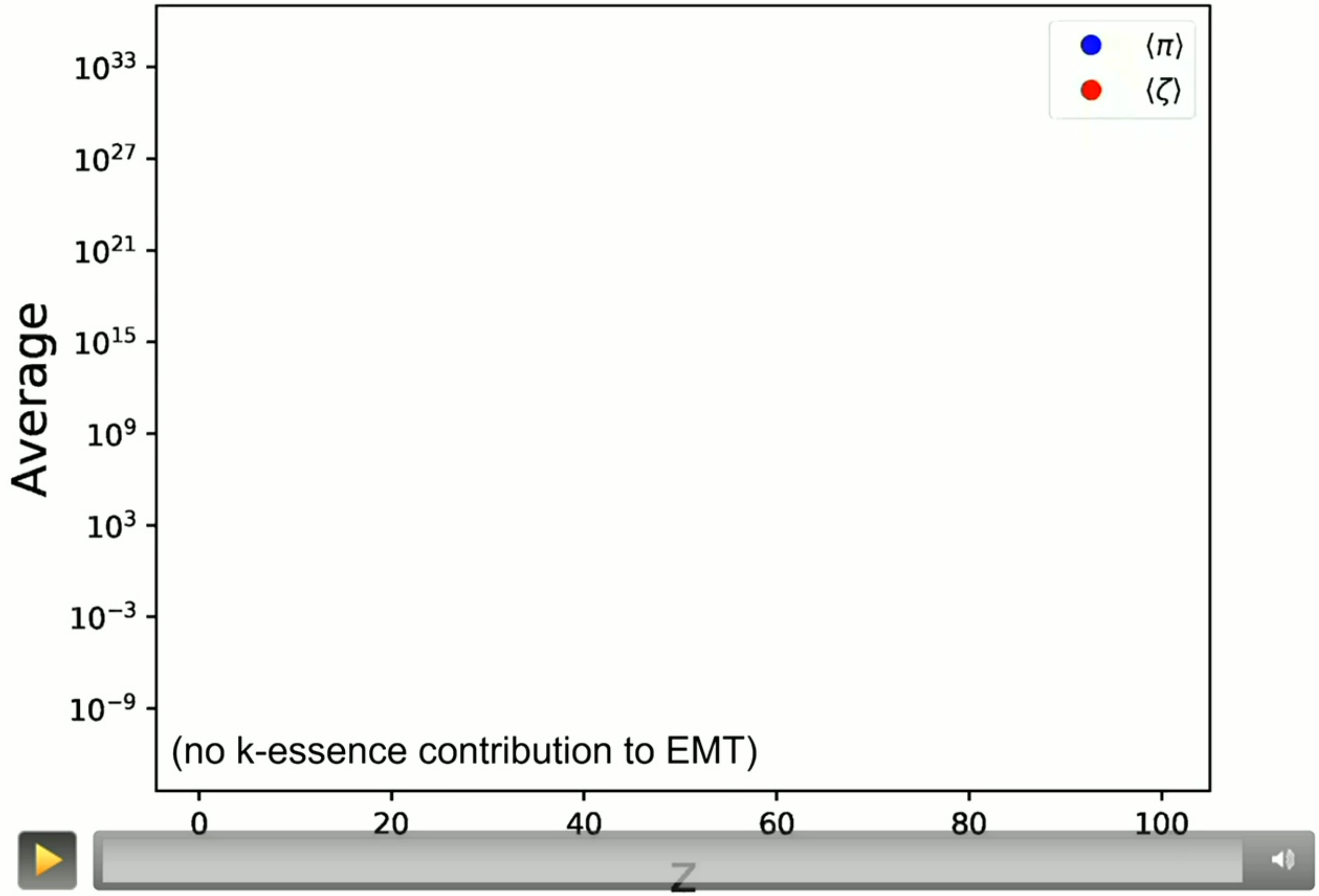
$$\mu^2(k,z) = \langle \mu(k,z)\mu(k,z)^* \rangle$$
$$= \frac{\bar{\rho}_{\rm DE}^2 \langle \Delta_{\rm DE}\Delta_{\rm DE}^* \rangle + \bar{\rho}_m^2 \langle \Delta_m \Delta_m^* \rangle + 2\bar{\rho}_{\rm DE}\bar{\rho}_m \langle \Delta_{\rm DE}\Delta_m^* \rangle}{\bar{\rho}_m^2 \langle \Delta_m \Delta_m^* \rangle}$$



This changes the μ needed to turn $P_m(k,z)$ into $P_\phi(k,z)$, and an accurate determination for low c_s^2 needs the slow sims including the DE field → build emulator → k-eμlator

Non-linear k-essence evolution

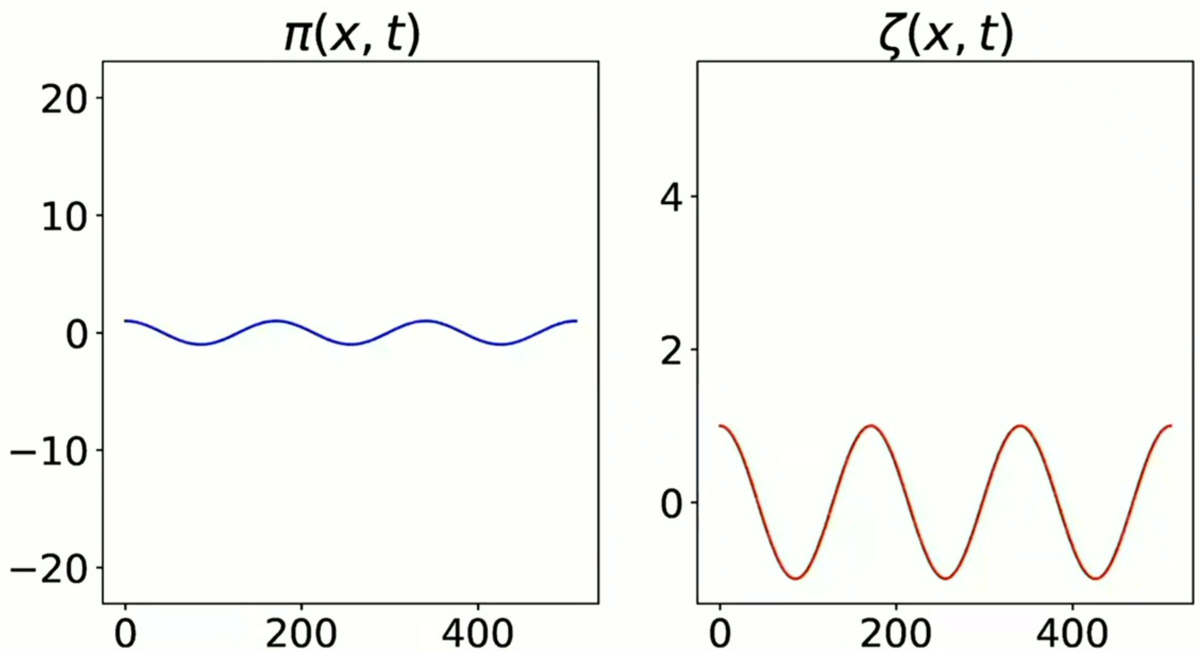
Hassani, Adamek, MK, Shi, Wittwer, arXiv:2204.13098



What just happened?!

Looking closer, there is a localized blow-up at the deepest minimum of the gravitational potential. The most important term seem to be $(\nabla\pi)^2$.

→ We look at $\partial_\tau{}^2\pi = (\partial_x\pi)^2$, a simple non-linear PDE that has never really been studied! (→ $c_s{}^2 = 0$ limit, else $c_s{}^2\Delta\pi$ term in addition)



"Peter's argument"

$$\partial_\tau{}^2\pi = (\partial_x\pi)^2$$

- The extrema don't move (if also $\partial_x\partial_\tau\pi = 0$ at the initial time).
- Particular solution near extremum: $\pi(\tau, r) = \kappa(\tau)r^2$ ($\to 2\kappa(\tau)$ is curvature)
- ODE for $\kappa(\tau)$: $\boldsymbol{\partial_\tau{}^2\kappa(\tau) = 4[\kappa(\tau)]^2}$ with given initial conditions
  - Like Newton's second law for potential $V(x) = -\frac{4}{3}x^3$
  - "Particle" (→ field curvature) will roll to infinity in finite time:
- Multiply both sides by $\partial_\tau\kappa = \kappa'$:

$$\frac{1}{2}\frac{d(\kappa'(\tau)^2)}{d\tau} = \frac{4}{3}\frac{d(\kappa(\tau)^3)}{d\tau}$$

- Integrate in time

$$\kappa'(\tau)^2 = \kappa'(0)^2 + \frac{8}{3}\kappa(\tau)^3 - \frac{8}{3}\kappa(0)^3$$

- Integrate again

$$\int_{\kappa(0)}^{\kappa(\tau)} \frac{d\kappa}{\sqrt{\kappa'(0)^2 + \frac{8}{3}\kappa^3 - \frac{8}{3}\kappa(0)^3}} = \int_0^\tau d\tau' = \tau$$

- Change $\kappa \to$ s: $s^3 = \frac{8\kappa^3}{3C}$, $C = \kappa'(0)^2 - \frac{8}{3}\kappa(0)^3$ →finite bound on blow-up time:

$$\tau_b = \left(\frac{3}{8}\right)^{\frac{1}{3}}\left(\frac{1}{C}\right)^{\frac{1}{6}}\int_{s(\kappa_0)}^{\infty}\frac{ds}{\sqrt{1+s^3}}$$

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- Particular solution near extremum: $\pi(\tau, r) = \kappa(\tau)r^2$ ($\rightarrow 2\kappa(\tau)$ is curvature)
- ODE for $\kappa(\tau)$: $\boldsymbol{\partial_\tau{}^2\kappa(\tau) = 4[\kappa(\tau)]^2}$ with given initial conditions
  - Like Newton's second law for potential $V(x) = -\frac{4}{3}x^3$
  - "Particle" (→ field curvature) will roll to infinity in finite time:
- Multiply both sides by $\partial_\tau\kappa = \kappa'$:

$$\frac{1}{2}\frac{d(\kappa'(\tau)^2)}{d\tau} = \frac{4}{3}\frac{d(\kappa(\tau)^3)}{d\tau}$$

- Integrate in time

$$\kappa'(\tau)^2 = \kappa'(0)^2 + \frac{8}{3}\kappa(\tau)^3 - \frac{8}{3}\kappa(0)^3$$

- Integrate again

$$\int_{\kappa(0)}^{\kappa(\tau)} \frac{d\kappa}{\sqrt{\kappa'(0)^2 + \frac{8}{3}\kappa^3 - \frac{8}{3}\kappa(0)^3}} = \int_0^\tau d\tau' = \tau$$

- Change $\kappa \rightarrow$ s: $s^3 = \frac{8\kappa^3}{3C}$, $C = \kappa'(0)^2 - \frac{8}{3}\kappa(0)^3$ →finite bound on blow-up time:

$$\tau_b = \left(\frac{3}{8}\right)^{\frac{1}{3}}\left(\frac{1}{C}\right)^{\frac{1}{6}}\int_{s(\kappa_0)}^{\infty}\frac{ds}{\sqrt{1+s^3}}$$

Near blow-up: $\kappa(\tau) = \dfrac{3}{2(\tau - \tau_b)^2}$

"Peter's argument"

$$\partial_\tau{}^2\pi = (\partial_x\pi)^2$$

- The extrema don't move (if also $\partial_x\partial_\tau\pi = 0$ at the initial time).
- Particular solution near extremum: $\pi(\tau, r) = \kappa(\tau)r^2$ ($\rightarrow 2\kappa(\tau)$ is curvature)
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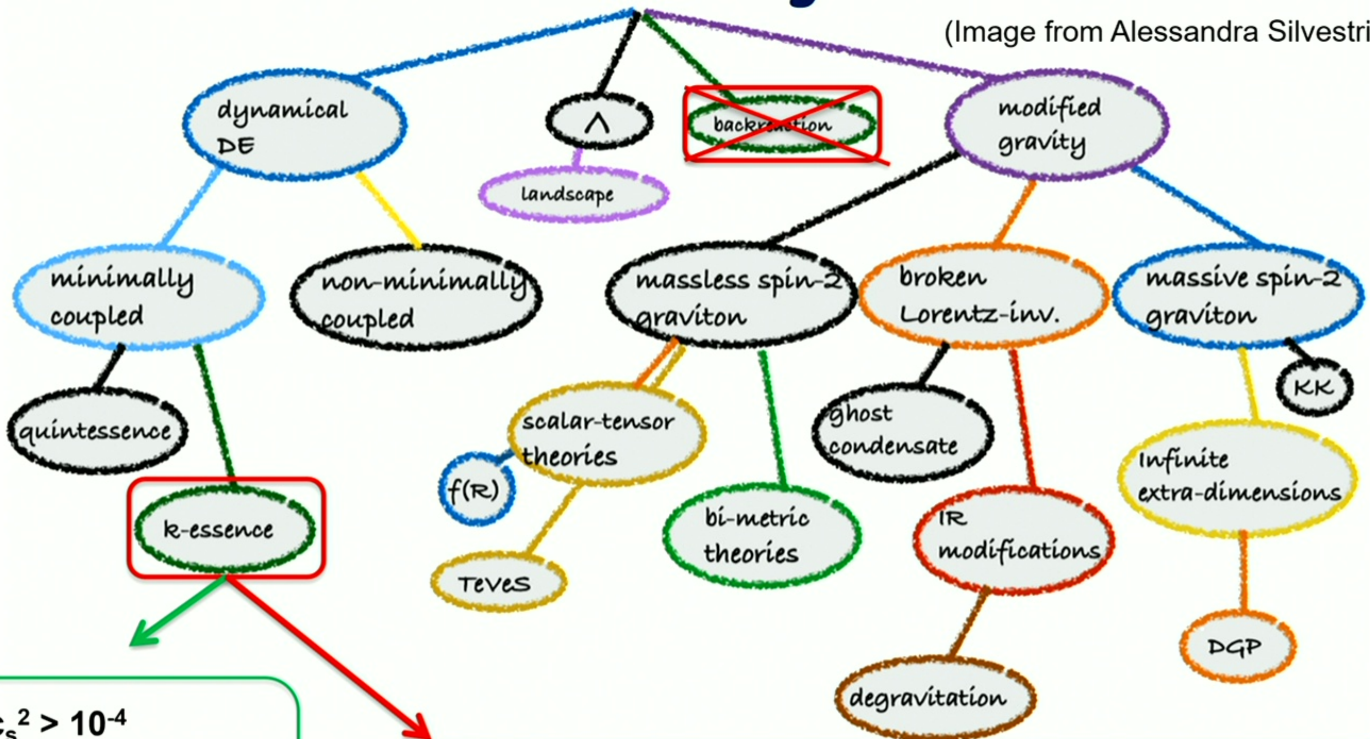
Near blow-up: $\kappa(\tau) = \dfrac{3}{2(\tau - \tau_b)^2}$

Summary

(Image from Alessandra Silvestri)



$c_s^2 > 10^{-4}$
k-essence 'linear', clustering on large scales only

$c_s^2 < 10^{-5}$: k-essence clusters non-linearly, need simulations
→ Localized blow-up [generic → affects all EFT?]
→ Form high-z SMBH? Or EFT breakdown? Large effects!

https://github.com/gevolution-code , include DE clustering with k-evolution & k-eμlator


Thank you!
esa
euclid