

Title: Conference Talk

Speakers:

Collection: 50 Years of Horndeski Gravity: Exploring Modified Gravity

Date: July 17, 2024 - 9:45 AM

URL: <https://pirsa.org/24070041>

# Beyond Horndeski theories

David Langlois  
(APC, Paris)

# Introduction

- **Horndeski theories:** most general family of scalar-tensor theories with a **single scalar** degree of freedom and at most **second-order** equations of motion.
- **Going beyond Horndeski:** most general family of scalar-tensor theories with a **single scalar** degree of freedom.  
Higher-order equations of motion are possible if the **Lagrangian is degenerate**.
- In this talk:
  1. DHOST (Degenerate Higher-Order Scalar-Tensor) theories
  2. Cosmological aspects
  3. Black hole perturbations

## Degenerate theories

- In general, higher-order theories with a Lagrangian of the form  $L(\ddot{q}, \dot{q}, q)$  contain an **extra dof**.
- Degenerate theories**: EOM higher order, **no extra dof**

$$L = \frac{1}{2}a\ddot{\phi}^2 + b\ddot{\phi}\dot{q} + \frac{1}{2}c\dot{q}^2 + \frac{1}{2}\dot{\phi}^2 - V(\phi, q) \quad \begin{cases} ac - b^2 \neq 0 : \mathbf{3 \text{ dof}} \\ ac - b^2 = 0 : \mathbf{2 \text{ dof}} \end{cases}$$

$$\longrightarrow L_{\text{deg}} = \frac{1}{2}c\left(\dot{q} + \frac{b}{c}\ddot{\phi}\right)^2 + \frac{1}{2}\dot{\phi}^2 - V(\phi, q) \quad [x \equiv q + \frac{b}{c}\dot{\phi} \text{ and } \phi]$$

- DHOST** (Degenerate Higher-Order Scalar-Tensor)

$$\begin{aligned} \phi(t) &\longrightarrow \phi(x^\mu) \\ q(t) &\longrightarrow g_{\mu\nu} \end{aligned}$$

DL & Noui '15

# DHOST theories

DL & Noui '15

Ben Achour, Crisostomi, Koyama, DL, Noui & Tasinato '16

- Action

$$S[g, \phi] = \int d^4x \sqrt{-g} \left[ F_{(2)}(X, \phi) {}^{(4)}R + P(X, \phi) + Q(X, \phi) \square \phi + \sum_{I=1}^5 A_I(X, \phi) L_I^{(2)} + F_{(3)}(X, \phi) G_{\mu\nu} \phi^{\mu\nu} + \sum_{I=1}^{10} B_I(X, \phi) L_I^{(3)} \right]$$

$$X \equiv \nabla_\mu \phi \nabla^\mu \phi$$

$$\phi_\mu \equiv \nabla_\mu \phi$$

$$\phi_{\mu\nu} \equiv \nabla_\nu \nabla_\mu \phi$$

with 5 **quadratic** elementary Lagrangians:

$$L_1^{(2)} = \phi_{\mu\nu} \phi^{\mu\nu}, \quad L_2^{(2)} = (\square \phi)^2, \quad L_3^{(2)} = (\square \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu$$

$$L_4^{(2)} = \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu, \quad L_5^{(2)} = (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2$$

and 10 **cubic** ones:

$$L_1^{(3)} = (\square \phi)^3, \quad L_2^{(3)} = (\square \phi) \phi_{\mu\nu} \phi^{\mu\nu}, \quad \dots, \quad L_{10}^{(3)} = (\phi_\mu \phi^{\mu\nu} \phi_\nu)^3$$

# DHOST theories

- Action [DL & Noui '15 ; Ben Achour, Crisostomi, Koyama, DL, Noui & Tasinato '16]

$$S[g, \phi] = \int d^4x \sqrt{-g} \left[ F_{(2)}(X, \phi) {}^{(4)}R + P(X, \phi) + Q(X, \phi) \square \phi + \sum_{I=1}^5 A_I(X, \phi) L_I^{(2)} \right. \\ \left. + F_{(3)}(X, \phi) G_{\mu\nu} \phi^{\mu\nu} + \sum_{I=1}^{10} B_I(X, \phi) L_I^{(3)} \right]$$

- Degeneracy conditions (3 for quadratic DHOST)

 **Full classification**

- Particular subsets:

- **Horndeski:**

$$F_{(2)} = G_4, \quad A_1 = -A_2 = 2G_{4,X}, \quad A_3 = A_4 = A_5 = 0$$

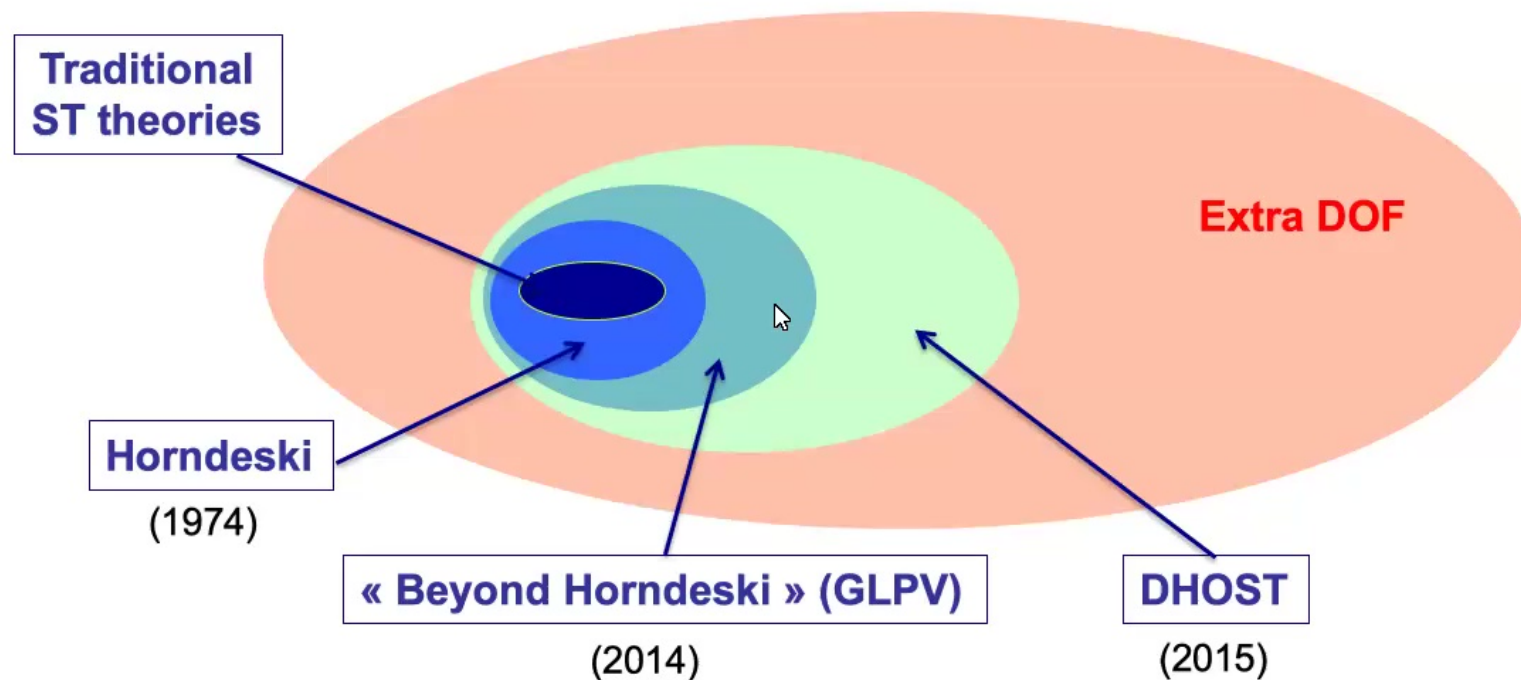
$$F_{(3)} = G_5, \quad 3B_1 = -B_2 = \frac{3}{2}B_3 = G_{5,X}, \quad B_I = 0 \quad (I = 4, \dots, 10)$$

- « Beyond Horndeski » (or GLPV) Gleyzes, DL, Piazza & Vernizzi '14



# Scalar-tensor lanscape

- Traditional theories:  $\mathcal{L}(\nabla_\lambda\phi, \phi)$
- Generalized theories:  $\mathcal{L}(\nabla_\mu\nabla_\nu\phi, \nabla_\lambda\phi, \phi)$   
**DHOST**: most general family with a **single scalar DOF**



# Disformal transformations

[Zumalacarregui & Garcia-Bellido '13; Ben Achour, DL & Noui '15]

- Transformation

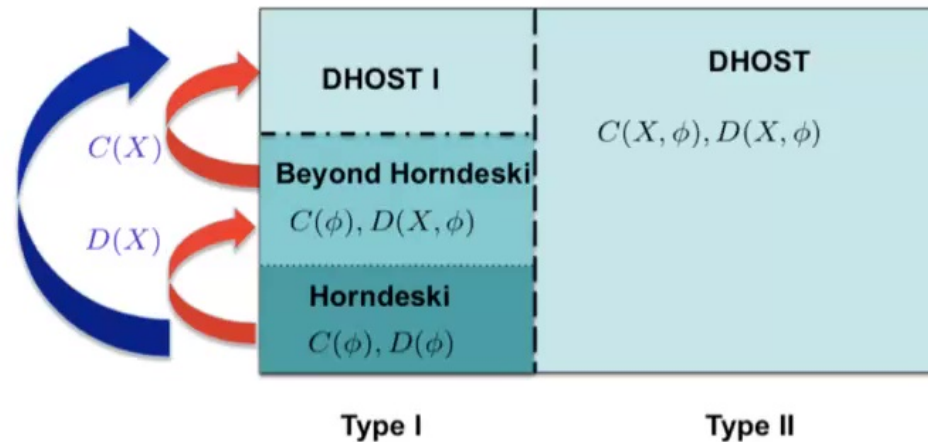
$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X, \phi) g_{\mu\nu} + D(X, \phi) \partial_\mu \phi \partial_\nu \phi$$

- From  $\tilde{S}[\phi, \tilde{g}_{\mu\nu}]$ , one gets the new action

[Bekenstein '92]

$$S[\phi, g_{\mu\nu}] \equiv \tilde{S}[\phi, \tilde{g}_{\mu\nu} = C g_{\mu\nu} + D \phi_\mu \phi_\nu]$$

- DHOST families are **closed** under these transformations



- When **standard fields** are (minimally) included, two disformally related theories are **physically inequivalent** !

$$S[g_{\mu\nu}, \phi] + S_m[\Psi_m, g_{\mu\nu}] \neq \tilde{S}[\tilde{g}_{\mu\nu}, \phi] + S_m[\Psi_m, \tilde{g}_{\mu\nu}]$$



# Dark energy & GW170817

- Constraint on the **speed of GW**:  $|c_{\text{gw}} - c|/c \lesssim 10^{-15}$

$$c_{\text{gw}} = c \text{ implies } A_1 = A_2 = 0 \quad (\text{and no cubic term})$$

- To avoid **GW decay** into dark energy

Creminelli, Lewandowski,  
Tambalo & Vernizzi '18

$$A_3 = 0$$

$$L_{c_g=1, \text{no decay}}^{\text{DHOST}} = F(X, \phi) R + P(X, \phi) + Q(X, \phi) \square\phi + 6 \frac{F_X^2}{F} \phi^\mu \phi_{\mu\nu} \phi^{\nu\sigma} \phi_\sigma$$

- However, constraints from GW170817 valid for  $f \sim 1 - 100$  Hz

**Not directly relevant for cosmological scales**

[ Moreover, at the limit of the regime of validity De Rham & Melville '18]

# Homogeneous cosmology of DHOST

Crisostomi, Koyama, DL, Noui & Steer '18; Boumaza, DL, Noui '20

- Quadratic DHOST theories:

$$L = F(X, \phi) R + P(X, \phi) + Q(X, \phi) \square \phi + \sum_{I=1}^5 A_I(X, \phi) L_I^{(2)}$$

- **Background:**  $ds^2 = -N^2(t)dt^2 + a^2(t) \delta_{ij} dx^i dx^j$ ,  $\phi = \phi(t)$



- Homogeneous action

$$\bar{S} = \int dt N a^3 \left\{ -6(F - X A_1) \left[ \frac{\dot{a}}{N a} - \mathcal{V} \frac{\dot{\phi}}{N^2} \frac{d}{dt} \left( \frac{\dot{\phi}}{N} \right) \right]^2 - 3(Q + 2F_\phi) \frac{\dot{a} \dot{\phi}}{N^2 a} - Q \frac{1}{N} \frac{d}{dt} \left( \frac{\dot{\phi}}{N} \right) + P \right\},$$

$$\text{with } \mathcal{V} \equiv \frac{4F_X + X A_3 - 2A_1}{4(F - X A_1)} \quad X = -\frac{\dot{\phi}^2}{N^2}$$

- Trick: auxiliary scale factor such that

$$\frac{\dot{b}}{b} = \frac{\dot{a}}{a} - \mathcal{V} \frac{\dot{\phi}}{N} \frac{d}{dt} \left( \frac{\dot{\phi}}{N} \right) + \dots \quad a = e^{\lambda(X, \phi)} b, \quad \lambda_X = -\frac{1}{2} \mathcal{V}$$



# Illustrative example

- Simple toy model

Boumazza, DL, Noui '20

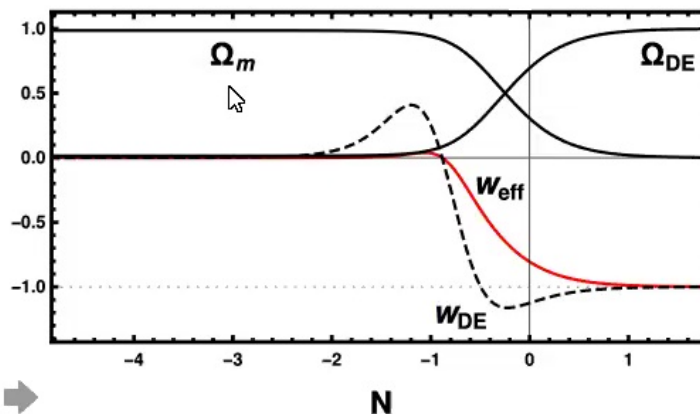
$$F = \frac{1}{2}, \quad P = \alpha X, \quad Q = 0, \quad A_1 = -A_2 = -\beta X, \quad \lambda = \frac{1}{2} s \beta X^2$$

Choice of  $\lambda$  determines  $A_3$ . Degeneracy imposes  $A_4$  and  $A_5$ .

- From matter era to dark energy era

$$3H^2 = \rho_m + \rho_{\text{DE}}$$

$$2\dot{H} + 3H^2 = P_m + P_{\text{DE}}$$



$$w_{\text{DE}} \equiv \frac{P_{\text{DE}}}{\rho_{\text{DE}}}$$

$$w_{\text{eff}} \equiv \frac{P_m + P_{\text{DE}}}{\rho_m + \rho_{\text{DE}}} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2}$$

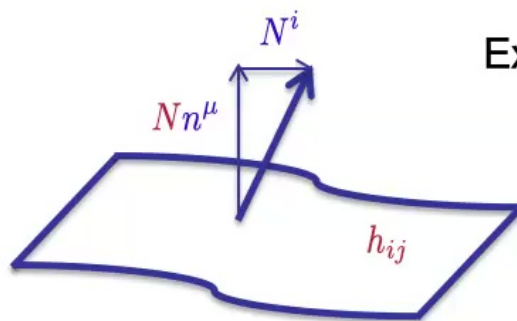
# Effective description of Dark Energy

[ Reviews: Gleyzes, DL & Vernizzi '14; DL '18]

- The scalar field defines a **preferred slicing**



- 3+1 decomposition** based on this uniform field slicing



Extrinsic curvature:  $K_{ij} = \frac{1}{2N} (\dot{h}_{ij} - D_i N_j - D_j N_i)$

Intrinsic curvature:  $R_{ij}$

$$X \equiv g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi = -\frac{\dot{\phi}^2(t)}{N^2}$$



# Effective description of Dark Energy

- **Action of the form:**  $S_g = \int d^4x N \sqrt{h} L(N, K_{ij}, R_{ij}; t)$

- **Homogeneous equations**

- FLRW metric:  $ds^2 = -\bar{N}^2(t) dt^2 + a^2(t) \delta_{ij} dx^i dx^j$

- Lagrangian

$$\bar{L}(a, \dot{a}, \bar{N}) \equiv L \left[ K_j^i = \frac{\dot{a}}{\bar{N}a} \delta_j^i, R_j^i = 0, N = \bar{N}(t) \right]$$

- **Perturbations:**  $\delta N \equiv N - \bar{N}$ ,  $\delta K_j^i \equiv K_j^i - H \delta_j^i$ ,  $\delta R_j^i \equiv R_j^i$

$$L(q_A) = \bar{L} + \frac{\partial L}{\partial q_A} \delta q^A + \frac{1}{2} \frac{\partial^2 L}{\partial q_A \partial q_B} \delta q_A \delta q_B + \dots$$

The quadratic action describes the dynamics of linear perturbations.



# DHOST cosmological perturbations

- **Quadratic action**

$$S_{\text{quad}} = \int d^3x dt a^3 \frac{M^2}{2} \left\{ \delta K_{ij} \delta K^{ij} - \delta K^2 + (1 + \alpha_T) \left( R \frac{\delta \sqrt{h}}{a^3} + \delta_2 R \right) \right. \\ \left. + H^2 \alpha_K \delta N^2 + 4H \alpha_B \delta K \delta N + (1 + \alpha_H) R \delta N + 4\beta_1 \delta K \delta \dot{N} + \beta_2 \delta \dot{N}^2 + \frac{\beta_3}{a^2} (\partial_i \delta N)^2 \right\}$$

	$\alpha_K$	$\alpha_B$	$\alpha_M$	$\alpha_T$	$\alpha_H$	$\beta_1$
K-essence	✓					
Kinetic braiding, DGP	✓	✓				
Brans-Dicke, f(R)	✓	✓	✓			
<b>Horndeski</b>	✓	✓	✓	✓		
<b>Beyond Horndeski</b>	✓	✓	✓	✓	✓	
<b>DHOST</b>	✓	✓	✓	✓	✓	✓

$$\alpha_M = \frac{1}{H} \frac{d}{dt} \ln M^2, \quad \beta_2 = -6\beta_1^2, \quad \beta_3 = -2\beta_1 [2(1 + \alpha_H) + \beta_1(1 + \alpha_T)]$$

# Physical degrees of freedom

- **Scalar** ( $\delta N$ ,  $N_i \equiv \partial_i \psi$ ,  $h_{ij} = a^2(t) e^{2\zeta} \delta_{ij}$ )

$$S_{\text{quad}}[\zeta] = \int d^3x dt a^3 \frac{M^2}{2} \left[ \mathcal{A}_\zeta \dot{\zeta}^2 - \mathcal{B}_\zeta \frac{(\partial_i \zeta)^2}{a^2} \right]$$

with  $\mathcal{A}_\zeta = \frac{1}{(1 + \alpha_B - \dot{\beta}_1/H)^2} \left[ \alpha_K + 6\alpha_B^2 - \frac{6}{a^3 H^2 M^2} \frac{d}{dt} (a^3 H M^2 \alpha_B \beta_1) \right]$

$$\mathcal{B}_\zeta = -2(1 + \alpha_T) + \frac{2}{aM^2} \frac{d}{dt} \left[ \frac{aM^2 (1 + \alpha_H + \beta_1(1 + \alpha_T))}{H(1 + \alpha_B) - \dot{\beta}_1} \right]$$

- **Tensor:**  $S_\gamma^{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[ \frac{M^2}{4} \dot{\gamma}_{ij}^2 - \frac{M^2}{4} (1 + \alpha_T) \frac{(\partial_k \gamma_{ij})^2}{a^2} \right]$
- **Stability** (neither ghost nor gradient instability)

$$M^2 > 0, \quad \mathcal{A}_\zeta > 0, \quad \mathcal{B}_\zeta > 0, \quad c_T^2 \equiv (1 + \alpha_T) > 0$$

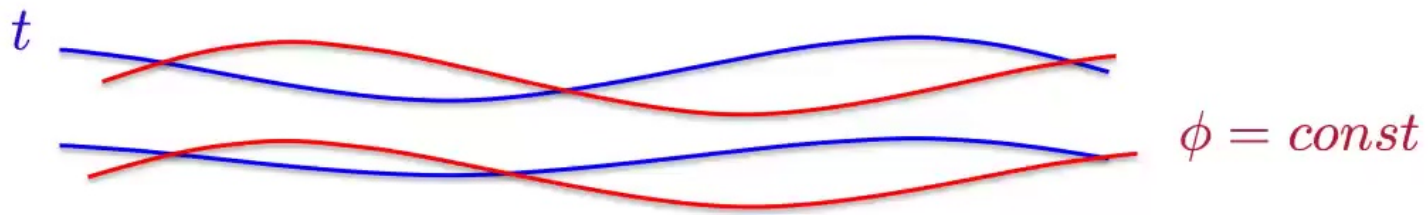


# Cosmological observations

- Use a traditional gauge, e.g. Newtonian gauge

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(t) (1 - 2\Psi) \delta_{ij} dx^i dx^j$$

- Description in an arbitrary slicing ?



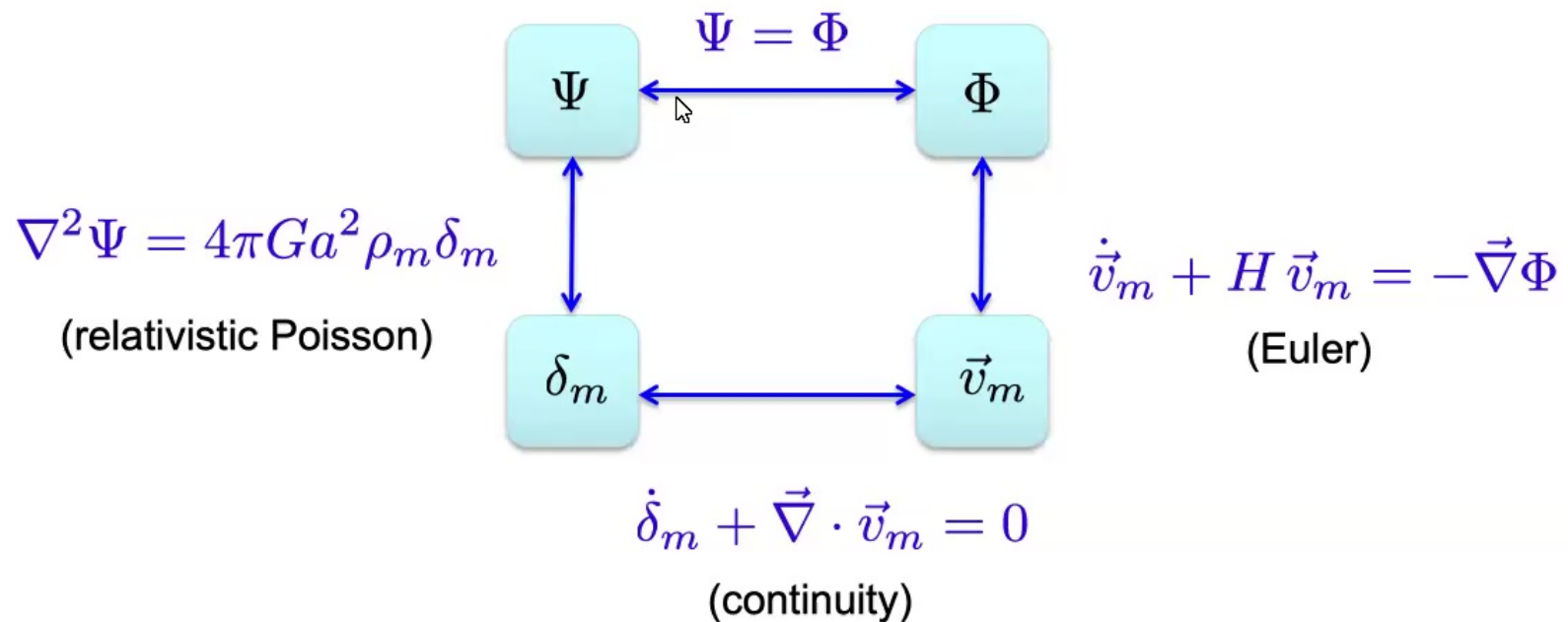
- Coordinate change  $t \rightarrow t + \pi(t, \vec{x})$
- Perturbations:  $\Phi, \Psi, \pi, \delta_m, \vec{v}_m$





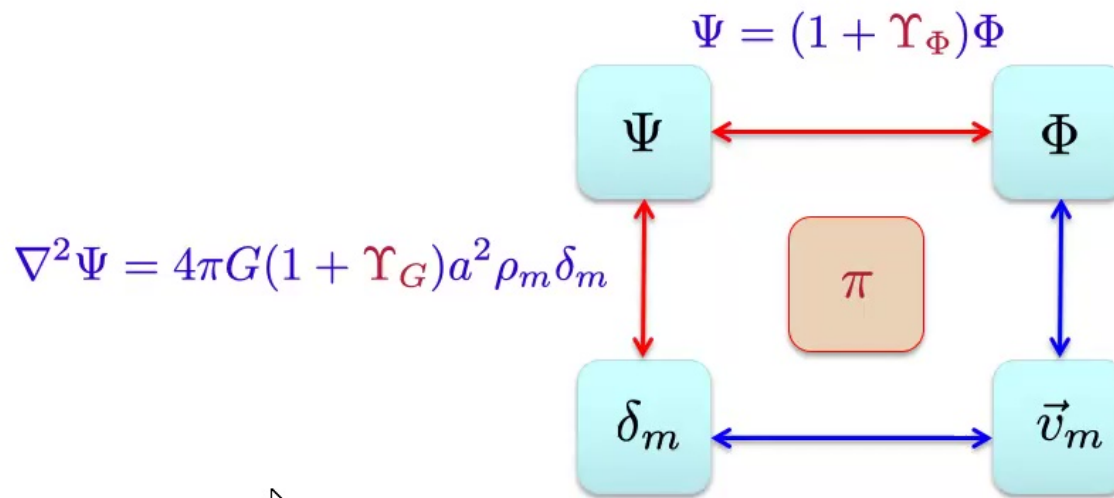
# Cosmological perturbations

- **Standard linear equations (in GR)**



# Cosmological perturbations

- **Modified equations (linear level)**



**Quasi-static approximation**  
(valid on scales  
 $kc_s \gg aH$ )

$$\Upsilon_G = \Upsilon_G(\alpha_i), \quad \Upsilon_\Phi = \Upsilon_\Phi(\alpha_i)$$

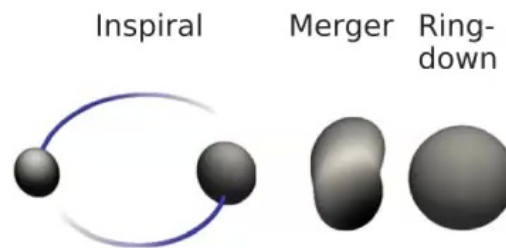
which can be confronted to observations (galaxy clustering, weak lensing...)



# Black hole perturbations

# Black hole perturbations

- **GW astronomy** provides a new window to **test GR**, in particular in the strong field regime.



- **Ringdown phase** of a BH merger is interesting for models of modified gravity, as it can be described by the formalism of BH perturbations.
- **Deviations** in the context of **DHOST** theories ?

# BH background solution

- Static spherically symmetric BH with a **nontrivial** scalar field

- Metric:  $ds^2 = -\mathcal{A}(r)dt^2 + \frac{dr^2}{\mathcal{B}(r)} + C(r)(d\theta^2 + \sin^2 \theta d\varphi^2)$

- Scalar field:  $\phi(t, r) = qt + \psi(r)$  [Babichev & Charmousis '13]

[  $q \neq 0$  possible in shift-symmetric theories ]

- **Examples:**

- « stealth » Schwarzschild:  $\mathcal{A} = \mathcal{B} = 1 - \frac{\mu}{r}$

- « BCL » [Babichev, Charmousis & Lehébel '17]  $\mathcal{A} = \mathcal{B} = 1 - \frac{\mu}{r} - \xi \frac{\mu^2}{2r^2}$

- « 4d Gauss-Bonnet » [Glavan & Lin '19, Lu & Pang '20]  $\mathcal{A} = \mathcal{B} = 1 - \frac{2\mu/r}{1 + \sqrt{1 + 4\alpha\mu/r^3}}$

- Several other solutions: cf Christos' talk

# Black hole perturbations

- In the frequency domain:  $f(t, r) = f(r) e^{-i\omega t}$

- Axial** (or odd) modes:  $h_0(r), h_1(r)$  [ Regge-Wheeler gauge ]

$$h_{\mu\nu} = \sum_{\ell, m} \begin{pmatrix} 0 & 0 & \frac{1}{\sin\theta} h_0^{\ell m} \partial_\varphi & -\sin\theta h_0^{\ell m} \partial_\theta \\ 0 & 0 & \frac{1}{\sin\theta} h_1^{\ell m} \partial_\varphi & -\sin\theta h_1^{\ell m} \partial_\theta \\ \text{sym} & \text{sym} & 0 & 0 \\ \text{sym} & \text{sym} & 0 & 0 \end{pmatrix} Y_{\ell m}(\theta, \varphi)$$

- Polar** (or even) modes:  $H_0, H_1, H_2, K$  (and  $\delta\phi$ )

$$h_{\mu\nu} = \sum_{\ell, m} \begin{pmatrix} A(r)H_0^{\ell m}(r) & H_1^{\ell m}(r) & 0 & 0 \\ H_1^{\ell m}(r) & A^{-1}(r)H_2^{\ell m}(r) & 0 & 0 \\ 0 & 0 & K^{\ell m}(r) r^2 & 0 \\ 0 & 0 & 0 & K^{\ell m}(r) r^2 \sin^2\theta \end{pmatrix} Y_{\ell m}(\theta, \varphi)$$



# Axial modes in GR

- The linearised metric eqs yield only 2 independent eqs

$$\frac{dY}{dr} = M(r) Y(r), \quad Y = \begin{pmatrix} h_0 \\ h_1/\omega \end{pmatrix}$$

or, in a **Schrodinger** form, [Regge & Wheeler '57]

$$\frac{d^2 \hat{Y}}{dr_*^2} + (\omega^2 - V(r)) \hat{Y} = 0$$

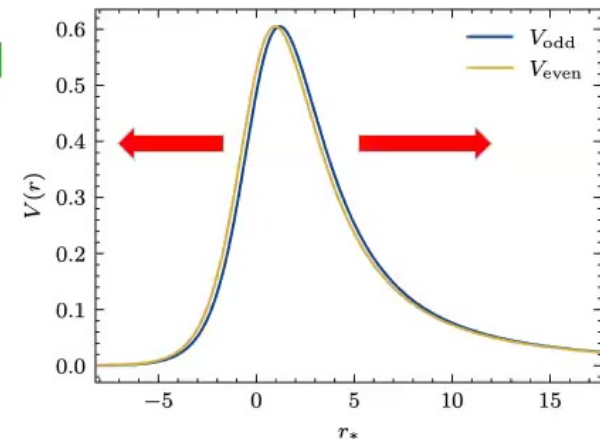
[  $r_*$  tortoise coordinate ]

- Asymptotically** ( $r_* \rightarrow -\infty, +\infty$ )

$$e^{-i\omega t} \hat{Y}(r) \approx a_+ e^{-i\omega(t-r_*)} + a_- e^{-i\omega(t+r_*)}$$

outgoing
ingoing

- Quasi-normal modes:**  $a_+^{\text{hor}} = 0$  and  $a_-^\infty = 0$



# Axial modes in DHOST

- The equations have a similar structure: [DL, Noui & Roussille '21,'22]

$$\frac{dY}{dr} = MY, \quad M \equiv \begin{pmatrix} 2/r + i\omega\Psi & -i\omega^2 + 2i\lambda\Phi/r^2 \\ -i\Gamma & \Delta + i\omega\Psi \end{pmatrix} \quad \lambda \equiv \frac{\ell(\ell+1)}{2} - 1$$

where  $\Psi, \Phi, \Gamma$  and  $\Delta$  depend on the Lagrangian's functions and on the background.

$$\mathcal{F} = \boxed{\mathcal{A}F_2} - (q^2 + \mathcal{A}X)A_1 - \frac{1}{2}\mathcal{A}\mathcal{B}\psi'X'F_{3X} - \frac{1}{2}\mathcal{B}\psi'(\mathcal{A}X)'B_2 - \frac{\mathcal{A}}{2\mathcal{B}}(\mathcal{B}\psi')^3X'B_6,$$

$$\mathcal{F}\Psi = q \left[ \psi'A_1 + \frac{1}{2}(\mathcal{B}\psi'^2)'F_{3X} + \frac{1}{2}\frac{(\mathcal{A}X)'}{\mathcal{A}}B_2 + \frac{1}{4}(\mathcal{B}^2\psi'^4)'B_6 \right],$$

$$\frac{\mathcal{F}}{\Phi} = \boxed{F_2} - XA_1 - \frac{1}{2}\mathcal{B}\psi'X'F_{3X} - \frac{1}{2}\mathcal{B}\psi'\frac{(CX)'}{C}B_2 - \frac{1}{2}\mathcal{B}\psi'XX'B_6,$$

$$\Gamma = \Psi^2 + \frac{1}{2\mathcal{A}\mathcal{B}\mathcal{F}} \left( 2q^2A_1 + \boxed{2\mathcal{A}F_2} + \mathcal{A}\mathcal{B}\psi'X'F_{3X} + q^2\frac{(\mathcal{A}X)'}{\mathcal{A}\psi'}B_2 + q^2\mathcal{B}\psi'X'B_6 \right),$$

$$\Delta = \boxed{-\frac{\mathcal{F}'}{\mathcal{F}} - \frac{\mathcal{B}'}{2\mathcal{B}} + \frac{\mathcal{A}'}{2\mathcal{A}}}$$

in GR



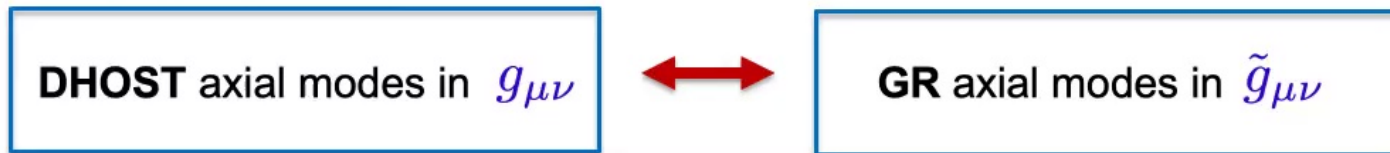
# Axial modes in DHOST

- The equations have a similar structure: [ DL, Noui & Roussille '21,'22 ]

$$\frac{dY}{dr} = MY, \quad M \equiv \begin{pmatrix} 2/r + i\omega\Psi & -i\omega^2 + 2i\lambda\Phi/r^2 \\ -i\Gamma & \Delta + i\omega\Psi \end{pmatrix} \quad \lambda \equiv \frac{\ell(\ell+1)}{2} - 1$$

where  $\Psi, \Phi, \Gamma$  and  $\Delta$  depend on the Lagrangian's functions and on the background.

- Correspondence**



with the **effective metric**

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = |\mathcal{F}| \sqrt{\frac{\Gamma\mathcal{B}}{\mathcal{A}}} \left( -\Phi(dt - \Psi dr)^2 + \Gamma\Phi dr^2 + \mathcal{C} d\Omega^2 \right)$$



# Polar modes

- The linearised metric equations yield
  - 2 independent equations in GR (1 dof)
  - **4 independent equations** in DHOST theories (2 dof)
- In **GR**: 2-dimensional system  $Y' = MY$ , which can be written in a Schroedinger form. [Zerilli '70]
- In **DHOST**, the system  $Y' = MY$  is now 4-dimensional, with
$$Y = {}^T(K \ \delta\phi \ H_1 \ H_0)$$
- It is convenient to do an **asymptotic analysis** of the first-order system.



# Asymptotics of a differential system

DL, Noui & Roussille '21

- Instead of a Schroedinger-like approach, one can use directly the initial first-order equations of motion and their asymptotic limit:

$$\frac{dY}{dz} = M(z) Y, \quad M(z) = M_r z^r + M_{r-1} z^{r-1} + \dots \quad (z \rightarrow \infty)$$

- The generic solution is of the form

$$Y(z) = e^{\Upsilon(z)} z^{\Delta} \mathbf{F}(z) Y_0, \quad (z \rightarrow \infty)$$

- There exists a well-defined algorithm to determine the diagonal matrices  $\Upsilon(z)$  and  $\Delta$ . [Balsler '99]

**Idea:** diagonalise, order by order, the matrix  $M$ , with  $Y(z) = P(z) \tilde{Y}(z)$

$$\frac{d\tilde{Y}}{dz} = \tilde{M}(z) \tilde{Y}, \quad \tilde{M}(z) \equiv P^{-1} M P - P^{-1} \frac{dP}{dz}$$



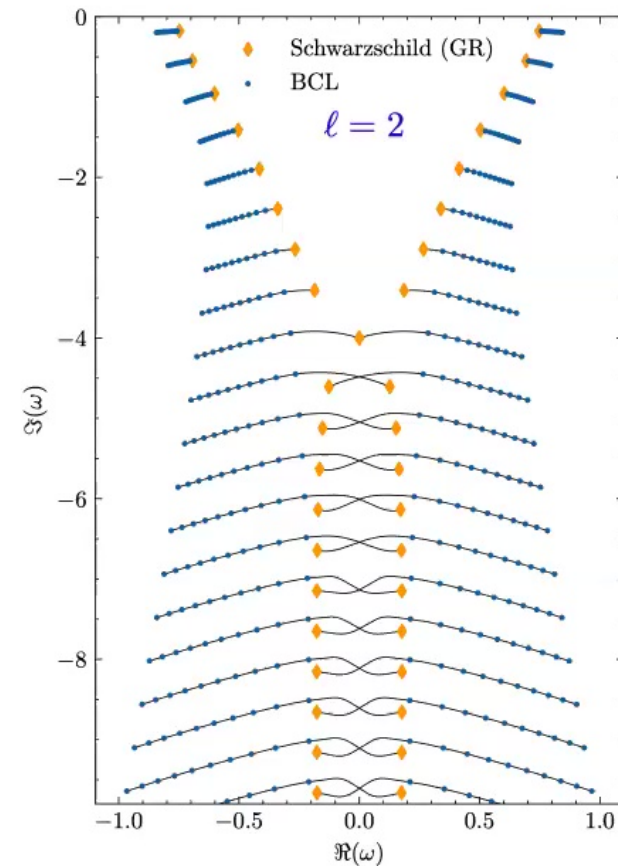
# Numerical axial modes for BCL

Roussille, DL & Noui '23

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$A(r) = \left(1 - \frac{r_+}{r}\right) \left(1 + \frac{r_-}{r}\right)$$

- Axial QNM modes for  $r_+ = 1$
- The parameter  $r_-$  varies between 0 (Schwarzschild) and 0.5.
- Migration of the modes in the complex plane



# Conclusion

- **DHOST theories:** most general framework for scalar-tensor theories propagating a single scalar dof.
- In cosmology, DHOST theories provide a **very rich phenomenology**, which can **parametrise deviations** from General Relativity and be tested in future cosmological observations.
- **Deviations from GR** can also be **explored** in compact objects, in particular **black holes**. **QNMs modes** are a natural target to **detect or constrain** deviations from GR in the strong gravity regime.

## Physical degrees of freedom

- **Scalar** ( $\delta N$ ,  $N_i \equiv \partial_i \psi$ ,  $h_{ij} = a^2(t) e^{2\zeta} \delta_{ij}$ )

$$S_{\text{quad}}[\zeta] = \int d^3x dt a^3 \frac{M^2}{2} \left[ \mathcal{A}_\zeta \dot{\zeta}^2 - \mathcal{B}_\zeta \frac{(\partial_i \zeta)^2}{a^2} \right]$$

with  $\mathcal{A}_\zeta = \frac{1}{(1 + \alpha_B - \dot{\beta}_1/H)^2} \left[ \alpha_K + 6\alpha_B^2 - \frac{6}{a^3 H^2 M^2} \frac{d}{dt} (a^3 H M^2 \alpha_B \beta_1) \right]$

$$\mathcal{B}_\zeta = -2(1 + \alpha_T) + \frac{2}{aM^2} \frac{d}{dt} \left[ \frac{aM^2 (1 + \alpha_H + \beta_1(1 + \alpha_T))}{H(1 + \alpha_B) - \dot{\beta}_1} \right]$$

- **Tensor:**  $S_\gamma^{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[ \frac{M^2}{4} \dot{\gamma}_{ij}^2 - \frac{M^2}{4} (1 + \alpha_T) \frac{(\partial_k \gamma_{ij})^2}{a^2} \right]$
- **Stability** (neither ghost nor gradient instability)

$$M^2 > 0, \quad \mathcal{A}_\zeta > 0, \quad \mathcal{B}_\zeta > 0, \quad c_T^2 \equiv (1 + \alpha_T) > 0$$

# Illustrative example

- Simple toy model

Boumazza, DL, Noui '20

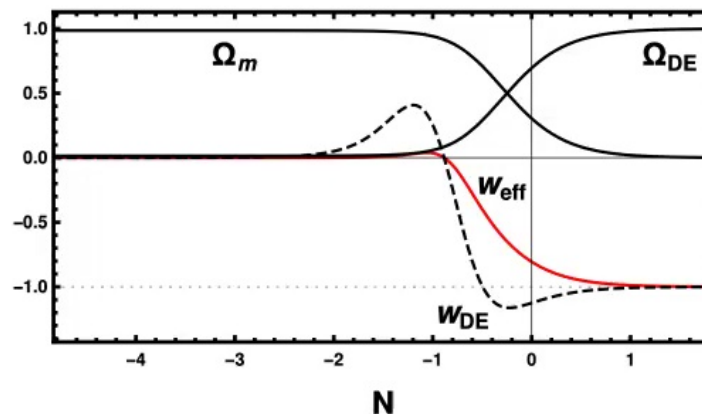
$$F = \frac{1}{2}, \quad P = \alpha X, \quad Q = 0, \quad A_1 = -A_2 = -\beta X, \quad \lambda = \frac{1}{2} s \beta X^2$$

Choice of  $\lambda$  determines  $A_3$ . Degeneracy imposes  $A_4$  and  $A_5$ .

- From matter era to dark energy era

$$3H^2 = \rho_m + \rho_{\text{DE}}$$

$$2\dot{H} + 3H^2 = P_m + P_{\text{DE}}$$



$$w_{\text{DE}} \equiv \frac{P_{\text{DE}}}{\rho_{\text{DE}}}$$

$$w_{\text{eff}} \equiv \frac{P_m + P_{\text{DE}}}{\rho_m + \rho_{\text{DE}}} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2}$$

# Illustrative example

- Simple toy model

Boumazza, DL, Noui '20

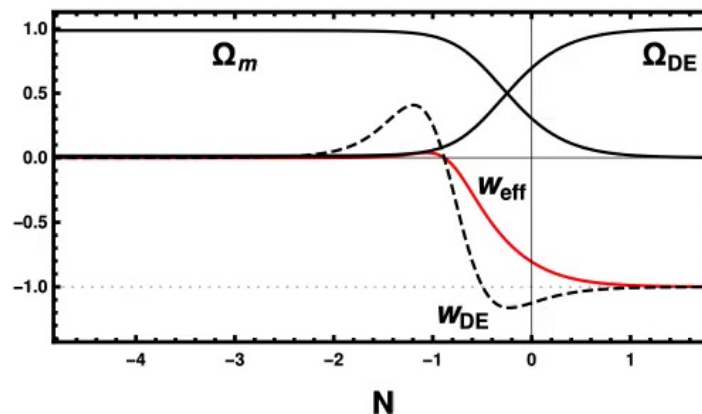
$$F = \frac{1}{2}, \quad P = \alpha X, \quad Q = 0, \quad A_1 = -A_2 = -\beta X, \quad \lambda = \frac{1}{2} s \beta X^2$$

Choice of  $\lambda$  determines  $A_3$ . Degeneracy imposes  $A_4$  and  $A_5$ .

- From matter era to dark energy era

$$3H^2 = \rho_m + \rho_{\text{DE}}$$

$$2\dot{H} + 3H^2 = P_m + P_{\text{DE}}$$



$$w_{\text{DE}} \equiv \frac{P_{\text{DE}}}{\rho_{\text{DE}}}$$

$$w_{\text{eff}} \equiv \frac{P_m + P_{\text{DE}}}{\rho_m + \rho_{\text{DE}}} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2}$$