Title: Black hole binaries in Einstein-scalar-Gauss-Bonnet gravity and their effective-one-body description

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Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

Subject: Cosmology, Strong Gravity, Mathematical physics

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Abstract:

I will show how to derive libraries of semi-analytic gravitational waveforms for coalescing “hairy” black hole binaries, focusing on the example of Einstein-scalar-Gauss-Bonnet gravity (ESGB). To do so, I will start from the state-of-the-art, effective-one-body waveform model “SEOBNRv5PHM” in general relativity, and deform it with ESGB corrections to infer inspiral-merger-ringdown waveform estimates.

Recent analytical progress on the modeling of black hole binaries in Einstein-scalar-Gauss-Bonnet gravity

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50 Years of Horndeski Gravity
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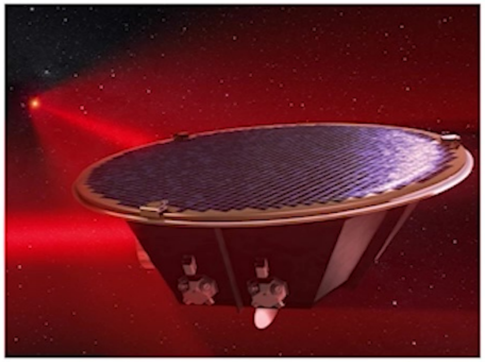
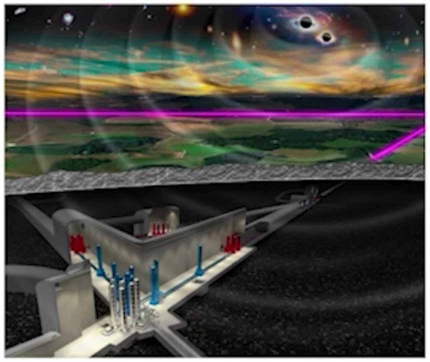
July 16th, 2024

Introduction

The era of gravitational wave astronomy

- GW150914 & GW170817: first observations of BBH and BNS coalescences by LIGO-Virgo
- O3: 90 candidate gravitational-wave event detections
- O4: started May 24, 2023 and lasting 18 months, joined by KAGRA
- June 2023: NANOGrav and EPTA new data release
- Next decade: LISA mission, 3rd generation ground detectors ET and CE





Opportunity for **new tests of gravity** in the strong-field regime of a compact binary coalescence.

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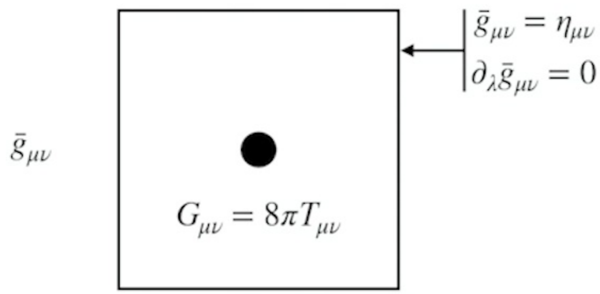
Introduction

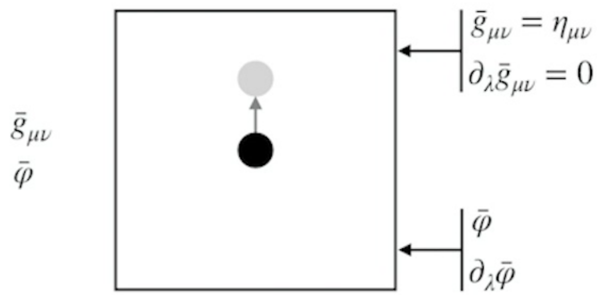
The strong equivalence principle: a thought experiment



GR *[Mathisson 1931, Infeld 1950,...]*

$$I_{\rm pp}[g_{\mu\nu}, z^\mu] = -\int m\, ds$$

with $ds = \sqrt{-g_{\mu\nu} dz^\mu dz^\nu}$.

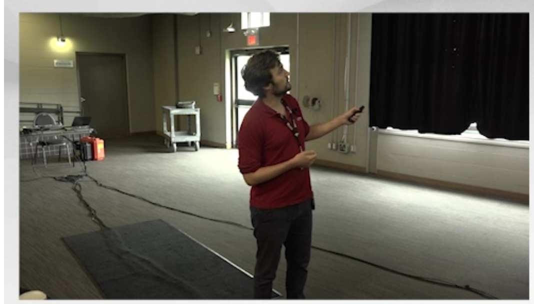


GR + scalar field *[Dicke 61, Eardley 75]*

$$I_{\rm pp}[g_{\mu\nu}, \varphi, z^\mu] = -\int m(\varphi)\, ds$$

Violation of the strong equivalence principle.

How to derive $m(\varphi)$ for a black hole, and how to measure it?



Introduction

Example: **Einstein-Scalar-Gauss-Bonnet gravity**

ESGB vacuum action ($G = c = 1$)

$$I_{\mathrm{ESGB}} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \ell_{\mathrm{GB}}^2 f(\varphi)\mathscr{R}_{\mathrm{GB}}^2\right)$$

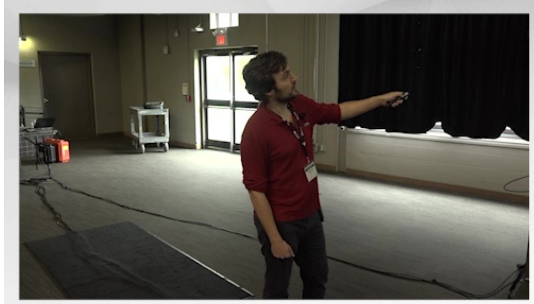
- Gauss-Bonnet scalar $\mathscr{R}_{\mathrm{GB}}^2 = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
- Fundamental coupling ℓ_{GB} with dimensions of length, and $f(\varphi)$ defines the ESGB theory
- $\int d^Dx\sqrt{-g}\,\mathscr{R}_{\mathrm{GB}}^2$ is a boundary term in $D \leqslant 4$
- Subclass of Horndeski theories *[Kobayashi 19]*, compactified 5D Lovelock theories *[Charmousis 15]*, low-energy limit of string theory *[Gross-Sloan 87]*,...

Second order field equations

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi - 4\ell_{\mathrm{GB}}^2\left(P_{\mu\alpha\nu\beta} - \frac{g_{\mu\nu}}{2}P_{\alpha\beta}\right)\nabla^\alpha\nabla^\beta f(\varphi)$$

$$\Box\varphi = -\frac{1}{4}\ell_{\mathrm{GB}}^2 f'(\varphi)\mathscr{R}_{\mathrm{GB}}^2$$

with $P_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - 2g_{\mu[\rho}R_{\sigma]\nu} + 2g_{\nu[\rho}R_{\sigma]\mu} + g_{\mu[\rho}g_{\sigma]\nu}R$



Hairy black holes in ESGB gravity

Analytical solutions in the small Gauss-Bonnet coupling $\ell_{\rm GB}$ limit

- **Einstein-dilaton-Gauss-Bonnet,** $f(\varphi) = e^{\varphi}$

  Mignemi-Stewart 93, Maeda et al. 97, Yunes-Stein 11, Pani et al. 11, Ayzenberg-Yunes 14, Maselli et al. 15
- **Shift-symmetric theories,** $f(\varphi) = \varphi$

  Sotiriou-Zhou 14
- **Generic ESGB theories**

  FLJ-Berti 19

Numerical solutions

- **Einstein-dilaton-Gauss-Bonnet,** $f(\varphi) = e^{\varphi}$

  Kanti et al. 95, Pani-Cardoso 09, Kleihaus 15, FLJ-Silva-Berti-Yunes 22
- **Shift-symmetric theories,** $f(\varphi) = \varphi$

  Antoniou et al. 18, Delgado et al. 20, FLJ-Silva-Berti-Yunes 22
- **Quadratic couplings,** $f(\varphi) = \varphi^2(1 + \lambda\varphi^2)$ and $f(\varphi) = -\,e^{-\lambda\varphi^2}$

  Doneva-Yazadjiev 17, Silva et al. 17, Minamitsuji-Ikeda 18, Macedo et al. 19, Dima et al. 20, Antoniou et al. 21, FLJ-Silva-Berti-Yunes 22, Lai et al. 23, FLJ 23

To be generalized to non-zero asymptotic values $\bar{\varphi}$ of the scalar field

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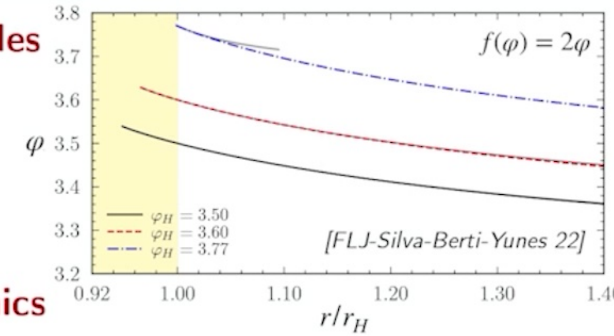
Hairy black holes and their thermodynamics

Static, spherically symmetric ESGB black holes



$$ds^2 = -A(r)dt^2 + B(r)^{-1}dr^2 + r^2 d\Omega^2\ , \quad \varphi = \varphi(r)$$

with $A(r_H) = B(r_H) = 0$ and $\varphi \underset{r \gg r_H}{\longrightarrow} \bar{\varphi}$

and checking their generalized thermodynamics

- **Temperature:** $\boxed{T = \dfrac{\kappa}{4\pi}}$ where $\kappa^2 = -\dfrac{1}{2}(\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu)_{r_H}$ is the surface gravity

- **Wald entropy:** $S_w = -8\pi \int_{r_H} d\theta d\phi \sqrt{\sigma}\, \dfrac{\partial \mathscr{L}}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu}\epsilon_{\rho\sigma}$ with $\epsilon_{\mu\nu} = n_{[\mu} l_{\nu]}$

  $\boxed{S_w = \dfrac{\mathscr{A}_H}{4} + 4\ell_{GB}^2 \pi f(\varphi_H)}$ in ESGB gravity.

- **Mass as a Nœther charge:**

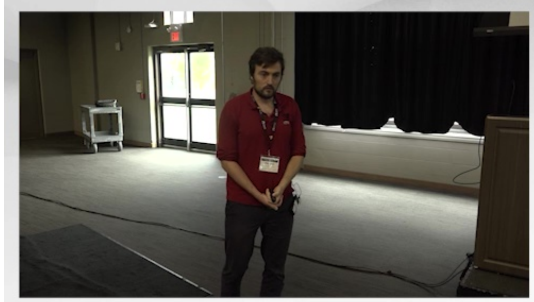
  $\boxed{M = M_{ADM} + \int D\, d\bar{\varphi}}$

  $$B = 1 - \frac{2M_{ADM}}{r} + \mathcal{O}(1/r^2)$$
  $$\varphi = \bar{\varphi} + \frac{D}{r} + \mathcal{O}(1/r^2)$$

  [Henneaux et al. 02, Cardenas et al. 16, Anabalon-Deruelle-FLJ 16]

The variations of S_w and M w.r.t. the BH's 2 integration constants satisfy: *[FLJ-Berti 19]*

$$T\delta S_w = \delta M$$



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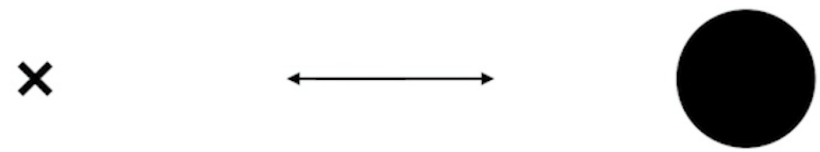
The sensitivity of black holes

$$I_{\rm pp}[g_{\mu\nu}, \varphi, z^\mu] = -\int m(\varphi)\, ds$$

Question: How to derive $m(\varphi)$ for an ESGB black hole?

Answer: by identifying the BH's fields to those sourced by the particle.



Fields of the particle, rest frame $z^i = 0$
Harmonic gauge $\partial_\mu(\sqrt{-g}g^{\mu\nu}) = 0$

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta_{\mu\nu}\left(\frac{2m(\bar{\varphi})}{\tilde{r}}\right) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

$$\varphi = \bar{\varphi} - \frac{1}{\tilde{r}}\frac{dm}{d\varphi}(\bar{\varphi}) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

Fields of the body in vacuum
Isotropic coordinates

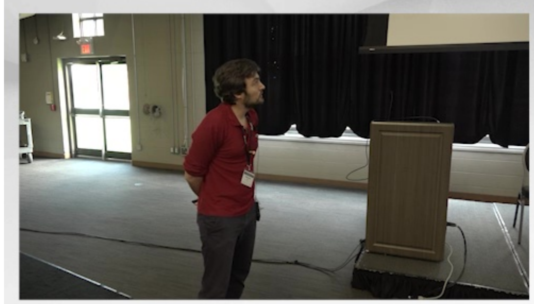
$$g_{\mu\nu} = \eta_{\mu\nu} + \delta_{\mu\nu}\left(\frac{2M_{\rm ADM}}{\tilde{r}}\right) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

$$\varphi = \bar{\varphi} + \frac{D}{\tilde{r}} + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)$$

Identification iff

(a) $m(\bar{\varphi}) = M_{\rm ADM}$
(b) $m'(\bar{\varphi}) = -D$

[FLJ 18, FLJ-Berti 19]



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The sensitivity of black holes

- Recall: ESGB first law of thermodynamics: $\boxed{T\delta S_{\rm w} = \delta M}$ where $\delta M = \delta M_{\rm ADM} + D\delta\bar{\varphi}$.

Matching conditions

(a) $m(\bar{\varphi}) = M_{\rm ADM}$
(b) $m'(\bar{\varphi}) = -D$

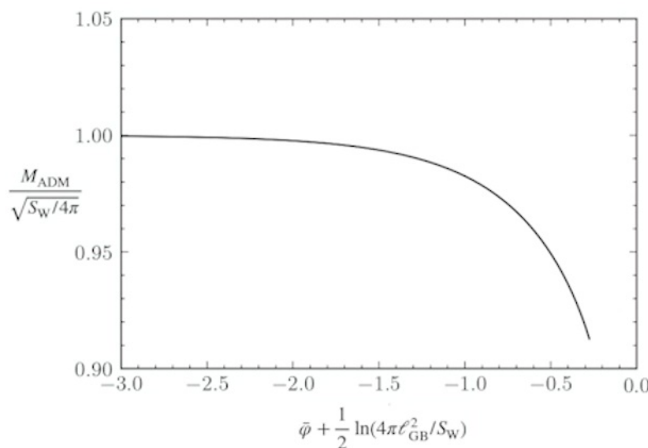
(a) and (b) $\Rightarrow$ $\delta M = \delta M_{\rm ADM} + D\delta\bar{\varphi} = 0 \Rightarrow \delta S_{\rm w} = 0$

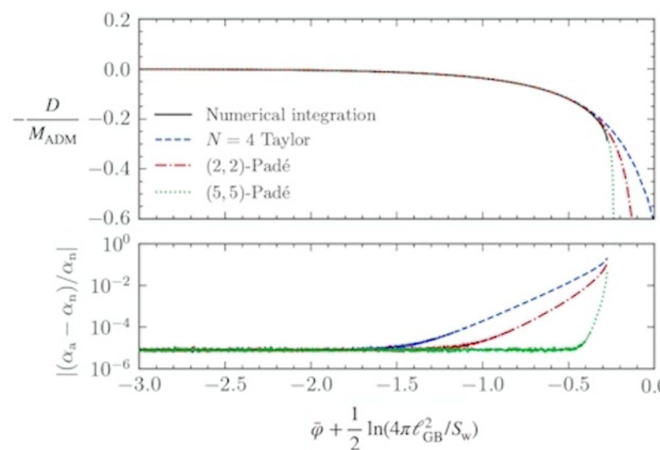
A black hole must be described by a sequence of constant Wald entropy equilibrium configurations.

[FLJ-Berti 19, Cardenas-FLJ-Deruelle 18, FLJ 18]

- **Example 1**: dilatonic theory $f(\varphi) = \frac{1}{4}\exp(2\varphi)$

[FLJ-Berti 19, FLJ-Silva-Berti-Yunes 22]





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The sensitivity of black holes

- Recall: ESGB first law of thermodynamics: $\boxed{T\delta S_{\rm w} = \delta M}$ where $\delta M = \delta M_{\rm ADM} + D\delta\bar{\varphi}$.

Matching conditions

(a) $m(\bar{\varphi}) = M_{\rm ADM}$
(b) $m'(\bar{\varphi}) = -D$

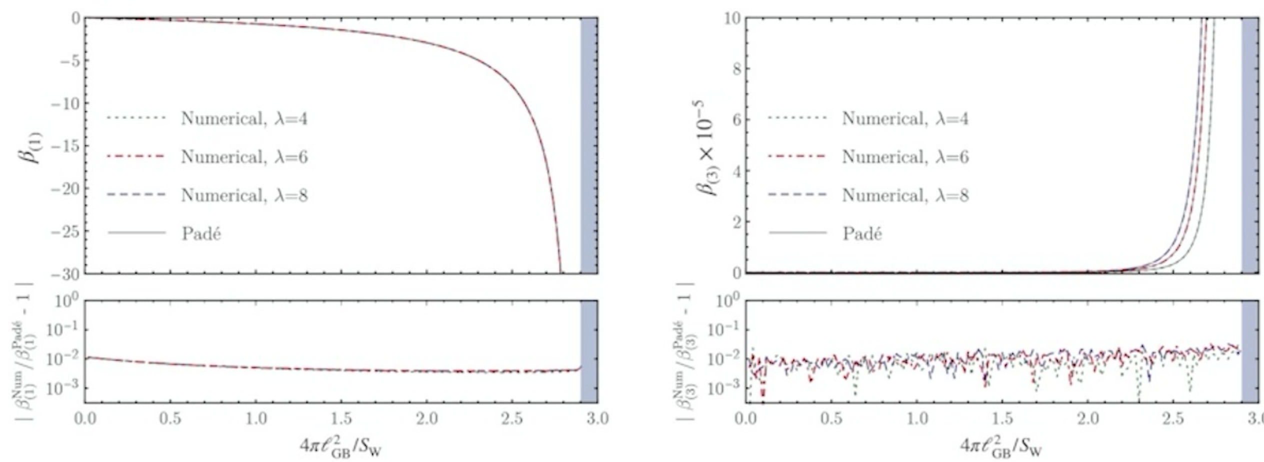
(a) and (b) $\Rightarrow$ $\delta M = \delta M_{\rm ADM} + D\delta\bar{\varphi} = 0$ $\Rightarrow$ $\delta S_{\rm w} = 0$

A black hole must be described by a sequence of constant Wald entropy equilibrium configurations.

[FLJ-Berti 19, Cardenas-FLJ-Deruelle 18, FLJ 18]

- **Example 2**: generic class of $\mathbb{Z}_2$-symmetric theories $f(\varphi) = \frac{1}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4 + \cdots$ with $\varphi \ll 1$ *[FLJ 23]*

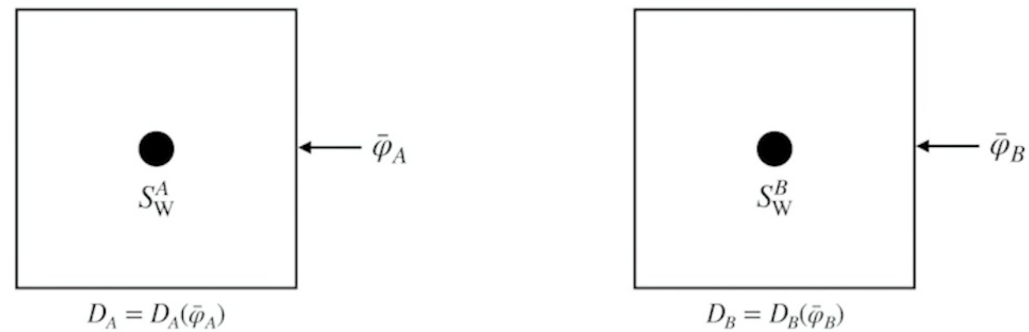
$$-\frac{D}{M_{\rm ADM}} = \beta_{(1)}(S_{\rm W})\,\bar{\varphi} + \frac{1}{6}\beta_{(3)}(S_{\rm W})\,\bar{\varphi}^3 + \cdots\ , \qquad \frac{M_{\rm ADM}}{\sqrt{S_{\rm W}/4\pi}} = 1 + \frac{1}{2}\beta_{(1)}(S_{\rm W})\,\bar{\varphi}^2 + \cdots$$



Dynamical scalarization in Schwarzschild binary inspirals

Setup: **black hole binary in the $\mathbb{Z}_2$-symmetric class**

$$f(\varphi) = \frac{1}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4 + \cdots$$



- **Recall:** when $\bar{\varphi}_A = 0$, BH A reduces to Schwarzschild, with mass $m_A^0 = \sqrt{S_{\mathrm{W}}^{A}/4\pi}$
- To **Coulomb order**, for simplicity:

$$\bar{\varphi}_A \simeq \frac{D_B(\bar{\varphi}_B)}{r} = \bar{\varphi}_A(\bar{\varphi}_B)$$

$$\bar{\varphi}_B \simeq \frac{D_A(\bar{\varphi}_A)}{r} = \bar{\varphi}_B(\bar{\varphi}_A)$$

2 equations for 2 unknowns $(\bar{\varphi}_A, \bar{\varphi}_B)$.

Insert one equation into the other *[FLJ 23]*

$$\bar{\varphi}_A \times \left[\left(\frac{\hat{r}^2}{\hat{r}_{DS}^2} - 1\right) - \mathscr{C}\,\bar{\varphi}_A^2 + \cdots\right] = 0$$

$$\hat{r}_{\mathrm{DS}} = \sqrt{\beta_{(1)}^{A}\beta_{(1)}^{B}\nu}$$

$$\mathscr{C} = \mathscr{C}(r, \beta_{(1,3)}^{A,B}) < 0$$

$$\hat{r} = \frac{r}{M}$$

$$M = m_A^0 + m_B^0$$

$$\nu = \frac{m_A^0 m_B^0}{M^2}$$

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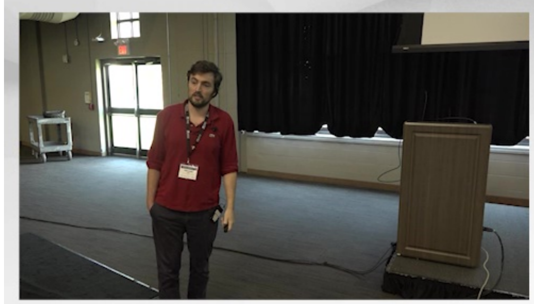
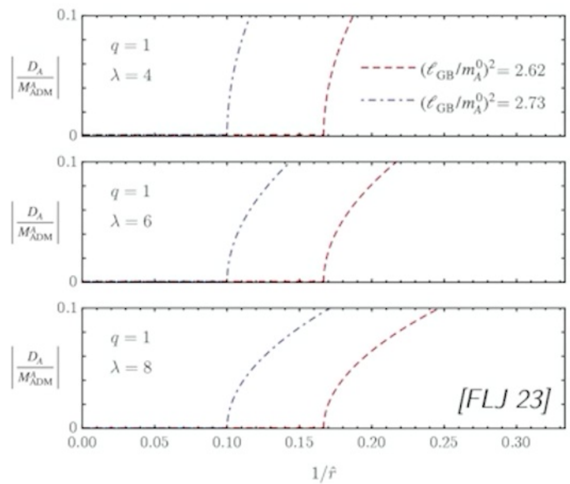


Dynamical scalarization in Schwarzschild binary inspirals

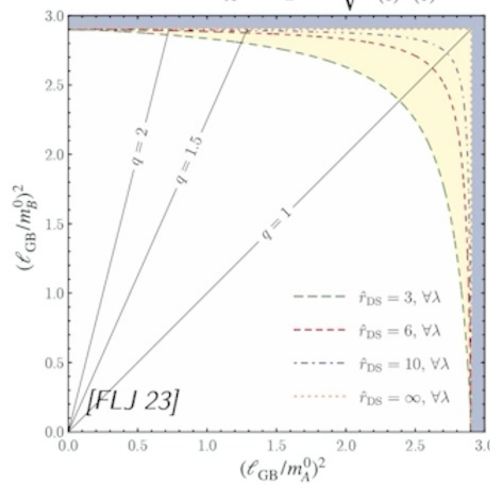
- The Schwarzschild configuration $\bar{\varphi}_A = 0$ is always a root.
- When $\hat{r} \leq \hat{r}_{\rm DS}$, two energetically favorable, equal and opposite roots $\bar{\varphi}_A \neq 0$ appear.

$$M^A_{\rm ADM} = m^0_A \left(1 + \frac{1}{2}\beta^A_{(1)}\bar{\varphi}^2_A + \cdots\right), \quad \beta^A_{(1)} \leq 0$$

- Reminiscent of NS binaries with fixed baryon number in scalar-tensor theories *[Barausse et al. 13 & 14, Shibata et al. 14, Khalil et al. 22]*



$$\hat{r}_{\rm DS} = r_{\rm DS}/(m^0_A + m^0_B) = \sqrt{\beta^A_{(1)}\beta^B_{(1)}\nu}$$



To leading order in $0 \leq \hat{r}^{-1} - \hat{r}^{-1}_{\rm DS} \ll 1$ *[FLJ 23]*

$$-\frac{D_A}{M^A_{\rm ADM}} = \pm\Gamma_A\left(\frac{1}{\hat{r}} - \frac{1}{\hat{r}_{\rm DS}}\right)^{1/2} + \cdots$$

$$\Gamma_A = \left[\frac{-2(\beta^A_{(1)})^2(\beta^A_{(1)}\beta^B_{(1)}\nu)^{1/2}}{\frac{1}{2}\beta^A_{(1)} + \frac{1}{6}\frac{\beta^A_{(3)}}{\beta^A_{(1)}} + q\left(\frac{1}{2}\beta^B_{(1)} + \frac{1}{6}\frac{\beta^B_{(3)}}{\beta^B_{(1)}}\right)\frac{\beta^A_{(1)}}{\beta^B_{(1)}}}\right]^{1/2} \quad \text{and } A \leftrightarrow B.$$

$$q = m^0_A/m^0_B$$

- The highlighted DS parameter space was not explored in NR simulations of $\mathbb{Z}_2$-symmetric ESGB theories

1 Hairy black holes and their thermodynamics

2 The sensitivity of black holes

3 Dynamical scalarization in Schwarzschild binary inspirals

4 Gravitational waveforms

- Post-Newtonian formalism
- The effective-one-body framework

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Post-Newtonian formalism

$$I_{\rm ESGB} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \ell_{\rm GB}^2 f(\varphi)\mathscr{R}_{\rm GB}^2\right) - \sum_A \int m_A(\varphi)ds_A$$

ESGB two-body post-Newtonian (PN) Lagrangian

- Conservative nPN dynamics: $\mathcal{O}\left(\frac{v}{c}\right)^{2n} \sim \mathcal{O}\left(\frac{GM}{r}\right)^n$ corrections to Newton
- Solve iteratively the field equations with point particle sources around

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu} \qquad \varphi = \varphi_0 + \delta\varphi$$

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi - 4\ell_{\rm GB}^2\left(P_{\mu\alpha\nu\beta} - \frac{1}{2}g_{\mu\nu}P_{\alpha\beta}\right)\nabla^\alpha\nabla^\beta f(\varphi) + 8\pi\sum_A\left(T_{\mu\nu}^A - \frac{1}{2}g_{\mu\nu}T^A\right)$$

$$\Box\varphi = -\frac{1}{4}\ell_{\rm GB}^2 f'(\varphi)\mathscr{R}_{\rm GB}^2 + 4\pi\sum_A \frac{ds_A}{dt}\frac{dm_A}{d\varphi}\frac{\delta^{(3)}(\mathbf{x} - \mathbf{z}_A(t))}{\sqrt{-g}}$$

- The sensitivities $m_A(\varphi)$ and $m_B(\varphi)$ are expanded around φ_0

$$\ln m_A(\varphi) = \ln m_A^0 + \alpha_A^0(\varphi - \varphi_0) + \frac{1}{2}\beta_A^0(\varphi - \varphi_0)^2 + \frac{1}{6}\beta'^0_A(\varphi - \varphi_0)^3 + \frac{1}{24}\beta''^0_A(\varphi - \varphi_0)^4 + \cdots$$

$$\ln m_B(\varphi) = \ln m_B^0 + \alpha_B^0(\varphi - \varphi_0) + \frac{1}{2}\beta_B^0(\varphi - \varphi_0)^2 + \frac{1}{6}\beta'^0_B(\varphi - \varphi_0)^3 + \frac{1}{24}\beta''^0_B(\varphi - \varphi_0)^4 + \cdots$$

Post-Newtonian formalism

Two-body Lagrangians and Hamiltonians in modified gravities: a state of the art

- **1PN:** Scalar-tensor — *Damour-Esposito-Farèse 92*
  Einstein-Maxwell-dilaton — *FLJ 18, Khalil et al. 18*
- **2PN:** Scalar-tensor — *Mirshekari-Will 13, FLJ-Deruelle 17, FLJ 18*
- **3PN:** Scalar-tensor — *Bernard 19, FLJ-Baibhav-Berti-Buonanno 23, Jain et al. 23*
  Einstein-scalar-Gauss-Bonnet — *FLJ-Berti 19, FLJ-Baibhav-Berti-Buonanno 23*

Scalar-tensor theories (i.e., ESGB in the limit $\ell_{\rm GB} = 0$)

$$I_{\rm ST} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right)$$

Einstein-Maxwell-dilaton theories

$$I_{\rm EMD} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - e^{-2a\varphi}F^{\mu\nu}F_{\mu\nu}\right)$$

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Post-Newtonian formalism

At 3PN *[ST: Bernard 19, ESGB: FLJ-Berti 19]*

$$L_{3\mathrm{PN}} = L_{3\mathrm{PN}}^{\mathrm{loc}} + L_{3\mathrm{PN}}^{\mathrm{tail}}$$

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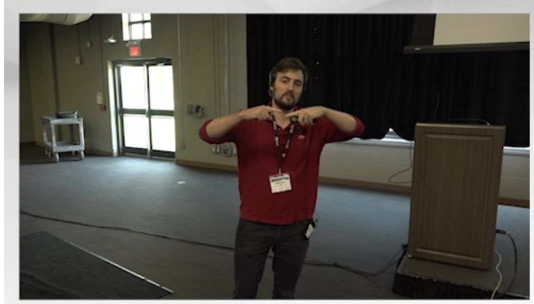


Post-Newtonian formalism

At 3PN *[ST: Bernard 19, ESGB: FLJ-Berti 19]*

$$L_{\mathrm{3PN}} = L^{\mathrm{loc}}_{\mathrm{3PN}} + L^{\mathrm{tail}}_{\mathrm{3PN}}$$

- Nonlocal-in-time tail term driven by the acceleration of the system's scalar dipole *[Bernard 19]*

$$L^{\mathrm{tail}}_{\mathrm{3PN}} = \frac{2M}{3}\ddot{D}^i(t)\left(\underset{2r(t)}{\mathrm{PF}}\int_{\mathbb{R}}\frac{d\tau}{|\tau|}\ddot{D}^i(t+\tau)\right) \quad \text{with} \quad D^i = \sum_A m_A^0 \alpha_A^0 x_A^i$$

- **Nota**: in general relativity, 4PN tail driven by the mass quadrupole over-acceleration

$$L^{\mathrm{tail}}_{\mathrm{GR,4PN}} = \frac{M}{5}\dddot{Q}^{ij}(t)\left(\underset{2r(t)}{\mathrm{PF}}\int_{\mathbb{R}}\frac{d\tau}{|\tau|}\dddot{Q}^{ij}(t+\tau)\right) \quad \text{with} \quad Q^{ij} = \sum_A m_A^0\left(x_A^i x_A^j - \delta^{ij} x_A^2/3\right)$$

ESGB is an ideal probe to model beyond-GR physics in the strong field regime of a hairy binary black hole coalescence.

The effective-one-body framework

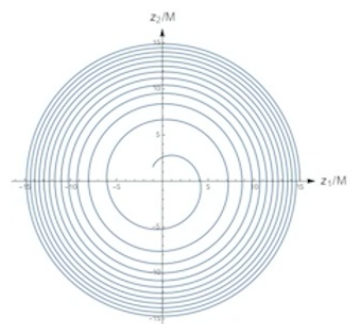
In general relativity, "effective-one-body" (EOB):

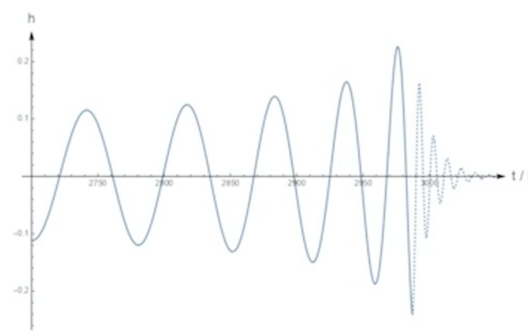
- Map the two-body PN dynamics to the motion of a **test particle** in an **effective static, spherically symmetric metric** *[Buonanno-Damour 98]*

$$H(q,p)\ , \quad \epsilon = \left(\frac{v}{c}\right)^2 \qquad \longrightarrow \qquad H_{\text{eff}}(Q,P)\ , \quad ds^2_{\text{eff}} = g^{\text{eff}}_{\mu\nu} dz^\mu dz^\nu$$

$$\boxed{H_{\text{eff}} = f_{\text{EOB}}(H)}$$

- Defines a resummation of the PN dynamics, hence describes the coalescence of 2 compact objects in **general relativity**, from inspiral **to merger**.





- Instrumental to build libraries of **several hundred thousand** semi-analytic **GW templates for LIGO-Virgo-KAGRA**.

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The effective-one-body framework

Step 1: resummed conservative sector

- Use canonical transformations $H(q,p) \to H(Q,P)$ to identify H with $H_{\rm EOB}$ order-by-order, where

EOB Hamiltonian

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

with $M = m_A^0 + m_B^0$, $\nu = \frac{m_A^0 m_B^0}{M^2}$, $\mu = \frac{m_A^0 m_B^0}{M}$

Effective Hamiltonian of a test particle μ

$$H_{\rm eff}(Q,P) = \sqrt{A\left(\mu^2 + AD\,P_R^2 + \frac{P_\Phi^2}{R^2}\right) + \mu^2 Q}$$

Motion in a static, spherically symmetric metric

in Schwarzschild-Droste coordinates:

$$ds_{\rm eff}^2 = -Adt^2 + \frac{dR^2}{AD} + R^2 d\Phi^2$$

with non-geodesic corrections Q which vanish for $P_R = 0$

- The potentials can be expanded through 3PN as

$$A(R) = 1 + \frac{a_1}{R} + \frac{a_2}{R^2} + \frac{a_3}{R^3} + \frac{a_4 + a_4^{\ln}\ln(R)}{R^4}$$

$$D(R) = 1 + \frac{d_1}{R} + \frac{d_2}{R^2} + \frac{d_3 + d_3^{\ln}\ln(R)}{R^3}$$

$$Q(R) = \frac{[q_1 + q_1^{\ln}\ln(R)]\,P_R^4}{R^2} + \frac{[q_2 + q_2^{\ln}\ln(R)]\,P_R^6}{R} + \cdots$$

- Yields a unique solution for the 1+2+2+8 coefficients

*[3PN (ST-ESGB): **FLJ et al. 23**, Jain et al. 23]*
*[2PN (ST): **FLJ** 18, **FLJ**-Deruelle 17]*
*[1PN (ST-EMD): **FLJ** 18, Khalil et al. 18]*
[General relativity at 3PN-4PN: Damour et al. 00-15]

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The effective-one-body framework

Step 2: resummed metric waveform

- Decomposition of h_+ and $h_\times$ on spherical harmonics, known at 2PN on circular orbits *[Lang 15, Sennett et al. 16]*

$$h_+ - ih_\times = \sum_{\ell=2}^{6} \sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\Theta, \Phi)\, h_{\ell m}(T, R)$$

- Rewrite each mode $h_{\ell m}$ as $h^{\mathrm{F}}_{\ell m}$

Factorized metric modes

$$h^{\mathrm{F}}_{\ell m} = h^{\mathrm{N}}_{\ell m} \hat{S}_{\mathrm{eff}}\, T_{\ell m}\, f_{\ell m}\, e^{i\delta_{\ell m}}$$

Effective source: radiation of a test particle μ

$$\hat{S}_{\mathrm{eff}} = \begin{cases} \dfrac{H_{\mathrm{eff}}}{\mu} & \ell + m \text{ even} \\[2ex] \dfrac{\sqrt{x}}{1 + \alpha_A^0 \alpha_B^0} \dfrac{p_\phi}{M\mu} & \ell + m \text{ odd} \end{cases}$$

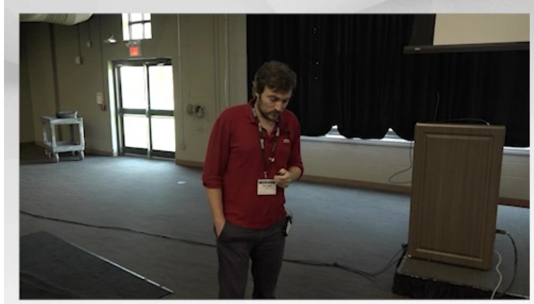
Kepler's 3rd law:
$x \equiv (G_{AB} M \dot{\phi})^{2/3} = \mathcal{O}(v^2)$

- $h^{\mathrm{N}}_{\ell m}$ is the mode at leading PN order ("Newtonian")

- $T_{\ell m}$ factorizes out tail effects for a Coulomb potential *[Asada-Futamase 97]*

$$T_{\ell m} = \frac{\Gamma\left(\ell + 1 - 2i\hat{k}\right)}{\Gamma(\ell + 1)} e^{\pi \hat{k}} e^{2i\hat{k}\ln(2m\dot{\phi}R_0)} \quad \hat{k} = m H_{\mathrm{EOB}} \dot{\phi}\,, \;\; \Gamma \text{ is Euler's Gamma function.}$$

- Absorb the remaining amplitude and phase in $f_{\ell m}$ and $\delta_{\ell m}$ *[2PN (ST-ESGB):* ***FLJ et al. 24****]*
[General relativity at 3PN: Damour-Iyer-Nagar 09]



The effective-one-body framework

Step 3: EOB dynamics including the dissipative sector

- Infer the metric flux, and add its scalar counterpart, known at 2.5PN beyond dipolar ("-1PN") order

*[**FLJ** 18, Khalil et al. 18, Yagi et al. 2012, Lang 15, Bernard et al. 22, Shiralilou et al. 22]*

$$\mathscr{F}^{\rm metric} = \frac{\dot\phi^2}{8\pi}\sum_{\ell=2}^{6}\sum_{m=1}^{\ell} m^2 \left| R h^{\rm F}_{\ell m}\right|^2 = \mathcal{O}\left(G_{AB}M\dot\phi\right)^{10/3}, \qquad \mathscr{F}^{\rm scalar} = \frac{\nu^2\left(G_{AB}M\dot\phi\right)^{8/3}}{\left(1+\alpha_A^0\alpha_B^0\right)^2}\left[\frac{1}{3}(\alpha_A^0-\alpha_B^0)^2 + \mathcal{O}\left(G_{AB}M\dot\phi\right)^{2/3} + \cdots\right]$$

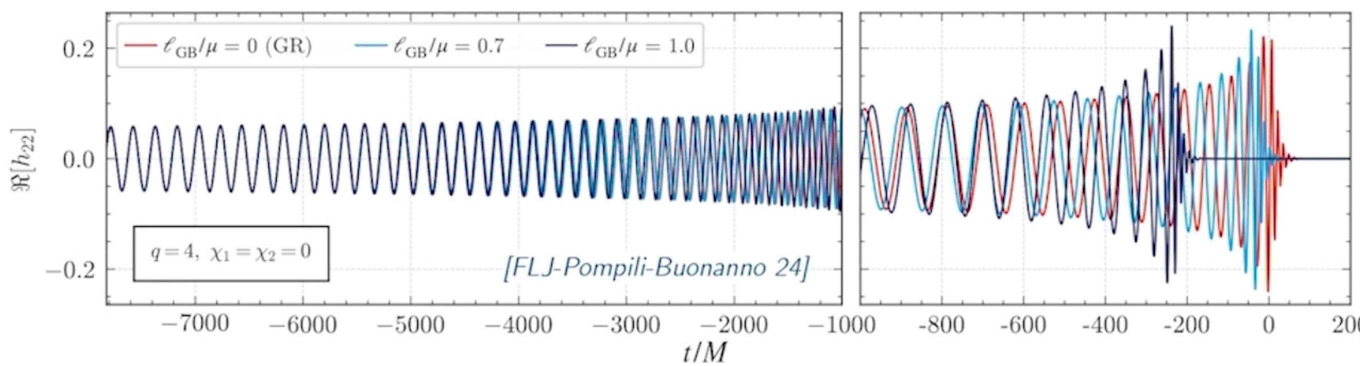
- On quasi-circular orbits: tangential force $F_\phi = -(\mathscr{F}^{\rm metric} + \mathscr{F}^{\rm scalar})/\dot\phi$

Kepler's 3rd law: $x \equiv (G_{AB}M\dot\phi)^{2/3} = \mathcal{O}(\nu^2)$

$$\dot r = \frac{\partial H_{\rm EOB}}{\partial p_r}, \qquad \dot p_r = -\frac{\partial H_{\rm EOB}}{\partial r} + \frac{p_r}{p_\phi}F_\phi, \qquad \dot\phi = \frac{\partial H_{\rm EOB}}{\partial p_\phi}, \qquad \dot p_\phi = F_\phi$$

- Include ESGB corrections in SEOBNRv5PHM, a state-of-the-art model in GR *[FLJ-Pompili-Buonanno 24]*

Example: EOB waveform for a binary black hole with $m_A^0/m_B^0 = 4$ in the dilatonic theory $f(\varphi) = e^{2\varphi}/4$



Conclusion

Recap

- Remarkably, the EOB approach can be extended to **ST** and **ESGB** at 3PN, and **EMD** at 1PN.
- Include other modified gravities such as **galileons** or **Horndeski**?
  [Van Aelst et al. 20, Babichev et al. 22-23,...]

Ongoing and future developments

- Model the dynamical scalarization of ESGB black holes at **post-adiabatic** order, and include it within EOB.
  [Khalil et al. 22]
- Our EOB model includes ESGB corrections to the **QNM spectrum** of the remnant, but the fractional change in the ringdown frequency is small (e.g., $\sim 10^{-5}$ for $q = 4$ and $\ell_{\rm GB}/\mu = 1$).
  [ESGB: Blázquez-Salcedo et al. 17 & 20, Pierini-Gualtieri 21, Langlois et al. 22, Chung-Yunes 24,...]
- ESGB corrections to the **black hole remnant's mass and spin** might yield **comparable** changes to the ringdown frequency. *[FLJ-Pompili-Buonanno 24]*
- Predict the final black hole's mass, spin and scalar charge, using NR simulations?

Thank you for your attention.