

**Title:** Scalarized black holes - from equilibrium models to nonlinear dynamics

**Speakers:** Daniela Doneva

**Collection/Series:** 50 Years of Horndeski Gravity: Exploring Modified Gravity

**Subject:** Cosmology, Strong Gravity, Mathematical physics

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**Abstract:**

Black holes in Horndeski theories of gravity are a perfect playground for exploring possible deviations from General Relativity in a theory-specific manner and studying their astrophysical manifestation. I will review the recent advances in constructing stationary hairy black hole models in Gauss-Bonnet theories. Special attention will be paid to their nonlinear dynamics when isolated or put in a binary, and the resulting astrophysical implications. The potential loss of hyperbolicity will also be discussed.



# Scalarized black holes

from equilibrium models to nonlinear dynamics

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**In collaboration with:**

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# Black holes with scalar hair in Gauss-Bonnet gravity



## scalar-Gauss-Bonnet gravity

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - 2\nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) + \lambda^2 f(\varphi) \mathcal{R}_{GB}^2 \right].$$

Gauss-Bonnet invariant:

$$\mathcal{R}_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$$

- Schwarzschild  $R_{GB}^2 = \frac{48M^2}{r^6}$  : relevant only **very close to the compact object** (for relation with cosmology see Anson et al (2019), Babichev et al (2024))
- Field equations :

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Gamma_{\mu\nu} = 2\nabla_\mu \varphi \nabla_\nu \varphi - g_{\mu\nu} \nabla_\alpha \varphi \nabla^\alpha \varphi - \frac{1}{2}g_{\mu\nu} V(\varphi),$$

$$\nabla_\alpha \nabla^\alpha \varphi = \frac{1}{4} \frac{dV(\varphi)}{d\varphi} - \frac{\lambda^2}{4} \frac{df(\varphi)}{d\varphi} \mathcal{R}_{GB}^2,$$

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## Scalar field coupling $f(\varphi)$ $\nabla_\alpha \nabla^\alpha \varphi = -\frac{\lambda^2}{4} \frac{df(\varphi)}{d\varphi} R_{GB}^2$

Expand  $f(\varphi)$  in series around  $\varphi = 0$ :

$$f(\varphi) = f_0 + f_1\varphi + f_2\varphi^2 + f_3\varphi^3 + f_4\varphi^4 + O(\varphi^5)$$

**Type I:**

- $f_1 \neq 0$ : **shift-symmetric** theory, Schwarzschild is not a solution,  $|\varphi| > 0$  always  
Kanti et al PRD(1996), Torii et al (1996), Pani&Cardoso PRD (2009)

**Type II:**

- $f_1 = 0, f_2 > 0, R_{GB}^2 > 0$ : **spontaneous** scalarization, Kerr unstable for **small masses** DD, Yazadjiev PRL (2018), Silva et al PRL (2018), Antoniou et al (2018)
- $f_1 = 0, f_2 < 0, R_{GB}^2 < 0$ : **spin-induced** scalarization, Kerr unstable for **large spins**  
Dima et al PRL (2020), DD et al RPD(2020), Berti et al PRL (2021), Herdeiro et al PRL (2021)

**Beyond Type II:**

- $f_1 = 0, f_2 = 0: \mu_{\text{eff}}^2 = 0$ , **nonlinear** scalarization, Kerr **linearly stable always**, **nonlinear scalarized phases can co-exist** DD, Yazadjiev, PRD Lett. (2021)

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## Scalar field coupling $f(\varphi)$

- Better numerically – consider an **exponential function**
  - ✓ Stable beyond-GR black holes
  - ✓ Additional parameter in the exponent controls the deviation from GR

$$f(\varphi) = \frac{1}{2\beta} e^{-\beta(\varphi^2 + \kappa\varphi^4)} \text{ (leading order } \sim \varphi^2 + \kappa\varphi^4 \text{)}$$

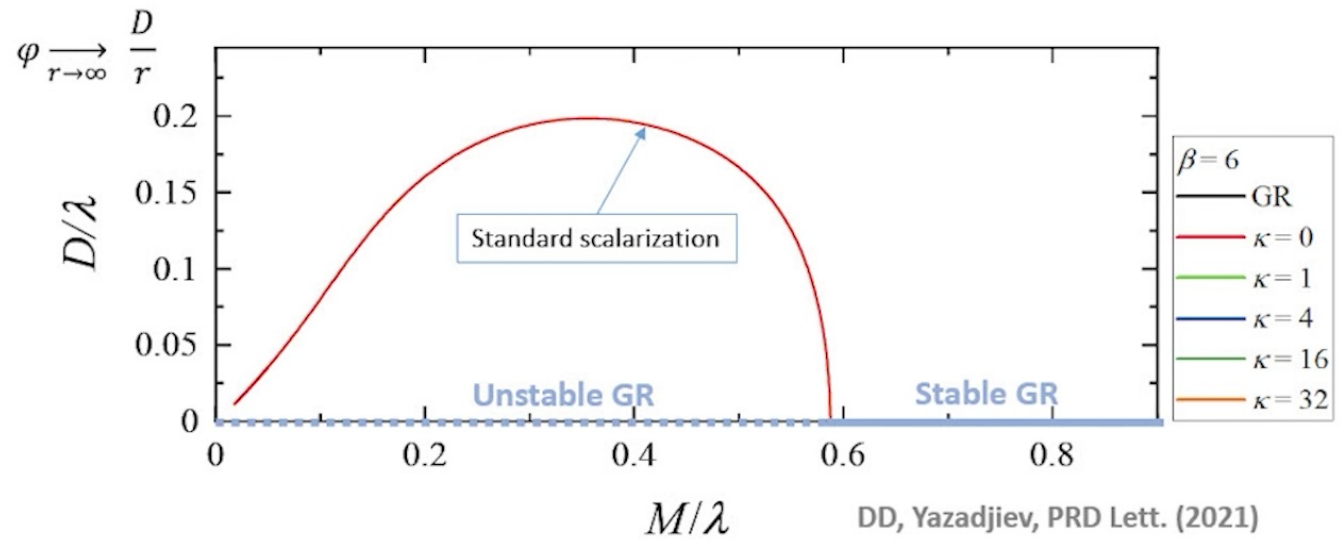
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## Standard scalarization with $\varphi^2$

$$f(\varphi) = \frac{1}{2\beta} e^{-\beta(\varphi^2 + \kappa\varphi^4)} \quad (\text{leading order } \sim \varphi^2 + \kappa\varphi^4)$$

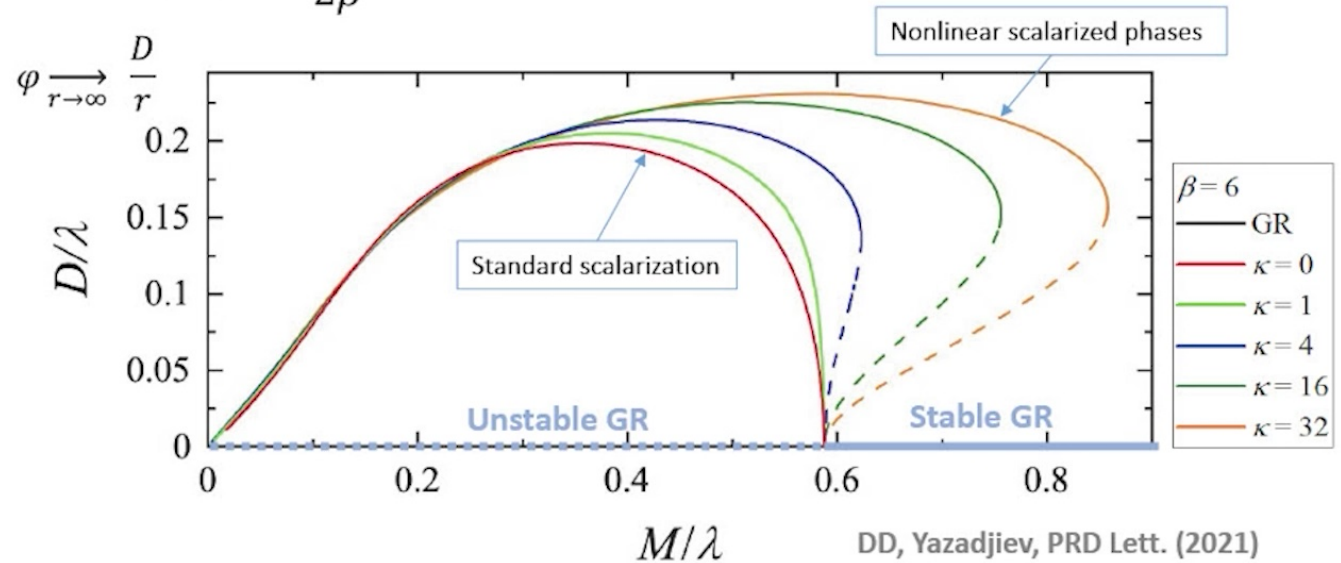


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## Standard + nonlinear scalarization

$$f(\varphi) = \frac{1}{2\beta} e^{-\beta(\varphi^2 + \kappa\varphi^4)} \quad (\text{leading order } \sim \varphi^2 + \kappa\varphi^4)$$



- Transition from **stable scalarized to GR** happens with a **jump**
- For a similar effect for charged BH see Blázquez-Salcedo et al. PLB (2020)

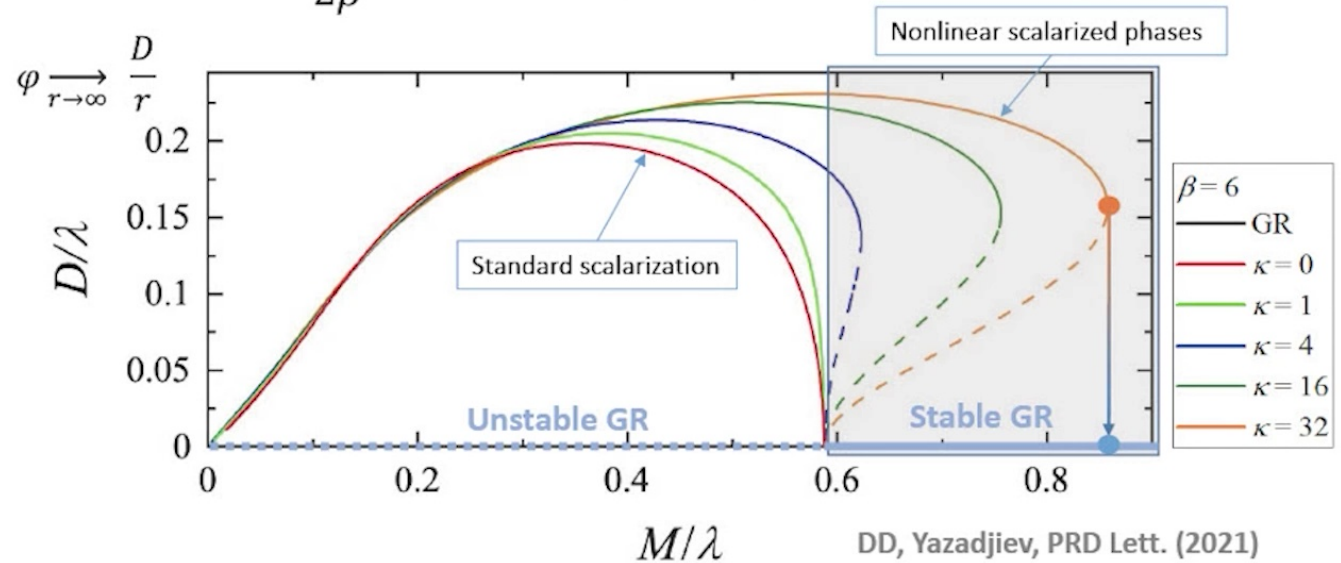
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## Well-posedness

- A solution exists;
- The solution is unique;
- It changes continuously with changes in the data.



## Gauss-Bonnet equations – significantly more complicated

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Gamma_{\mu\nu} = \frac{1}{2}\nabla_{\mu}\varphi\nabla_{\nu}\varphi - \frac{1}{4}g_{\mu\nu}\nabla_{\alpha}\varphi\nabla^{\alpha}\varphi - \frac{1}{2}g_{\mu\nu}V(\varphi)$$

$$\nabla_{\alpha}\nabla^{\alpha}\varphi = \frac{dV(\varphi)}{d\varphi} - \frac{\lambda}{4}\frac{df(\varphi)}{d\varphi}\mathcal{R}_{GB}^2,$$

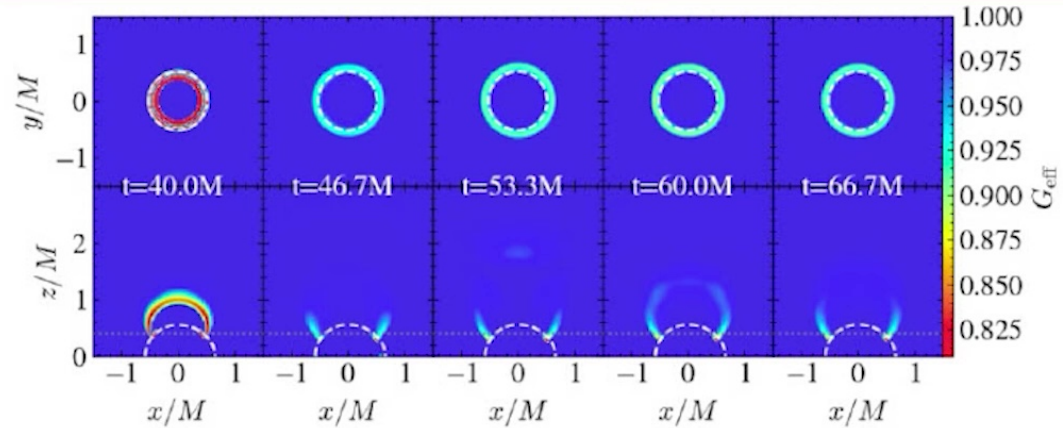
$$\Gamma_{\mu\nu} = -\frac{1}{2}R\Omega_{\mu\nu} - \Omega_{\alpha}^{\alpha}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + 2R_{\alpha(\mu}\Omega_{\nu)}^{\alpha} - g_{\mu\nu}R^{\alpha\beta}\Omega_{\alpha\beta} + R^{\beta}_{\mu\alpha\nu}\Omega_{\beta}^{\alpha}$$

$$\Omega_{\mu\nu} = \lambda\nabla_{\mu}\nabla_{\nu}f(\varphi).$$



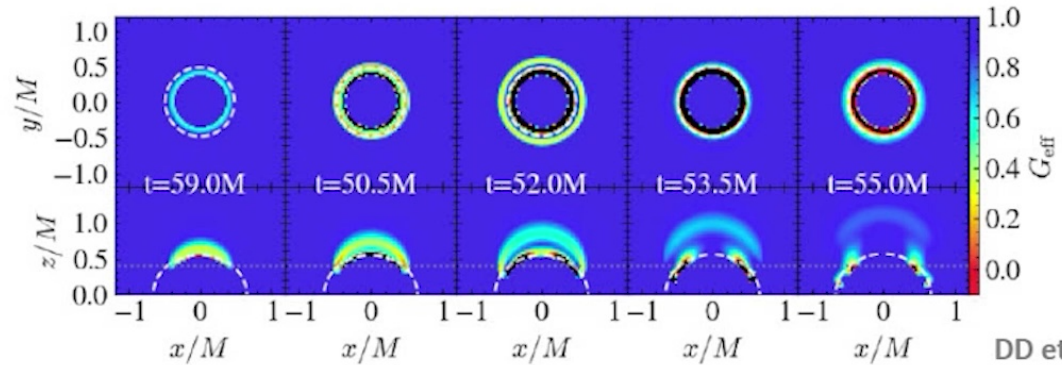
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## Normalized determinant – spin-induced black hole



Hyperbolic evolution

VS.



Hyperbolicity loss

DD et al. PRD (2023)

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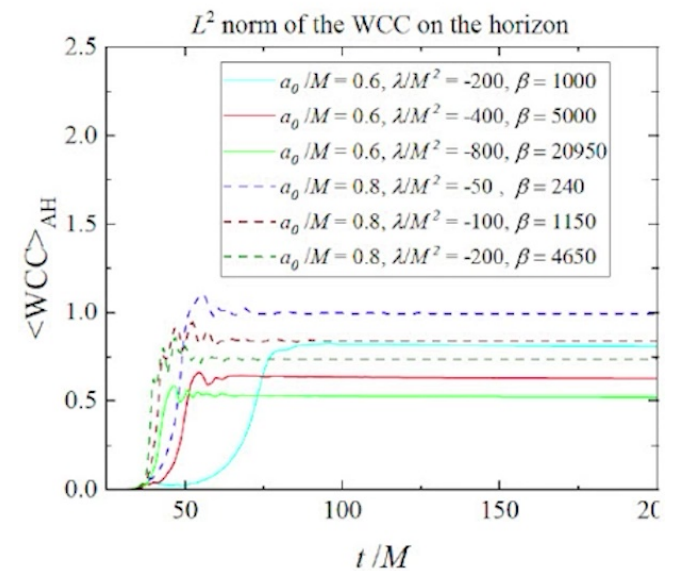
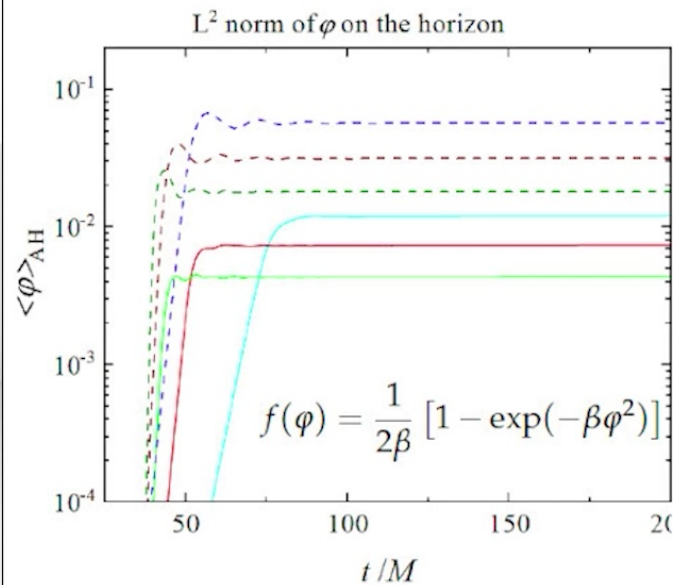


## Limiting models and weak coupling condition

- Weak coupling condition

$$\sqrt{|\lambda f'(\varphi)|}/L \ll 1$$

$$L^{-1} = \max\{|R_{ij}|^{1/2}, |\nabla_\mu \varphi|, |\nabla_\mu \nabla_\nu \varphi|^{1/2}, |\mathcal{R}_{GB}^2|^{1/4}\}$$



DD et al. PRD (2023)

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## Resolving the problem

- **Gauge change** – not very likely to help, hyperbolicity loss due to eigenvalues of physical modes becoming imaginary Areste-Salo et al PRL (2022), PRD (2022), DD et al. PRD (2023)
- **Fixing approach** Franchini et al PRD (2022), Cayuso et al PRL (2023)
  - ✓ A prescription to control the high frequency behaviour of an EFT
  - ✓ Modify in an *ad hoc* way the higher-order contributions to the field equations
  - ✓ Add a driver equation to let the solution relax to its correct value
- **Additional interactions** in the action can mitigate the hyperbolicity loss

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi + \frac{1}{4} \lambda^{GB} f(\varphi) R^{GB} - \beta(\varphi) R \right]$$

Ricci scalar coupling



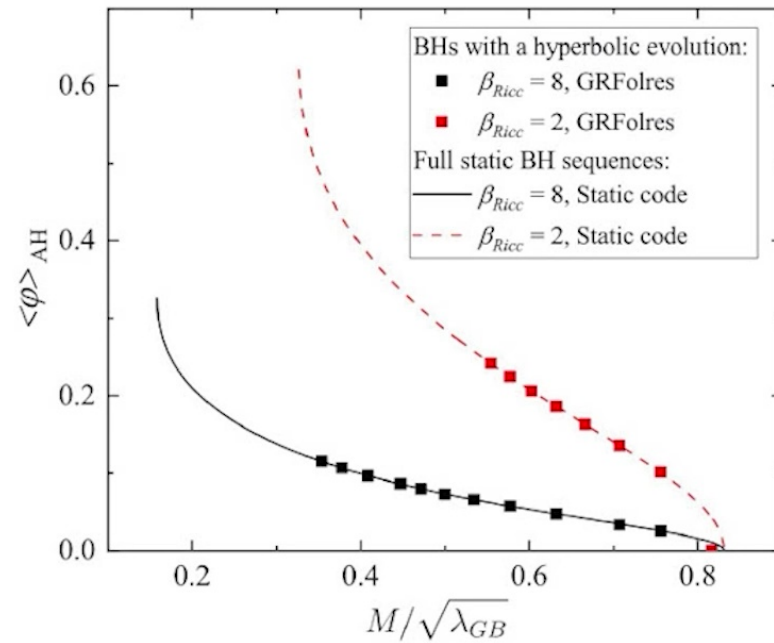
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## Ricci scalar coupling

$$\lambda(\varphi) = \lambda_{\text{GB}} \varphi^2 \quad \beta(\varphi) = \beta_{\text{Ricci}} \varphi^2$$

- Effective metric:  $g_{\text{eff}}^{\mu\nu} = g^{\mu\nu} (1 - \beta(\varphi)) - \Omega^{\mu\nu}$
- **3+1 simulations** – hyperbolicity maintained only for “weak” scalar field



DD, Areste-Salo, Yazadjiev PRD (2024)

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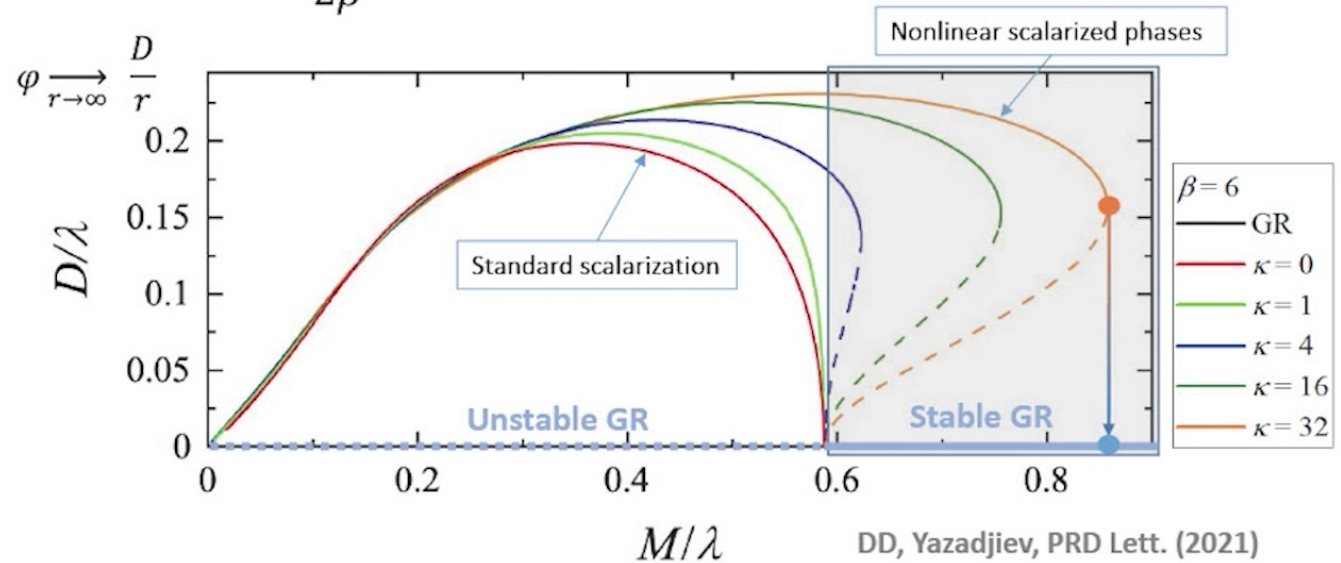
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# Binary mergers



## Standard + nonlinear scalarization

$$f(\varphi) = \frac{1}{2\beta} e^{-\beta(\varphi^2 + \kappa\varphi^4)} \quad (\text{leading order } \sim \varphi^2 + \kappa\varphi^4)$$



DD, Yazadjiev, PRD Lett. (2021)

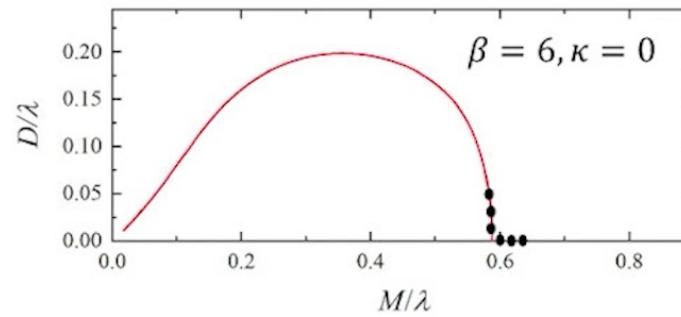
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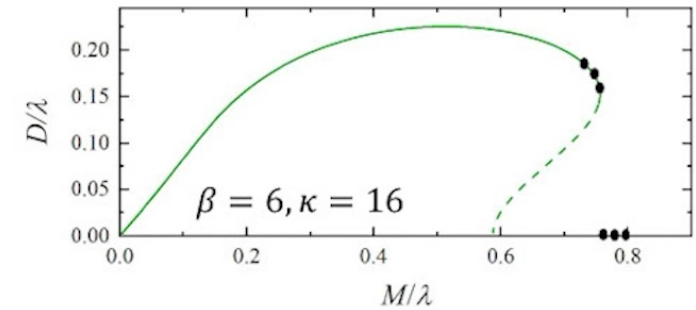


## Decoupling limit – jump in the solutions

$$f(\varphi) = \frac{1}{2\beta} (1 - e^{-\beta(\varphi^2 + \kappa\varphi^4)})$$



VS.

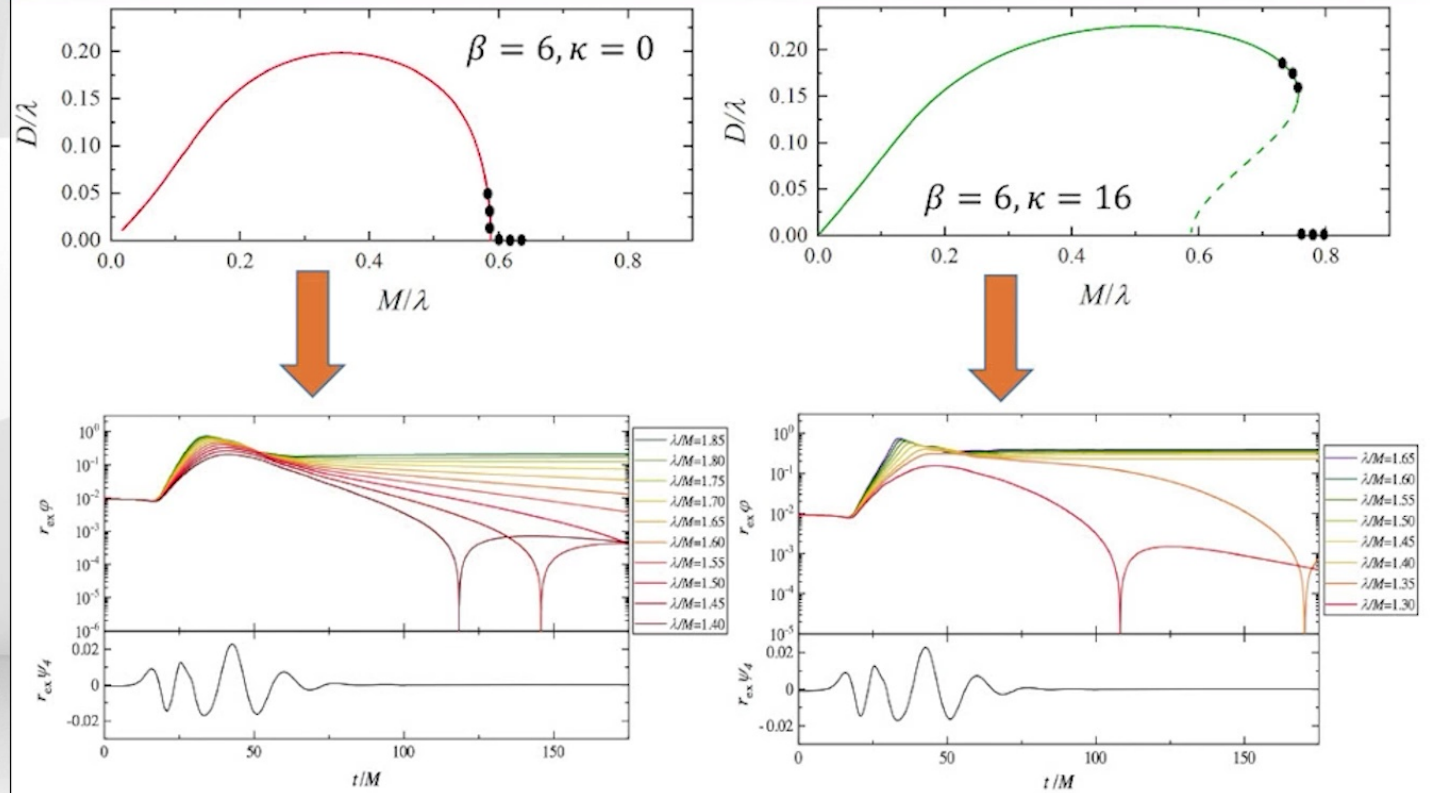


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## Head on collision: Jump vs. No Jump



DD, Vano-Vinuales, Yazadjiev (2022)

- IMR (in)consistency in modified gravity?

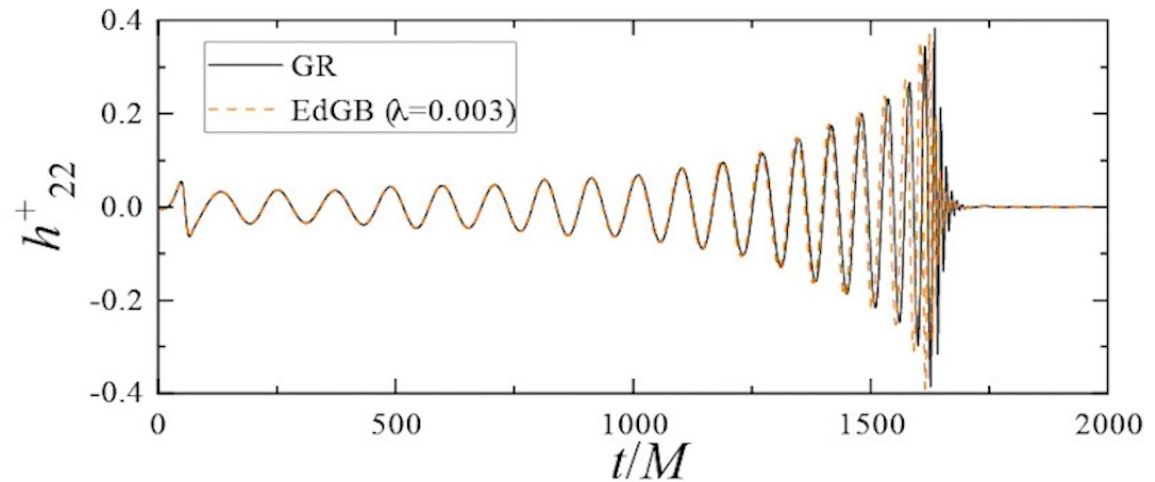
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## Full problem – inspiral of two equal mass BHs

- Solving the full problem with **GRFolres + GRChombo** in a **modified puncture gauge** Kovacs&Real PRL (2021), Areste-Salo et al PRL (2023) (for other codes see *Max's talk* East&Ripley PRL (2021), Corman et al. PRD (2023), and *Felix's talk* for EOB approach )
- **EdGB gravity** (no scalarization)
- **Scalar dipole radiation – zero** for equal mass binary

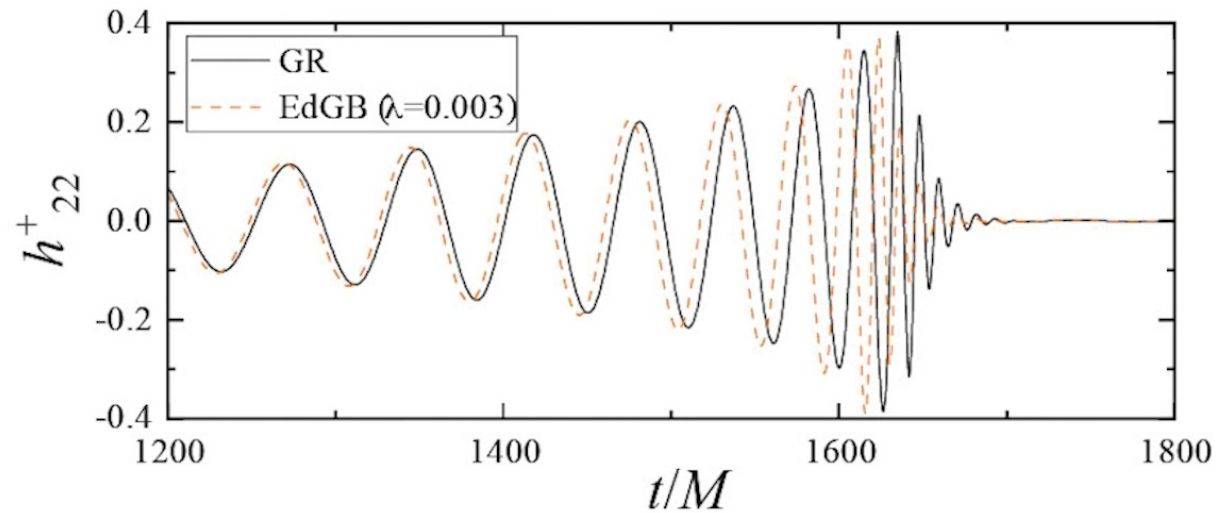
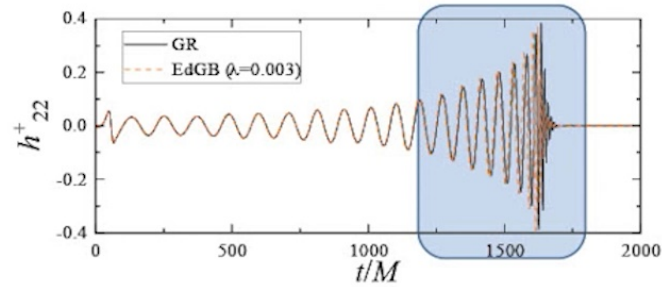


DD, Aresté Saló, Clough, Figueras, Yazadjiev, in preparation

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## Inspiral of two equal mass BHs – zoom of the merger



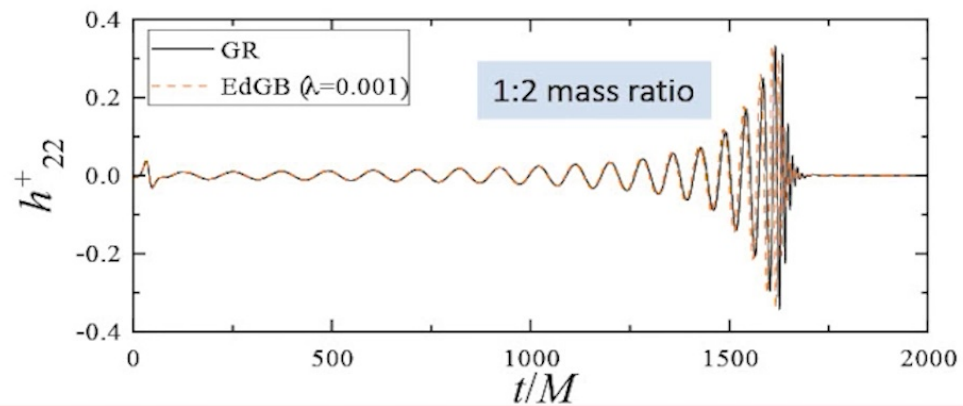
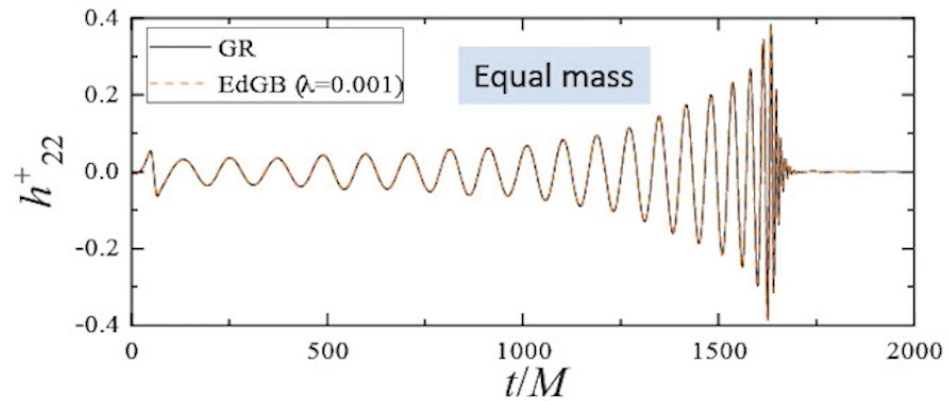
DD, Aresté Saló, Clough, Figueras, Yazadjiev, in preparation

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## Equal vs unequal mass

- Smaller coupling – no difference in the equal case

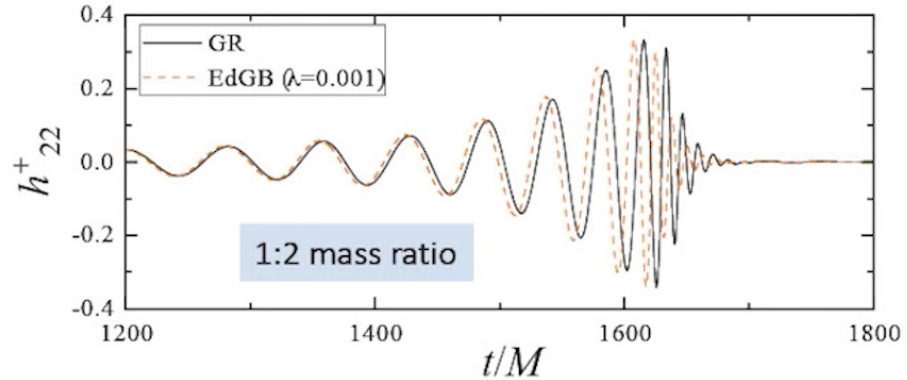
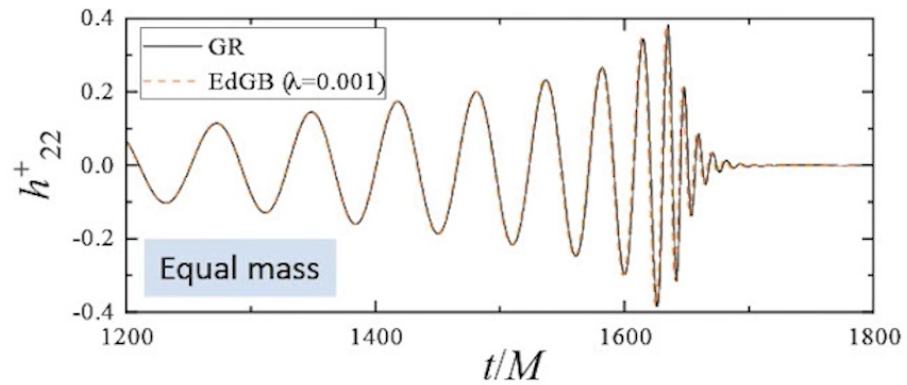


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## Equal vs unequal mass - zoomed

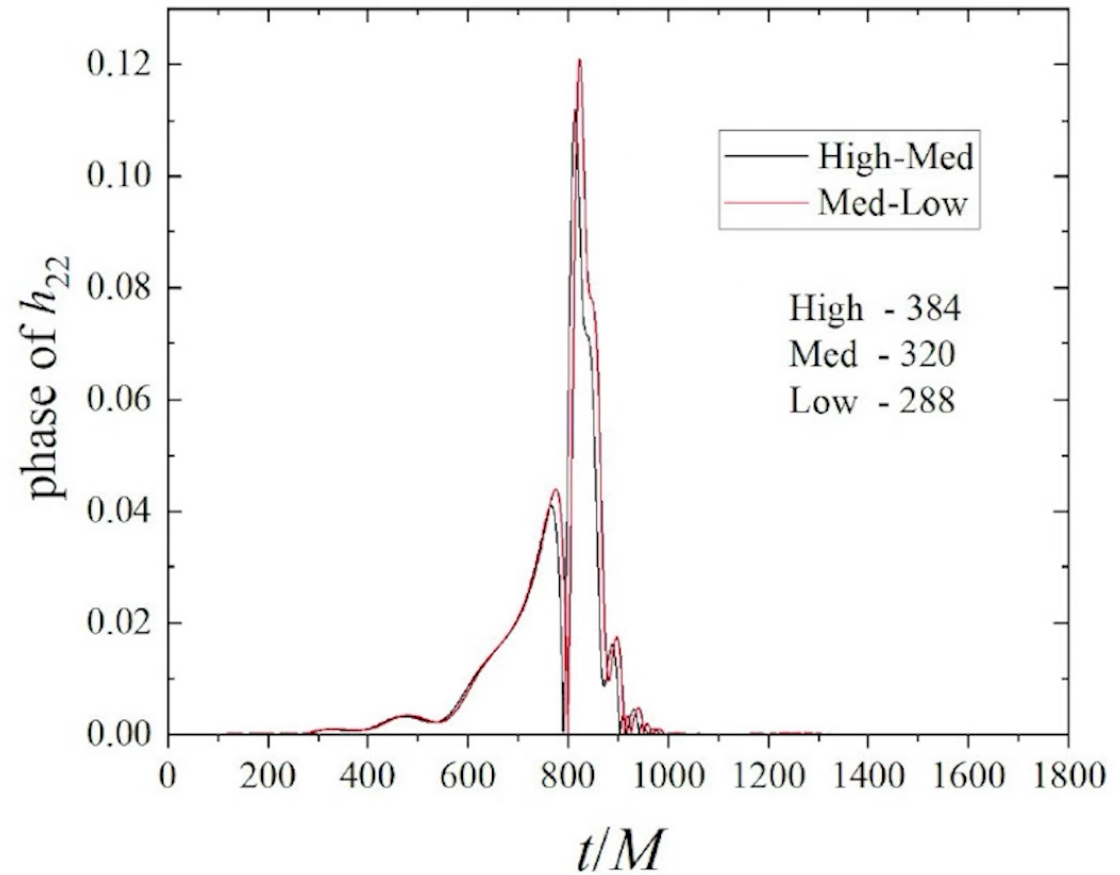
- Smaller coupling – no difference in the equal case



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## Unequal mass 1/3 – convergence



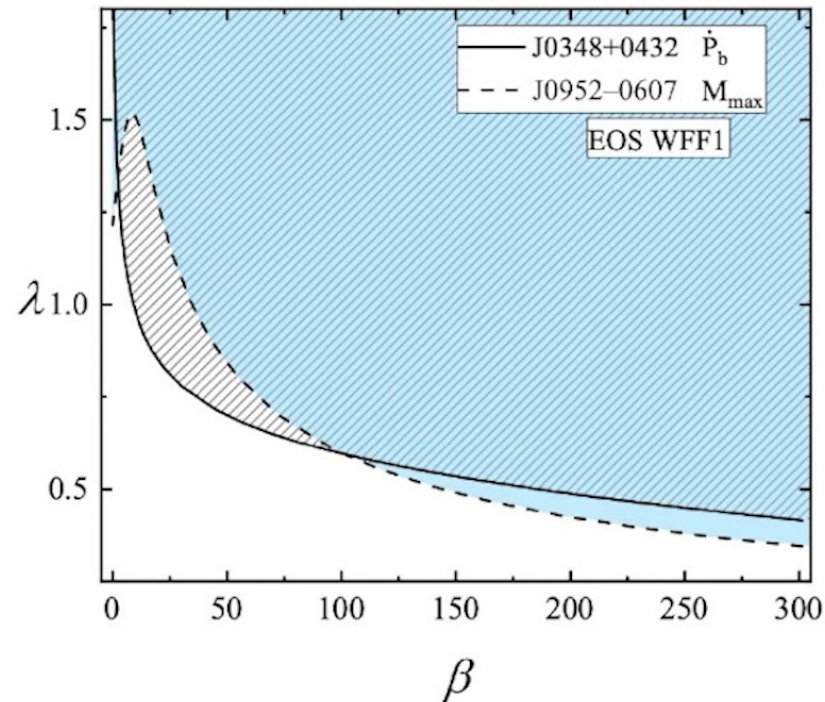
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## Binary pulsars – Einstein-dilaton-Gauss-Bonnet theory

- Constraints from J0348 + 0432 and the maximum mass of J0952 – 0607



- EdGB gravity with coupling

$$f(\varphi) = \frac{\lambda^2}{2\beta} e^{2\beta\varphi}$$

(leading order  $\sim \lambda^2\varphi$ )

- Constrains from **binary black hole mergers**  
 $\lambda < 3.01$

Yordanov, Staykov, DD, Yazadjiev (2024)

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## Conclusions

- Black holes in Horndeski theories offer a **very interesting venue of exploration.**
- Binary mergers can possess a **qualitatively new phenomenology**
- **Well-posedness** more subtle than in GR
- **Weak coupling condition** should always be obeyed

## Outlook

- A deeper investigation **of binary mergers** and their **astrophysics signatures**

**Final goal:** understand which exotics are physically motivated and constrain them via GWs.

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**THANK YOU!**



## Studying hyperbolicity

- **Principle symbol** – a matrix assembled by the coefficients in front of the leading (2<sup>nd</sup>) order derivative in the differential equation
- The contribution from **the Gauss-Bonnet sector to the principal part** of the evolution equations **only affects the physical modes** (purely gravitational modes and mixed scalar-gravitational ones).
- The eigenvalues from the purely gravitational sector lie on **the null cone of the effective metric** Real PRD (2021), Areste-Salo et al PRL (2022), PRD (2022)

$$g_{\text{eff}}^{\mu\nu} = g^{\mu\nu} - \Omega^{\mu\nu}$$

$$\Omega_{\mu\nu} = \lambda \nabla_{\mu} \nabla_{\nu} f(\varphi)$$

- **Hyperbolicity loss:** the normalized determinant of the effective metric (with respect to GR)  $< 0$  (East&Ripley PRL (2021), Areste-Salo et al PRL (2022), Hegade et al PRD (2023))



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