

Title: Contributed Talks

Speakers:

Collection: 50 Years of Horndeski Gravity: Exploring Modified Gravity

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# Horndeski goes non-linear

**Tessa Baker**

Institute of Cosmology & Gravitation  
University of Portsmouth

50 Years of Horndeski Gravity, 15-19th July 2024 @ PI



# Outline

- A little history
- Simulating non-linear structure in Horndeski gravity with Hi-COLA
- Selection of example results
- [If time permits: constraints from GWs]

## The Team

Ashim  
Sen Gupta



Bartolomeo  
Fiorini



Eric Kwon



Konstantin Leyde



Charlie Dalang



Anson Chen



Stefano Zazzera

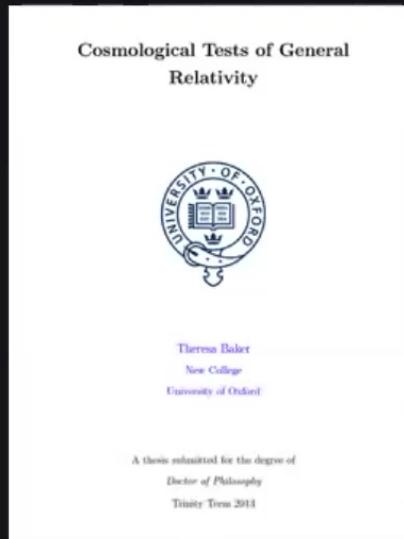


Elena Colangeli



+ Anna Balaudo  
(Leiden) & Krishna  
Naidoo (UCL)  
joining in Autumn

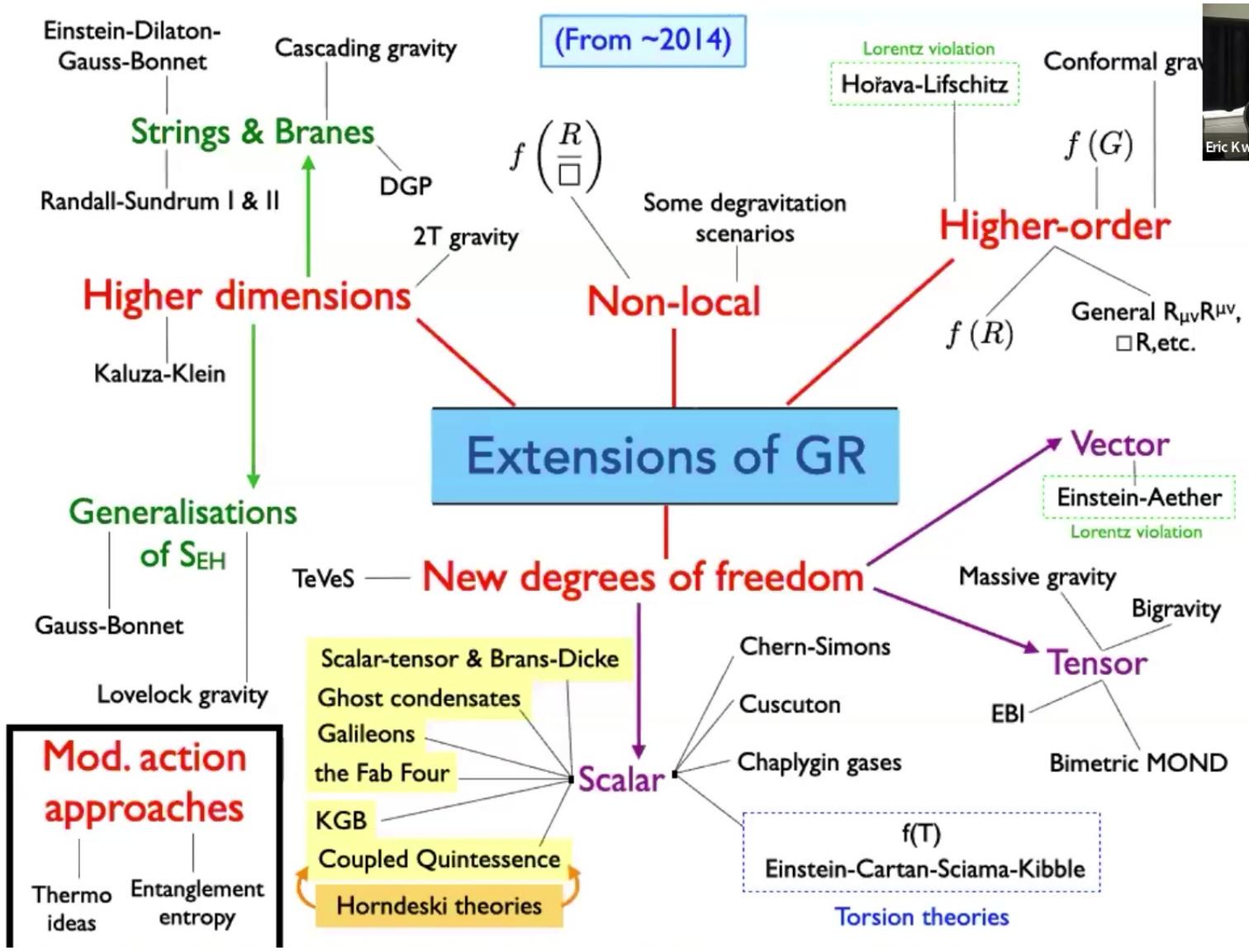
# Back to 2013...



I'd been building frameworks for unified modified gravity theories...

...mine had 22 (functional) coefficients!

Horndeski gravity seemed beautifully efficient.



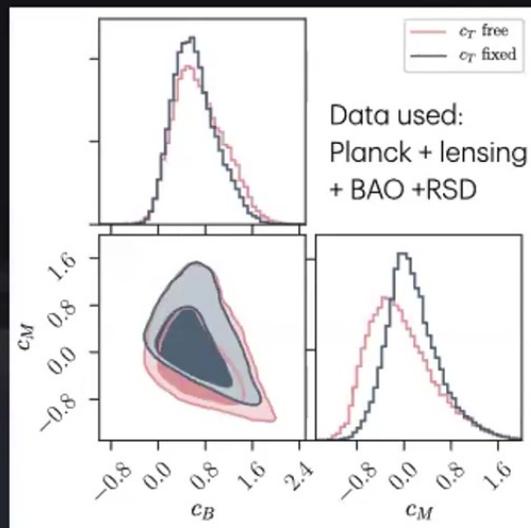
Eric Kwon

# A decade later...



## Large-scale Structure (LSS)

- Constraints in the linear regime:  
(Noller & Nicola 2018, see also Bellini+ 2016)



- Today we head into the **non-linear** regime.

## Gravitational Waves (GWs)



- Strong constraints on GW propagation speed
- Constraints on GW 'friction' with dark sirens
- Preparing for future LVK, LISA & 3G events

# Simulating Structure Formation in Horndeski Gravity

# The (luminal) Horndeski Action



For now, we'll deal with *luminal* Horndeski gravity, i.e. where  $c_{\text{GW}} = 1$ .

Motivated by GW170817 results (see Alessandra's talk).

These are **not** watertight...perhaps see Claudia's talk(?)

$$S = \int d^4x \sqrt{-g} [G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\square\phi] + S_M$$

where  $X =$  kinetic term of scalar field

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E.g.  $f(R)$  gravity uses these two

where  $X = \text{kinetic term of scalar field}$

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$$S = \int d^4x \sqrt{-g} [G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\square\phi] + S_M$$

E.g. cubic Galileon uses these two  
 $G_3$  is key in Vainshtein screening

where  $X =$  kinetic term of scalar field

# The Hi-COLA Code

Hi-COLA = Horndeski in COLA (thanks Hi-CLASS for the inspiration)

COLA = COmoving Lagrangian Acceleration (simulation technique, next slide)



Sales pitch: Hi-COLA is flexible, efficient and consistent.

And yours for \$0 from <https://github.com/Hi-COLACode/Hi-COLA> !!!

## i) Flexible:

No more hard-coded models – Hi-COLA takes any *user-specified* form of  $K$ ,  $G_3$  and  $G_4^*$ .

$$S = \int d^4x \sqrt{-g} [G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\square\phi] + S_M$$

\*but, Garbage In  $\Rightarrow$  Garbage Out.

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Ashim Sen Gupta  
(SymPy wizardry)



# The COLA Method

## ii) Efficient:

The COLA method hybridises Lagrangian Perturbation Theory (LPT) and a N-body code.

➔ Allows to trade no. of timesteps for small-scale accuracy *without* losing accuracy on large scales.



(Tassev, Zaldarriaga & Eisenstein, 2013)

Hi-COLA is built ontop of the COLA solver in the FML software library by Hans Winther.

Run times ~ 100-1000 faster than full N-body.

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<https://www.wintherscoming.no>

# Hi-COLA Components



ii) Consistent:

## Background

$$H, \dot{H}, \dot{\phi}, \Omega_M, \Omega_\phi$$

## 2LPT growth

Linear growth factor,  $D_1$

2nd-order growth,  $D_2$

## Screening

Inter-particle forces

$$F_{\text{tot}} = F_N + F_\phi$$

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Screening  $\rightarrow F_\phi$  must 'hide' on small scales / dense or in dense environments

(See also Martin's and Kazuya's talks on Wed.)

# Hi-COLA Components



Eric Kwon

Hi-COLA

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Currently Hi-COLA can do:

- Vainshtein screening
- K-mouflage screening

No Chameleon yet, send me your PhD applicants

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## Screening

Inter-particle forces

$$F_{\text{tot}} = F_N + F_\phi$$

## + Initial conditions

Back-scaled from  $z=0$  with appropriate growth factors

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No Chameleon yet, send me your PhD applicants

# Force Expression in Hi-COLA



- Start with: i) spherically symmetric mass distribution  
ii) Quasi-static approximation (drop time derivatives of metric potentials and  $\phi$ )
- The force experienced outside the mass is of the form: (details in arXiv 2209.01666)

$$F_{\text{tot}} = F_{\text{N}} \frac{\overbrace{G G_4}^{\text{Effective G}}}{G_{\text{N}}} \left[ 1 + \underbrace{\beta(z)}_{\text{Coupling}} \underbrace{S(z, \delta_m)}_{\text{Screening factor}} \right]$$

Gives overall strength of fifth force (function of time)      Modulates fifth force between 0 and 1 depending on environment

# Force Expression in Hi-COLA



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Effective  $G$  — NB: unscreened, modifies Newtonian force

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Gives overall strength of fifth force (function of time)      Modulates fifth force between 0 and 1 depending on environment

- Full expressions for  $\beta(z)$  and  $S(z, \delta_M)$  are given in terms of  $K, G_3, G_4$  evaluated at background level.
- $\beta(z)$  only depends on background solution, can pre-compute [ $\beta(z)$  gives the linear modification].
- $S(z, \delta_M)$  also depends on density field in simulation; we pre-compute what we can.

# Force Expression in Hi-COLA



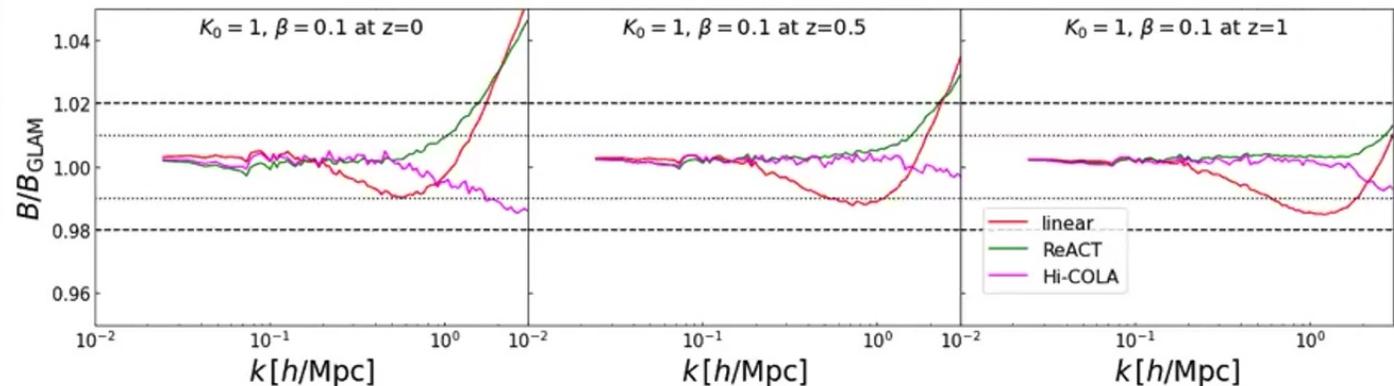
You **can** apply spherically symmetric force prescriptions in a N-body simulation.

First introduced in [Winther & Ferreira \(2015\)](#)

- Removes need to solve e.o.m. for  $\phi$  everywhere  $\rightarrow$  major speed-up ( $\sim$ same speed as LCDM).
- Introduces a well-characterised error on small scales ( $k \gtrsim 1$  h/Mpc)
- Recent improvements to push further ([Brando, Koyama & Winther 2023](#))

Bose+, arXiv 2406.1366

Recent comparison of approximate simulation methods with N-body results:  
(this plot is for K-mouflage)



# Code Diagram



Eric Kwon

Hi-COLA

$$= \int d^4x \sqrt{-g} [G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\square\phi] + S_M$$

## Hi-COLA front-end (~ seconds)

User specifies forms  
for  $K$ ,  $G_3$ ,  $G_4$

Symbolic algebra  
routines

Dynamic  
equations

Background solver

Background  
solutions

Outputs, e.g. matter  
power spectrum

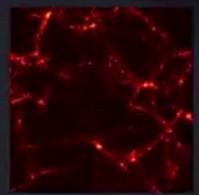
COLA simulation in  
MG theory

Forces between ptls

Density field

Fifth force expression

## Hi-COLA simulation ( $\approx O(1)$ hr on small cluster)

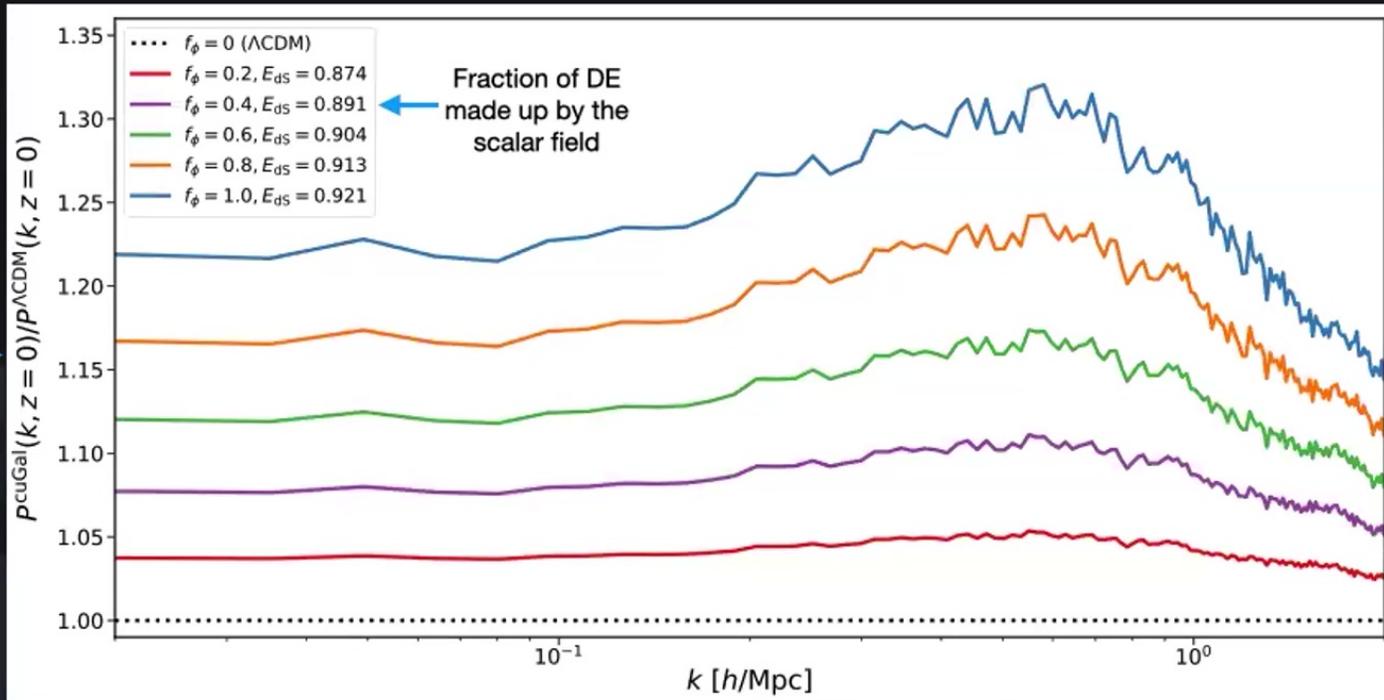




# Hi-COLA Results

# Results — Cubic Galileon

- $K \propto X$ ,  $G_3 \propto X$ ,  $G_4 = M_p^2$  ( $\Rightarrow$  no change to Newtonian forces).
- Validated against the N-body simulations of [Barreira et al. \(2014\)](#).

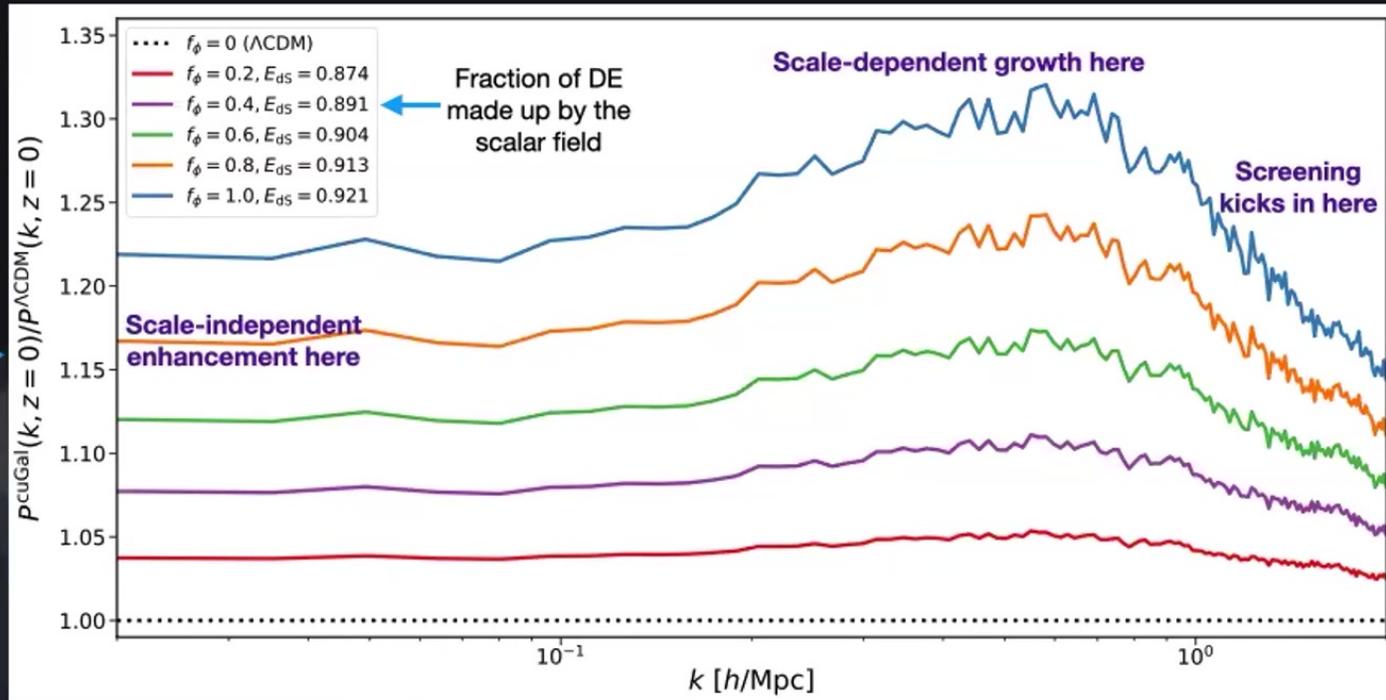


NB: matter power spectrum as a ratio to LCDM predictions, called the BOOST (B)

This plateau + bump shape is classic Vainshtein behaviour.

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# Results — Extended Shift Symmetric (ESS)



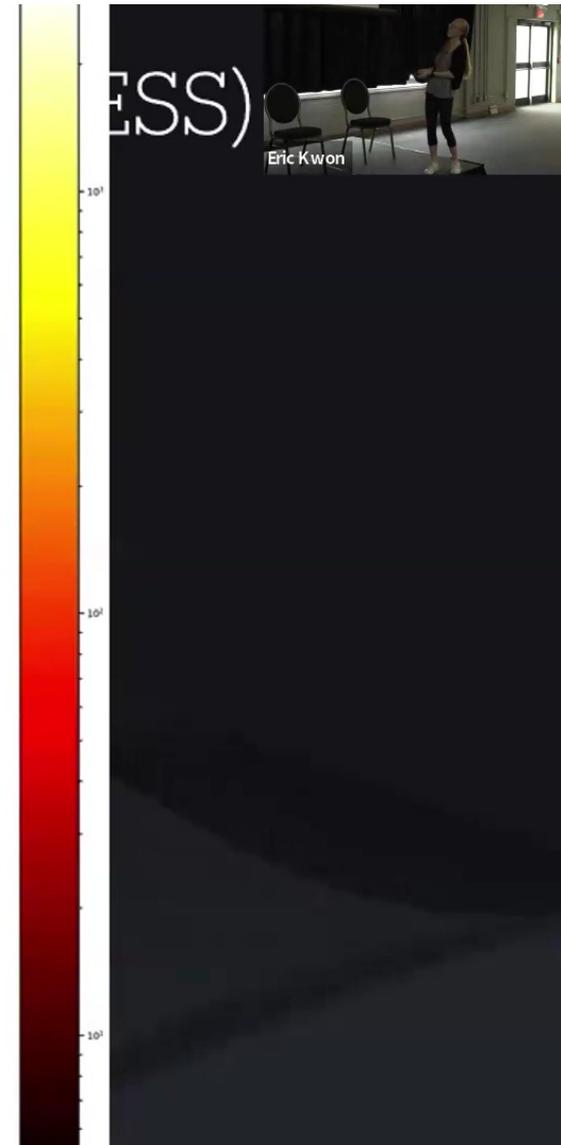
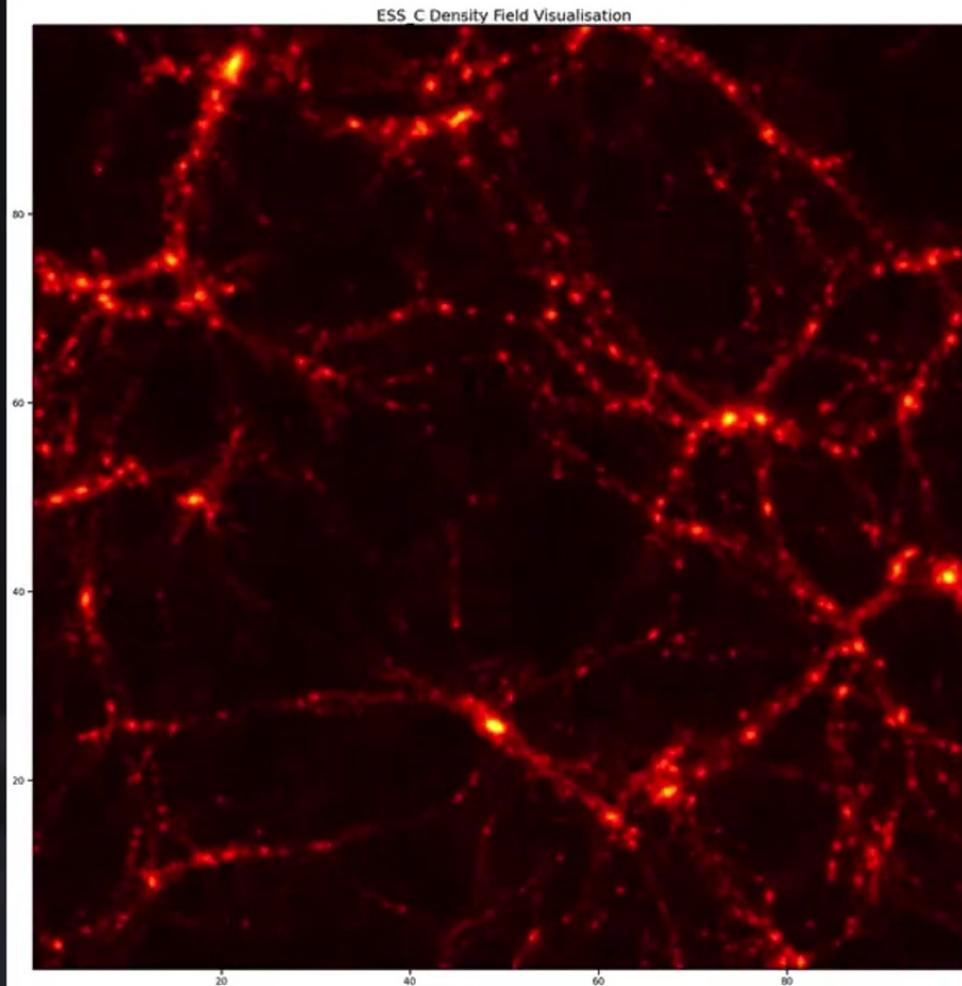
- $K = k_1 X + k_2 X^2$
- $G_3 = g_1 X + g_2 X^2$
- $G_4 = M_{\text{P}}^2$

Traykova et al. (2021)

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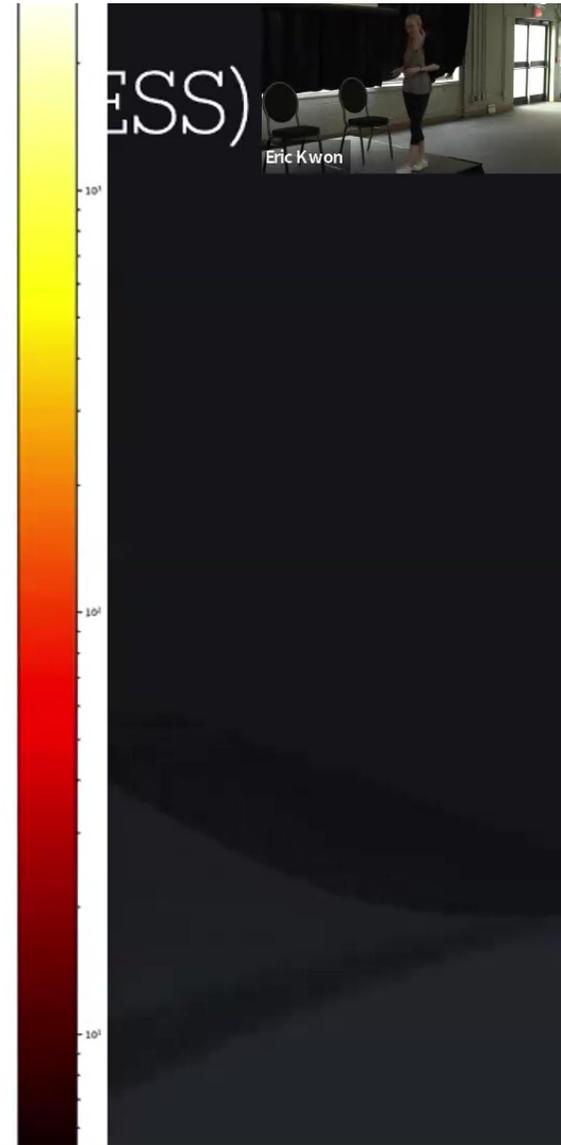
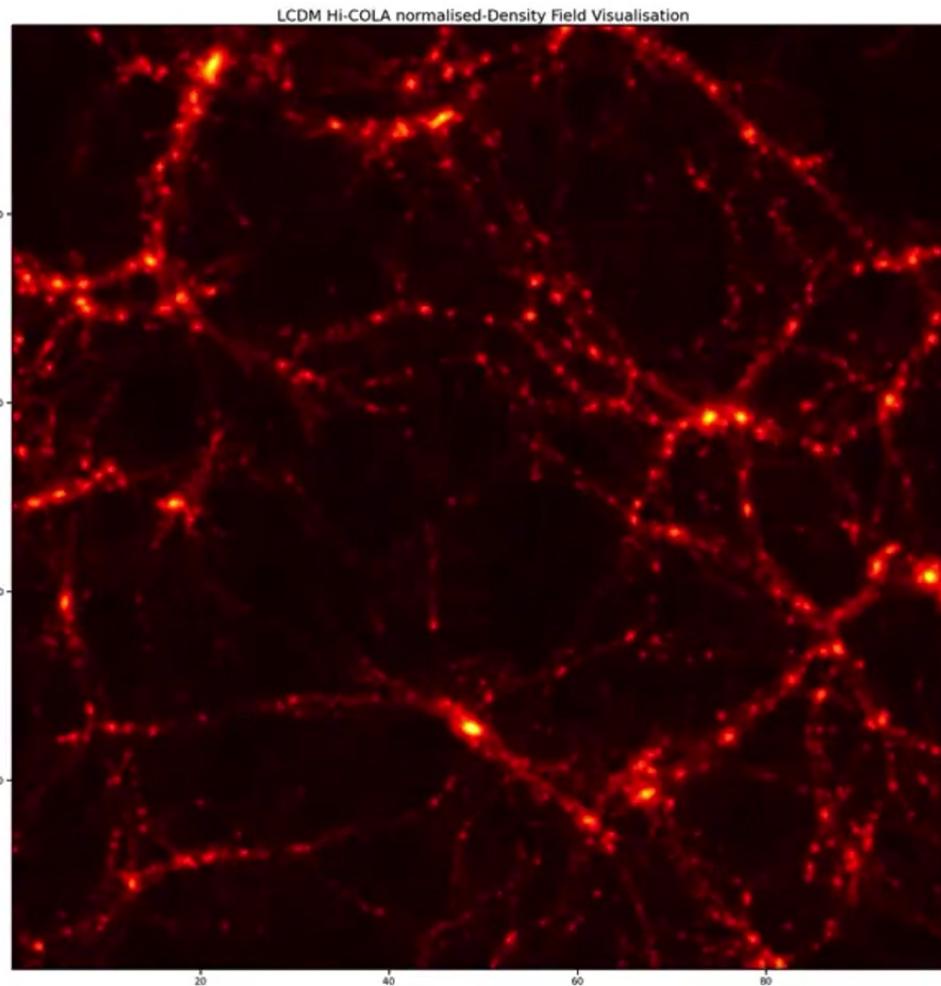
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- $G_4 = M_P^2$

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# K-mouflage screening



- Hi-COLA's force law works 'out the box' for Vainshtein screening, but K-mouflage requires more thought.

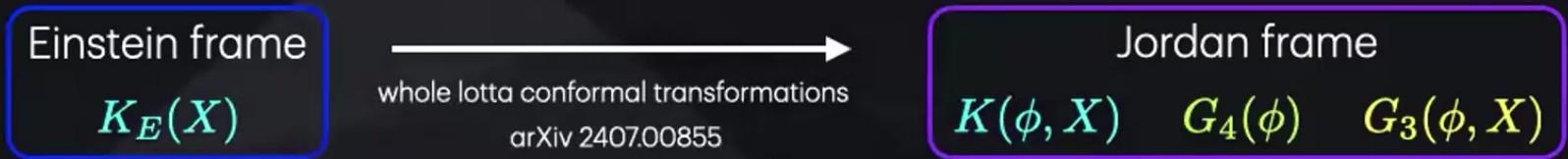
- We will study the example model: (Brax & Valageas 2014)

In the Einstein frame:  $K_E(X) = -1 + X + K_0 X^n$

Bart Fiorini  
(Conformal transformation hero)

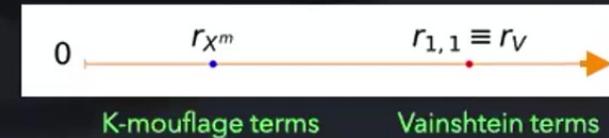


- But...Hi-COLA is a Jordan-frame code.



- This potentially mixes K-mouflage & Vainshtein behaviour! In the example theory we study here, this doesn't happen.

If present, Vainshtein terms dominate, as  $r_V \gg r_K$ .



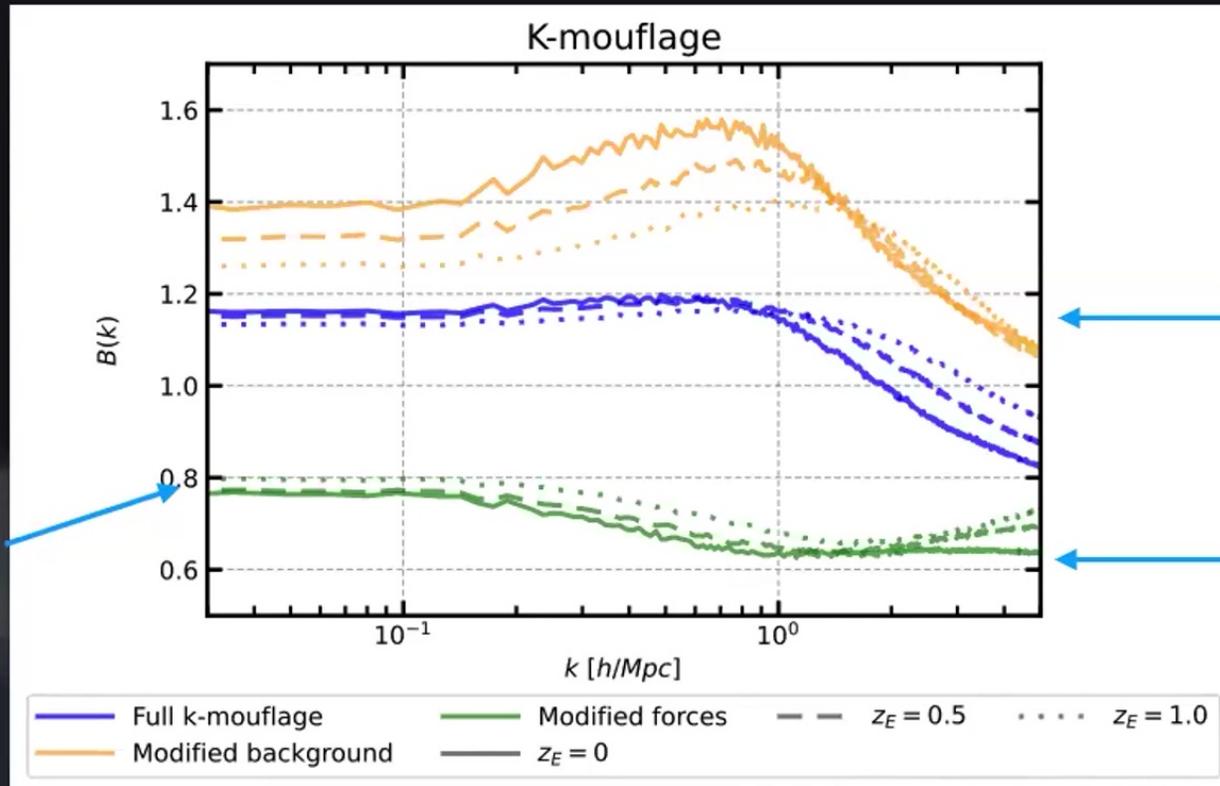
# Results — K-mouflage



- We can tease out the contributions from background and forces.

$$B(k) = \frac{P_{\text{kmou}}(z_J, k)}{P_{\text{LCDM}}(z_E, k)}$$

Suppression below LCDM due to modified  $G_{\text{eff}}$



Most of the small-scale drop off comes from the background

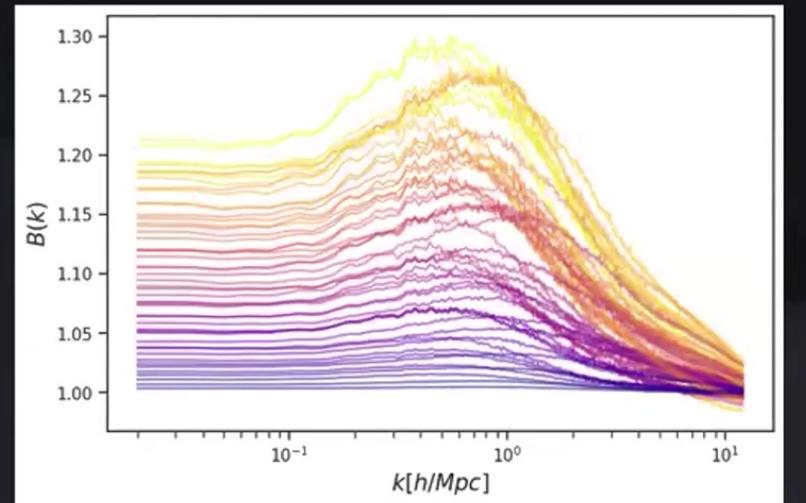
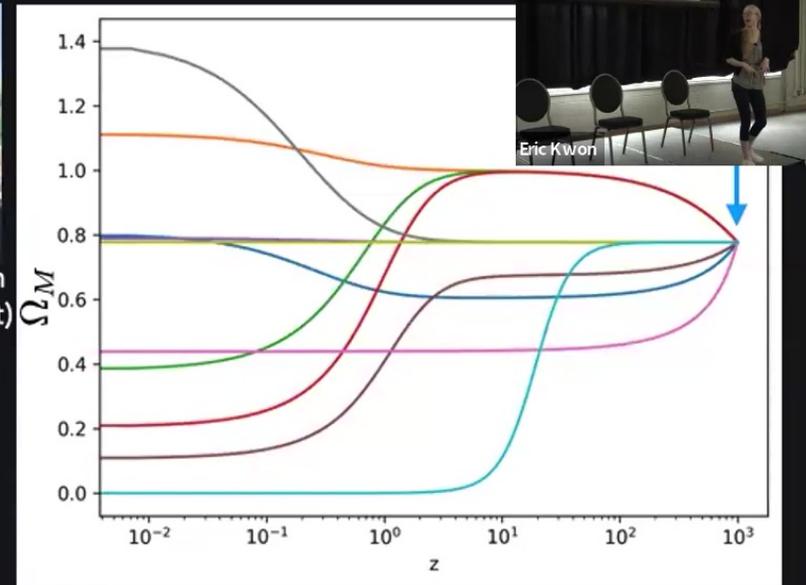
K-mouflage screening is weak on cosmological scales

# Next Steps

- Lots of fun throwing in random forms for  $K$ ,  $G_3$ ,  $G_4$  to explore Horndeski space.  
(See also Matteo Cataneo's talk?)
- For 'serious' models, we can generate lots of simulation boxes and train an emulator for the boost.  
Cubic Galileon emulator under construction by Carola Zanoletti (Newcastle), Nesar Ramachandra (Argonne) & the Hi-COLA team →
- Idea is to develop tools useable for clustering/lensing surveys, e.g. Rubin/LSST-DESC.
- Plenty more development work to: chameleon screening, interface with Hi\_Class, study haloes...



Duncan Bowden  
(summer student)





# Conclusions

- If you **don't** run LSS simulations:

▸ Horndeski Lagrangian  $\longrightarrow$  Nonlinear dark matter structure  $\xrightarrow{\text{work in progress}}$  Observations

It does *not* require mega resources.

- Tell us what Horndeski models are interesting for you.
- What motivations/bounds can guide us through the large space of Horndeski models?

- If you **do** run LSS simulations:



<https://github.com/Hi-COLACode/Hi-COLA>

Email [team.hicola@gmail.com](mailto:team.hicola@gmail.com)

Main publications: arXiv 2209.01666, arXiv 2407.00855

Comparison/validation exercise: 2406.13667