Title: Black holes in Horndeski theories

Speakers: Christos Charmousis

Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

Subject: Cosmology, Strong Gravity, Mathematical physics

Date: July 15, 2024 - 11:45 AM

URL: https://pirsa.org/24070030

Abstract:

We will discuss black hole solutions in Horndeski and beyond Horndeski theories. Starting from the no hair paradigm in GR we will elaborate on one of the first black hole solutions with secondary hair. We will then start by introducing stealth solutions, in other words GR metrics endowed with a non trivial scalar field, their regularity properties, shortcomings etc. We will then go on to construct black holes with primary scalar hair which can be regular black holes and modify usual GR static metrics. We will discuss their properties and the status of explicit stationary metrics to conclude.

Black holes with primary hair

Christos Charmousis

IJCLab-CNRS

50 years of Horndeski gravity : Exploring modified gravity

Collaborators : E. Babichev [1312.3204 [gr-qc]]

A. Bakopoulos, N. Lecoeur, P. Kanti [2203.14595 [gr-qc]], T. Nakas [2310.11919 [gr-qc]]

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

cnrs dépasser les frontières

Horndeski and Fab 4

arXiv:...2000v2 [hep-th] 16 Jun 2011

General second order scalar-tensor theory, self tuning, and *the Fab Four*

Christos Charmousis,[1,2] Edmund J. Copeland,[3] Antonio Padilla,[3] and Paul M. Saffin[3]

[1]*LPT, CNRS UMR 8627, Université Paris Sud-11, 91405 Orsay Cedex, France.*
[2]*LMPT, CNRS UMR 6083, Université François Rabelais-Tours, 37200, France*
[3]*School of Physics and Astronomy, University of Nottingham, Nottingham NG7 2RD, UK*

(Dated: June 17, 2011)

Starting from the most general scalar-tensor theory with second order field equations in four dimensions, we establish the unique action that will allow for the existence of a consistent self-tuning mechanism on FLRW backgrounds, and show how it can be understood as a combination of just four base Lagrangians with an intriguing geometric structure dependent on the Ricci scalar, the Einstein tensor, the double dual of the Riemann tensor and the Gauss-Bonnet combination. Spacetime curvature can be screened from the net cosmological constant at any given moment because we allow the scalar field to break Poincaré invariance on the self-tuning vacua, thereby evading the Weinberg no-go theorem. We show how the four arbitrary functions of the scalar field combine in an elegant way opening up the possibility of obtaining non-trivial cosmological solutions.

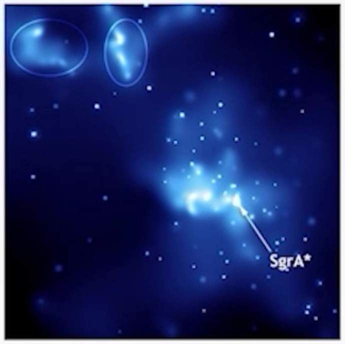
In a little known paper published in 1974, G.W. Horndeski presented the most general scalar-tensor theory with second order field equations in four dimensions [1]. Given the amount of research into modified gravity over the last ten years or so (see [2] for a review), it seems appropriate to revisit Horndeski's work. Scalar tensor models of modified gravity range from Brans-Dicke gravity [3] to the recent models [4, 5] inspired by galileon theory [6], the latter being examples of higher order scalar tensor Lagrangians with second order field equations. Each of these models represent a special case of Horndeski's panoptic theory.

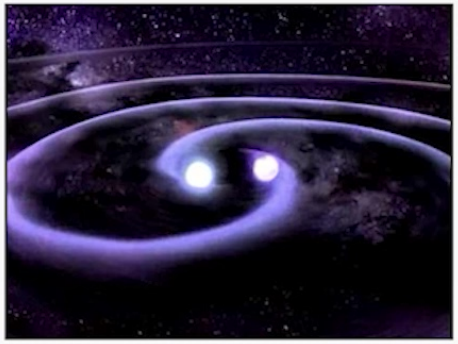
In this letter, we study Horndeski's theory on FLRW backgrounds. In particular we ask whether or not there are subclasses of [1] giving a viable self-tuning mecha-

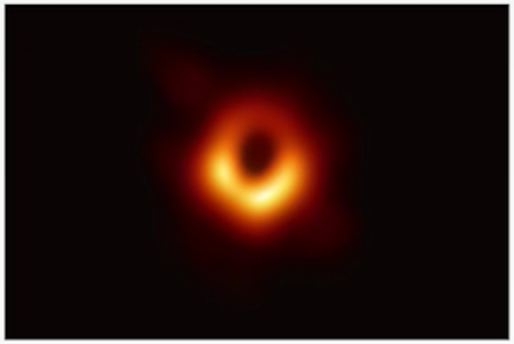
where $\hat{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss-Bonnet combination, $\varepsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor and $P^{\mu\nu\alpha\beta} = -\frac{1}{4}\varepsilon^{\mu\nu\lambda\sigma}\, R_{\lambda\sigma\gamma\delta}\, \varepsilon^{\alpha\beta\gamma\delta}$ is the double dual of the Riemann tensor [9].

Our results prove that any self tuning scalar-tensor theory (satisfying EP) must be built from the Fab Four. The weakest of the four is Ringo since this cannot give rise to self-tuning without "a little help from [its] friends", John and Paul. When this is the case, Ringo does have a non-trivial effect on the cosmological dynamics but does not spoil self-tuning. George also has difficulties in going solo: when V_{george} = const., we just have GR and no self-tuning, whereas when $V_{george} \neq$ const., we have Brans-Dicke gravity with $w = 0$, which does self-tune but is immediately ruled out by solar system constraints.

Breakthrough in observational data concerning compact objects

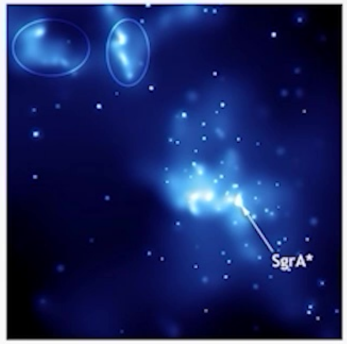


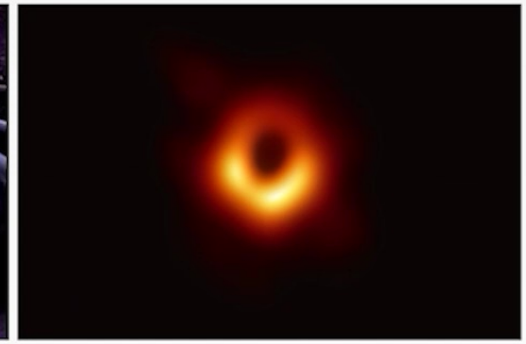




- GW signals from compact binaries : for example the GW170817 neutron star merger, GW190814 and the large mass secondary at $2.59^{+0.08}_{-0.09}\ M_0$
- Array of radio telescopes, EHT : for example image of M87 black hole with its light ring
- GRAVITY VLT: observation of star trajectories orbiting SgrA central black hole : orbit characteristics give us tests of strong gravity which get better as precision increases.
- What is the maximal mass of neutron stars ? What is their equation of state? How rapid can their rotation be before instability?
- eg.: Is the compact secondary of GW190814 the heaviest neutron star or the lightest astrophysical black hole? How does this fit with GR?
- eg. Can we find pulsars in the vicinity of SgrA and follow them around the central black hole?

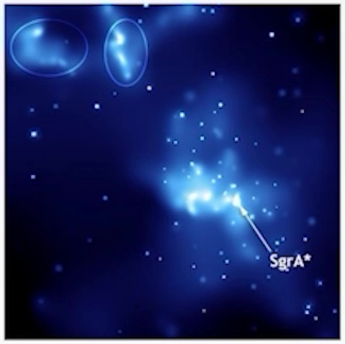
Breakthrough in observational data concerning compact objects

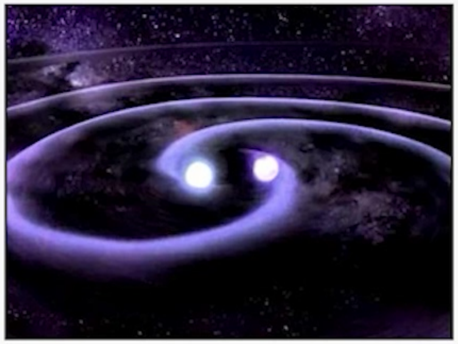




- GW signals from compact binaries : for example the GW170817 neutron star merger, GW190814 and the large mass secondary at $2.59^{+0.08}_{-0.09}\ M_{0}$
- Array of radio telescopes, EHT : for example image of M87 black hole with its light ring
- GRAVITY VLT: observation of star trajectories orbiting SgrA central black hole : orbit characteristics give us tests of strong gravity which get better as precision increases.
- Can we find alternatives to GR black holes and neutron stars as precise rulers of departure from GR?

Plan and keywords





- No hair paradigm : a black hole with secondary hair
- Horndeski theories and beyond... as a measurable departure from GR
- Stealth black holes and the role of spacetime geodesics as scalar hair
- Constructing black holes with primary hair
- Concluding properties, stability and stationarity?

Black holes... have no hair

In GR black holes are "unique" and characterised by a finite number of charges, essentially, J, M

They are relatively simple solutions-they have no hair, eg. quadrapole is fixed, $Q^2 = -J^2/M$

- During collapse, black holes lose their hair and relax to some stationary state of large symmetry. They are (mostly) vacuum solutions of Einstein's eqs, $G_{\mu\upsilon} = 0$
- In GR : Static and spherically symmetric is Schwarzschild solution parametrised my mass M
- Unique rotating black hole in GR : Kerr solution parametrised by M mass and a angular momentum per unit mass
- We will seek black holes with primary or secondary hair in S-T theories.

Hairy black hole Example: the BBMB solution

- Consider a *conformally coupled scalar field* φ:

$$S[g_{\mu\nu}, \varphi, \psi] = \int_M \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}\partial_\alpha\varphi\partial^\alpha\varphi - \frac{1}{12}R\varphi^2\right) d^4x + S_m[g_{\mu\nu}, \psi]$$

- **Invariance of the EOM of φ under the local conformal transformation**

$$g_{\alpha\beta} \mapsto \tilde{g}_{\alpha\beta} = \Omega^2 g_{\alpha\beta}$$
$$\varphi \mapsto \tilde{\varphi} = \Omega^{-1}\varphi$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair. The BBMB solution [N. Bocharova et al.-70 , J. Bekenstein-74]
- Static and spherically symmetric solution

$$ds^2 = -\left(1 - \frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1 - \frac{m}{r}\right)^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

with **secondary** scalar hair

$$\varphi = \sqrt{\frac{3}{4\pi G}}\,\frac{m}{r - m}$$

- Geometry is that of an extremal RN. The scalar field is **unbounded** at $(r = m)$.
- To go further we have to consider higher order theories

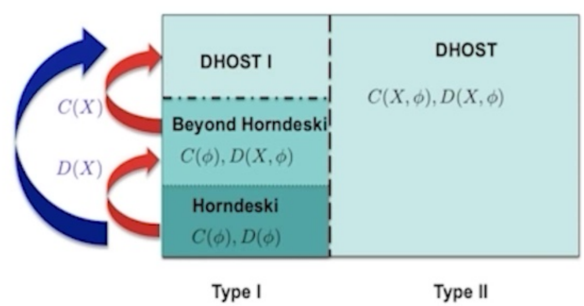
Scalar tensor theories : a robust measurable departure from GR

Simplest modified gravity theory with a single scalar degree of freedom

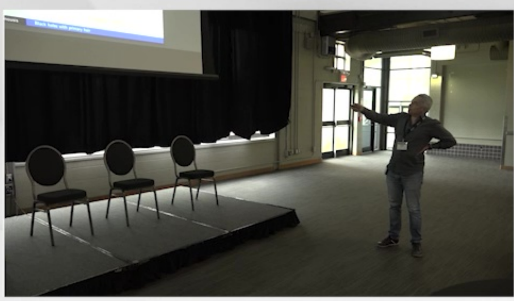
BD theory,..., Horndeski (or generalised Galileon [Deffayet, Deser, Esposito-Farèse,...]),..., beyond Horndeski,..., DHOST theories [Achour, Crisostomi, Koyama, Langlois, Noui, et.al.]

- *Nothing fundamental* about ST theories, they are just measurable departures from GR which are robust with a single additional degree of freedom
- They are limits of more complex fundamental theories (massive gravity, braneworld models, EFT from string theory, Lovelock theory etc.)
- Horndeski is parametrized by 4 functions of scalar and its kinetic energy, $G_i = G_i(\varphi, X)$ with $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$
  BBMB example : $G_i = G_i(\varphi, X)$, eg.: $G_2 = X$, $G_4 = 1 - \frac{1}{12}\varphi^2$.
- Beyond Horndeski or DHOST are parametrised by two more functions corresponding to conformal and dysformal transformations of Horndeski

$$g_{\mu\nu} \dashrightarrow \tilde{g}_{\mu\nu} = C(\varphi, X)g_{\mu\nu} + D(\varphi, X)\nabla_\mu\varphi\nabla_\nu\varphi$$



[Langlois, 2018]



Scalar tensor theories : a robust measurable departure from GR

Simplest modified gravity theory with a single scalar degree of freedom

BD theory,..., Horndeski (or generalised Galileon [Deffayet, Deser, Esposito-Farèse,...]),..., beyond Horndeski,..., DHOST theories [Achour, Crisostomi, Koyama, Langlois, Noui, et.al.]

$$S_H = d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$L_2 = G_2(\varphi, X),$$
$$L_3 = -G_3(\varphi, X)\Box\varphi,$$
$$L_4 = G_4(\varphi, X)R + G_{4X}\left[(\Box\varphi)^2 - (\nabla_\mu\nabla_\nu\varphi)^2\right],$$
$$L_5 = G_5(\varphi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\varphi - \frac{G_{5X}}{6}\left[(\Box\varphi)^3 - 3\Box\varphi(\nabla_\mu\nabla_\nu\varphi)^2 + 2(\nabla_\mu\nabla_\nu\varphi)^3\right]$$

Stealth solutions with secondary black hole hair

Scalar tensor theories

Example in Horndeski with $G_2 = \Lambda + \eta X$, $G_4 = 1 + 2\beta X$

$$S = \int d^4x \sqrt{-g}\left[R - 2\Lambda_b - X + \beta G^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right],$$

- Kinetic term is $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ $(= -\frac{1}{2}g^{\mu\nu}\varphi_\mu\varphi_\nu)$.
- The theory has global shift and parity symmetry. Conserved current $\nabla_\mu J^\mu = 0$ for the scalar field equation
- One simple (stealth) solution reads

$$f = h = 1 - \frac{2\mu}{r} + \frac{\eta}{3\beta}r^2$$

,

$$\varphi = qt \pm \int dr \, \frac{q}{f}\sqrt{1-f}$$

with secondary hair $q^2 = \frac{\zeta\eta + \Lambda_b\beta}{\beta\eta}$ relating the action couplings.

Shift symmetry allows for linear time dependence.

The associated energy-momentum tensor for the scalar must have the same symmetries as the metric

C. Charmousis | Black holes with primary hair

Stealth solutions with secondary black hole hair

Scalar tensor theories

Example in Horndeski with $G_2 = \Lambda + \eta X$, $G_4 = 1 + 2\beta X$

$$S = \int d^4x \sqrt{-g}\left[R - 2\Lambda_b - X + \beta G^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right],$$

- Kinetic term is $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \; (= -\frac{1}{2}g^{\mu\nu}\varphi_\mu\varphi_\nu)$.
- The theory has global shift and parity symmetry. Conserved current $\nabla_\mu J^\mu = 0$ for the scalar field equation
- One simple (stealth) solution reads

$$f = h = 1 - \frac{2\mu}{r} + \frac{\eta}{3\beta}r^2$$

,

$$\varphi = qt \pm \int dr \frac{q}{f}\sqrt{1-f}$$

with secondary hair $q^2 = \frac{\zeta\eta + \Lambda_b\beta}{\beta\eta}$ relating the action couplings.

- $X = g^{\mu\nu}\varphi_\mu\varphi_\nu = -\frac{q^2}{h} + q^2\frac{f(1-h)}{h^2} = -q^2$ is constant and stealth solutions are quite generic [Kobayashi, Tanahashi]
- Scalar field is regular at the (future) event horizon for all q.
  $\varphi = qv - q\int \frac{dr}{1+\sqrt{1-f}}$, $dv = dt + \frac{dr}{f}$ in advanced EF coordinates.
- The scalar φ is associated to the regular HJ potential for geodesics of the spacetime [Gregory]

C. Charmousis Black holes with primary hair

Stealth solutions with secondary black hole hair

Scalar tensor theories

Example in Horndeski with $G_2 = \Lambda + \eta X$, $G_4 = 1 + 2\beta X$

$$S = \int d^4x \sqrt{-g}\left[R - 2\Lambda_b - X + \beta G^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right],$$

- Kinetic term is $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ $(= -\frac{1}{2}g^{\mu\nu}\varphi_\mu\varphi_\nu)$.
- The theory has global shift and parity symmetry. Conserved current $\nabla_\mu J^\mu = 0$ for the scalar field equation
- One simple (stealth) solution reads

$$f = h = 1 - \frac{2\mu}{r} + \frac{\eta}{3\beta}r^2$$

,

$$\varphi = qt \pm \int dr\,\frac{q}{f}\sqrt{1-f}$$

with secondary hair $q^2 = \frac{\zeta\eta + \Lambda_b\beta}{\beta\eta}$ relating the action couplings.

- Using the integrability of geodesics for Kerr spacetime and properties of DHOST $c_g = 1$ theories we can construct stealth Kerr solutions [CC, Crisostomi, Gregory, Stergioulas]
- via disformal transformation we can construct disformal Kerr metrics which are non trivial stationary regular geometries [Anson, Babichev, CC, Hassaine]
- The geometries are interesting but scalar fluctuations are strongly coupled

From stealth to primary hair [Bakopoulos, CC, Kanti, Lecoeur, Nakas]

- Consider shift symmetric beyond Horndeski theory parametrised G_2, G_4, F_4 as functions of X
- The theory reads,

$$S[g_{\mu\nu},\varphi] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\Big\{G_2(X) + G_4(X)R + G_{4X}\left[(\Box\varphi)^2 - \varphi_{\mu\nu}\varphi^{\mu\nu}\right] + F_4(X)\,\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma}{}_{\sigma}\,\varphi_\mu\varphi_\alpha\varphi_{\nu\beta}\varphi_{\rho\gamma}\Big\}.$$

- We seek spherically symmetric solutions

$$ds^2 = -h(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \qquad \varphi = qt + \psi(r),$$

- Once we consider the substitution,

$$Z(X) = 2XG_{4X} - G_4 + 4X^2F_4,$$

- we get an exactly solvable (integrable) system,

$$\frac{f}{h} = \frac{\gamma^2}{Z^2}$$

$$r^2(G_2Z)_X + 2(G_4Z)_X\left(1 - \frac{q^2\gamma^2}{2Z^2X}\right) = 0$$

$$2\gamma^2\left(hr - \frac{q^2r}{2X}\right)' = -r^2G_2Z - 2G_4Z\left(1 - \frac{q^2\gamma^2}{2Z^2X}\right) + \frac{q^2\gamma^2X'r}{ZX^2}(2XG_{4X} - G_4)$$

Primary hair black holes

[A. Bakopoulos, C.Cs, P. Kanti, N. Lecoeur., T. Nakas, 2023]

- Take $Z = \gamma$ therefore $f = h$ and consider analytic functions $G_2, G_4 - 1 \sim X^n$ with $n \in N$
  [Baake et al., Bakopoulos et al.]
- For $n = 2$ and $Z = \gamma$. We have,

$$G_2 = -\frac{8\eta}{3\lambda^2} X^2, \quad G_4 = 1 - \frac{4\eta}{3} X^2, \quad F_4 = \eta,$$

  where $\eta, \lambda > 0$ are coupling constants of the theory
- We seek spherically symmetric solutions

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \mathrm{d}\Omega^2, \qquad \varphi = qt + \psi(r),$$

- We find X which is not constant but regular everywhere and the scalar as a function of the metric :

$$X = \frac{q^2/2}{1 + (r/\lambda)^2} \quad \psi'(r)^2 = \frac{q^2}{f^2(r)} \left[1 - \frac{f(r)}{1 + (r/\lambda)^2}\right],$$

  Note that when $q = 0$ the scalar is trivial. X is constant at $r = 0$ and goes to zero monotonically at asymptotic infinity. Finally we solve for the metric,
-

$$h(r) = f(r) = 1 - \frac{2M}{r} + \eta q^4 \left[\frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1 + (r/\lambda)^2}\right],$$

  $q = 0$: Schwarzschild, $q \neq 0$: departure from Schwarzschild. The solution has two independent integration constants : M and q
- Scalar is always regular on the horizon independently of the value of q

Horizons

[A. Bakopoulos, C. Charmousis, P. Kanti, N. L., T. Nakas, 2023]

Far away the black hole behaves much like RN but with the scalar playing the role of EM charge

$$f(r) = 1 - \frac{2M}{r} + \eta q^4 \left(\frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1+(r/\lambda)^2} \right)$$

$$= 1 - \frac{2M}{r} + 2\lambda^2 \frac{\eta q^4}{r^2} + O\left(\frac{1}{r^4}\right), \quad r \to \infty$$

while close to the origin we get,

$$f(r) = 1 - \frac{2M - \pi \eta q^4 \lambda/2}{r} - \frac{2\eta q^4 r^2}{3\lambda^2} + O\left(r^4\right).$$

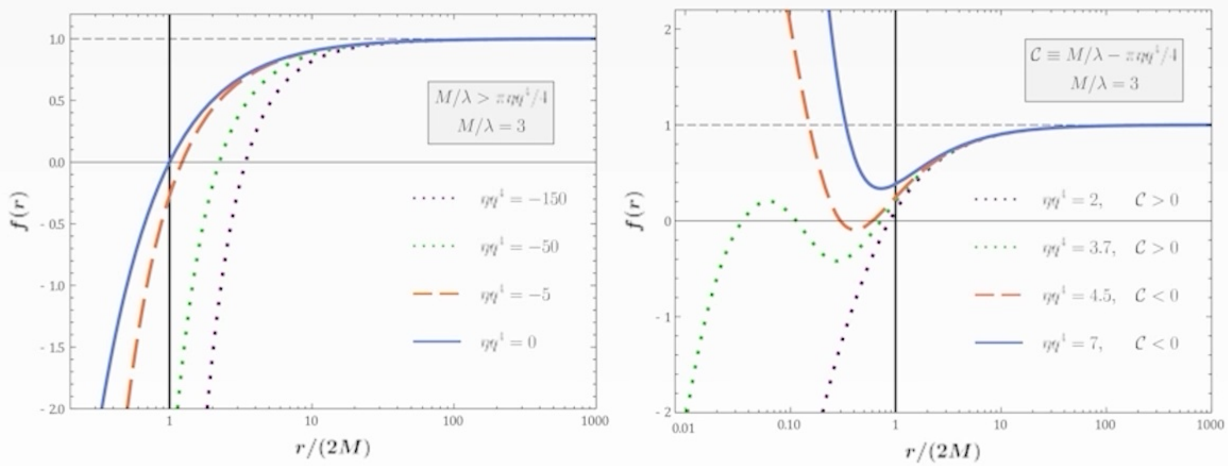


Figure: Left: $\eta < 0$, unique horizon greater than the Schwarzschild radius $r_S = 2M$. Right: $\eta > 0$, one, two, three or zero horizons, horizon smaller than Schwarzschild.

Regular spacetime (black hole or soliton)

For $M = \pi\eta q^4\lambda/4$, the central singularity disappears and all curvature invariants become infinitely regular:

$$f(r) = 1 - \frac{4M}{\pi\lambda}\left(\frac{\arctan(r/\lambda)}{r/\lambda} - \frac{1}{1+(r/\lambda)^2}\right)$$

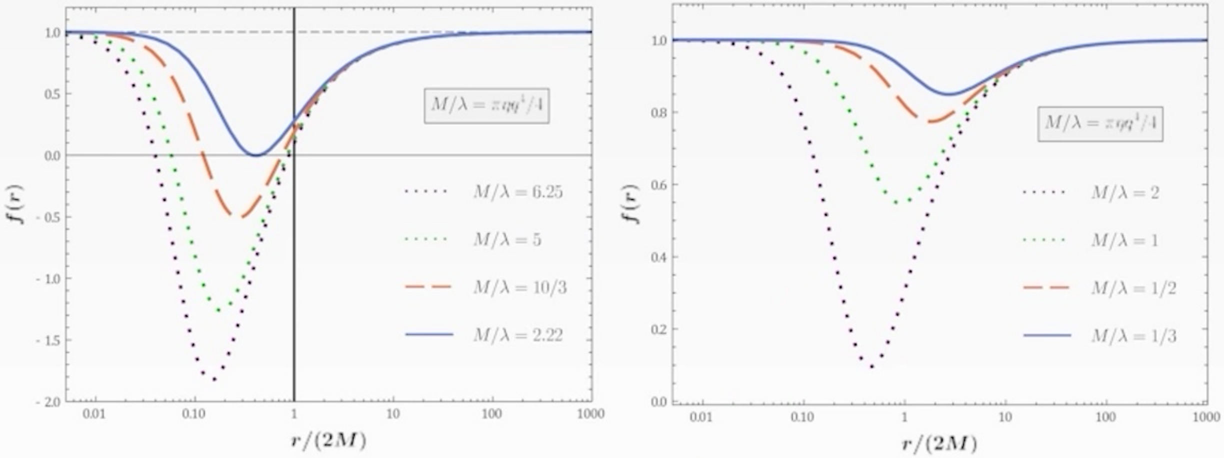


Figure: Left: Regular BH solutions. Right: regular solitonic solutions.

We have a regular black hole or soliton

Regular scalar field and other special cases

- The scalar field, $\varphi = qt + \psi(r)$, is regular at all future (or past) event or inner horizons.
- Far away the scalar is linear in the v coordinate. The action is everywhere regular. The norm associated to the conserved current of shift symmetry is also everywhere regular.
- Scalar hair q is a Noether charge associated to $\nabla_\mu J^\mu = 0$ [Bakopoulos, Chatzifotis, Nakas]
  For $n = 2$ we have $Q = \int_\Sigma d^3x \sqrt{-\gamma}\, n_\mu J^\mu$ and $Q \sim q^3$
- Particular cases that stand out are $n = 1$ and $n = 1/2$.
- for $n = 1$ we have a canonical kinetic term $G_2 \sim X$. The solution is locally asymptotically flat. This is a typical geometry as that of a gravitational monopole. The solution is again regular at the center but one has to find a different Noether charge (for example).
- for $n = 1/2$ we have absence of an F_4 term i e., we have pure Horndeski theory $G_2 = 2\eta\sqrt{X}$, $G_4 = 1 + \lambda X$. Solution is then stealth Schwarzschild but G_4 is only C^1 at $X = 0$

Conclunding remarks

- We have constructed static black holes with primary hair. The solutions are generic and regular in shift symmetric DHOST theories. Key is the presence of a time dependent scalar, that provides scalar charge and renders the solutions regular.
- Solutions become regular at the origin for a certain scalar charge without any fine tuning of theory parameters.
- Varying $Z = Z(X)$ from a constant to a non trivial function of X we get from $f = h$ to f /= h solutions. This defines a class of equivalence of solutions related modulo Z (or modulo a disformal transformation). For example $D = X$ takes our $n = 2$ solution from beyond Horndeski to a pure Horndesky theory. The solution has f /= h but is again a black hole with primary hair.
- Study perturbations of the black holes and determine windows of stability.
- Axial perturbations are well behaved with a well defined Schrodinger potential as long as theory functions are C^2. Scalar perturbations are more involved but there is real hope here to understand how we bifurcate from GR (work in progress)
- Can we find rotating counterparts to these solutions? Is there some geometrical interpretation for the precise form of X bifurcating in between timelike and null geodesics