Title: Horndeski Gravity in Cosmology

Speakers: Alessandra Silvestri

Collection/Series: 50 Years of Horndeski Gravity: Exploring Modified Gravity

Subject: Cosmology, Strong Gravity, Mathematical physics

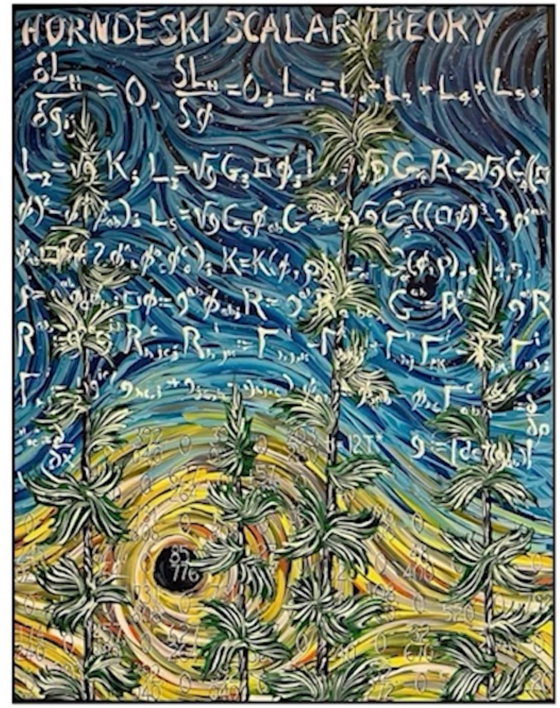
Date: July 15, 2024 - 9:30 AM

URL: https://pirsa.org/24070028

Abstract:

In this talk I will review how Horndeski gravity made its way in Cosmology and why it became a very popular framework for tests of gravity on cosmological scales.

Horndeski Gravity in Cosmology



Alessandra Silvestri
Leiden University

50 Years of Horndeski Gravity: Exploring Modified Gravity, Perimeter Institute & University of Waterloo, July 15-19 2024

Dear Gregory, it's been a true pleasure!

International Journal of Theoretical Physics (2024) 63:38
https://doi.org/10.1007/s10773-024-05558-2

REVIEW

50 Years of Horndeski Gravity: Past, Present and Future

Gregory W. Horndeski[1] · Alessandra Silvestri[2]

Received: 15 January 2024 / Accepted: 16 January 2024 / Published online: 7 February 2024

Abstract
An essay on Horndeski gravity, how it was formulated in the early 1970s and how it was 're-discovered' and widely adopted by Cosmologists more than thirty years later.

Keywords Scalar-tensor field theories · Horndeski gravity · Lagrangians · Modified gravity · Dark energy

1 Introduction

ı September, 2023, Prof. Andreas Wipf, the Editor in Chief of the International Journal f Theoretical Physics, contacted me (Gregory Horndeski) concerning my paper entitled Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space." That paper riginally appeared in his Journal back in 1974, and he thought it would be appropriate for ıe to write an essay, to appear in the International Journal of Theoretical Physics in 2024, to elebrate the 50th anniversary of that paper's publication. At first I was a little apprehensive bout accepting his proposal because he wanted the essay to cover how the theory came into eing and how it has been used. Well I certainly was familiar with the theory's inception, ut I really was not qualified to discuss how it was being used. To allay my concerns Prof. /ipf said that he would not mind if the essay had a co-author who could explain the theories alue to the Physics community. Well given that I would be permitted to have a colleague › do the hard work, I agreed to participate in this essay project. My next task was to find a o-author. To that end I contacted a colleague of mine at the University of Waterloo, Prof. hazal Geshnizjani, who has worked with my scalar-tensor equations, to see if she could rovide me with a list of names of physicists who might be able to assist me in my endeavor. he provided me with the names of a handful of physicists, all of whom were familiar to me om what little I knew about applications of my equations. At the top of the list was Prof. lessandra Silvestri, with whom I had prior contact concerning my work as an artist. So I sked her if she would be interested in co-authoring an essay with me and she said yes.

B Gregory W. Horndeski
ghorndeski@uwaterloo.ca; horndeskimath@gmail.com

Alessandra Silvestri
silvestri@lorentz.leidenuniv.nl

1 Applied Math Department, University of Waterloo, 200 University Avenue, Waterloo N2L 3G1, ON, Canada

2 Institute Lorentz, Leiden University, PO Box 9506, Leiden 2300 RA, The Netherlands

The process

Gregory Horndeski 11 October 2023 at 12:38 PM
Re: Essay for International J. of Theo. Phys on Horndeski Theory
To: Alessandra Silvestri

Dear Alessandra:

Typing has been going OK so far today, but not perfect. The problem is that I somehow get into italics after typing certain functions or Greek variables. See the epic line 118, that eventually starts to turn into italics, and also lines 128 and 143, where I have an italics problem and trouble getting the words "functions of" instead of ""functionso f"." Please help.

As for numbering equations I just put the numbers into the text. It seems to be easier for me than typing out the symbols that represent the equations.

Your Confused Santa Fe Sidekick,

Gregory

The process

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128 and 14
Please help

As for num
symbols tha

Your Confus

Gregory

Gregory Horndeski 22 December 2023 at 10:21 AM
Re: Essay for International J. of Theo. Phys on Horndeski Theory
To: Alessandra Silvestri

Dear Alessandra:

Sounds like a plan. It looks like we should be able to get it done by the end of this year. I'll look through my section again to see if I missed any index changes.

Yours truly,

Gregory

See More from Alessandra Silvestri

The process

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Yours truly,

Gregory

See More from Alessa

Gregory Horndeski 16 January 2024 at 10:48 AM
Re: Essay for International J. of Theo. Phys on Horndeski Theory
To: Alessandra Silvestri

Dear Alessandra:

Hurray! Our essay has been accepted.

Now we should start working on the one to celebrate the 100th anniversary of my paper's publication. Hopefully we won't have to use LaTex for that one.

Yours truly,

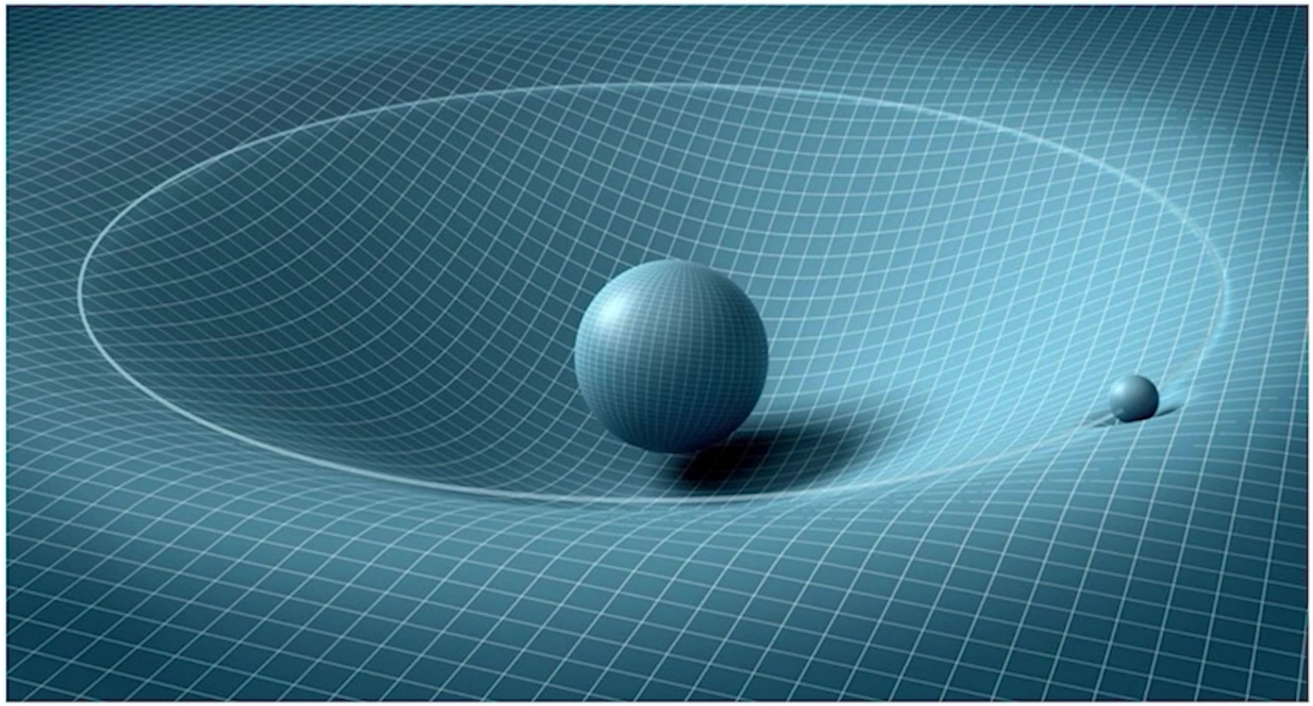
Gregory

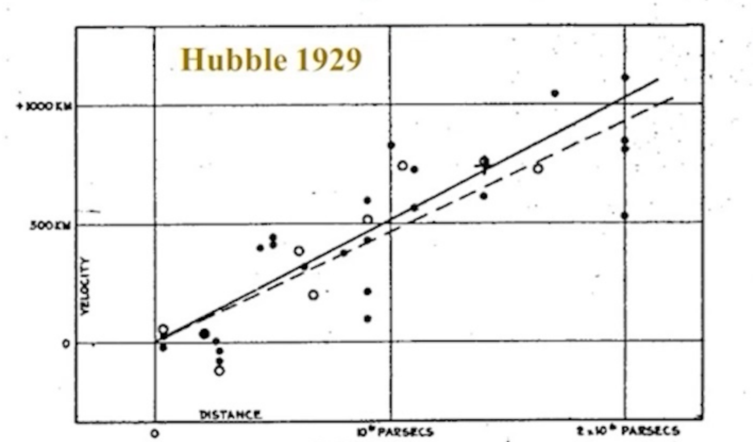
Let us go in order !

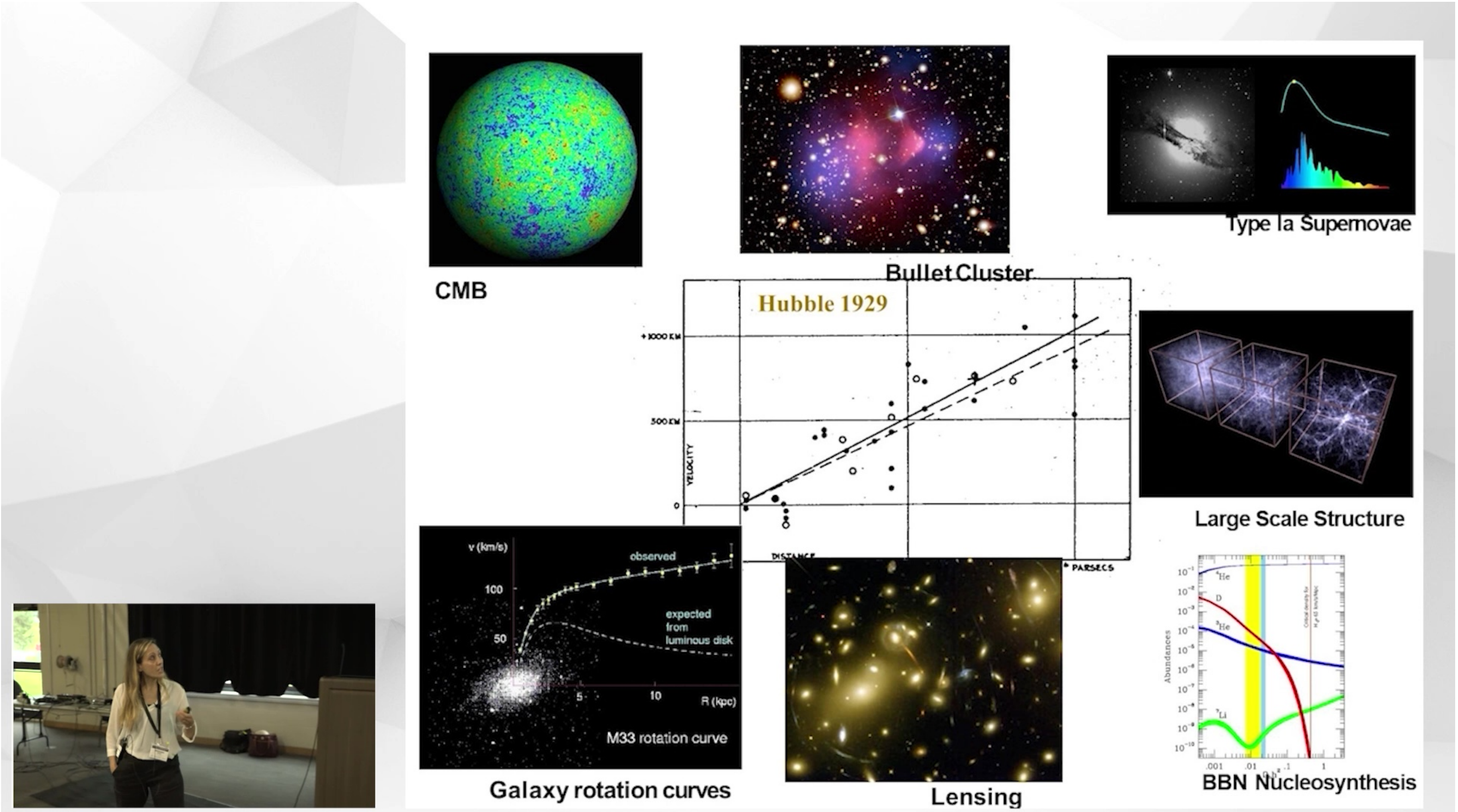
General Relativity

Einstein 1915: The theory of General Relativity

A theory of gravity that ties time and space into a dynamical 4-dimensional entity:
SPACETIME !



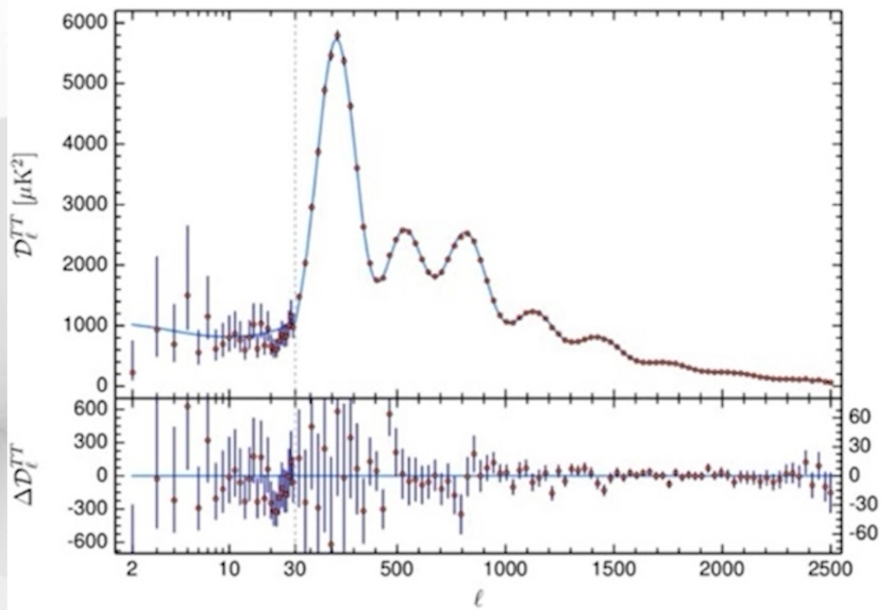

Hubble 1929
+1000 KM
500 KM
0
VELOCITY
DISTANCE
0
10⁶ PARSECS
2 x 10⁶ PARSECS
FIGURE 1


CMB
Bullet Cluster
Type Ia Supernovae
Hubble 1929
+1000 KM
500 KM
0
VELOCITY
DISTANCE
PARSECS
Large Scale Structure
v (km/s)
100
50
observed
expected
from
luminous disk
5
10
R (kpc)
M33 rotation curve
Galaxy rotation curves
Lensing
Abundances
BBN Nucleosynthesis

The standard model of Cosmology

LCDM: 6 parameters to describe it all!

based on General Relativity

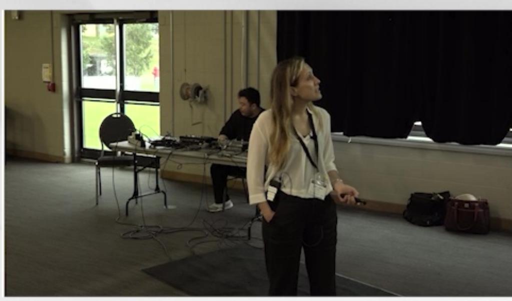


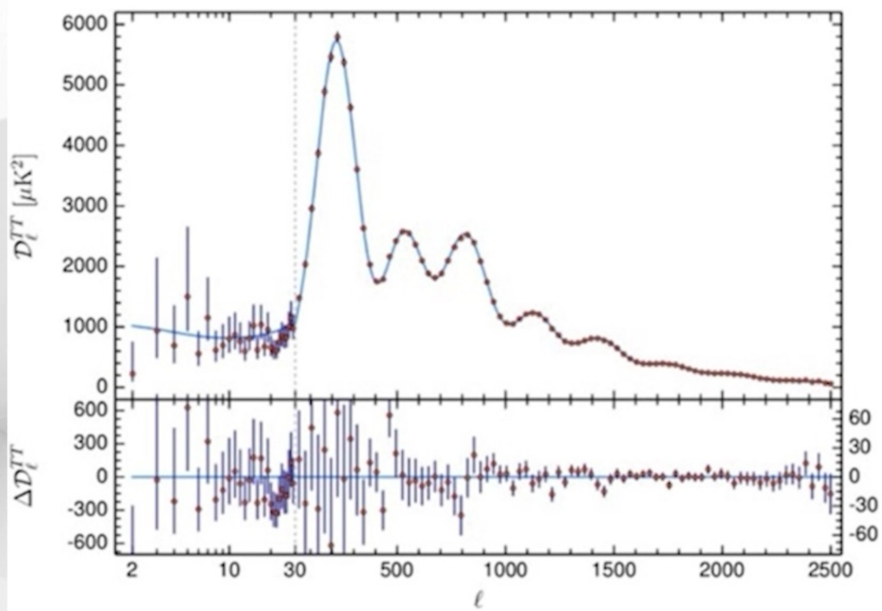
Parameter	TT+lowE 68% limits
$\Omega_b h^2$	0.02212 ± 0.00022
$\Omega_c h^2$	0.1206 ± 0.0021
$100\theta_{MC}$	1.04077 ± 0.00047
τ	0.0522 ± 0.0080
$\ln(10^{10} A_s)$	3.040 ± 0.016
n_s	0.9626 ± 0.0057

The standard model of Cosmology

LCDM: 6 parameters to describe it all!

based on General Relativity





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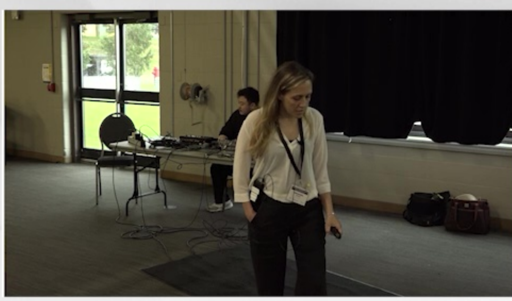
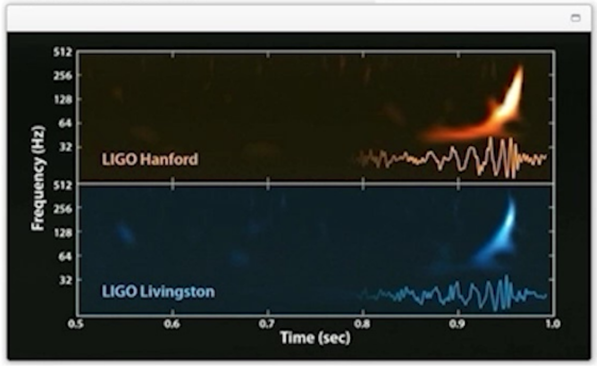
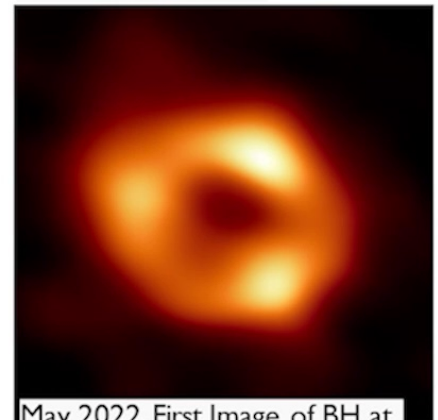
Black holes & Gravitational Waves



GW150914 LVK collaboration

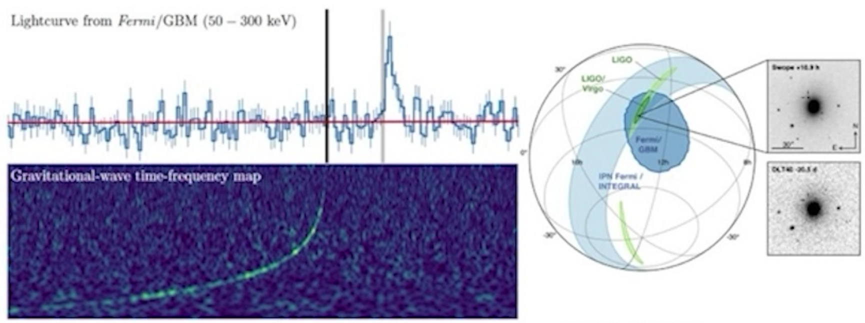


EHT Collaboration



May 2022, First Image of BH at the centre of the Milky Way

GW170817 +GRB170817A



LIGO,Virgo, Fermi,-GBM, INTEGRAL, Astrophys.J. 848 (2017)

Open questions

What is the nature of dark matter?

What is the physics of dark energy?

How about inflation?

Open questions

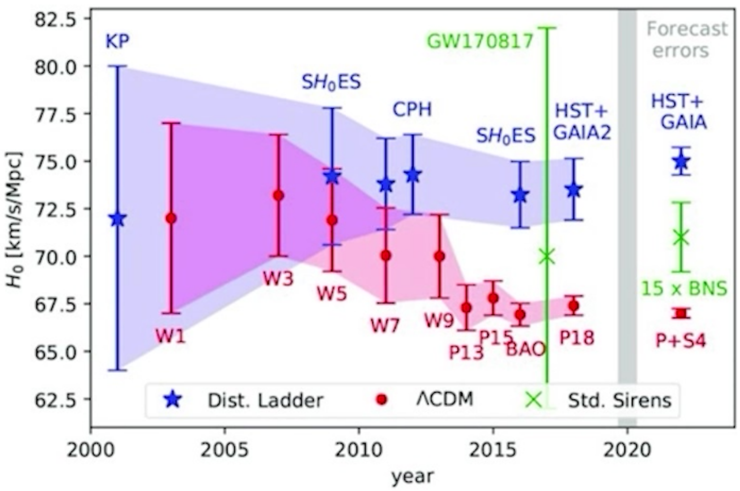
What is the nature of dark matter?

What is the physics of dark energy?

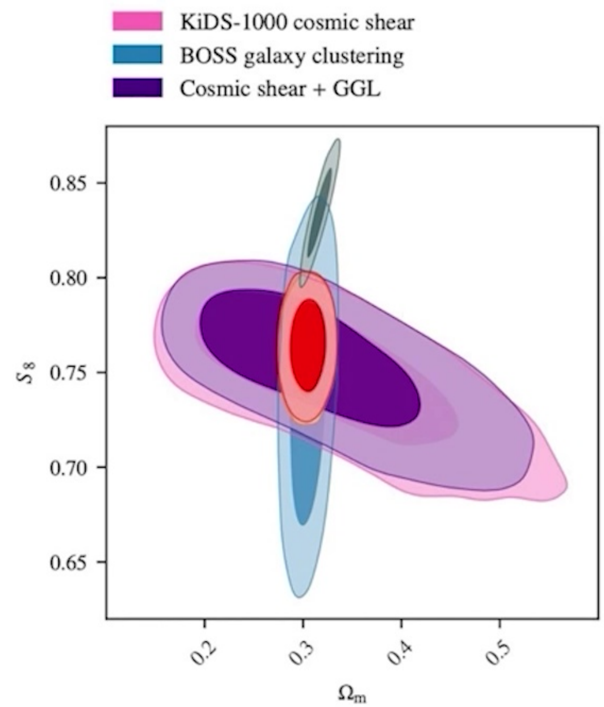
How about inflation?

Are we really sure that GR correctly describes gravity on cosmological scales?

Cosmological Tensions



Ezquiaga et al., Front.Astron.Space Sci. (2018)



C. Heymans et al., "KiDS-1000 Cosmology: Multi-probe weak gravitational lensing and spectroscopic galaxy clustering constraints," 7, 2020

If not GR, then what?

Weinberg-Deser theorem: A Lorentz invariant theory of a massless spin-2 particle must be GR at low energies.

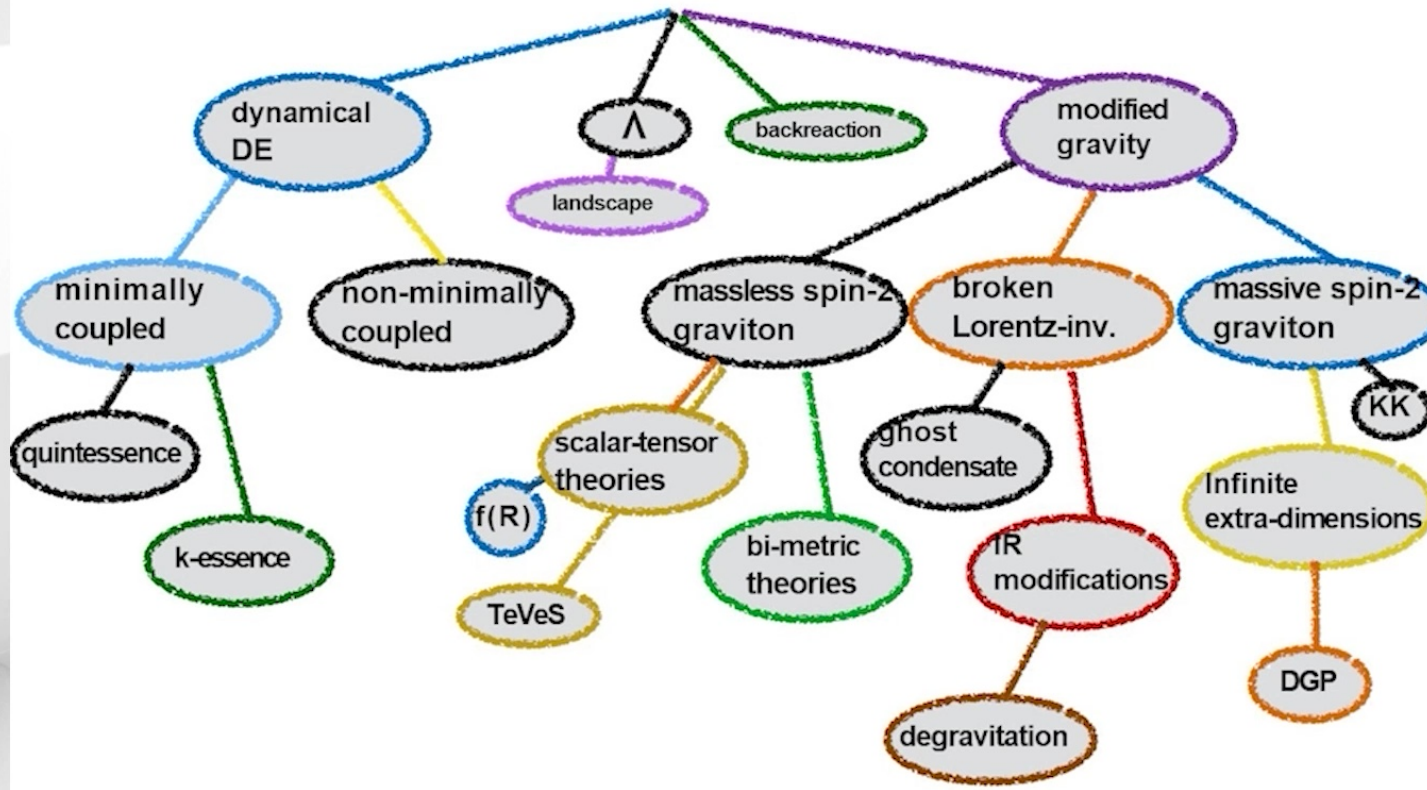
Lovelock's theorem: (1971) The only possible second-order, Euler-Lagrange equations obtainable in a 4D spacetime from an action containing solely the 4D metric and its derivatives are the Einstein field equations

↓

So, to modify GR we can either give mass to the graviton, introduce new DOF or break Lorentz invariance: extra (scalar) dynamical DOF !

Clifton et al., Phys.Rept. 513 (2012)

Gravitational landscape



If not GR, then what?

In identifying the most general metric theory of gravity with 2nd order EOMs in D dimensions, Lovelock worked with the so-called Lovelock scalars (e.g. Ricci scalar, Gauss-Bonnet). A nice property of these scalars is that any Lagrangian written in terms of Lovelock scalars, *including non linear functions of them*, will not contain extra tensorial DOF.

In 4D

$$L = \sqrt{-g}(\alpha_0 + \alpha_1 R + \alpha_2 G)$$ 1 massless spin-2 DOF

$$L = \sqrt{-g}(\alpha_1 f(R) + \alpha_2 f(G))$$ 1 massless spin-2 DOF + 2 scalar DOF: f_G , f_R

$$\nabla_\mu \nabla_\nu f_R \,, \; \nabla_\mu \nabla_\nu f_G$$

More generally, a theory of gravity which maintains second order EOMs for the tensor and has a single additional propagating scalar DOF in 4D is:

$$L = \sqrt{-g} f(R, G) \quad \text{with} \quad f_{RR} f_{GG} \quad f_{RG}^2 = 0$$

A. de Felice, S.Tsujikawa, Living Rev.Rel. 13 (2010) 3

Galileons

In Minkowski space there is something analogous to Lovelock argument that allows us to identify the most general Lagrangian for a scalar DOF with at most 2nd order EOMs in D dimensions.

Studying the decoupling limit of the higher-dimensional DGP model, it was realized that the scalar field corresponding to the bending mode of the brane obeyed a galilean shift symmetry, inherited from the higher-dimensional Lorentz invariance : $\phi \to \phi + b + c^{\mu}x_{\mu}$

The corresponding Lagrangian contained derivative coupling terms of cubic order in the scalar field $\Box\phi\,\partial\phi\cdot\partial\phi$ Models of massive gravity in 4D contributes similar terms but of quartic and quintic order.

Interestingly, requiring a theory for a scalar field to be galilean invariant and to have EOMs at most of 2nd order, identifies a finite number of terms !

In particular, we have D+1 galileon terms for a Lagrangian in D dimensions, following the general structure:

$$\partial\phi\cdot\partial\phi(\partial^2\phi)^{n-2} \qquad n = 1 \cdots D+1$$

All this, in Minkowski space…

A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D79, 064036 (2009).

Covariant Galileons

If we extend the analysis to curved spacetime, the Lagrangians need to be promoted to a covariant form, and some non-minimal coupling between gravity and the scalar field needs to be introduced, breaking the galilean symmetry. The result is the Covariant Galileons action:

$$S_{CG} = \int d^4x\sqrt{-g}\left\{\frac{M_P^2}{2}R - \frac{1}{2}c_1 M^3\phi + c_2 X + \frac{2c_3}{M^3}X\Box\phi + \frac{c_4}{M^6}X^2 R + \frac{2c_4}{M^6}X\left[(\Box\phi)^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right] - \frac{3c_5}{M^9}X^2 G_{\mu\nu}\phi^{\mu\nu} + \frac{c_5}{M^9}X\left[(\Box\phi)^3 - 3\Box\phi\,\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi^{\mu\nu}\phi_{\mu\sigma}\phi^{\nu}_{\sigma}\right]\right\},$$

There are 5 blocks inside of the Lagrangian, each contributing separately equations of motion of second order. The two non-minimal coupling terms and the secific linear combinations of opetaror within each block are fixed to guarantee this.

C. Deffayet, G. Esposito-Farese and A.Vikman, Phys. Rev. D79, 084003 (2009).

Generalized Galileons

Since we have lost Galilean invariance, there are more terms that we can include. A survey of such terms, take us to the 'most general' scalar-tensor theory having second-order field equations is described by the following Galileon Lagrangian:

$$L = \overset{5}{\underset{i=2}{X}} L_i ,$$

where

$$L_2 = K(<, X),$$

$$L_3 = -\, G_3(<,X) \leftarrow <,$$

$$L_4 = G_4(<,X)\, R + G_{4,X}\, [(\leftarrow <)^2 - (r_\mu r_{\otimes} <)(r^\mu r^{\otimes} <)],$$

$$L_5 = G_5(<,X)\, G_{\mu\otimes}(r^\mu r^{\otimes} <) - \frac{1}{6} G_{5,X}\, [(\leftarrow <)^3 - 3(\leftarrow <)(r_\mu r_{\otimes} <)(r^\mu r^{\otimes} <)$$

$$+ 2 (r^\mu r_{\lrcorner} <)(r^{\lrcorner} r_{/3} <)(r^{/3} r_\mu <)]$$

$$X \;\text{⌘}\!- \frac{1}{2} @^{\lrcorner} ¢ @_\mu ¢$$

C. Deffayet, X. Gao, D.A. Steer, and G. Zahariade, Phys. Rev. D 84, 064039 (2011)

Galileon models gained significant interest in Cosmology because they allow for self accelerating solutions that could describe both the inflationary epoch and the late time accelerated expansion.

Generalized Galileons

In 2011, *C. Charmousis et al. (Phys. Rev. Lett. 108, 051101)*, revisited Horndeski gravity, to look for subclasses that would display a self-tuning mechanism on the FLRW background as an approach to the cosmological constant. At that point, T.Kobayashi et al. noticed the equivalence with the Generalized Galileons that they were studying.

$$L_2 = K(<, X),$$
$$L_3 = - G_3(<, X) \leftarrow <,$$
$$L_4 = G_4(<, X)\, R + G_{4,X}\, [(\leftarrow <)^2 - (r_\mu r_{\otimes} <)(r^\mu r^{\otimes} <)],$$
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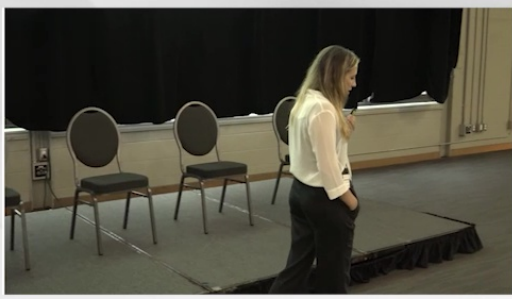
$$+ 2 (r^\mu r_{\lrcorner} <)(r^{\lrcorner} r_{/3} <)(r^{/3} r_\mu <)]$$

$$G_2 = \kappa_9 + 4X \int^X dX'(\kappa_{8\phi} - 2\kappa_{3\phi\phi}),$$
$$G_3 = 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X dX'(\kappa_8 - 2\kappa_{3\phi})$$
$$G_4 = 2F - -4X\kappa_3$$
$$G_5 = -4\kappa_1$$

$$\mathcal{L} = \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\left[\kappa_1 \phi^\mu_\alpha R_{\beta\gamma}{}^{\nu\sigma} + \frac{2}{3}\kappa_{1X}\phi^\mu_\alpha\phi^\nu_\beta\phi^\sigma_\gamma + \kappa_3\phi_\alpha\phi^\mu R_{\beta\gamma}{}^{\nu\sigma} + 2\kappa_{3X}\phi_\alpha\phi^\mu\phi^\nu_\beta\phi^\sigma_\gamma\right]$$
$$+ \delta^{\alpha\beta}_{\mu\nu}\left[(F + 2W)R_{\alpha\beta}{}^{\mu\nu} + 2F_X\phi^\mu_\alpha\phi^\nu_\beta + 2\kappa_8\phi_\alpha\phi^\mu\phi^\nu_\beta\right]$$
$$- 6\left(F_\phi + 2W_\phi - X\kappa_8\right)\Box\phi + \kappa_9.$$

What makes Horndeski so special in Cosmology ?

Before discussing that, let me take a brief detour to discuss what we will happen from the observational point of view in the upcoming years.



Testing Gravity on Cosmological Scales

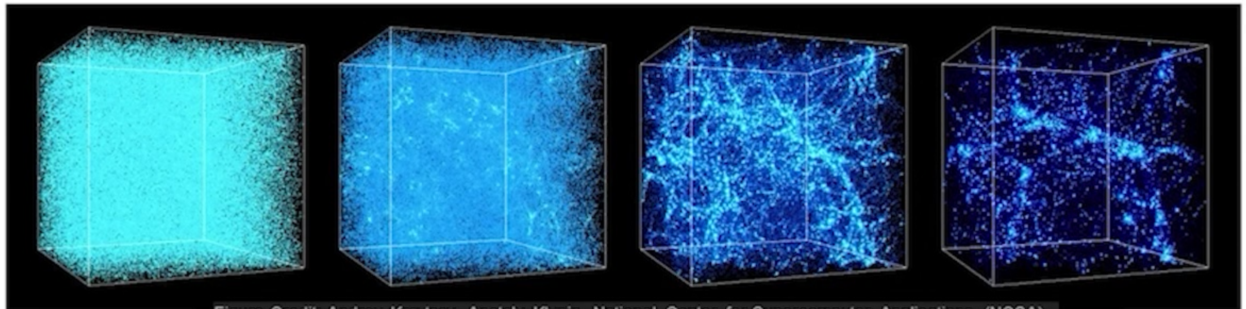
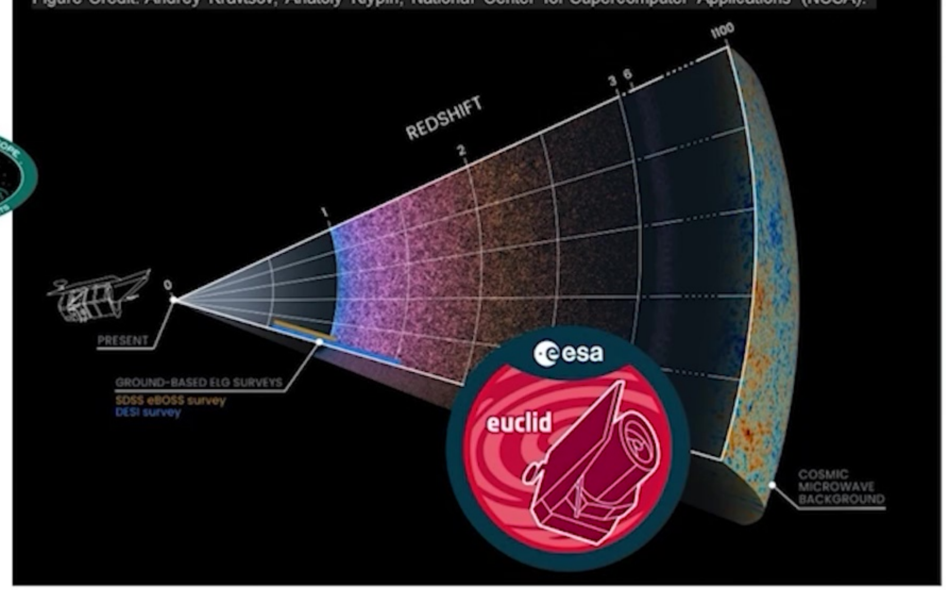


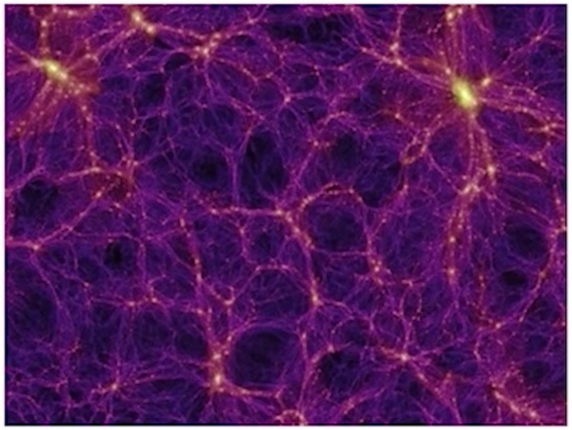
Figure Credit: Andrey Kravtsov, Anatoly Klypin, National Center for Supercomputer Applications (NCSA).



Euclid

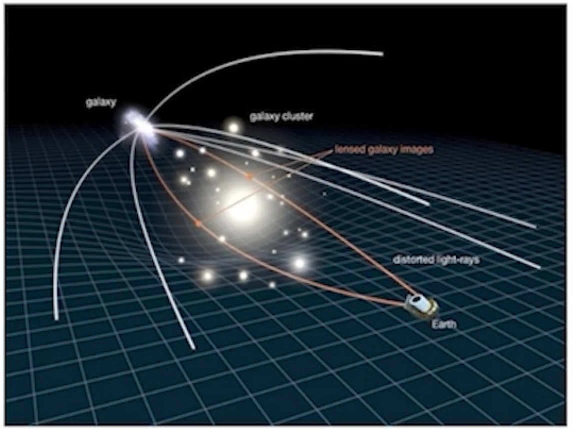
15,000 deg^2 — shape and photometry for 10^9 galaxies — spectroscopy for 10^7 galaxies

Galaxy clustering



G_{matter}

Weak gravitational lensing



G_{light}

LSS Phenomenology

$$ds^2 = - [1 + 2 \quad (t, \vec{x})]\, dt^2 + a^2(t)\, [1 + 2{<}I(t, \vec{x})]\, d\vec{x}^2$$

We can capture the background and large scale structure dynamics with few phenomenological functions:

Expansion: $$\frac{H^2}{H_0^2} = \frac{\boxtimes_r}{a^4} + \frac{\boxtimes_M}{a^3} + \boxtimes_{DE} a^{-3 \, R \, da(1+w_{DE}(a))}$$

Clustering: $$k^2 \quad = - \mu(a, k) \frac{a^2}{2M_P^2} \cdots b$$ G_{matter}

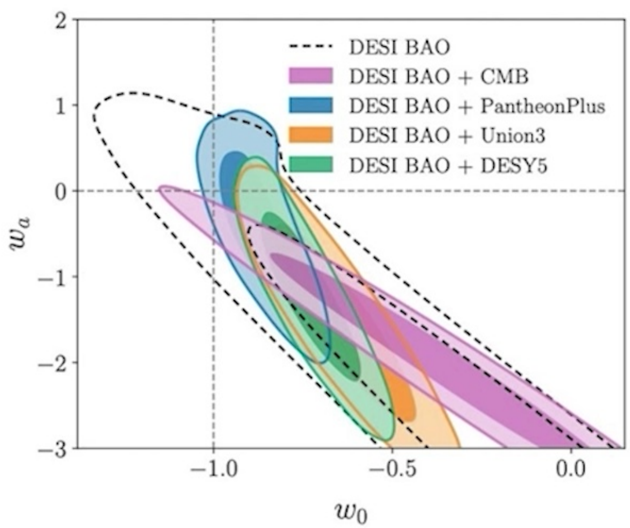
Lensing: $$k^2 ({<}I + \quad) = - \hat{}\,(a, k) \frac{a^2}{2M_P^2} \cdots b$$ G_{light}

Pogosian et al., Phys.Rev. D81 (2010)

Popular Parametrizations

expansion

$w(a) = w_0 + w_a(1 - a)$

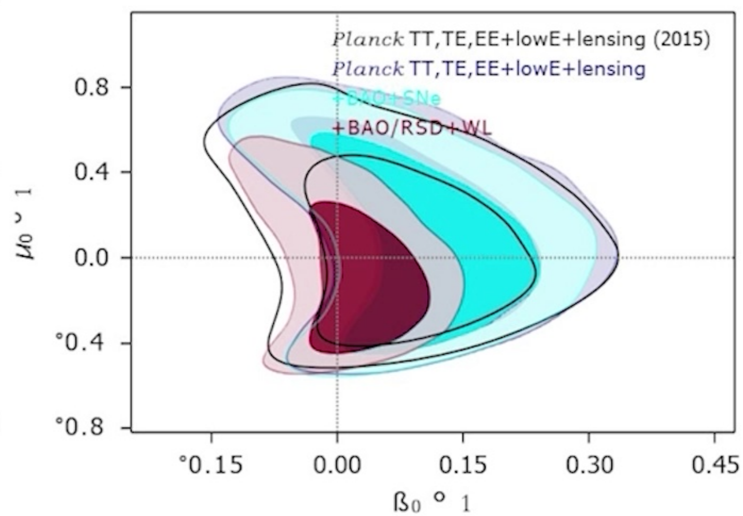


DESI 2024

clustering and lensing

$\mu(a) = \mu_0 + \mu_a(1 - a)$

$\wedge(a) = \wedge_0 + \wedge_a(1 - a)$



Planck 2018

Will we be able to extract the correct physics out of these wonderful measurements?

And how about the tensors?

EFT of Cosmological Perturbations

$$S = \int d^4x \sqrt{-g} \left[\frac{m_0^2}{2}[1+\Omega(t)]R + \Lambda(t) - c(t) a^2 \delta g^{00} + \gamma_1(t)\frac{m_0^2 H_0^2}{2}(a^2\delta g^{00})^2 - \gamma_2(t)\frac{m_0^2 H_0}{2}(a^2\delta g^{00})\delta K^\mu_\mu - \gamma_3(t)\frac{m_0^2}{2}\left(\delta K^\mu_\mu\right)^2 \right] + S_m[g_{\mu\nu}, \chi_i].$$

We consider the cosmological background as a state spontaneously breaking time-translations, and perturbations as the corresponding Nambu-Goldstone modes.

The action is built out of all geometrical quantities that are invariant under the time-dependent 3D spatial diffeomorphisms.

It is written in Jordan frame, and the functions in front of the operators guarantee that the resulting EOMs will be of second order.

UNITARY GAUGE → $n_\mu \equiv \frac{\partial_\mu \phi}{\sqrt{(\partial_\mu \phi)^2}}$

Stability Analysis

Expanding the given action up to second order in the perturbations, and removing spurious DOFs, we can inspect the dynamics of perturbations; in this case one scalar field, i.e. ξ, and the tensor:

$$S^{(2)}_{\xi,h^T} = \frac{1}{(2\pi)^3}\int d^3k\, dt\, a^3 \left[L_{\xi\xi}\dot{\xi}^2 - k^2 G \xi^2 + \frac{A_T}{8}\left[\dot{h}^{T\,2}_{ij} - \frac{c_T^2}{a^2}\left(h^T_{ij}\right)^2 \right]\right]$$

E.g., avoidance of ghost and gradient instabilities translate into the following set of conditions:

$$L_{\xi\xi} > 0$$

$$c_s^2 \equiv \frac{G}{L_{\xi\xi}} > 0$$

$$A_T > 0$$

$$c_T^2 > 0$$

⇒ very general conditions that constitute a stability check to be run at the very beginning!

Frusciante, Papadomanolakis, AS JCAP 1607 (2016).
De Felice et al., JCAP 1703 (2017); J. Gleyzes et al., Int.J.Mod.Phys. D23 (2015); R. Kase, S.Tsujikawa, Int.J.Mod.Phys. D23 (2015)

Sampling the Theory Space

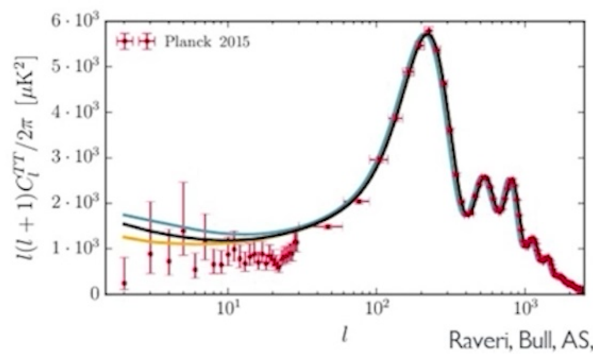
$$S = \int d^4x \sqrt{-g}\, \frac{m_0^2}{2}\left[1 + \Omega(\tau)\right] R + \Lambda(\tau) - c(\tau) a^2 \delta g^{00} + \gamma_1(\tau) \frac{m_0^2 H_0^2}{2} (a^2 \delta g^{00})^2 - \gamma_2(\tau) \frac{m_0^2 H_0}{2} (a^2 \delta g^{00}) \delta K^{\mu}_{\mu} - \gamma_3(\tau) \frac{m_0^2}{2} \left(\delta K^{\mu}_{\mu}\right)^2 + S_m[g_{\mu\nu}, \chi_i].$$

$$f(a) = \frac{\sum_{n=0}^{N} \alpha_n (a - a_0)^n}{1 + \sum_{n=0}^{M} \beta_m (a - a_0)^m}$$

$$\begin{cases} a_0 = 0, 1 \\ \alpha_n, \beta_m \in [-1, 1] \\ M + N = 9 \end{cases}$$

Saving only **viable** models:

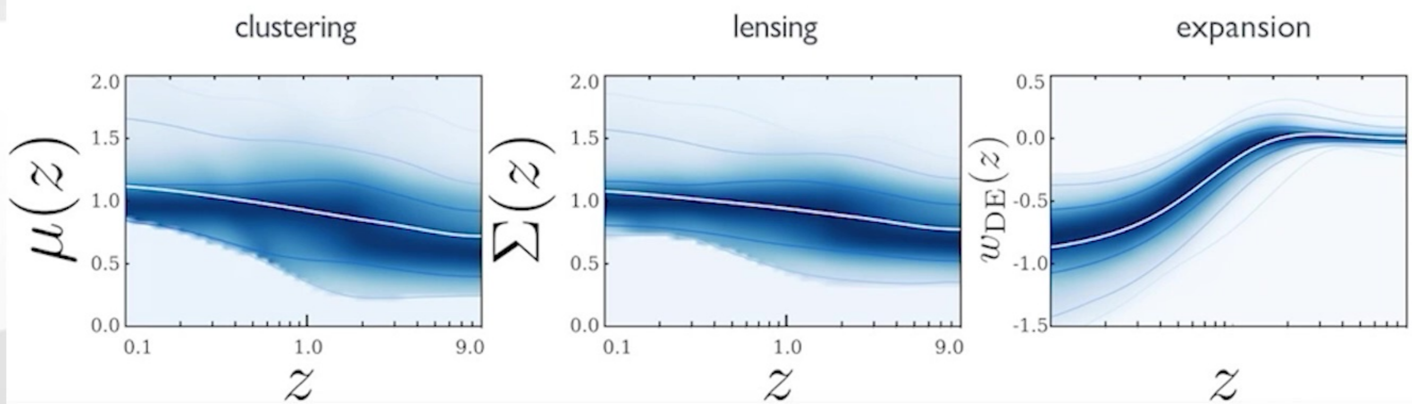
10^4 - 10^6 models →

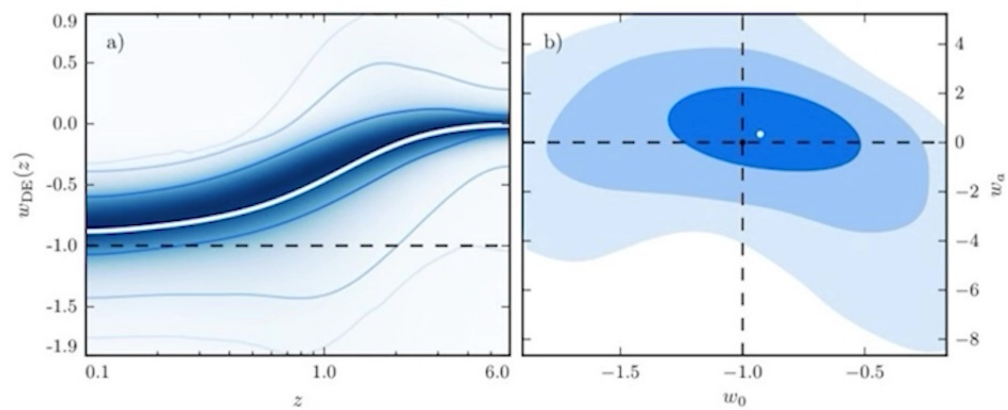


EffectiveFieldTheory with CAMB

Raveri, Bull, AS, Pogosian, PRD 2017

LSS Phenomenology of viable Horndeski

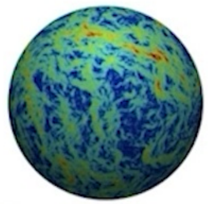


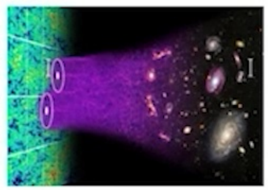


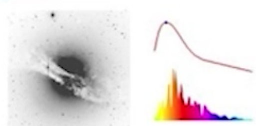
Espejo et al., PRD 2019

Reconstructed Gravity

Baseline

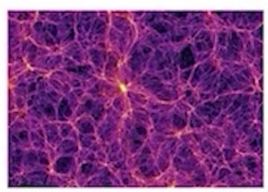




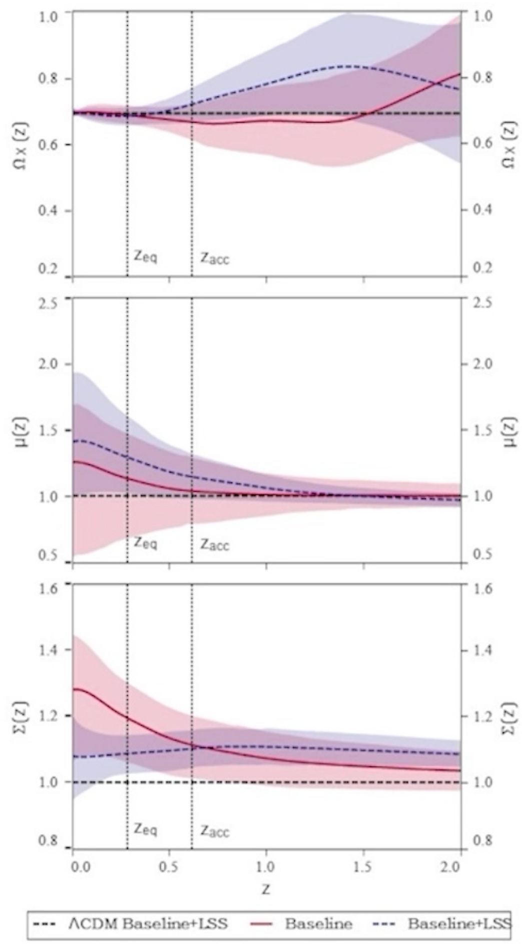


+

LSS

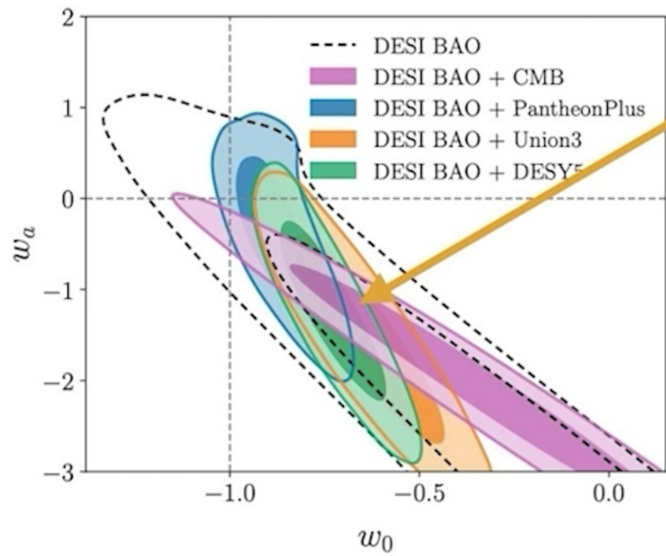


Pogosian et al., Nature Astronon

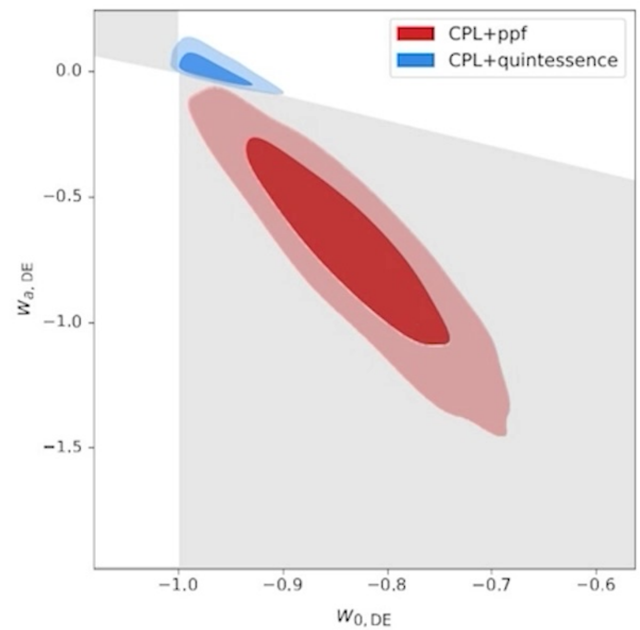


Stability and Phantom Crossing

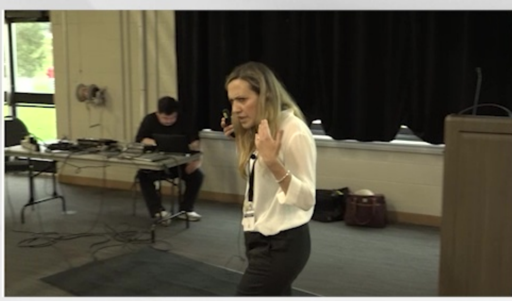
$$w = w_0 + w_a(1 - a)$$



WHICH DE IS IT?

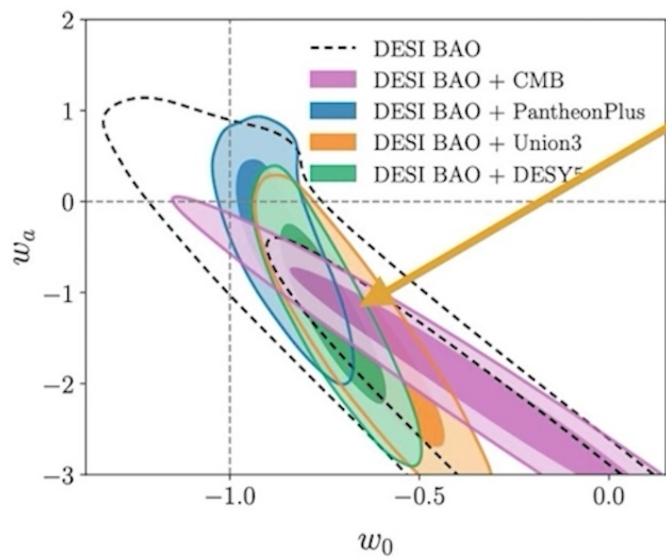


DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations

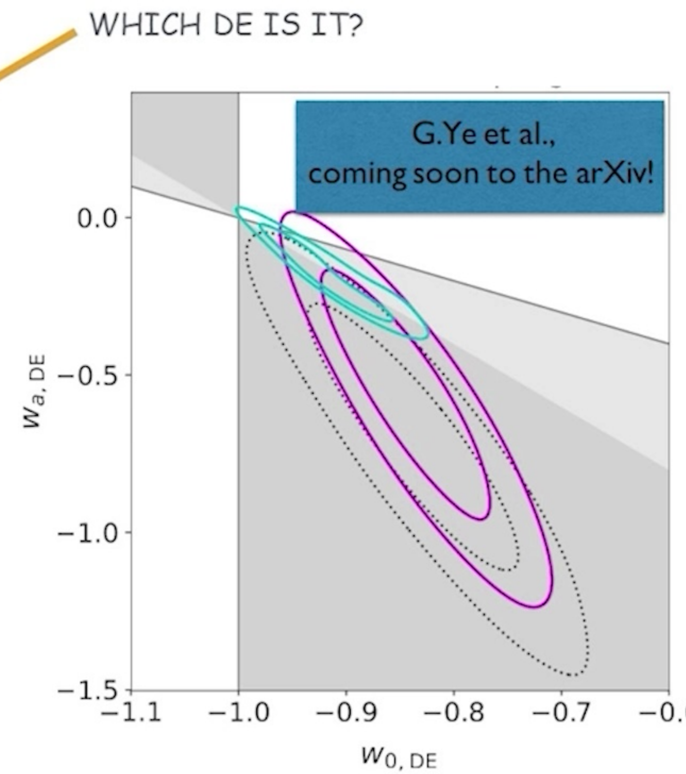


Stability and Phantom Crossing

$$w = w_0 + w_a(1 - a)$$



DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations



EFT of Cosmological Perturbations- v2

$$S^{(2)} = \int d^3x\, dt\, a^3 \frac{M^2}{2}\left[\delta K_{ij}\delta K^{ij} - \left(1+\frac{2}{3}\alpha_L\right)\delta K^2 + (1+\alpha_T)\left(R\frac{\delta\sqrt{h}}{a^3} + \delta_2 R\right) + \alpha_K H^2 \delta N^2 + 4\alpha_B H \delta K \delta N \right.$$
$$\left. + \alpha_H R \delta N + 4\beta_1 \delta K \delta\dot{N} + \beta_2 \delta\dot{N}^2 + \frac{\beta_3}{a^2}(\partial_i \delta N)^2\right]$$

* with some degeneracy conditions on the α_L and β_i parameters

DHOST

Langlois et al., JCAP(2016)
Crisostomi et al., JCAP(2019)

EFT of Cosmological Perturbations- v2

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DHOST

Langlois et al., JCAP(2016)
Crisostomi et al., JCAP(2019)

Dark-Energy Instabilities induced by Gravitational Waves,
Creminelli et al. arXiv:1910.14035 [gr-qc]

Resonant Decay of Gravitational Waves into Dark Energy,
Creminelli et al. JCAP(2018), JCAP(2019)

EFT of Cosmological Perturbations- v2

$$S^{(2)} = \int dx^3\, dt\, a^3 \frac{M_*^2}{2} \left[\delta K_{ij}\delta K^{ij} - \left(1 + \frac{2}{3}\alpha_L\right)\delta K^2 + (1+\alpha_T)\left(\delta_2 \sqrt{h} R/a^3\right) + \alpha_K H^2 \delta N^2 + 4\alpha_B H \delta K \delta N + \alpha_H R \delta N + 4\beta_1 \delta K \delta \dot{N} + \beta_2 \delta \dot{N}^2 + \frac{\beta_3}{a^2}(\partial_i \delta N)^2 \right]$$

* with some degeneracy conditions on the α_L and β_i parameters

DHOST

Langlois et al., JCAP(2016)
Crisostomi et al., JCAP(2019)

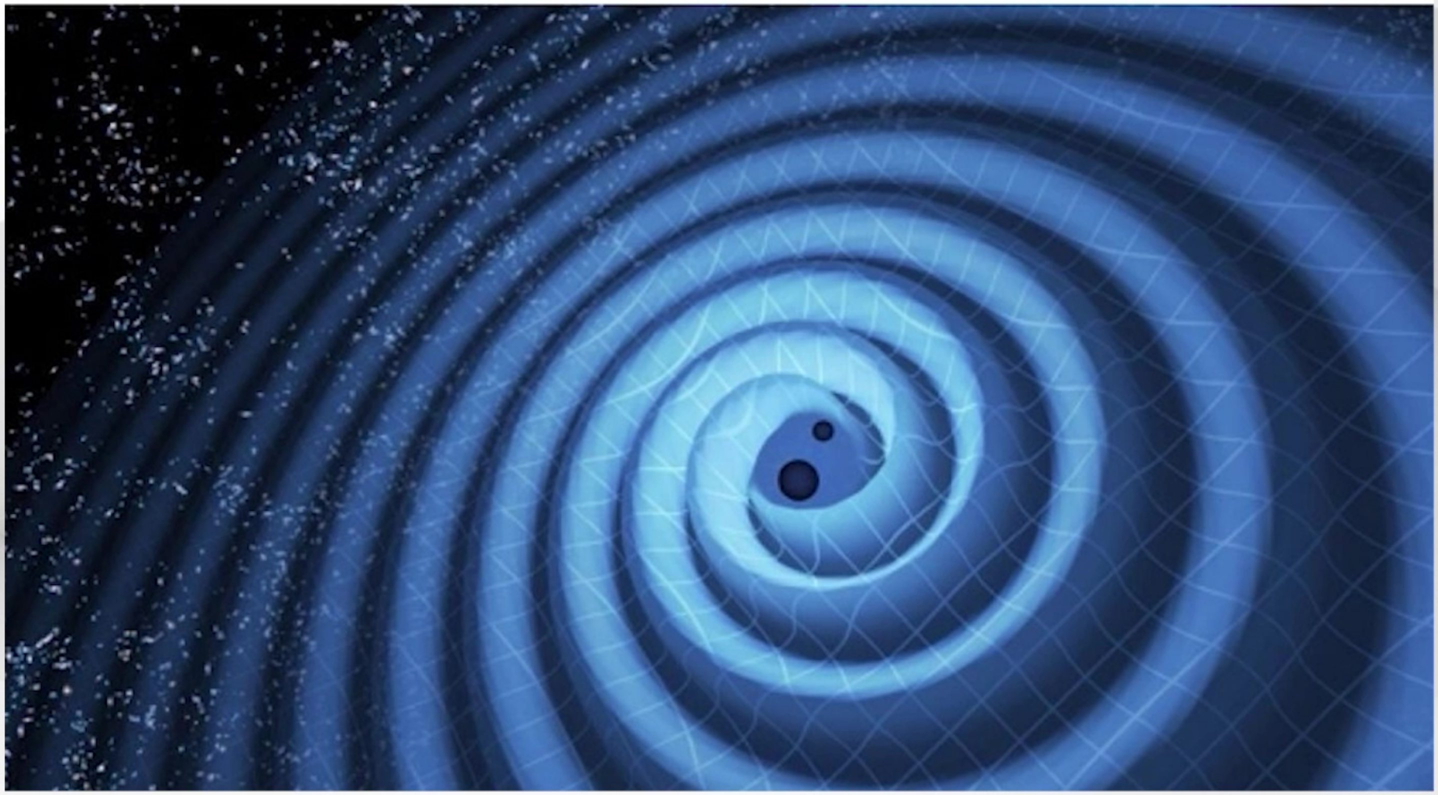
α_M Running of Planck's constant, generated by non-minimal coupling

α_T Deviation of speed of GWs from unity; non-zero whenever there is a non-linear derivative coupling of the scalar field to the metric. Same non-linearity is responsible for non-zero anisotropic stress.

α_K Quantifies the independent dynamics of the scalar-field

α_B Signals a coupling between the metric and the scalar-field

Gravitational Waves Frontier



GW Phenomenology in Horndeski

From the same unified action, we can now study also the propagation of tensors:

$$\ddot{h}_{ij} + (3 + \alpha_M) H \dot{h}_{ij} + (1 + \alpha_T) k^2 h_{ij} = 0$$

Amplitude

$$h \propto \frac{1}{\tilde{a}} \qquad \frac{\dot{\tilde{a}}}{\tilde{a}} = \frac{3 + \alpha_M}{2} H$$

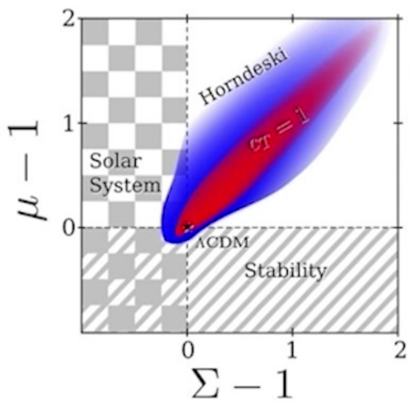
$$d_L^{GW}(z) = \frac{a(z)}{\tilde{a}(z)} d_L^{EM}(z)$$

$$d_L^{GW}(z) = \frac{M_*(0)}{M_*(z)} d_L^{EM}(z)$$

Amendola et al., JCAP 08 (2018) 030;
Belgacem et al., Phys.Rev.D 98 (2018)

Speed of propagation

Using GW170817, in particular the observed time-delay btw GRB and GW, a very stringent limit was placed on the speed of gravity effectively amounting to



Creminelli & Vernizzi, PRL 2017
Ezquiaga, Zumalacarregui, PRL 2017
Baker et al., PRL 2017

Pogosian & Silvestri PRD 2016

GW Phenomenology in Horndeski

On the perturbed Universe, while crossing LSS we pick up additional, theory-dependent corrections from inhomogeneities along the line of sight:

$$\frac{d_L^{SN}}{\bar{d}_L^{SN}} = - \; - \; (\; + \;) + \frac{1}{x} \int_0 d\tilde{x} (\; + \;) + \left(\frac{1}{xH} \right) + v_k \left(1 - \frac{1}{Hx} \right)$$

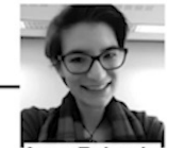
$$- \left(1 - \frac{1}{xH} \right) \int_0 d\tilde{x} ({}^0 + {}^0)$$

$$\frac{d_L^{GW}}{\bar{d}_L^{GW}} = - \; - \; (¢ + \;) + \frac{1}{x} \int_0 d\tilde{x} (¢ + \;) + ¢ \left(\frac{1}{xH} - \frac{M_P^0}{HM_P} \right) + v_k \left(1 - \frac{1}{Hx} + \frac{M_P^0}{HM_P} \right)$$

$$- \left(1 - \frac{1}{xH} + \frac{M_P^0}{HM_P} \right) \int_0 d\tilde{x} (¢^0 + {}^0) + \frac{M_{P,'}}{M_P} b' + \frac{M_{P,X}}{M_P} bX$$

A.Garoffolo, AS et al., PRD 2021

GW Number Count in Lum. Dist. Space



For scalar-tensor theories:

$$
\begin{aligned}
\hat{\Delta}(D,\vec{n}) = \; & \delta_{\rm N}^{\rm gw} + \left[1 + \frac{\gamma}{\mathcal{H}}(\dot{\zeta} - \zeta\mathcal{H}) - \zeta(\beta+1)\right]\boldsymbol{v}\cdot\boldsymbol{n} - \left[\frac{\gamma}{\mathcal{H}}\zeta\right]\partial_{\bar{\chi}}(\boldsymbol{v}\cdot\boldsymbol{n}) + \\
& \int_0^{\bar{\chi}} d\chi' \left[\left(\frac{\beta-1}{2}\right)\frac{\bar{\chi}-\chi'}{\bar{\chi}\chi'} + \frac{\gamma}{2\mathcal{H}\bar{\chi}^2}\right]\nabla^2_{\Omega}(\Phi+\Psi) + \left[\frac{1-\beta}{\bar{\chi}} + \frac{\gamma}{\mathcal{H}\bar{\chi}^2}\right]\int_0^{\bar{\chi}} d\chi'(\Phi+\Psi) + \\
& + \left[\zeta(\beta+1) - \frac{\gamma}{\mathcal{H}}\dot{\zeta}\right]\int_0^{\bar{\chi}} d\chi'(\dot{\Phi}+\dot{\Psi}) + \left[\beta - 1 - \frac{\gamma}{\bar{\chi}\mathcal{H}}\right]\Phi + \frac{\gamma}{\mathcal{H}}\partial_{\bar{\chi}}\Phi + \\
& + \left[\frac{\gamma}{\mathcal{H}}(\zeta-1)\right]\dot{\Phi} + \left[1 - \frac{\gamma}{\mathcal{H}}\left(\frac{1}{\bar{\chi}} + \dot{\zeta}\right) + \zeta(\beta+1)\right]\Psi + \\
& + \gamma\frac{\alpha_M}{2}\left(\frac{\dot{\delta\varphi}}{\varphi}\right) + \gamma\left[\frac{\dot{\alpha_M}}{2} + \frac{\alpha_M}{2}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}} - \beta - 1\right)\right]\frac{\delta\varphi}{\varphi}
\end{aligned}
$$

where

$$
\gamma = \left[1 + \frac{1}{\bar{\chi}\,\mathcal{H}} - \frac{\alpha_M}{2}\right]^{-1}
$$

$$
\begin{aligned}
\beta & \equiv \gamma\left(\frac{2}{\bar{\chi}\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{\gamma}{a^2\bar{D}}\frac{d^2\bar{D}}{d\bar{z}^2} - 1 - \frac{\partial\ln\bar{n}}{\partial\ln a}\right) \\
& = \gamma\left\{\frac{2}{\bar{\chi}\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \gamma\left[\frac{1}{\bar{\chi}\,\mathcal{H}}\left(1 + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right) - \frac{\alpha_M}{2}\left(1 + \frac{2}{\bar{\chi}\,\mathcal{H}}\right) + \frac{\alpha_M^2}{4} + \frac{\dot{\alpha}_M}{2\mathcal{H}}\right] - 1 - \frac{\partial\ln\bar{n}}{\partial\ln a}\right\}
\end{aligned}
$$

A. Balaudo, M. Pantiri, A.S., JCAP 02 (2024) 023

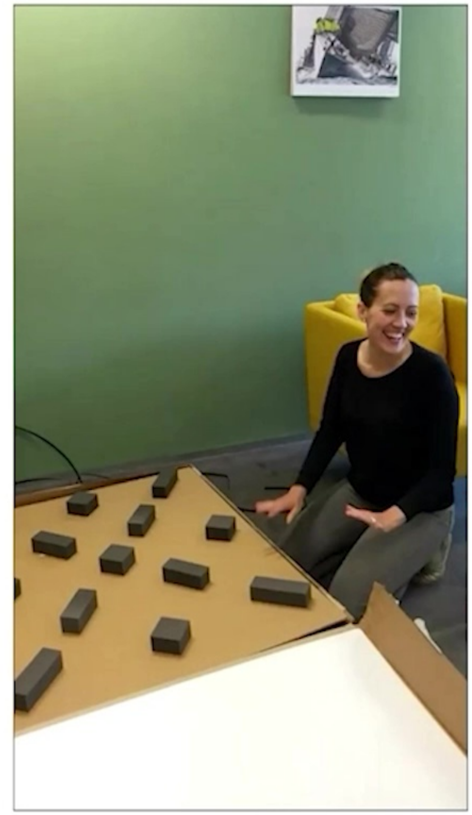
Synergy will be the key!

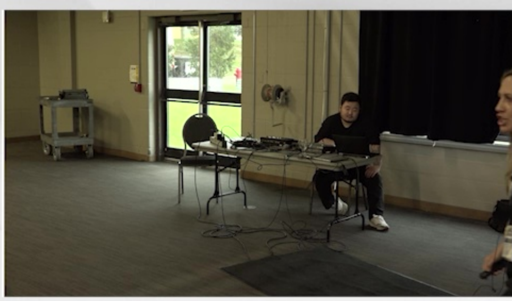
(Over)simplified parametrizations are prone to missing out important information in the data and to misleading us into wrong conclusions.

Precise *and* accurate cosmological inference requires an efficient, encompassing and *physically informed modeling*. Horndeski gravity, in the form of EFT of Cosmological Perturbations, provides just that !

We have very exciting years in front of us !
Stay tuned for Euclid DR1 in 2026 !!

Unwrapping Horndeski





Unwrapping Horndeski

THANK YOU !