

Title: Lecture - Twisted Holography b

Speakers:

Collection: Celestial Holography Summer School 2024

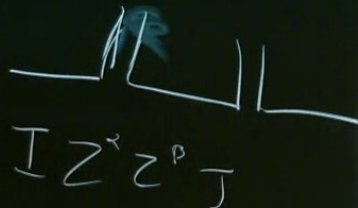
Date: July 26, 2024 - 11:30 AM

URL: <https://pirsa.org/24070023>

$$\Delta=1 \oint I^A J_B(z) dz = m_{\substack{(1) A \\ B}}^{\hat{gl}_n/\mathfrak{u}} \leftarrow \rightarrow \begin{matrix} P \\ 1 \end{matrix}$$

$$\Delta=\frac{3}{2} \oint I^A z^c J_B z^{m=0,1} dz = m_{\substack{(1) A \alpha \\ B n}} \leftarrow \rightarrow \begin{matrix} P_m^{\alpha \in SU(z)} \\ SL(z) \end{matrix}$$

$$\Delta=2 \quad I z^c z^B \quad \frac{1}{q} I^A z^c \overbrace{J_B(z) \frac{1}{z} I^C}^{\overline{m}} z^B J_D(w)$$



$$P_m^\alpha = P_m^\beta =$$

$$I Z^{\alpha_1} \dots Z^{\alpha_m} J \rightarrow P^{(m)} \begin{pmatrix} \alpha_1 & \dots & \alpha_m \\ m_1 & \dots & m_m \end{pmatrix} = P_{(m_1)} \dots P_{(m_m)}$$

$$P_{\alpha}^{\beta} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$ad - bc = \lambda \quad SL_2(\mathbb{C})$$

$$P_{\alpha}^{\beta} = P_{\beta}^{\alpha} = P^{(\alpha, \beta)} \begin{pmatrix} \alpha & \beta \\ n & m \end{pmatrix} + \lambda \epsilon^{\alpha\beta} \epsilon_{mm}$$

$$\det p = \lambda$$

$$\lambda \in \mathbb{C} \setminus \{0\}$$

$$I z^{\alpha} \dots z^{\beta} J \rightarrow$$



$$p_{\alpha}^{\alpha} = p_{\beta}^{\beta} = \begin{pmatrix} p_{\alpha\alpha} & (\alpha, \beta) \\ & p_{\beta\beta} \end{pmatrix} + \lambda e^{\alpha+\beta} e_{\alpha+\beta}$$

$$\det p = \lambda$$

$$P = \mathcal{O}(SL_2(\mathbb{C}))$$

→ SPACE-FILLING → D-BRANES

$$\text{hCS} \int A \bar{\partial} A \leftarrow \Omega^{4p}$$

$$\bar{\partial} A + A \wedge A = 0$$

$$A + A + \bar{\partial} \Lambda + [A, \Lambda]$$

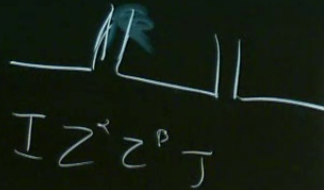
$$A \in^{+\beta} \epsilon_{mm}$$

$$\Delta=1 \oint I^A J_B(z) dz = m_{B \quad A}^{(1)} \leftarrow \rightarrow P$$

$$\lambda = kN$$

$$\Delta = \frac{3}{2} \oint I^A z^c J_B z^{m=0,1} dz = m_{B \quad n}^{(1) A \quad \alpha} \leftarrow \rightarrow P_m \begin{matrix} \leftarrow \leftarrow \leftarrow \\ \leftarrow \leftarrow \leftarrow \end{matrix} \begin{matrix} SU(2) \\ SL(2) \end{matrix}$$

$$\Delta=2 \quad IZ^2 J \quad \frac{1}{2} I^A z^c \overbrace{J_B \frac{1}{z} I^C} \quad z^p J_D(w)$$



$$I z^{\dots} z^{\dots}$$

$$P_m^2 = P_m^3 = P_m^{(2)} \begin{matrix} (n) \\ (-n) \end{matrix}$$

$$\det p = \dots$$

$$P = O(SU)$$

$$\left[\frac{1}{2} x^3, I Y^2 J \right]$$

$$\lambda = \pm N$$

$$I Z^{n_1} \dots Z^{n_m} J$$

$SL(2)$

$$[a_{m,s}, P_m^a] = f_{m,s}(P) \frac{\partial}{\partial P_m^a}$$

$$P_m^a = P_m^b = \begin{pmatrix} \alpha & \beta \\ -\gamma & \delta \end{pmatrix} +$$

$$\det p = \lambda$$

$$P = \mathcal{O}(SL_2(\mathbb{C}))$$

$$\lambda = \det P$$

$$I(z^{\alpha_1} \dots z^{\alpha_n}) \rightarrow$$

$$P_{\alpha}^{\beta} = P_{\alpha\beta}^{\beta} = \begin{pmatrix} p_{\alpha\beta} & \dots \\ \dots & \dots \end{pmatrix} + \lambda e^{\alpha\beta} e_{\alpha\beta}$$

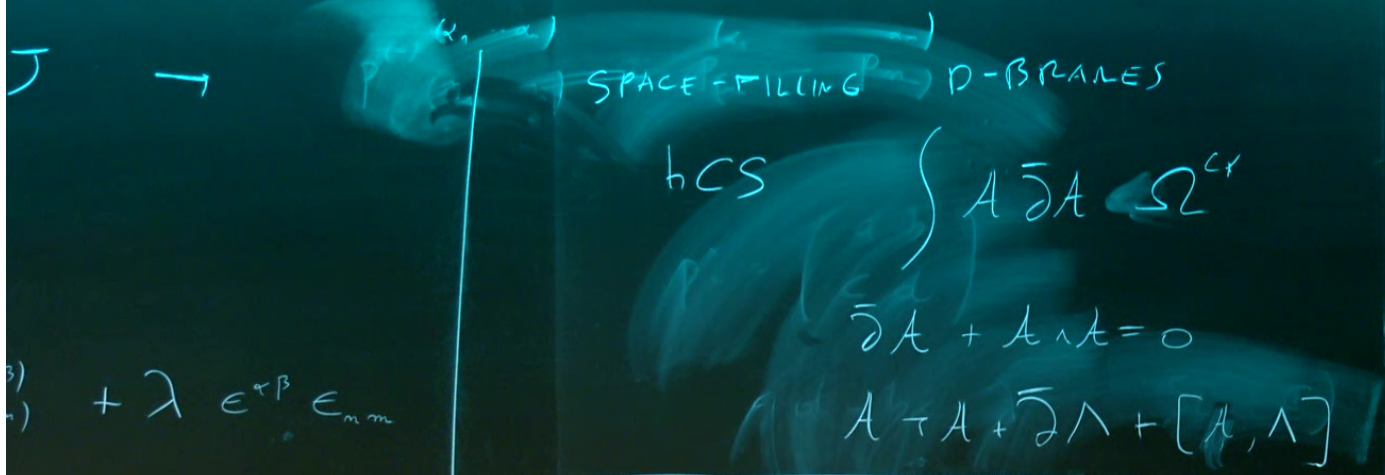
$$\det P = \lambda$$

$$P = \mathcal{O}(SL_2(\mathbb{C}))$$

$$[a_{m,s}, P_m^{\alpha}] = f_{m,s}^{\alpha}(P)$$

$$f_{m,s}^{\alpha}(P) \frac{\partial}{\partial P_m^{\alpha}}$$

HOL VECTOR FIELD
DIV-FREE



$+ \lambda \epsilon^{\alpha\beta} \epsilon_{mn}$

(C)

KS THEORY $\quad \beta$ DIV-FREE

$\bar{\partial} \beta + \{ \beta, \beta \} = 0$

$\beta^{-1} \beta \nearrow \bar{\partial} \gamma + \{ \beta, \gamma \}$

\uparrow
 VECTOR FIELD

$(\bar{\partial} + \{ \beta, \cdot \})^2 = 0$



bCS $\left\{ \begin{array}{l} A \bar{\partial} A \Omega^{4,1} \\ \bar{\partial} A + A \wedge A + \{B, A\} = 0 \\ A \mp A + \bar{\partial} \Lambda + [A, \Lambda] \end{array} \right.$

$$(\bar{\partial} + \{B, \cdot\})^2 = 0$$

$$+ \lambda \epsilon^{\alpha\beta} \epsilon_{mn}$$

KS THEORY β DIV-FREE

$$\bar{\partial} \beta + \{B, \beta\} = 0$$

$$\beta \rightarrow \beta + \bar{\partial} \gamma + \{B, \gamma\}$$

\uparrow
VECTOR FIELD

(C)

$\lambda = kN$

$[Y^2]$

$[a, a]$

$SL(2)$

$[a_{m,s}, P_m^\alpha] = f_{m,s}(P)_m^\alpha$

$f_{m,s}(P)_m^\alpha \frac{\partial}{\partial P_m^\alpha}$

HOL VECTOR FIELD
DIV-FREE

$[Z^{\alpha_1} \dots Z^{\alpha_n}] \rightarrow$

\sum

$P_m^\alpha = P_m^{\alpha\beta} + \lambda e^{\alpha\beta} e_{mm}$

$\det p = \lambda$

$\mathcal{P} = \mathcal{O}(SL_2(\mathbb{C}))$

SPACE-F

hCS

KS T

$\bar{\partial} P$

β

$$\lambda = kN$$

$$[a, a]$$

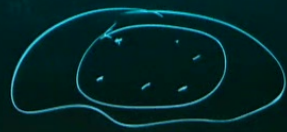
SL(2)

$$[a_{m,n}, P^-] = f_{m,n}(P^-)$$

$$f_{m,n}(P^-) \sim \frac{1}{\partial P^-}$$

HOL VECTOR FIELD
DIV-FREE

$$[Z^{1,0}, Z^{0,1}] \rightarrow$$



$$P^- = P_{m,n}^+ = P_{(m,n)}^{(k,P)} + \lambda e^{+P} e_{m,n}$$

$$\det P = \lambda$$

$$P = \mathcal{O}(SL_2(\mathbb{C}))$$



SPACE-FILLING D-BRANES

$$A \bar{\partial} A \sim \Omega^{CP^1}$$

$$\bar{\partial} A + A \wedge A + \{B, A\} = 0$$

$$A + A + \bar{\partial} \Lambda + [A, \Lambda]$$

KS THEORY

B

DIV-FREE

$$\bar{\partial} B + \{B, B\} = 0$$

$$B + B + \bar{\partial} X + \{B, X\}$$

VECTOR FIELD

$$(\bar{\partial} + \{B, \cdot\})^2 = 0$$



$$\lambda = kN$$

$$[a, \alpha]$$

\mathcal{L}

$SL(2)$

$$[a_{m,s}, P_m^a] = f_{m,s}(P)_m^a$$

$$P_m^a = P_m^b = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

$$\det p = \lambda$$

$$P = \mathcal{O}(SL_2)$$

HOL VECTOR FIELD
DIV-FREE

$$T_{\partial} X^a - Y^a$$

Handwritten mathematical notes on a chalkboard. The board is divided into two panels by a vertical line.

Top left: A diagram showing a circle with several lines inside, possibly representing a Riemann surface or a domain in the complex plane.

Top right: A diagram showing a circle with a shaded region inside, possibly representing a domain or a boundary.

Center: A large equation:
$$\int d\bar{z} dz e^{\bar{\psi} X(z) \psi + \bar{\psi} \psi} = \det X(z)$$

Below this, on the left panel, is another equation:
$$\langle \prod_{i=1}^n \det (m_i + X(z_i, u_i)) \rangle$$

Below that, on the left panel, is a third equation:
$$\langle \prod e^{\bar{\psi} X(z) \psi} \rangle$$

On the right panel, there are several smaller equations and diagrams:

- Top right: $\psi \rightarrow \psi C(z)$
- Middle right: $\langle \bar{\psi} \psi \rangle$
- Bottom right: $\int d\bar{z} dz e^{\bar{\psi} \psi}$
- Bottom right: $\langle \bar{\psi} \psi \rangle$
- Bottom right: $\langle \bar{\psi} \psi \rangle$
- Bottom right: $\langle \bar{\psi} \psi \rangle$

There are also some faint notes and diagrams scattered around the main equations.



$z = z$

$\bar{\psi}(x, y, z)$

$$\int d\bar{\psi} d\psi e^{-\bar{\psi} X(z) \psi} = \det X(z)$$

$\psi \rightarrow \psi C(z)$



SPACE-FILLING
bCS

$\lambda_z = 0$

$$\left\langle \prod_{l=1}^n \det m_l + X(z, u_l) \right\rangle$$

$$\langle \bar{\psi}^n \psi^n \rangle = e^{-f(e)}$$

$$\left\langle \prod_{l=1}^n \int d\bar{\psi}_l d\psi_l e^{-\bar{\psi}_l X(z, u_l) \psi_l} \right\rangle$$

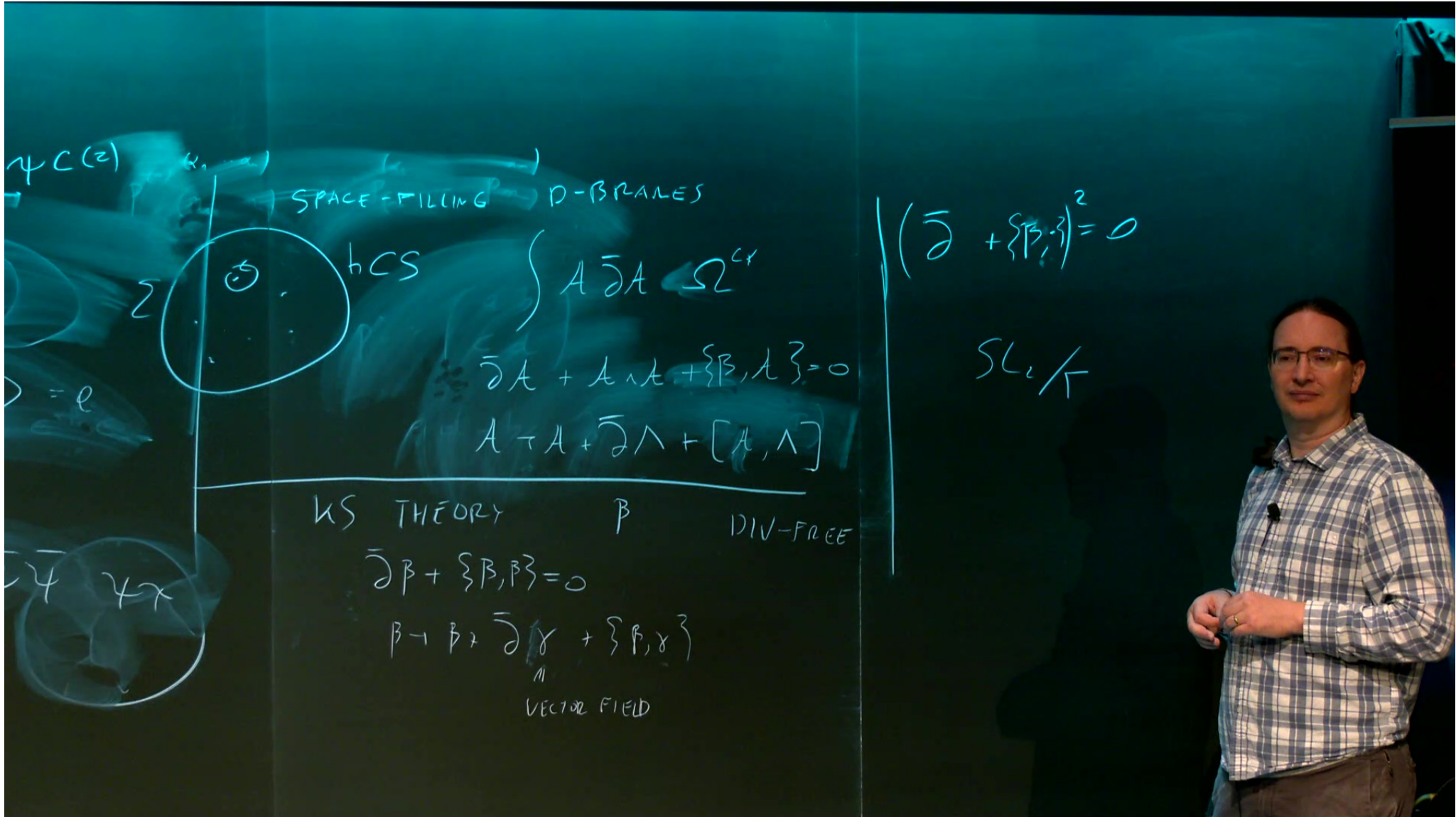
$$= \int \prod d\bar{\psi}_l d\psi_l e^{-\bar{\psi}_l X(z, u_l) \psi_l}$$

DOUBLES
 x, y, z, c

KS THEO
 $\bar{\partial} \beta + \dots$
 $\beta \rightarrow \bar{\beta}$

$\psi \rightarrow \psi C(z)$
 $\bar{\psi} X(z) \psi + \bar{\psi} \psi = \det X(z)$
 $\int d\bar{\psi} d\psi e^{\dots} = \det X(z)$
 $\langle \prod_{i=1}^n \det m_i + X(z_i, u_i) \rangle = \int d\bar{\psi} d\psi e^{\bar{\psi} X(\dots) \psi}$
 $\langle \bar{\psi} \psi \rangle = e$
 $f(e) = 0$
 $\int d\bar{\psi} d\psi e^{\bar{\psi} \psi \psi \chi}$

SPACE-FILLING D-BRANES
 $\frac{1}{2} \partial_2 \partial_4$
 $\bar{\partial} A + A \wedge A + \{B, A\} = 0$
 $A + A + \bar{\partial} \Lambda + [A, \Lambda]$
 hCS
 $\bar{\partial} B + \{B, B\} = 0$
 $B \rightarrow \bar{B} + \bar{\partial} \gamma + \{B, \gamma\}$
 VECTOR FIELD
 KS THEORY
 DIV-FREE



$\psi C(z)$
 $\psi = e$
 $\psi \quad \psi \gamma$

SPACE-FILLING D-BRANES
 hCS
 $A \bar{\partial} A \sim \Omega^{4,1}$
 $\bar{\partial} A + A \wedge A + \{B, A\} = 0$
 $A \bar{\partial} A + \bar{\partial} \Lambda + [A, \Lambda]$

KS THEORY β DIV-FREE
 $\bar{\partial} \beta + \{B, \beta\} = 0$
 $\beta \mapsto \beta + \bar{\partial} \gamma + \{B, \gamma\}$
 \uparrow
 VECTOR FIELD

$$(\bar{\partial} + \{B, \cdot\})^2 = 0$$

$$SL_2/\mathbb{R}$$