

Title: Lecture - Twisted Holography a

Speakers:

Collection: Celestial Holography Summer School 2024

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# TWISTED HOLOGRAPHY

AdS / CFT

QFT  $T_N \xrightarrow{N \rightarrow \infty} \frac{1}{N}$  EXPANSION  $\equiv$  PERTURBATIVE EXPANSION OF

HODFT-STYLE

$T_N$  IS  $U(N)$  GAUGE THEORY

$$\mathcal{L}_N = \frac{1}{k} \text{Tr} \mathcal{L}(\Phi) \quad \Phi \in N \times N \text{ MATRICES}$$

$k \sim (2N)^2$

$\Rightarrow$  STRING THEORY

T

$\rightarrow \infty$

$\frac{1}{N}$  EXPANSION  $\equiv$

PERTURBATIVE EXPANSION OF

TGR ON  $X$   
 $N \sim$  RAYS IN PLANE  
AdS  $\times$

GAUGE THEORY

$\implies$  STRING THEORY (1)  $l = g_s^2$

$\Phi$   $N \times N$  MATRICES

$N = \lambda$

$$\omega = \frac{\mathcal{L}}{T} = \int dt \int d^3x (dn^0 + A dn)$$

# TWISTED HOLOGRAPHY

AdS / CFT

$\mathcal{N}=4$  2d HOLONOMIC CFT

BETTER 2d CHIRAL ALGEBRA

$$O_i(z)$$

$$O_i(z) O_j(u) \sim \sum c_{ij}^k \frac{1}{(z-u)^h} O_k(u)$$

$$\langle O_{i_1}(z_1) \dots O_{i_n}(z_n) \rangle$$

$$T(z)$$

# AdS / CFT

$d = 4$   $\mathcal{N} = 4$  2d HoloMORPHIC CFT

BETTER 2d CHIRAL ALGEBRA

FREE  
2d CFT

$$O_i(z)$$

$$O_i(z) O_j(u) \sim \sum_k \frac{c_{ij}^k}{(z-u)^h} O_k(u)$$

$$\langle O_{i_1}(z_1) \dots O_{i_n}(z_n) \rangle$$

$$T(z)$$

# TWISTED HOLOGRAPHY

4d N=4  
SYM

AdS / CFT

2d HOLONORPHIC CFT

FREE  
2d CFT

BETTER

2d CHIRAL ALGEBRA

$$O_i(z)$$

$$O_i(z) O_j(w) \sim \sum \frac{c_{ij}^k}{(z-w)^h} O_k(w)$$

$$\langle O_{i_1}(z_1) \dots O_{i_n}(z_n) \rangle$$

$$T(z)$$

IIB ON  $AdS_5 \times S^5$

U

HOLOMORPHIC

GRAVITY

ON

3d CY

KODAIRA SPENCER

/BCOV

NON-RENORMALIZABLE

BUT UNITARY

B-MODEL

TOP

STRINGS

IF ASSUME

$$SL_2(\mathbb{C}) \simeq \frac{SL_2(\mathbb{C})}{SO(2)} \simeq \frac{SU(2)}{SO(2)} \simeq AdS_3 \times S^3$$

$D_2(u)$

RING THEORY (1)

# TWISTED HOLOGRAPHY

GAUGE-FIX  
 $A_z \rightarrow 0$

$\Delta_x = \Delta_y = \frac{1}{2}$

$\Delta_b = 1 \quad \Delta_c = 0$

$e^{\frac{1}{\hbar}} \int X \bar{\partial} Y$   
 $\Rightarrow$

$\frac{1}{\hbar} \int X \bar{\partial} Y + T_2 b \bar{\partial} c$

$X(z) Y(u) \sim \frac{\hbar}{z-u}$

$Q = \frac{1}{\hbar} \int T_2 b [c, c] + Xc$

- $Q_0$ :
- $Q_0 c \rightarrow [c, c]$
  - $Q_0 X \rightarrow [c, X]$
  - $Q_0 Y \rightarrow [c, Y]$
  - $Q_0 b = [c, b] \neq \mu(X, Y)$

$Q = Q_0 + \hbar Q_1$



$$x = \Delta y = \frac{1}{2}$$

$$a_b = 1 \quad a_c = 0$$

$$x \bar{y} + \frac{1}{2} b \bar{c}$$

$x, y \in \mathbb{S}^N$

$b, c \in \mathbb{R}$

$$\frac{1}{2} x^2$$

$$= \frac{1}{2} \left( \frac{1}{2} b [c, c] + x c y \right) \circledast x, y$$

$$Q = Q_0 + k Q_1$$

$X, Y \in \mathfrak{sl}_N$

$b, c \in \mathbb{C}$

$T_2 X^2$

$T_2 X^3$

$T_2 X^5 T_2 X^9$

$$\begin{pmatrix} X \\ Y \end{pmatrix} = Z^a \quad T_2 (X + uY)^m = \sum u^k \binom{m}{k} T_2 X^{m-k} Y^k$$

$$A_m(z, u) = \frac{1}{k} T_2 (X + uY)^m$$

$$Z^\alpha(z) Z^\beta(u) \sim \frac{e^{\alpha\beta}}{z-u}$$

slv

$$T + \epsilon \partial \bar{X} \bar{Y}$$

$$T_2 X^3$$

$$T_2 X^5 T_2 X^9$$

$$(X + uY)^n = \sum u^k \binom{n}{k} T_2 X^{n-k} Y^k$$

$$A_n(z, u) = \frac{1}{k} T_2 (X + uY)^n$$

$$T = \frac{1}{2} T_2 (X \partial \bar{Y} - Y \partial \bar{X}) + T_2 b \partial c$$

$$D_n(z; u)$$

$$\langle A_{m_1}(z_1, u_1) \rightarrow \dots \rightarrow A_{m_n}(z_n, u_n) \rangle_{\text{CP1}} \stackrel{\text{CONNECTED}}{=} \frac{1}{k} \text{PLAN}$$

$$a_{m,s}(u) = \int \sum_s A_n(z; u)$$

$$s = 0, \dots, m-2$$

$\mathcal{L}$

# GLOBAL SYMMETRIES

$b, c$   
PHIC

$$[a, a] = \alpha + \text{NON-PLANAR}$$

$$T + E \in \partial \Sigma_X$$

$$a_{m,s}(0) \in$$

SPIN  $\frac{m}{2}$  FOR  $SU(2)$

SPIN  $\frac{m}{2}$  FOR  $SL(2)$  CONFORMAL

$$f(z) = \frac{1}{z} + \dots$$

$$T = T(XDY - YDX) - \dots$$

# TWISTED

# HOLOGRAPHY

$$A_{\bar{z}} \xrightarrow{\text{GAUGE-FIX}} 0$$

$gl(u|u)$

$$Q^2 = 0$$

$$\left(\frac{1}{k}\right) \left( T_X X \bar{\partial} Y + T_b b \bar{\partial} c + I^A \bar{\partial} J_A \right)$$

$$Q_0 b = [X, Y] + J_A I^A$$

$$I = N \times (k|k)$$

$$J = (u|u) \times N$$

$$I^A z z z J_B = M^A_B(z; u)$$

# GLOBAL SYMMETRIES

(0)  $[a, a] = a + \text{NON-PLANAR}$

$\left( \int_D^A m_B^C - \int_B^C m_D^A \right) \in$

SPIN  $\frac{m}{2}$  FOR  $SU(2)$   
 SPIN  $\frac{m}{2}$  FOR  $SL(2)$  CONFORMAL

$gl_{4|4}(P)$

$[a, m] = m$

$A_{\bar{z}}$  GAUGE-FIX  $\rightarrow 0$

$gl(u|u)$

$$\frac{1}{k} \left( \int_N X \bar{\partial} Y + \int_N b \bar{\partial} c + \int_N I^A \bar{\partial} J_A \right)$$

$$Q_0 b = [X, Y] + J_A I^A$$

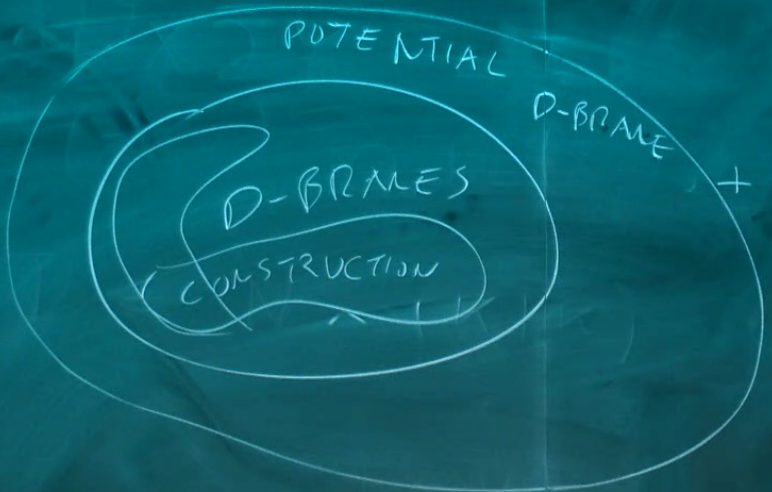
$$I = N \times (k|k)$$

$$J = (u|u) \times N$$

$$I^A \underbrace{\quad}_{\sim ZZZ} J_B =$$



$$Q = 0$$



$$\text{HOM}(\text{TEST D-BRANE}, D)$$

(SPACE-FILLING, DEFORMATION)

$$Q \rightarrow Q + \int e^{-I} [zzz]$$