Title: Lecture - IR S-matrix b

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Infrared Divergences in QFT

(Some) References

S. Caron-Huot, M. Giroux, HSH, S. Mizera 2308.02125

HSH, M. Schwartz 1911.06821

A. Strominger 1703.05448

D. Kapec, M. Perry, A.-M. Raclariu, A. Strominger 1705.04311

Review from NRQM:

- Short-range potentials

$$\psi^{in} \xrightarrow[|\vec{x}|=r\to\infty]{} e^{i\vec{p}\cdot\vec{x}} + \underbrace{f(p,\cos\theta)}_{\text{scattering amplitude}} \frac{e^{ipr}}{r}$$

- Coulomb potential:

$$\psi^{in} \xrightarrow[|\vec{x}|=r\to\infty]{} e^{i\vec{p}\cdot\vec{x} + \frac{i\gamma}{p}\log[pr(1-\cos\theta)]} - \tilde{f}(p,\cos\theta)\frac{e^{ipr - \frac{i\gamma}{p}\ldots}}{r}$$

...nces in QFT

Giroux, HSH, S. Mizera
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...tz 1911.06821
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...ry, A.M. Raclariu,
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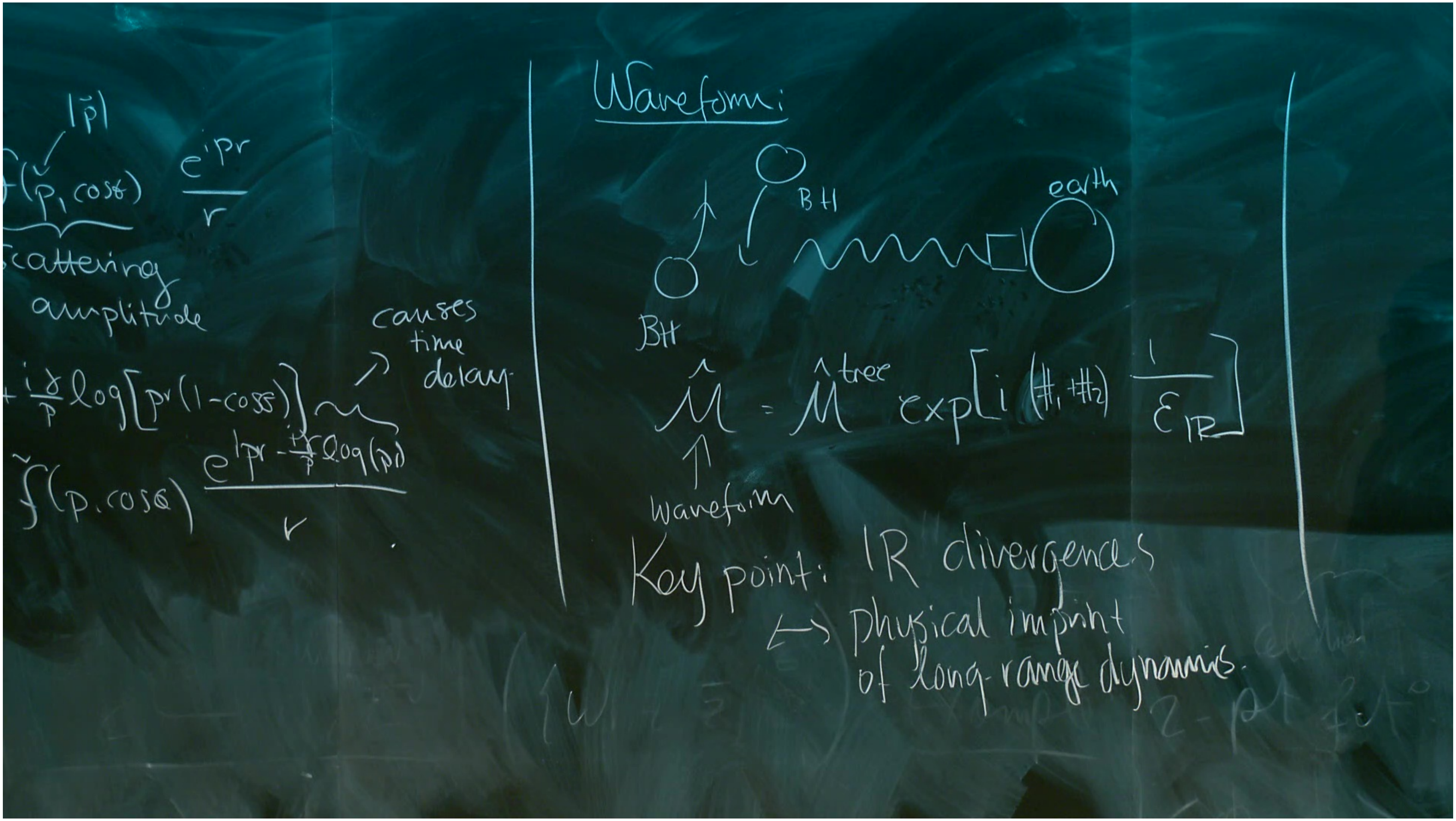
causes time delay


Waveform:
BH
earth
BH
$\hat{\mathcal{M}} = \hat{\mathcal{M}}^{\text{tree}} \exp\left[i \left(\#_1 \cdot \#_2 \right) \frac{1}{\varepsilon_{IR}} \right]$
waveform
Key point: IR divergences
$\longleftrightarrow$ physical imprint
of long-range dynamics.
$|\vec{p}|$
$f(p, \cos\theta) \, \frac{e^{ipr}}{r}$
scattering
amplitude
$+ \frac{i\gamma}{p} \log\left[pr(1-\cos\theta)\right]$
causes
time
delay
$\tilde{f}(p, \cos\theta) \, \frac{e^{ipr - \frac{i\gamma}{p}\log(pr)}}{r}$

Complications in QFT:

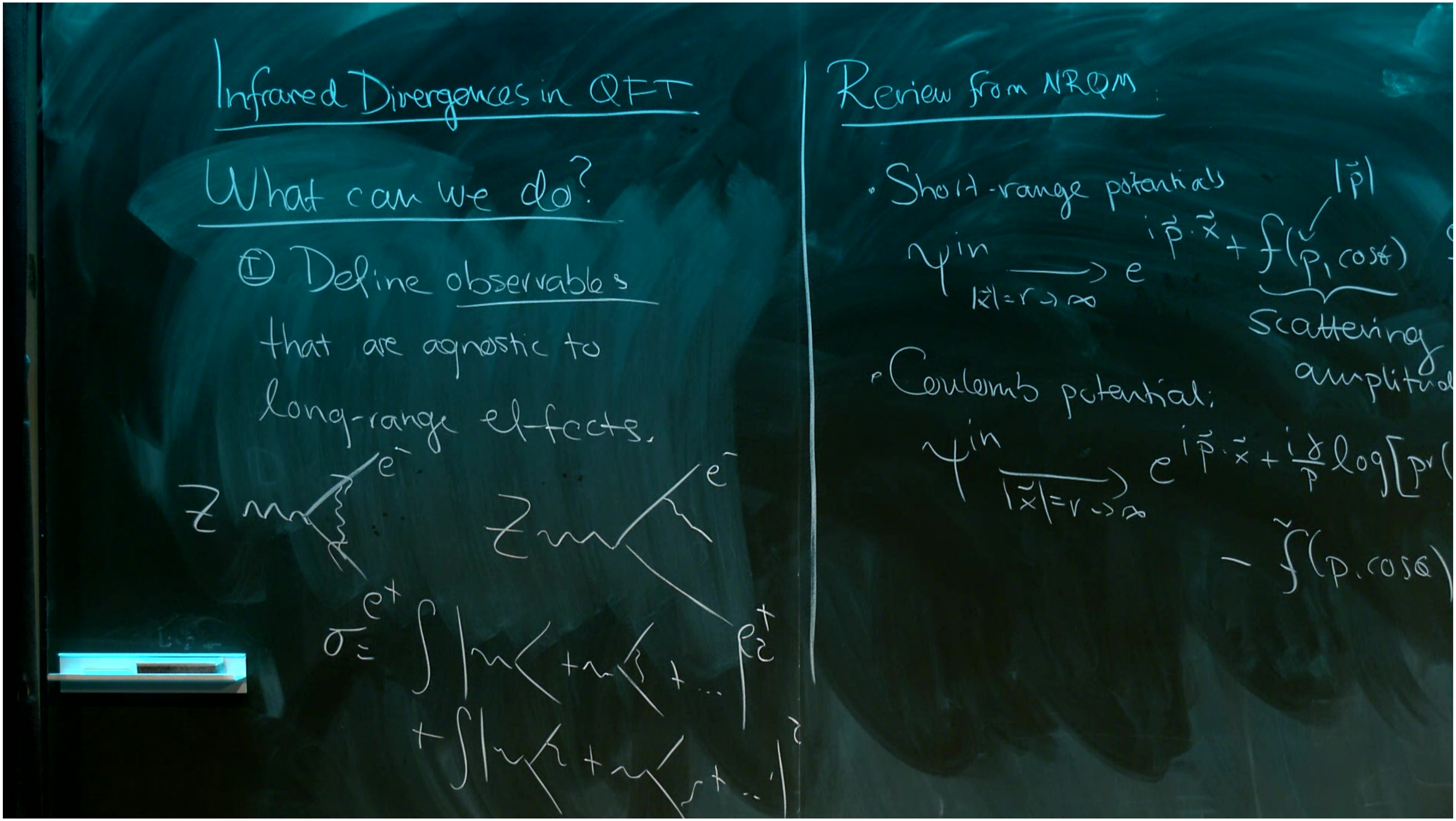
- IR divergence is not just a phase.
- IR divergences do not need to exponentiate.
- IR divergences not only from low-energy particles.

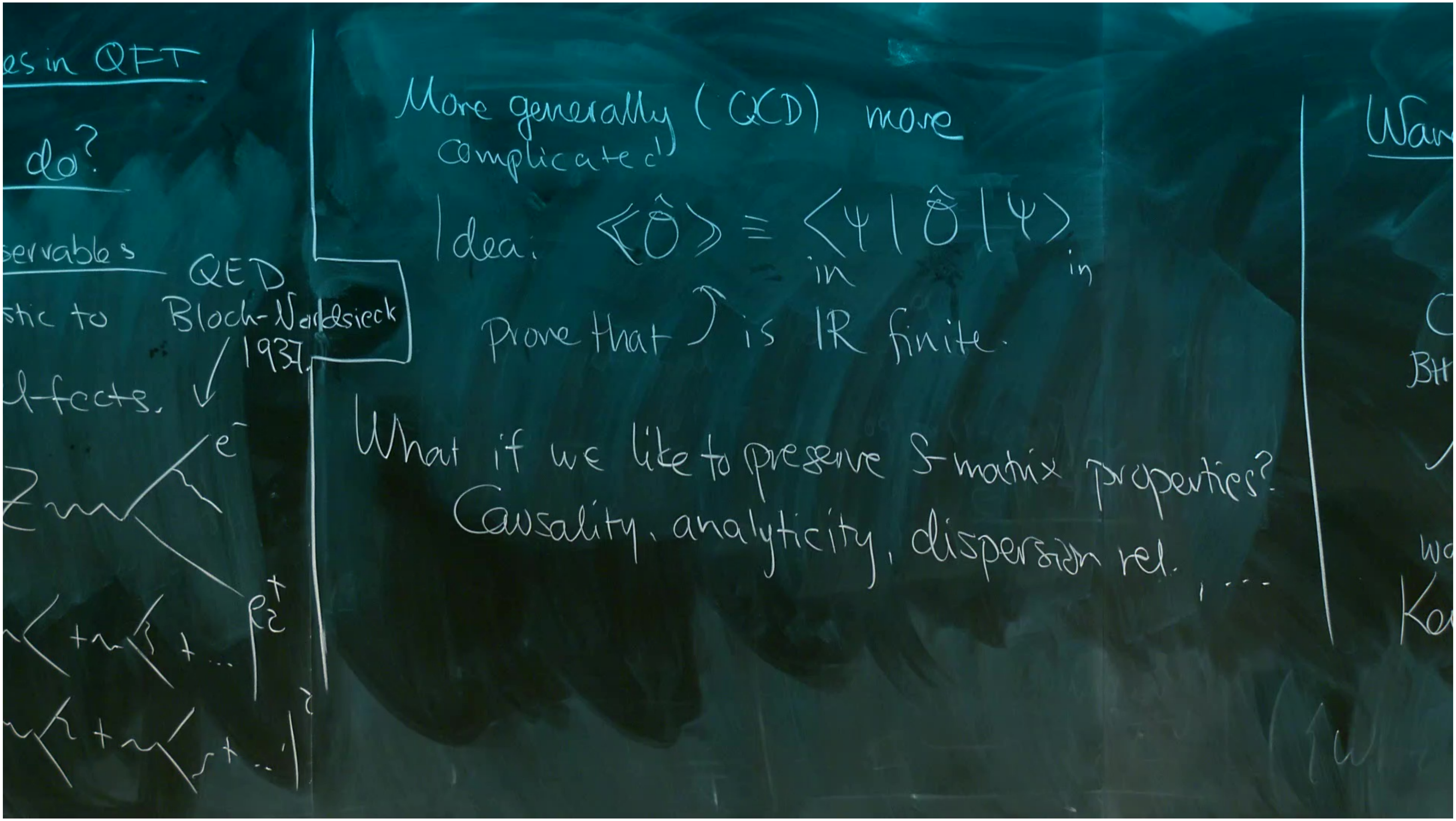
$$\frac{1}{(p+k)^2-m^2+i\epsilon} \longrightarrow \frac{1}{2p\cdot k+k^2+i\epsilon}$$

$k \to 0$ } divergence.

propagator: $\dfrac{p\cdot}{(p+k)^2+i\epsilon} \sim \dfrac{1}{2|k|(1-\cos\theta)+i\epsilon}$

divergences $\begin{cases} |k| \to 0 & \text{soft} \\ \theta \to 0 & \text{collinear.} \end{cases}$


Infrared Divergences in QFT
What can we do?
① Define observables
that are agnostic to
long-range effects.
Review from NRQM:
• Short-range potentials
Coulomb potential:
Scattering
amplitude


es in QFT
do?
servables
QED
stic to Bloch-Nordsieck
1937
More generally (QCD) more complicated
Idea: ⟪Ô⟫ ≡ in⟨Ψ | Ô | Ψ⟩in
Prove that ∫ is IR finite.
What if we like to preserve S-matrix properties?
Causality, analyticity, dispersion rel., ...

$|\psi\rangle_{in}$

...te.

...matrix properties?

...spersion rel., ...

(II) Redefine / modify

$\underbrace{\text{S-matrix}}_{\text{Heisenberg picture}}$ $\underbrace{\text{scattering states.}}_{\text{Schrödinger picture.}}$

Recall QM case:

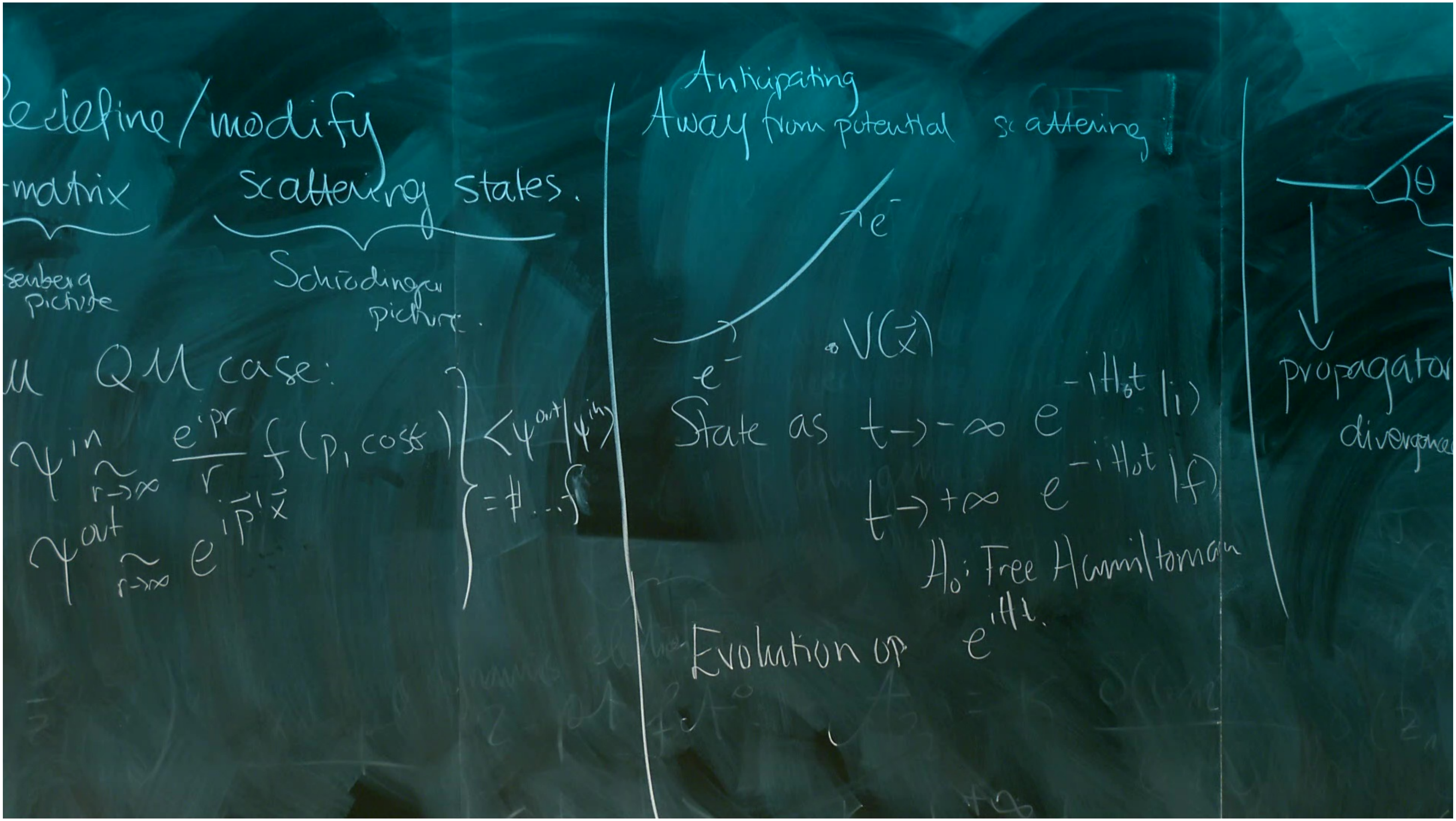
$$\psi^{in} \underset{r\to\infty}{\sim} \frac{e^{ipr}}{r} f(p, \cos\theta)$$

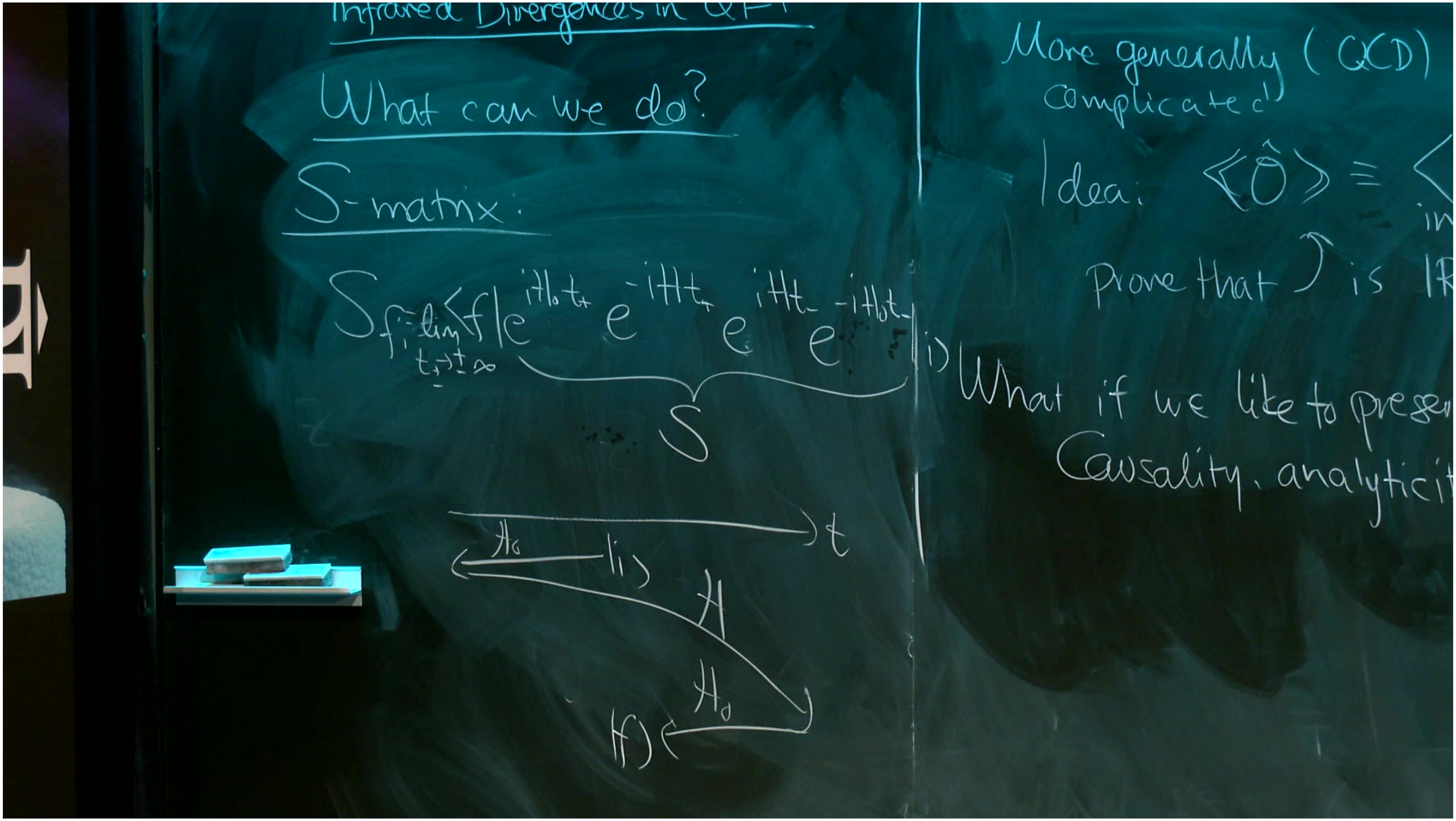
$$\psi^{out} \underset{r\to\infty}{\sim} e^{i\vec{p}'\cdot\vec{x}}$$

$\langle \psi^{out} | \psi^{in} \rangle = \# \ldots f$

Complications

- IR divergen... not just...
- IR divergen... need to ...
- IR divergen... from low-...


edefine / modify
matrix
scattering states.
senberg picture
Schrödinger picture.
QM case:
Anticipating
Away from potential scattering
V(x)
State as t -> -∞
t -> +∞
H0: Free Hamiltonian
Evolution op
propagator
divergence


Infrared Divergences in QFT
What can we do?
S-matrix:
$S_{fi} = \lim_{t_\pm \to \pm\infty} \langle f | e^{iH_0 t_+} e^{-iHt_+} e^{iHt_-} e^{-iH_0 t_-} | i \rangle$
S
t
H_0
$|i\rangle$
H
H_0
$|f\rangle$
More generally (QCD)
complicated
Idea: $\langle\langle \hat{O} \rangle\rangle \equiv \langle$
Prove that is IR
What if we like to prese
Causality, analyticit

...gences in QFT

...we do?

$$\underbrace{\ldots e^{-iHt_+} e^{iHt_-} e^{-iH_0 t_-} |i\rangle}_{S}$$

$\longrightarrow t$

$|i\rangle \xrightarrow{H}$, $|f\rangle \xleftarrow{H_0}$

- Free theory: $H = H_0$

$$S = \mathbb{1}, \quad S_{fi} = \langle f | i \rangle \ \checkmark$$

- Short-range potential scattering $\checkmark$
- Constant potential: $H = H_0 + V_0$

$$S_{fi} = \langle f | i \rangle \lim_{T \to \infty} e^{-i 2 V_0 T}$$

- QED: $S_{fi} = \left(1 - \frac{\alpha \#}{\varepsilon} + \ldots\right) \hat{S} = \exp\left[-\frac{\alpha \#}{\varepsilon}\right] \hat{S} \xrightarrow[\varepsilon \to 0]{} 0$

II

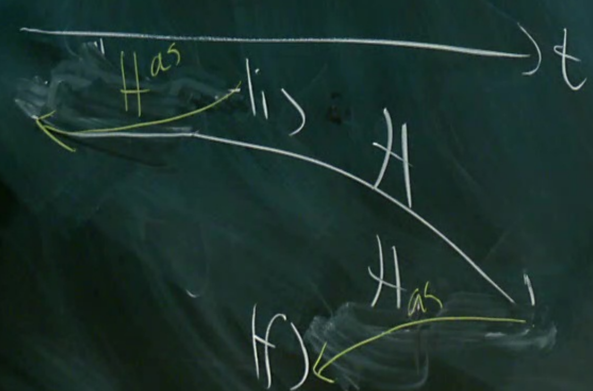
Re...

Infrared Divergences in QFT

What can we do?

S-matrix:

$$S^{H}_{fi} = \lim_{t_\pm \to \pm\infty} \langle f| \underbrace{e^{i \cancel{H_0}^{H_{as}} t_+} e^{-iHt_+} e^{iHt_-} e^{-i\cancel{H_0}^{H_{as}} t_-}}_{S} |i\rangle$$



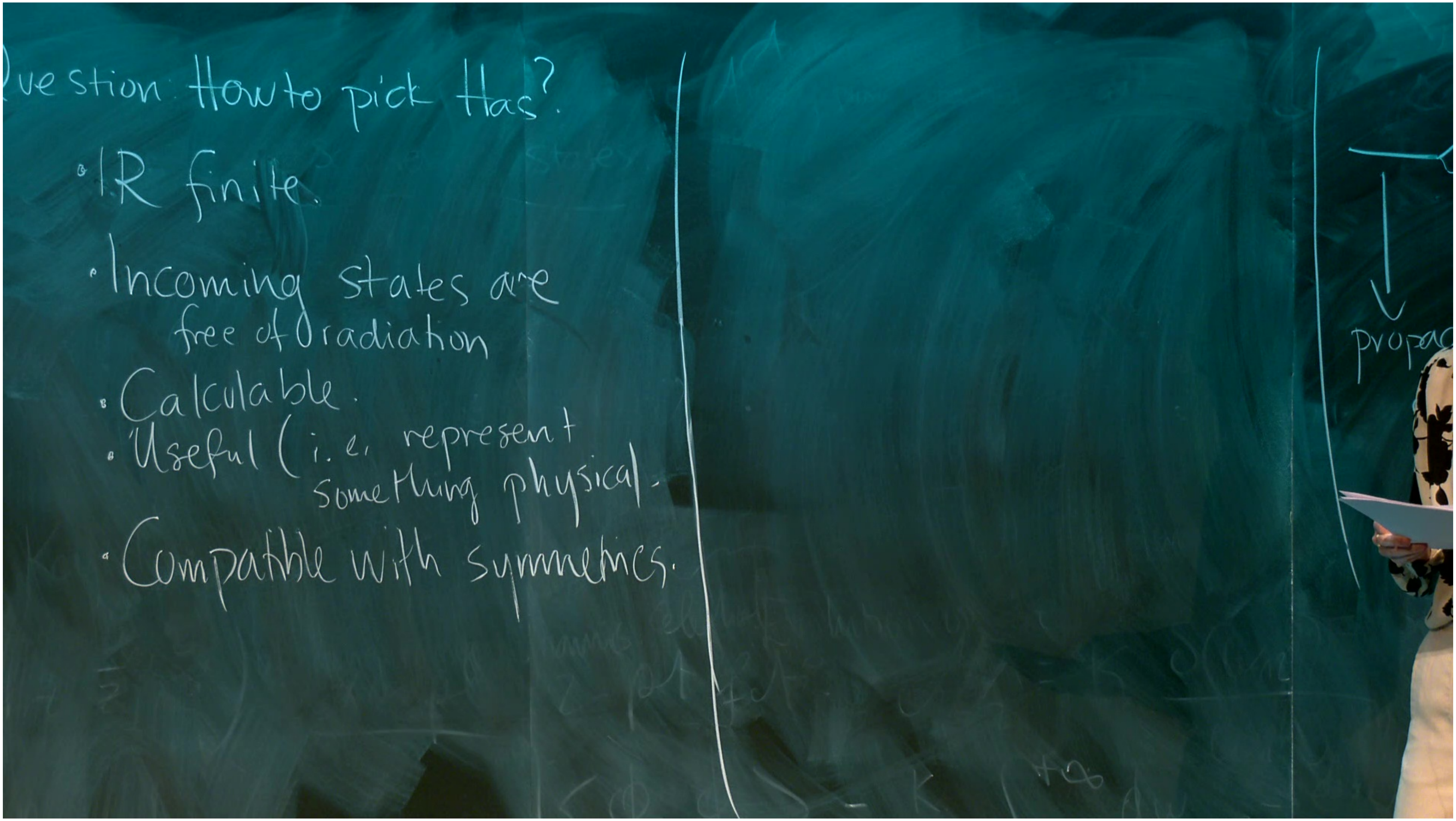
- Free theory: $H = H_0$

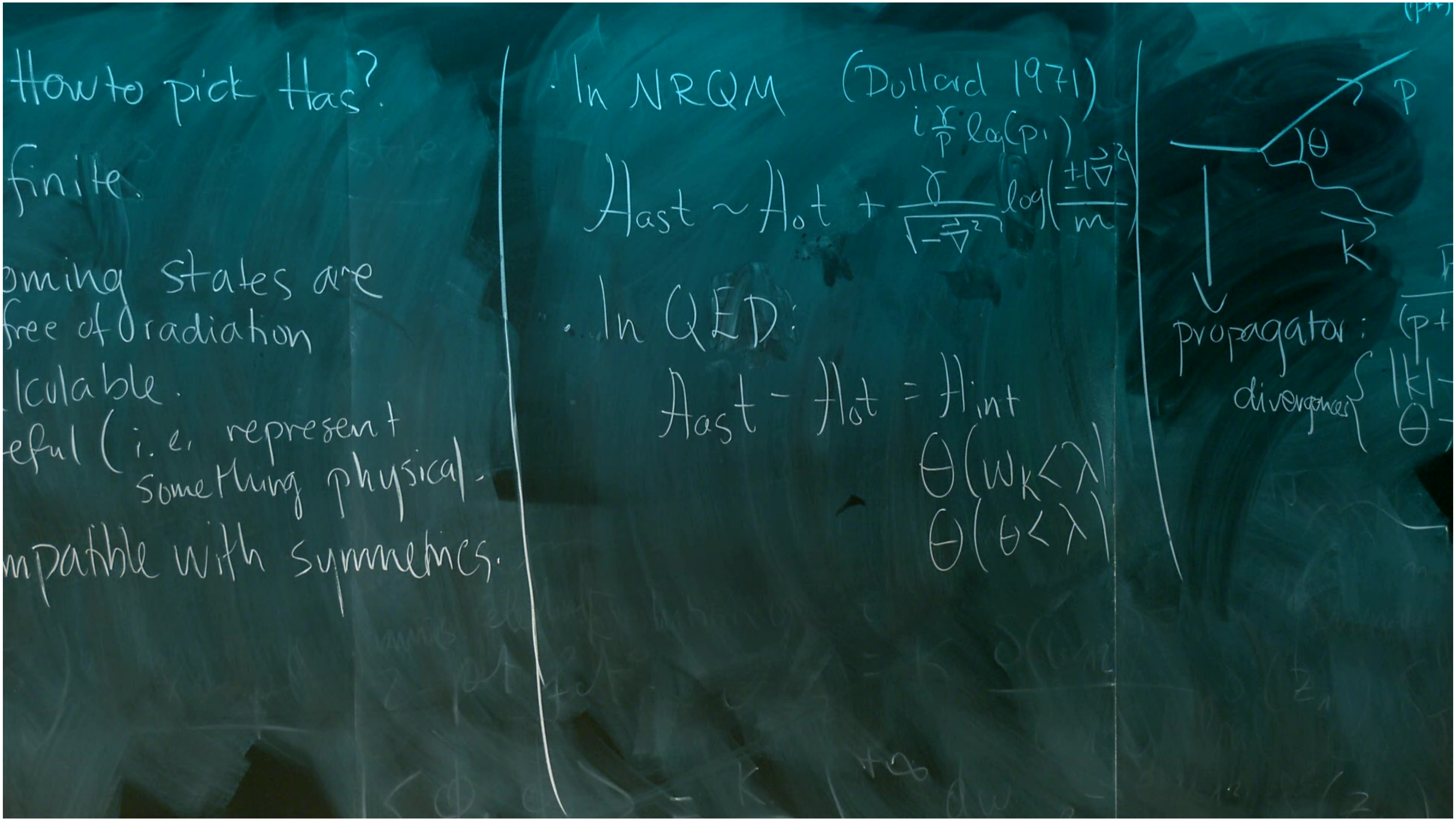
$$S = 1, \quad S_{fi} = \langle f|i$$

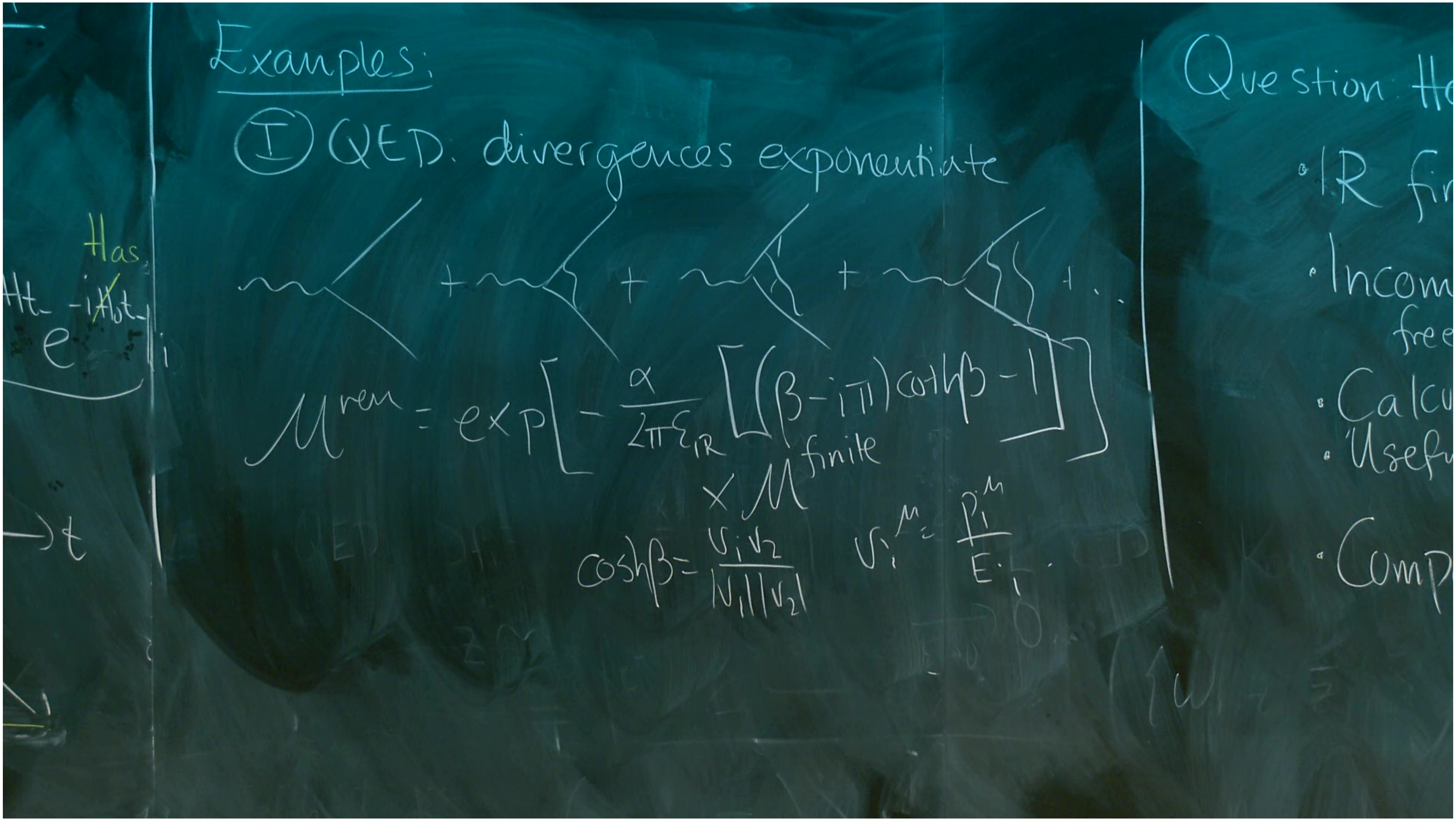
- Short-range potential sca
- Constant potential: H

$$S_{fi} = \langle f|i\rangle \lim_{T\to\infty}$$

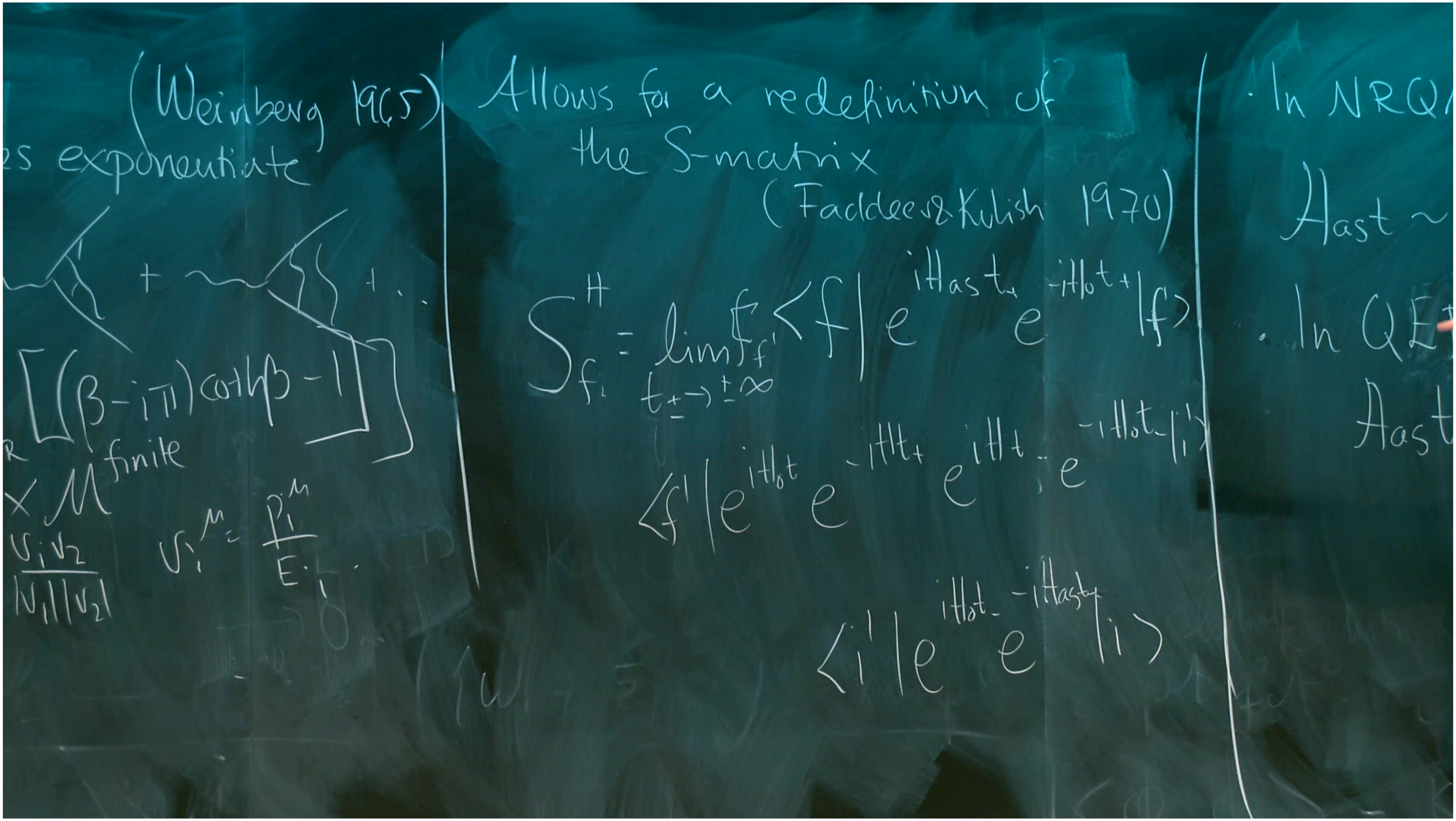
- QED: $S_{fi} = \left(1 - \frac{\alpha \#}{\varepsilon} + \dots\right)$


Question: How to pick Has?
IR finite.
Incoming states are free of radiation
Calculable.
Useful (i.e. represent something physical.
Compatible with symmetries.


How to pick Has?
finite.
oming states are
free of radiation
lculable.
eful (i.e. represent something physical.
mpatible with symmetries.
In NRQM (Dollard 1971)
i γ/p log(p t)
Hast ~ H0 t + γ/√(-∇²) log(±i∇/m)
In QED:
Hast - H0 t = Hint
Θ(ωk < λ)
Θ(θ < λ)
p
θ
k
propagator:
divergences

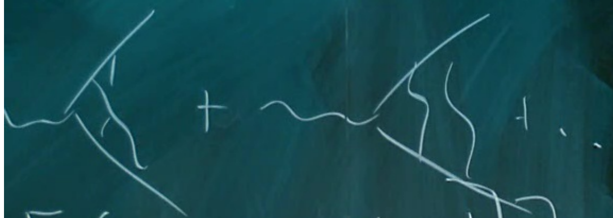


Examples:
I QED: divergences exponentiate
$\mathcal{M}^{ren} = \exp\left[-\frac{\alpha}{2\pi \varepsilon_{IR}} \left[(\beta - i\pi)\coth\beta - 1 \right] \right] \times \mathcal{M}^{finite}$
$\cosh\beta = \frac{v_1 \cdot v_2}{|v_1||v_2|}$
$v_i^\mu = \frac{p_i^\mu}{E_i}$
Question:
· IR
· Incom
free
· Calcu
· Usef
· Comp
Has


(Weinberg 1965)
es exponentiate
$[(\beta - i\pi)\coth\beta - 1]$
finite
$\times \mathcal{M}$
$\frac{v_1 \cdot v_2}{|v_1||v_2|}$
$v_1^\mu = \frac{p_1^\mu}{E_1}$
Allows for a redefinition of the S-matrix
(Faddeev & Kulish 1970)
$S^H_{fi} = \lim_{t_\pm \to \pm\infty} \langle f | e^{iH_{as}t_+} e^{-iH_0 t_+} | f' \rangle$
$\langle f' | e^{iH_0 t} e^{-iHt_+} e^{iHt_-} e^{-iH_0 t_-} | i' \rangle$
$\langle i' | e^{iH_0 t_-} e^{-iH_{as} t_-} | i \rangle$
· In NRQ
$A_{ast} \sim$
· In QE
A_{ast}

(Weinberg 1965)

...es exponentiate



$$\ldots\left[\left[(\beta - i\pi)\coth\beta - 1\right]\right]$$

$\ldots_R \times \mathcal{M}^{\text{finite}}$

$\frac{v_1 \cdot v_2}{|v_1||v_2|} \qquad v_i^\mu = \frac{p_i^\mu}{E_i}$.

Allows for a redefinition of the S-matrix

(Faddeev & Kulish 1970)

$$S^H_{fi} = \lim_{t_\pm \to \pm\infty} \sum_{f'} \langle f| e^{iH_{as} t_+} e^{-iH_0 t_+} |f'\rangle$$

$$\underbrace{\langle f'| e^{iH_0 t} e^{-iHt_+} e^{iHt_-} e^{-iH_0 t_-} |i'\rangle}_{\text{usual } S}$$

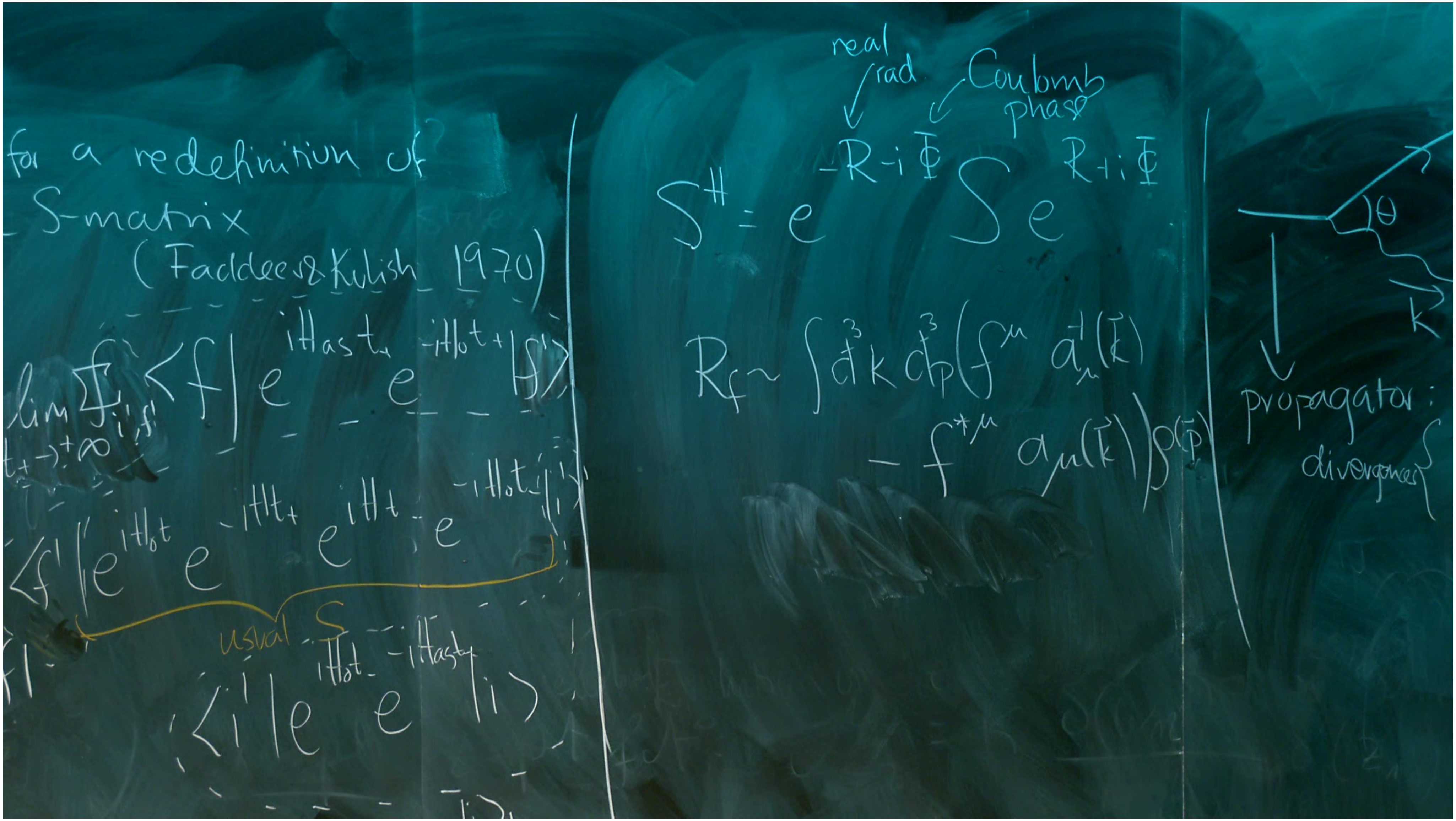
$$\langle i'| e^{iH_0 t_-} e^{-iH_{as} t_-} |i\rangle$$

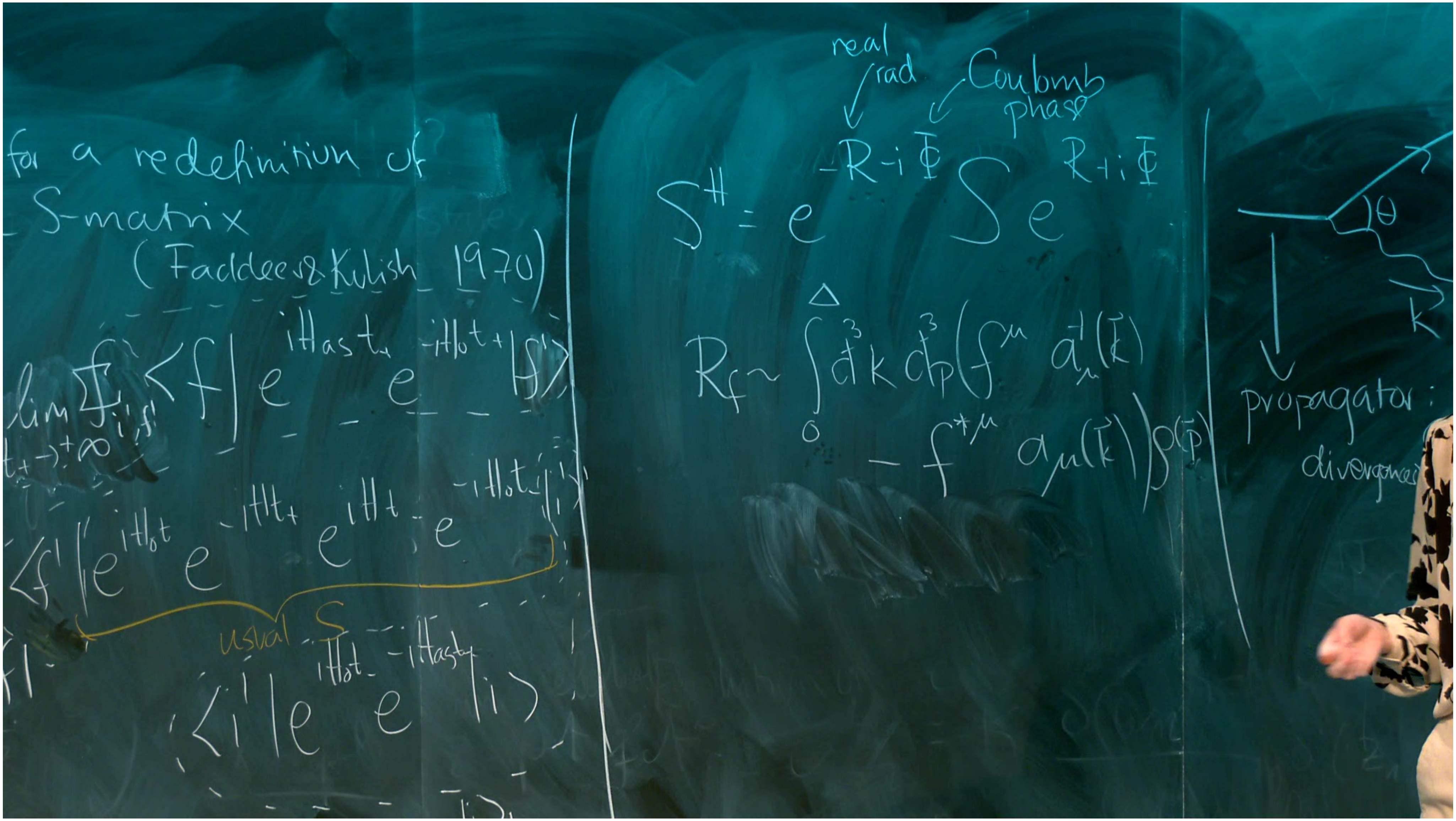
· In NRQ...

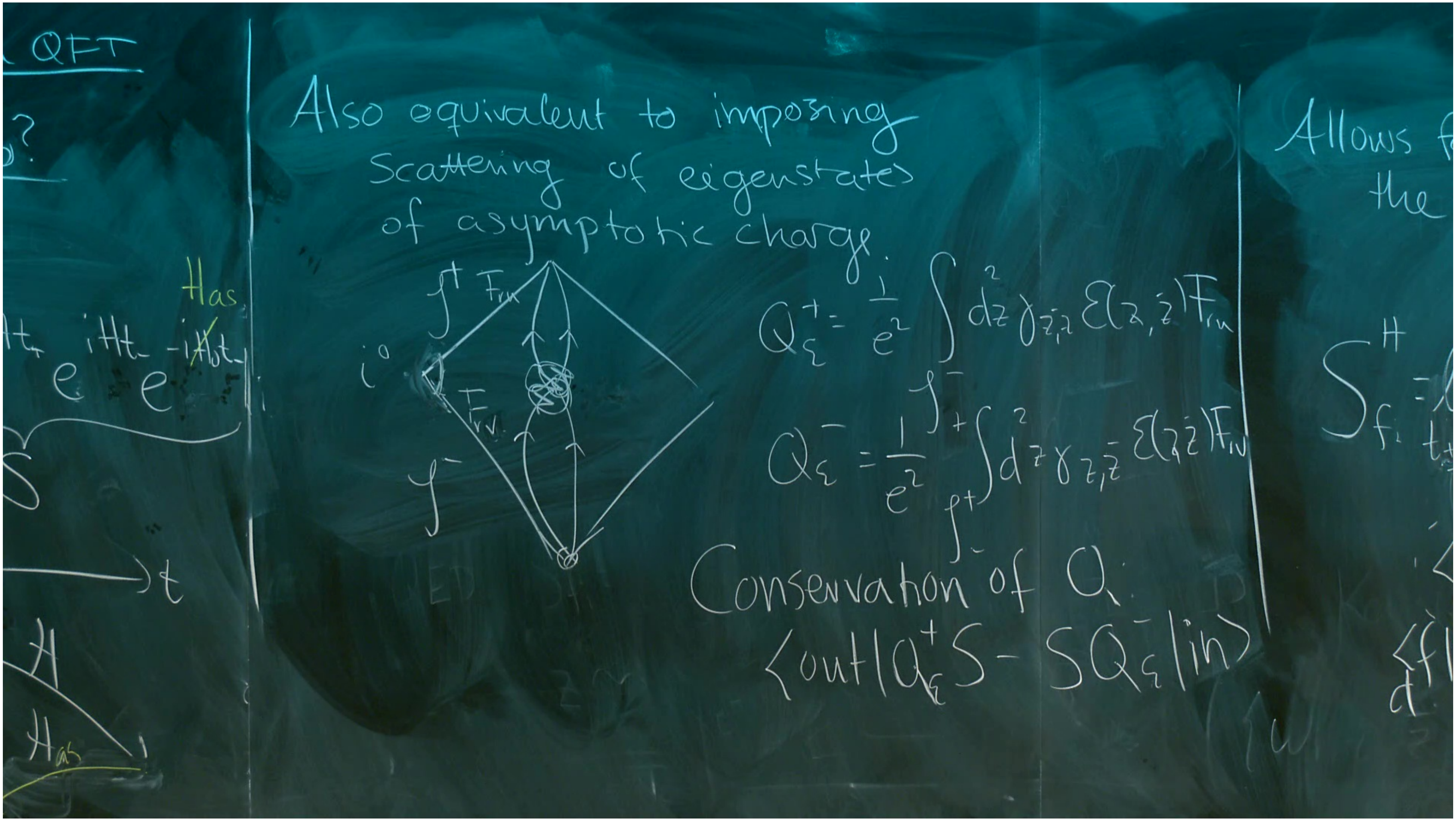
$H_{ast} \sim$ ...

· In QE...

H_{ast} ...


for a redefinition of S-matrix
(Faddeev & Kulish 1970)
usual S
real rad
Coulomb phase
propagator:
divergences


for a redefinition of?
S-matrix
(Faddeev & Kulish 1970)
usual S
real rad
Coulomb phase
propagator:
divergence


QFT
Also equivalent to imposing
Scattering of eigenstates
of asymptotic charge:
$Q_\varepsilon^+ = \frac{1}{e^2}\int_{\mathcal{I}^+_-} d^2z\, \gamma_{z\bar{z}}\, \varepsilon(z,\bar{z}) F_{ru}$
$Q_\varepsilon^- = \frac{1}{e^2}\int_{\mathcal{I}^-_+} d^2z\, \gamma_{z\bar{z}}\, \varepsilon(z,\bar{z}) F_{rv}$
Conservation of Q:
$\langle out | Q_\varepsilon^+ S - S Q_\varepsilon^- | in \rangle$
Allows
the

ock basis:

on thm (Weinberg)

e eigenstates

Q^- and Q^+

& Kulish.

real rad — Coulomb phase

$$S^\dagger = e^{-R - i\Phi}\, S\, e^{R + i\Phi}$$

$$R_f \sim \int_0^{\Delta} d^3k\, d^3p \left(f^\mu\, a^\dagger_\mu(\vec{k}) - f^{*\mu}\, a_\mu(\vec{k}) \right) \rho(\vec{p})$$

$$f^\mu \to \frac{p^\mu}{p\cdot k}$$

$$\frac{1}{(p+k)^2 - m^2 + i\varepsilon}$$

$k \to 0$

propagator: $\dfrac{p\cdot}{(p+k)^2 + i\varepsilon} \sim$

divergences $\begin{cases} |k| \to 0 & \text{so} \\ \theta \to 0 & \text{col} \end{cases}$

nberg)

tes

real rad

Coulomb phase

$$S^H = e^{-R - i\Phi} \, S \, e^{R + i\Phi}$$

$$R_f \sim \int_0^{\Delta} d^3k \, d^3p \left(f^\mu a^\dagger_\mu(\vec{k}) - f^{*\mu} a_\mu(\vec{k}) \right) \rho(\vec{p})$$

$$f^\mu = \frac{p^\mu}{p \cdot k} \phi$$

What do we take as Δ and f^μ? ($\phi \to 1$ $|\vec{k}| \to 0$)

1) Connect to physical scales?

2) Keep analyticity properties?

$$\mathcal{A} = \langle 0 | W_1 \ldots W_n | 0 \rangle \times \mathcal{A}^{Hard}$$

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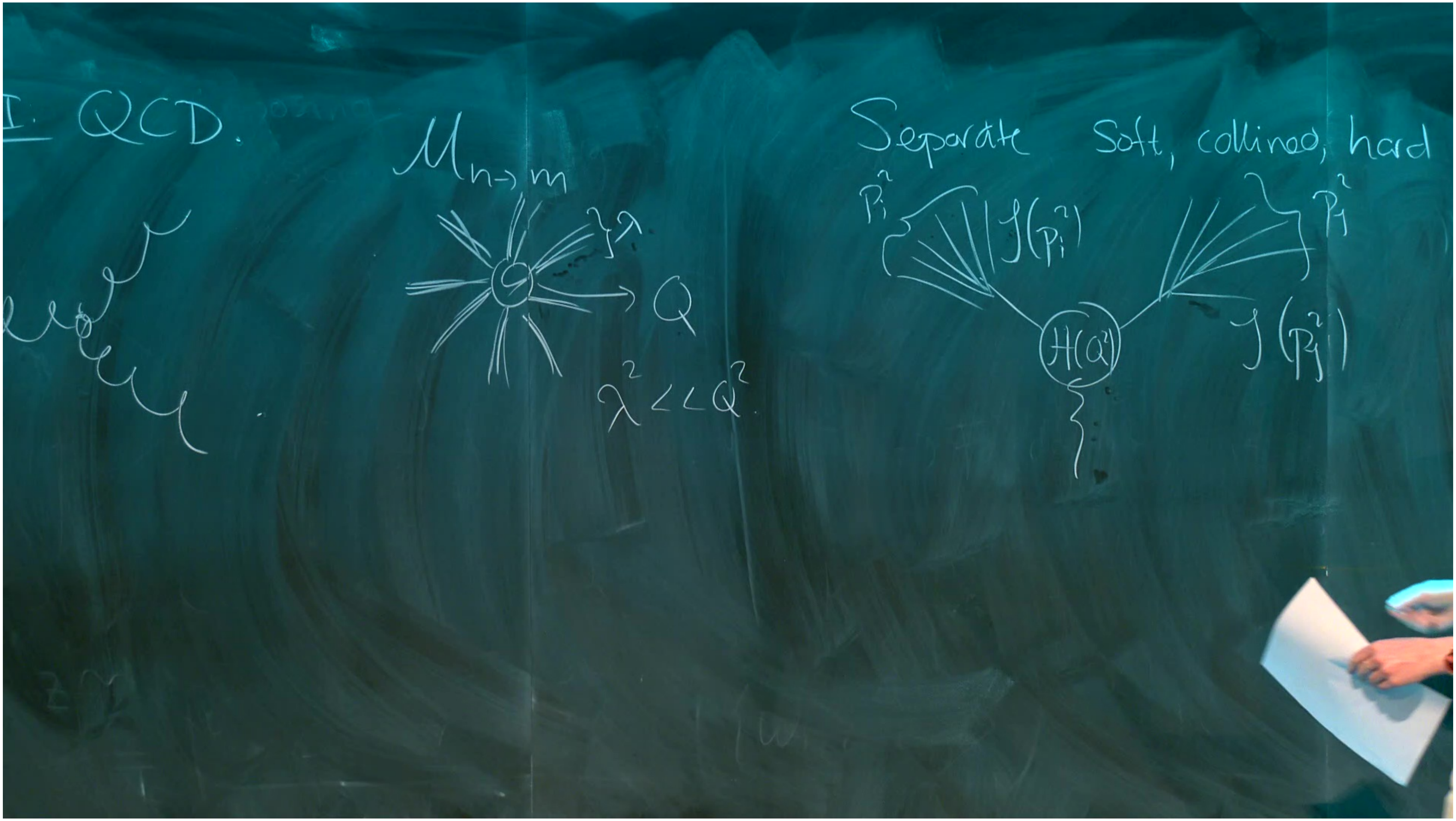
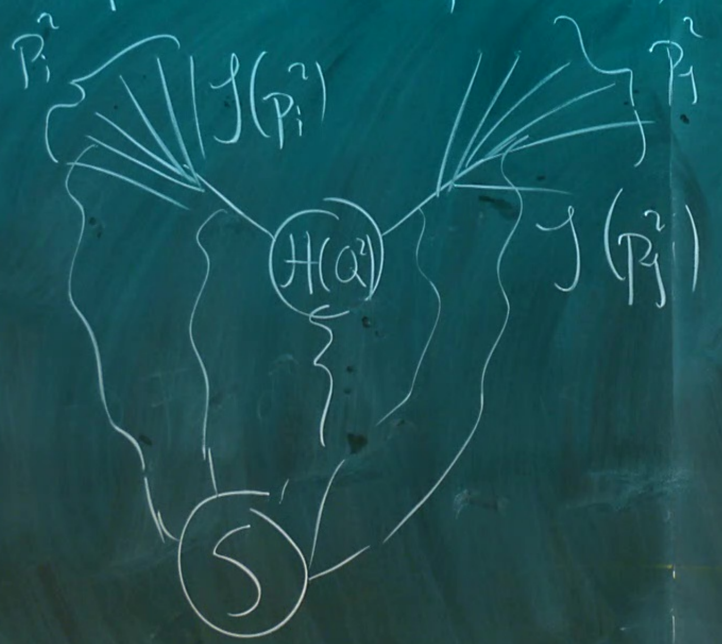

I. QCD.
M n→m
λ
Q
λ² << Q²
Separate Soft, collinear, hard
p_i²
J(p_i²)
H(Q²)
J(p_j²)
p_j²


I. QCD.
M n->m
λ
Q
λ² << Q²
Separate soft, collinear, hard
p_i²
J(p_i²)
p_j²
J(p_j²)
H(Q²)
S

Separate soft, collinear, hard

Framework: Soft-collinear effective theory 1410.1892.

Separate soft, collinear, hard



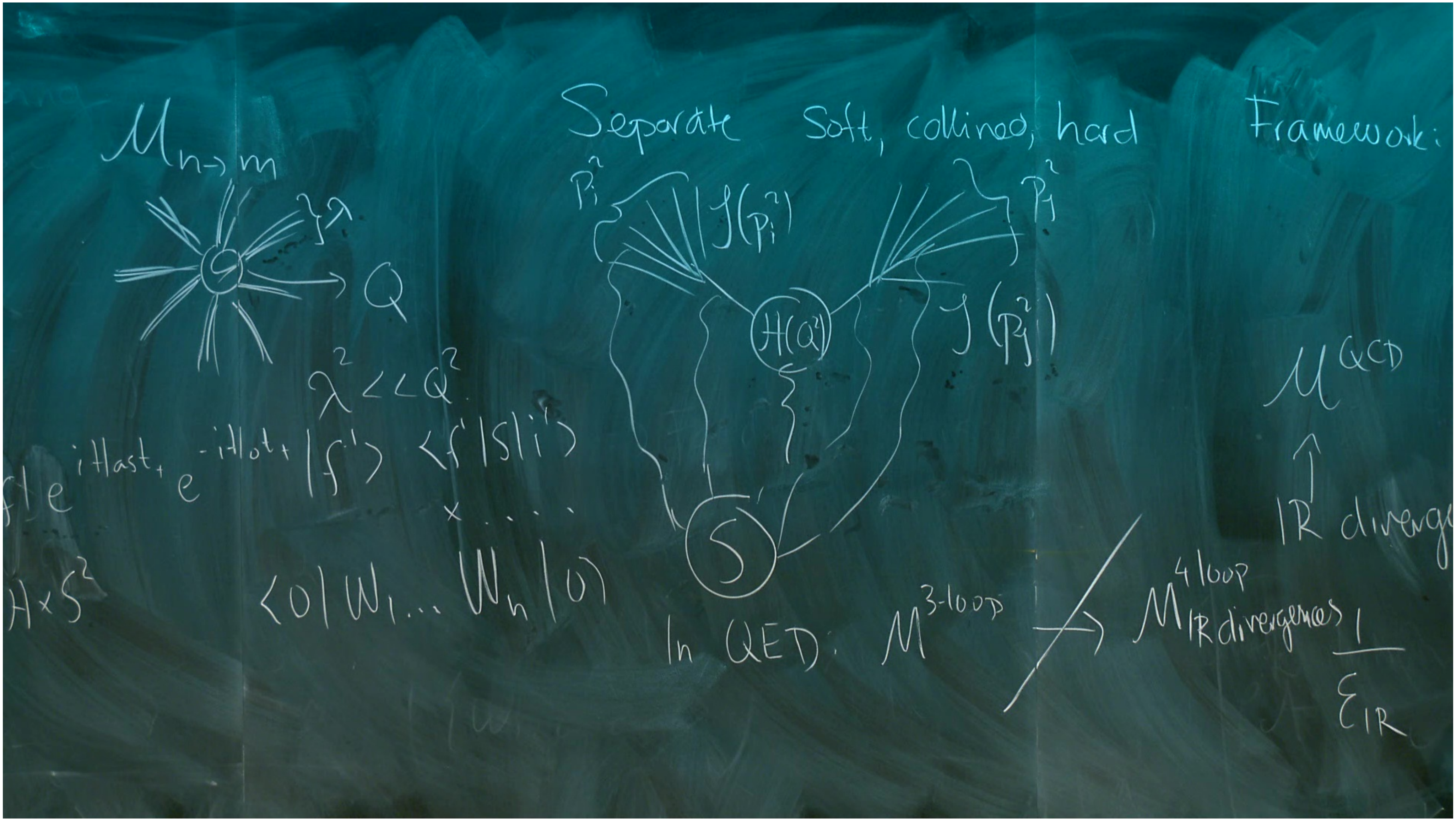
Framework: Soft-collinear effective theory 1410.1892.

$$\mathcal{M}^{QCD} = C^{SCET}$$

IR divergent

Wilson coefficient UV divergent

$$\frac{1}{\epsilon_{IR}} = \frac{1}{\epsilon_{UV}} + \left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right)$$


Separate Soft, collinear, hard
Framework:
M n->m
λ
Q
λ² << Q²
J(p_i²)
H(Q²)
J(p_j²)
S
⟨0| W_1 ... W_n |0⟩
In QED: M^3-loop ↛ M^4 loop IR divergences
1/ε_IR
M^QCD
IR divergences