Title: Lecture - Carrollian Physics b

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Collection/Series: Celestial Holography Summer School 2024

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Carrollian geometry and Symmetries

$X^a = (u, X^A)$

* $ds^2 = \eta_{ab}\, dx^a dx^b = -c^2 du^2 + \delta_{AB}\, dX^A dX^B$

$$\xrightarrow[c\to 0]{} 0\, du^2 + \delta_{AB}\, dX^A dX^B \equiv q_{ab}\, dx^a dx^b$$

* $-c^2 \eta^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -c^2 \delta^{AB} \end{pmatrix} \xrightarrow{c\to 0} n^a n^b$, $n^a = \delta^a_u$

* Carrollian geometry (q_{ab}, n^c) emerges naturally in the limit $c \to 0$

  ↳ Same structure than the one appearing at $\mathcal{I}$.

$q_{ab} n^b = 0$

$c \to 0$

$* \; -c^2 \eta^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -c^2 \delta^{AB} \end{pmatrix} \xrightarrow{c \to 0} n^a n^b \quad , \; n^a = \delta^a_n$

* Carrollian geometry (q_{ab}, n^c) emerges naturally in the limit $c \to 0$
  - Same structure than the one appearing at $\mathcal{I}$.

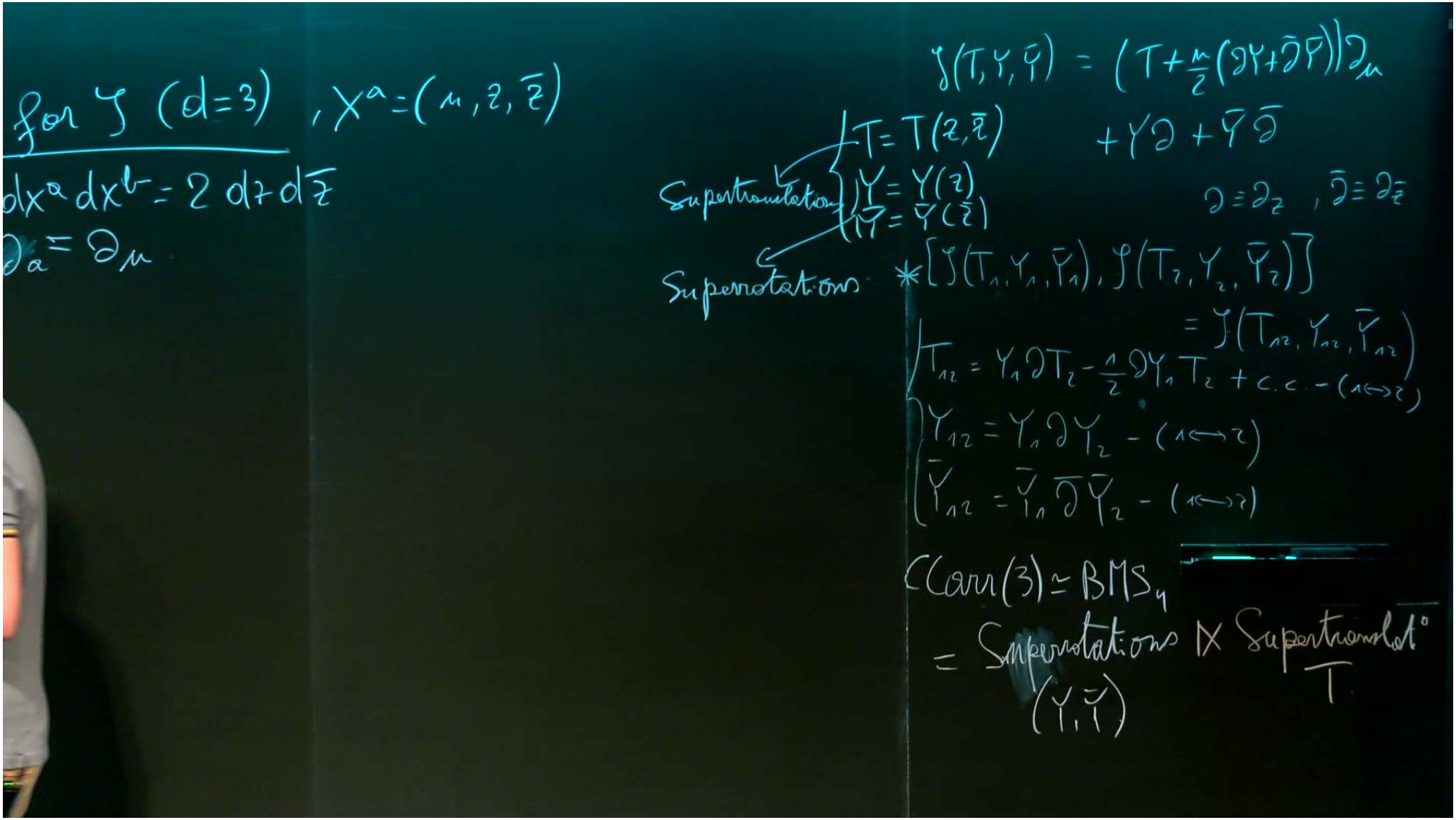
Carrollian algebra: $\mathcal{L}_\xi q_{ab} = 0 = \mathcal{L}_\xi n^a$.

($\parallel \mathcal{L}_\xi \eta_{ab} = 0$ Poincaré)

Conformal Carrollian algebra $\mathfrak{CCarr}(d)$

$\mathcal{L}_\xi q_{ab} = 2\alpha q_{ab}$ ($\parallel$ conformal algebra $\mathcal{L}_\xi \eta_{ab} = 2\alpha \eta_{ab}$)

$\mathcal{L}_\xi n^a = -\alpha n^a$.


for $\mathcal{Y}$ $(d=3)$, $X^a = (u, z, \bar{z})$
$dx^a dx^b = 2\, dz\, d\bar{z}$
$\partial_a = \partial_u$
$\mathcal{Y}(T, Y, \bar{Y}) = \left(T + \frac{u}{2}(\partial Y + \bar{\partial}\bar{Y})\right)\partial_u$
$+ Y\partial + \bar{Y}\bar{\partial}$
$\partial \equiv \partial_z$, $\bar{\partial} \equiv \partial_{\bar{z}}$
Supertranslations $\begin{cases} T = T(z,\bar{z}) \\ Y = Y(z) \\ \bar{Y} = \bar{Y}(\bar{z}) \end{cases}$
Superrotations
$*\left[\mathcal{Y}(T_1, Y_1, \bar{Y}_1), \mathcal{Y}(T_2, Y_2, \bar{Y}_2)\right]$
$= \mathcal{Y}(T_{12}, Y_{12}, \bar{Y}_{12})$
$T_{12} = Y_1 \partial T_2 - \frac{1}{2}\partial Y_1 T_2 + c.c. - (1 \leftrightarrow 2)$
$Y_{12} = Y_1 \partial Y_2 - (1 \leftrightarrow 2)$
$\bar{Y}_{12} = \bar{Y}_1 \bar{\partial} \bar{Y}_2 - (1 \leftrightarrow 2)$
$\mathbb{C}Carr(3) \simeq BMS_4$
$=$ Superrotations $(Y, \bar{Y})$ $\ltimes$ Supertranslat° T

$x^a dx^b = z\, dz + d\bar{z}$

$\partial_a = \partial_\mu$

Supertranslations $\begin{cases} T = T(z) \\ \bar{T} = \bar{T}(\bar{z}) \end{cases}$

Superrotations

$\partial = \partial_z$, $\bar{\partial} = \partial_{\bar{z}}$

$SO(d,2)$

$\downarrow c \to 0$

Global $CCarr(d)$

$* [\mathcal{J}(T_1, Y_1, \bar{Y}_1), \mathcal{J}(T_2, Y_2, \bar{Y}_2)]$

$= \mathcal{J}(T_{12}, Y_{12}, \bar{Y}_{12})$

$\begin{cases} T_{12} = Y_1 \partial T_2 - \frac{1}{2} \partial Y_1 T_2 + c.c. - (1 \leftrightarrow 2) \\ Y_{12} = Y_1 \partial Y_2 - (1 \leftrightarrow 2) \\ \bar{Y}_{12} = \bar{Y}_1 \bar{\partial} \bar{Y}_2 - (1 \leftrightarrow 2) \end{cases}$

$CCarr(3) \simeq BMS_4$

$= \underset{(Y, \bar{Y})}{\text{Superrotations}} \ltimes \underset{T}{\text{Supertranslat°}}$

Global $CCarr(3) \simeq Poincaré_4$

$\begin{cases} T = 1, z, \bar{z}, z\bar{z} \\ Y = 1, z, z^2 \\ \bar{Y} = 1, \bar{z}, \bar{z}^2 \end{cases}$

Ward identities:

$\mathcal{C}(u,z,\bar{z})\,\phi_{k,\bar{k}}(u,z,\bar{z})$

* Carrollian primary of Weights $(k,\bar{h})$:

$$\delta_{(T,Y,\bar{Y})}\,\phi_{(k,\bar{k})}(u,z,\bar{z}) = \left(T+\frac{u}{2}\left(\partial Y+\bar{\partial}\bar{Y}\right)\right)\partial_u\phi_{k,\bar{k}} + \left(Y\partial + k\,\partial Y + \bar{Y}\bar{\partial} + \bar{k}\,\bar{\partial}\bar{Y}\right)\phi_{k,\bar{k}} \quad (*)$$

* Quasi primary if transforms as $(*)$ under Global $CConf(3)$

Remark. $\partial_u\phi_{k,\bar{k}}$ is also a primary of shifted Weights $\left(k+\frac{1}{2},\bar{k}+\frac{1}{2}\right)$

(∂_u-descendants are Carrollian primaries).

Carrollian Ward identities: $\sum_{i=1}^{n}\mathcal{C}(u_i,z_i,\bar{z}_i)\left\langle\phi_{k_1,\bar{k}_1}(u_1,z_1,\bar{z}\right.$

$\mathcal{C}(u,z,\bar z)\,\phi_{k,\bar k}(u,z,\bar z)$, $X^a=(u,z,\bar z)$

$\bar Y))\,\partial_u\,\phi_{k,\bar k}$

$+\bar Y\bar\partial+\bar k\,\bar\partial\bar Y)\,\phi_{k,\bar k}$ (*)

obal CCarr(3)

fted Weights $(k+\frac{1}{2},\bar k+\frac{1}{2})$

$,z_i,\bar z_i)\,\langle\phi_{k_1,k_1}(u_1,z_1,\bar z_1)\cdots\phi_{(k_i,\bar k_i)}(u_i,z_i,\bar z_i)\cdots\phi_{(k_n,\bar k_n)}(u_n,z_n,\bar z_n)\rangle=0$

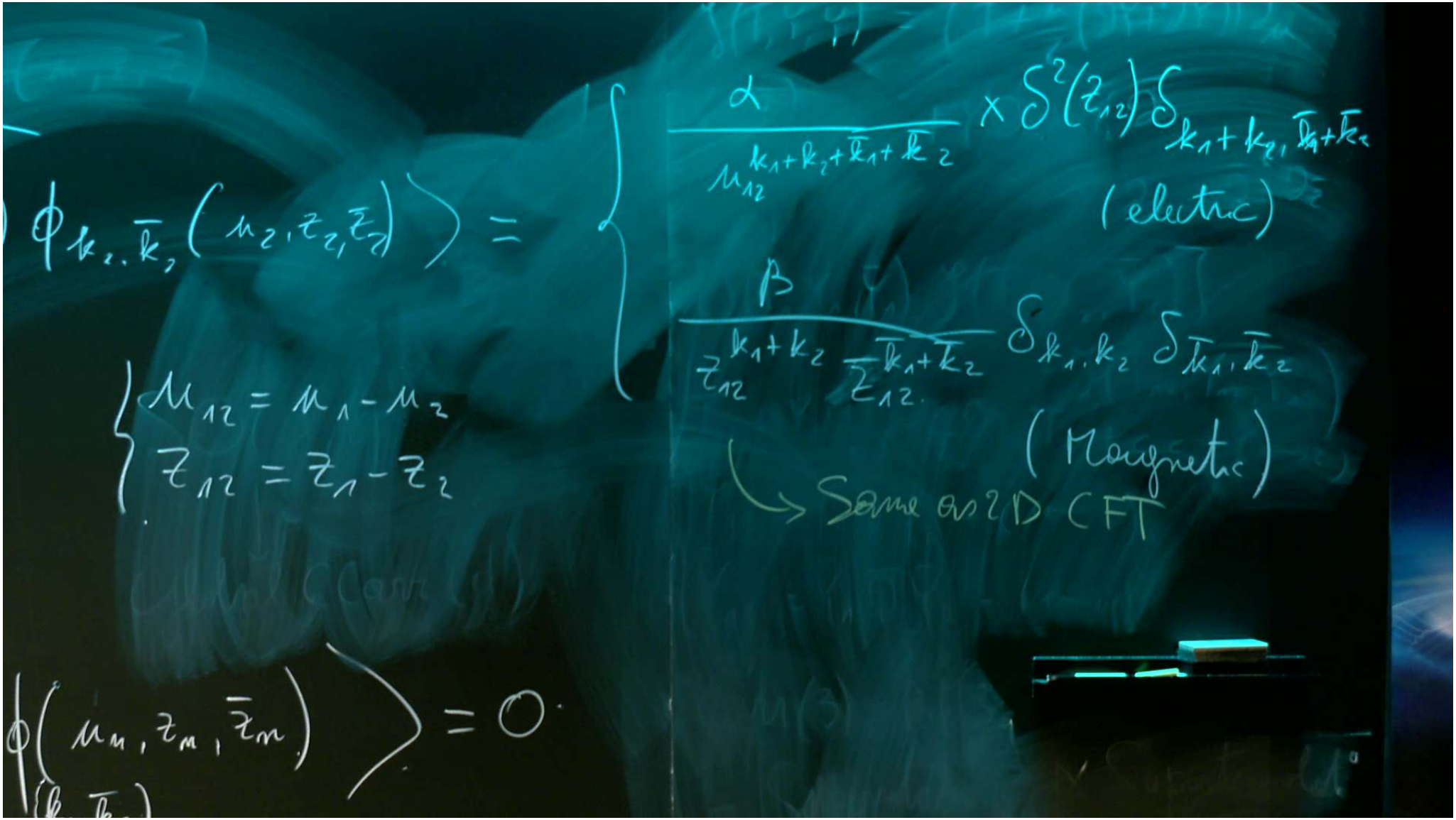
Supertranslations

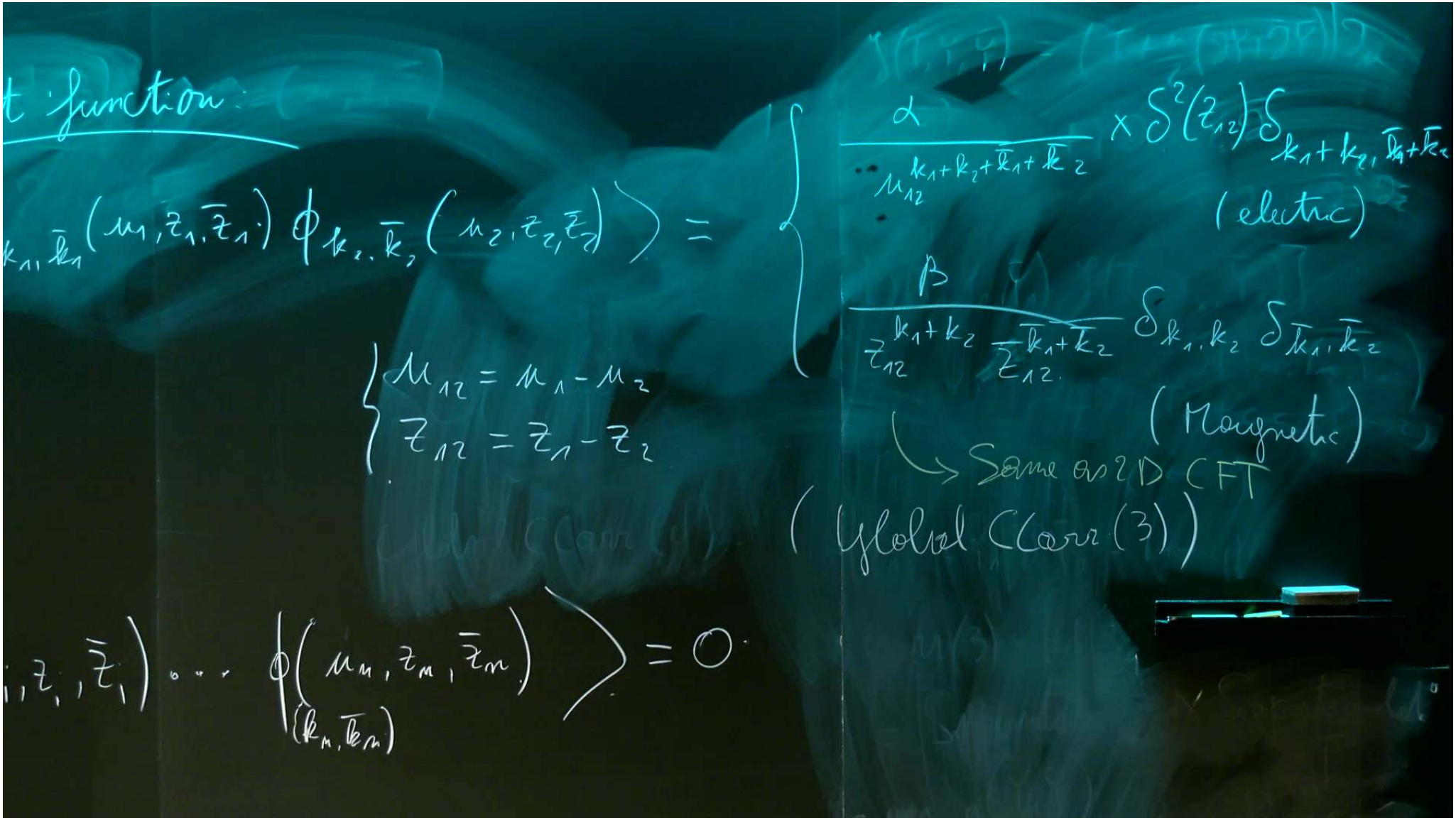
Superrotations

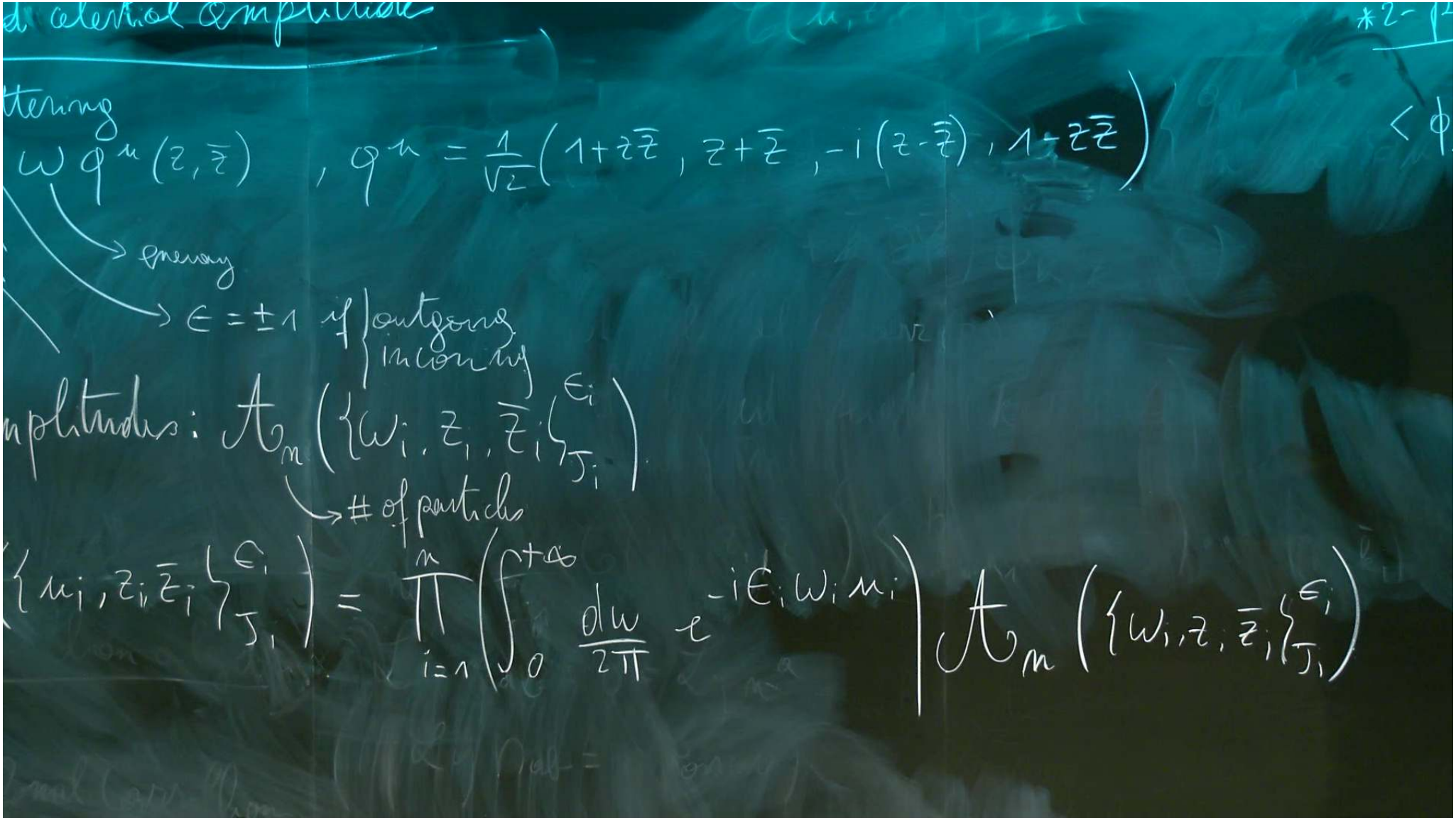
$SO(d,2)$

$\downarrow c\to 0$

Global CCarr(d)


$\langle \phi_{k_i,\bar{k}_i}(u_i,z_i,\bar{z}_i) \rangle =$
$\begin{cases} u_{12} = u_1 - u_2 \\ z_{12} = z_1 - z_2 \end{cases}$
$\frac{\alpha}{u_{12}^{k_1+k_2+\bar{k}_1+\bar{k}_2}} \times \delta^2(z_{12})\,\delta_{k_1+k_2,\bar{k}_1+\bar{k}_2}$
(electric)
$\frac{\beta}{z_{12}^{k_1+k_2}\,\bar{z}_{12}^{\bar{k}_1+\bar{k}_2}}\,\delta_{k_1,k_2}\,\delta_{\bar{k}_1,\bar{k}_2}$
(Magnetic)
↳ Same as 2D CFT
$\langle \phi(u_m,z_m,\bar{z}_m) \rangle = 0$


t function:
$\phi_{k_1,\bar{k}_1}(u_1,z_1,\bar{z}_1)\,\phi_{k_2,\bar{k}_2}(u_2,z_2,\bar{z}_2)\rangle =$
$\begin{cases} u_{12} = u_1 - u_2 \\ z_{12} = z_1 - z_2 \end{cases}$
$\frac{\alpha}{u_{12}^{k_1+k_2+\bar{k}_1+\bar{k}_2}} \times \delta^2(z_{12})\,\delta_{k_1+k_2,\bar{k}_1+\bar{k}_2}$
(electric)
$\frac{\beta}{z_{12}^{k_1+k_2}\,\bar{z}_{12}^{\bar{k}_1+\bar{k}_2}}\,\delta_{k_1,k_2}\,\delta_{\bar{k}_1,\bar{k}_2}$
(Magnetic)
↳ Same as 2D CFT
(Global Conf(3))
$\ldots\,\phi_{(k_m,\bar{k}_m)}(u_m,z_m,\bar{z}_m)\rangle = 0$



d celestial amplitude

ttering

$\omega\, q^\mu(z,\bar z)$, $q^\mu = \frac{1}{\sqrt{2}}\left(1+z\bar z,\ z+\bar z,\ -i(z-\bar z),\ 1-z\bar z\right)$

$\omega \to$ energy

$\to \epsilon = \pm 1$ if $\begin{cases}\text{outgoing}\\ \text{incoming}\end{cases}$

mplitudes: $\mathcal{A}_n\left(\{\omega_i, z_i, \bar z_i\}_{J_i}^{\epsilon_i}\right)$

$n \to$ # of particles

$$\left(\{u_i, z_i, \bar z_i\}_{J_i}^{\epsilon_i}\right) = \prod_{i=1}^{n}\left(\int_0^{+\infty}\frac{d\omega}{2\pi}\, e^{-i\epsilon_i\omega_i u_i}\right)\mathcal{A}_n\left(\{\omega_i, z_i, \bar z_i\}_{J_i}^{\epsilon_i}\right)$$

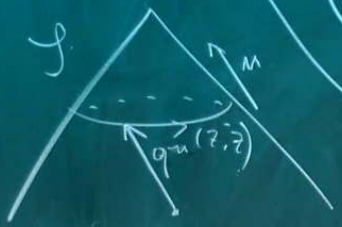
*2-

$\langle \phi$

Carrollian and celestial amplitudes

Rindler scattering

$$p^\mu = \epsilon \omega q^\mu(z,\bar z) \quad , \quad q^\mu = \frac{1}{\sqrt{2}}\left(1+z\bar z \, , \, z+\bar z \, , \, -i(z-\bar z) \, , \, 1-z\bar z\right)$$



energy

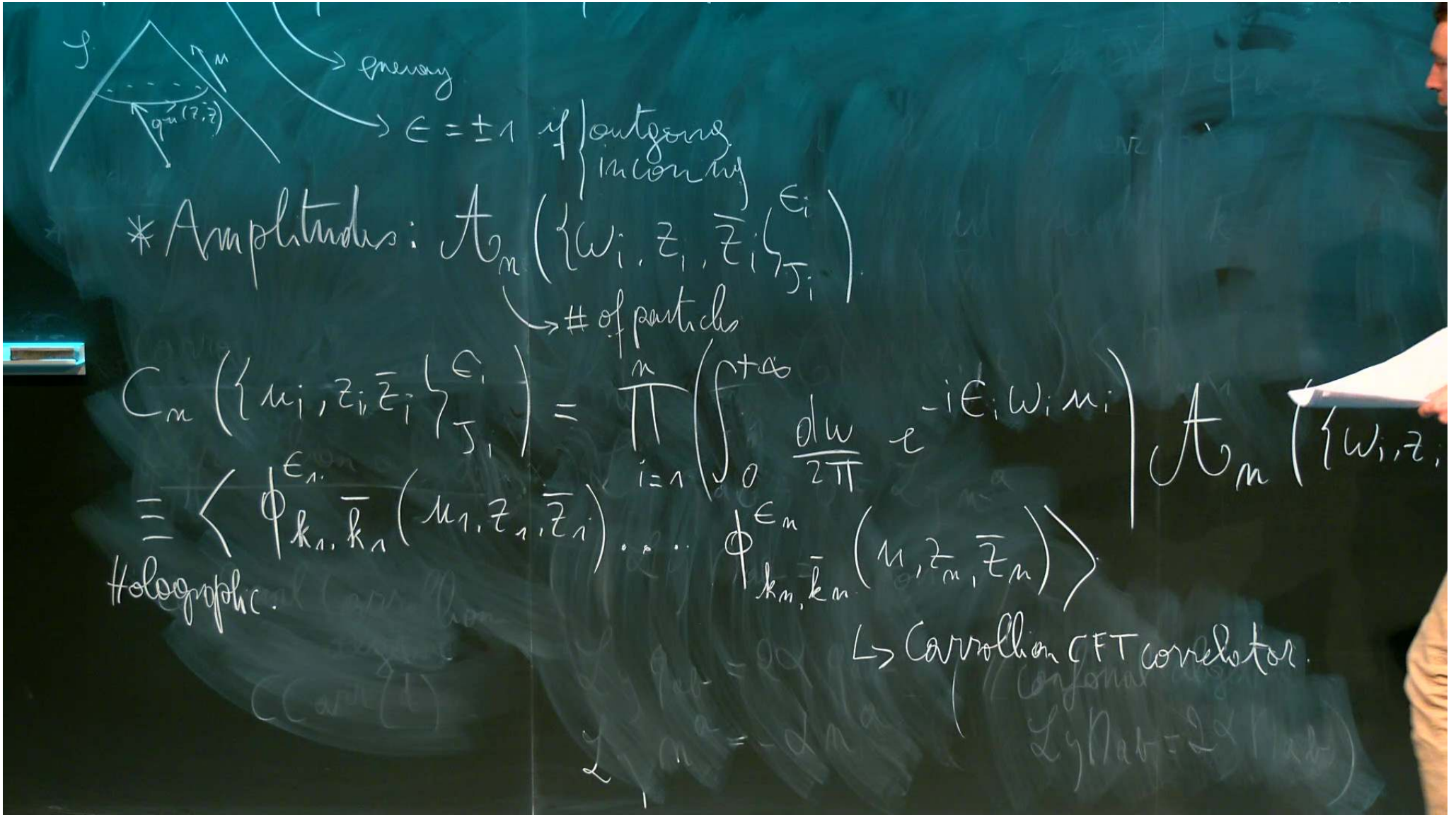
$\epsilon = \pm 1$ if $\begin{cases} \text{outgoing} \\ \text{incoming} \end{cases}$

* Amplitudes: $\mathcal{A}_n\left(\{\omega_i, z_i, \bar z_i\}^{\epsilon_i}_{J_i}\right)$

$n \to$ # of particles

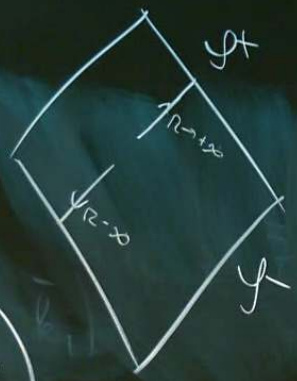
$$C_n\left(\{\omega_i, z_i \bar z_i\}^{\epsilon_i}_{J_i}\right) = \prod_{i=1}^{n}\left(\int_0^{+\infty} \frac{d\omega}{2\pi} e^{-i\epsilon_i \omega_i u_i}\right) \mathcal{A}_n\left(\{\omega_i, z\right.$$

$=$

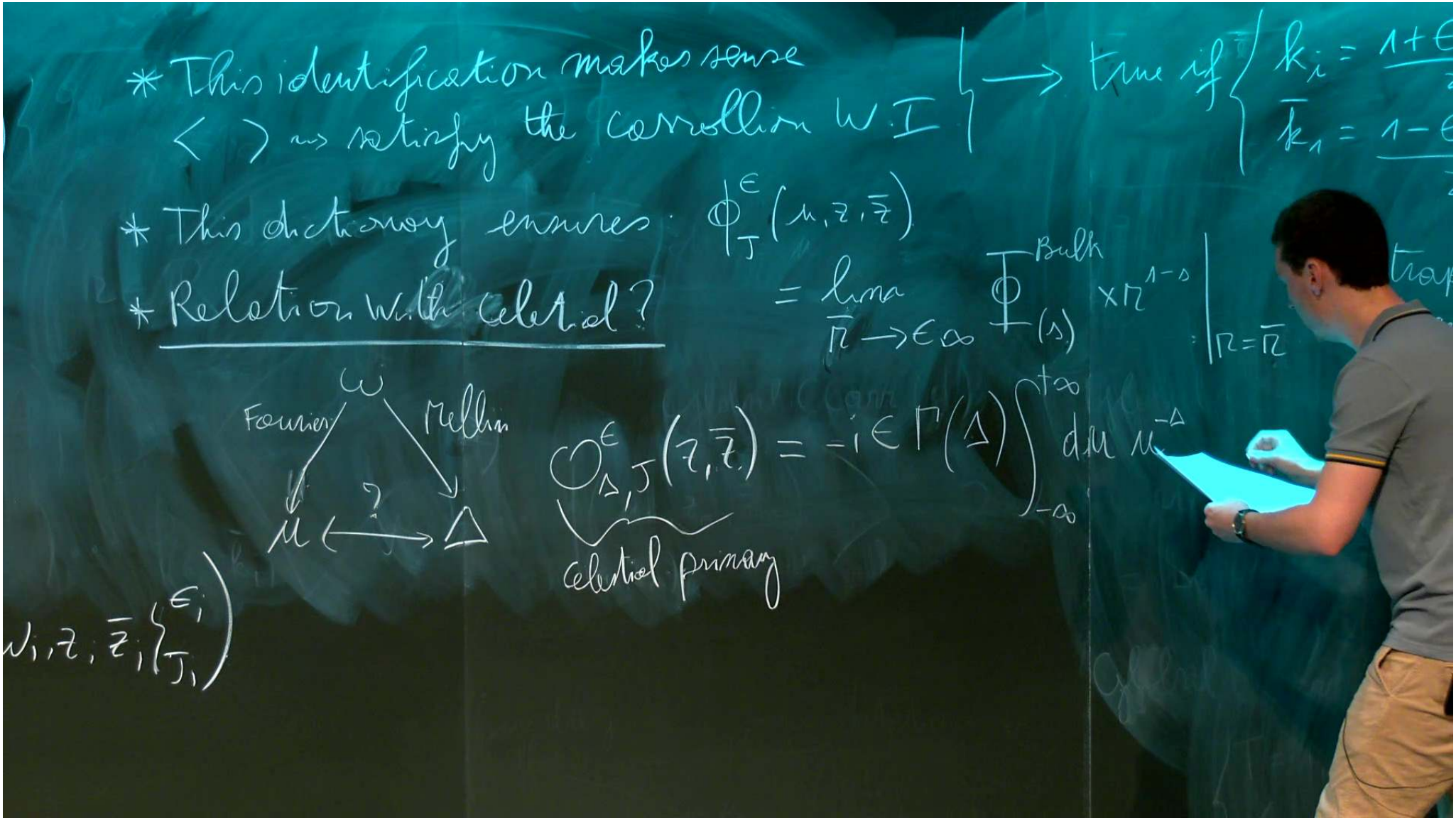

$\mathscr{I}^-$
u
$q_{AB}(z,\bar{z})$
energy
$\epsilon = \pm 1$ if $\begin{cases} \text{outgoing} \\ \text{incoming} \end{cases}$
* Amplitudes: $\mathcal{A}_n\left(\{\omega_i, z_i, \bar{z}_i\}_{J_i}^{\epsilon_i}\right)$
$\hookrightarrow$ # of particles
$C_n\left(\{u_i, z_i, \bar{z}_i\}_{J_i}^{\epsilon_i}\right) = \prod_{i=1}^{n}\left(\int_0^{+\infty} \frac{d\omega}{2\pi} e^{-i\epsilon_i \omega_i u_i}\right) \mathcal{A}_n\left(\{\omega_i, z_i$
$= \langle \phi_{k_1,\bar{k}_1}^{\epsilon_1}(u_1, z_1, \bar{z}_1) \ldots \phi_{k_n,\bar{k}_n}^{\epsilon_n}(u_n, z_n, \bar{z}_n) \rangle$
Holographic.
$\hookrightarrow$ Carrollian CFT correlator.
conformal
$\mathcal{L}_Y q_{AB} = 2\alpha q_{AB}$

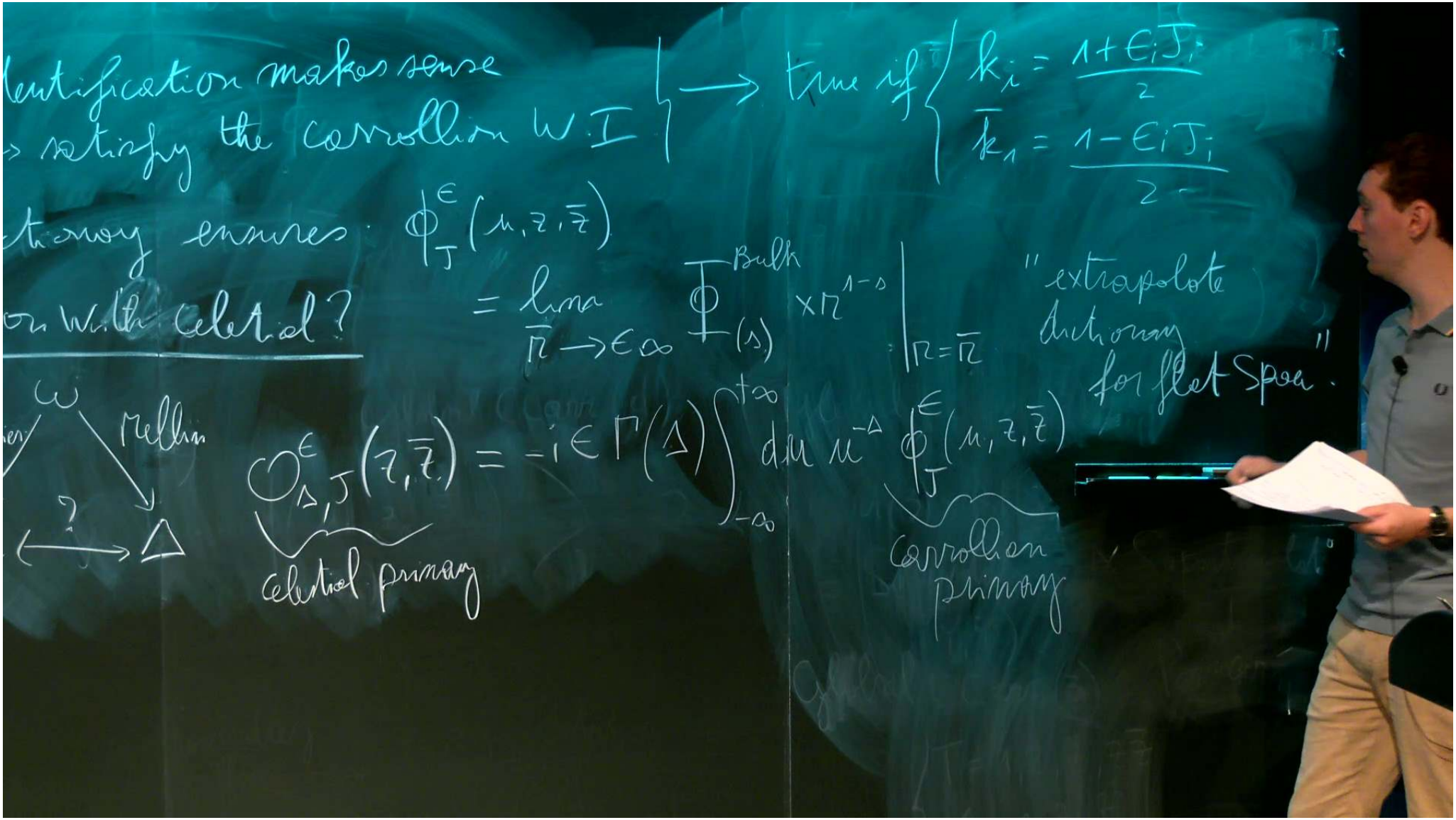
$$ds^2 = -2\,du\,dr + 2r^2\,dz\,d\bar{z}$$

$r \in \mathbb{R}$

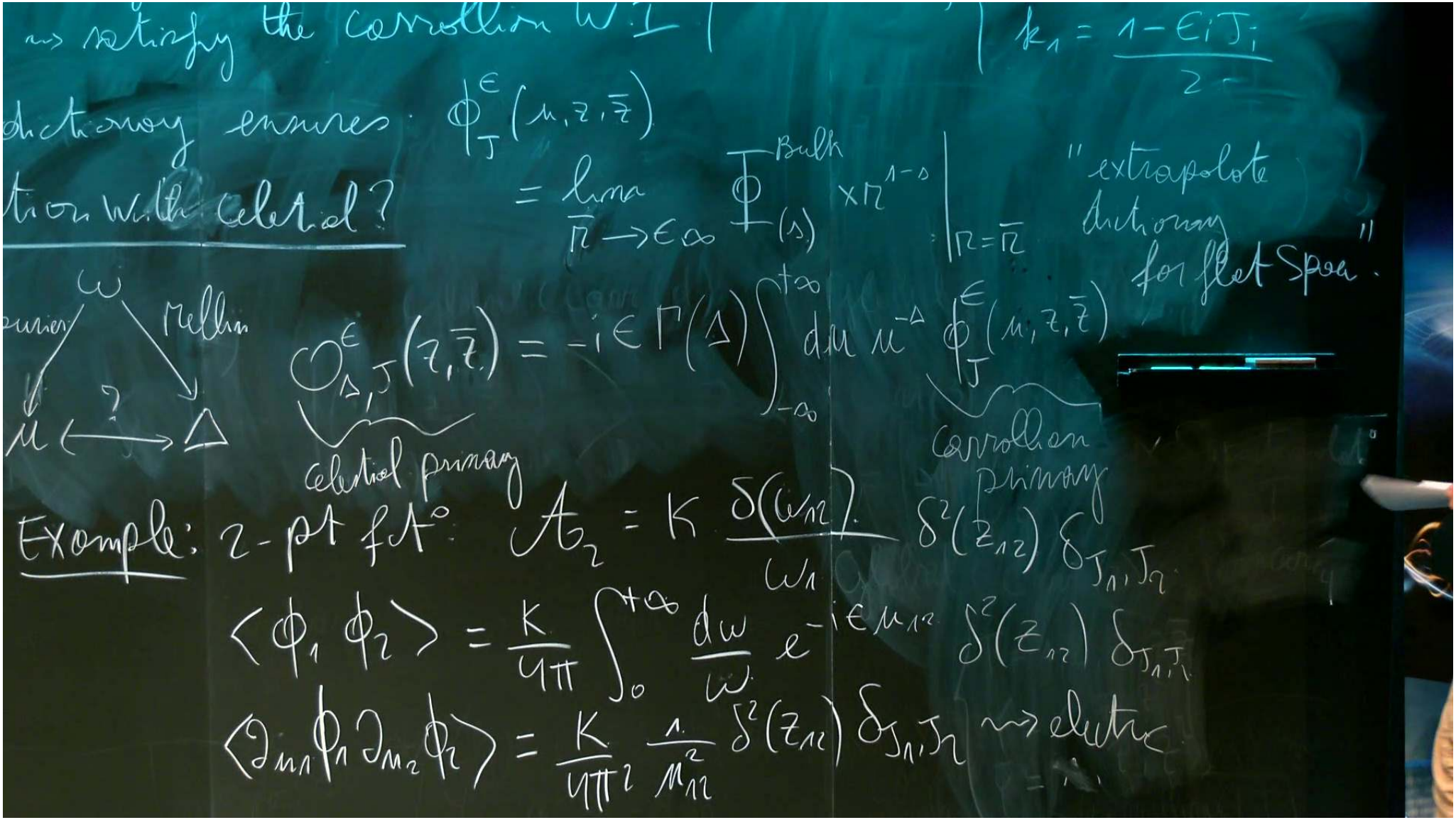


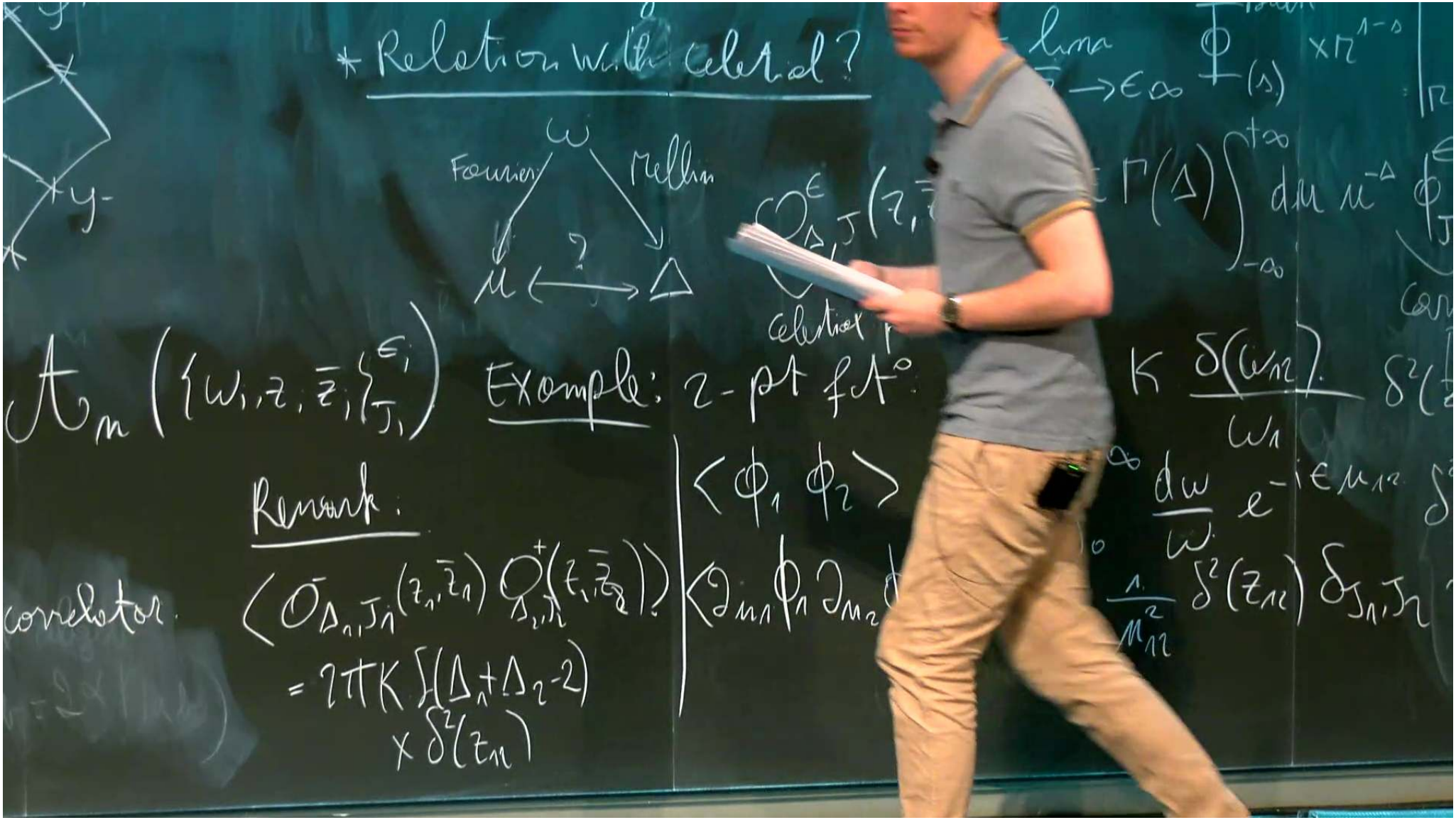
$$A_n\left(\{\omega_i, z_i, \bar{z}_i\}_{J_i}^{\epsilon_i}\right)$$


* This identification makes sense
< > ... satisfy the correlation W.I
* This dictionary ensures
* Relation with celestial?
Fourier
Mellin
celestial primary



lentification makes sense
→ satisfy the carrollian W.I
→ true if
$k_i = \frac{1+\epsilon_i J_i}{2}$
$\bar{k}_1 = \frac{1-\epsilon_i J_i}{2}$
tionary ensures: $\phi_J^\epsilon(u,z,\bar{z})$
on with celestial?
$= \lim_{\bar{r}\to\epsilon\infty} \Phi^{Bulk}_{(s)} \times r^{1-s} \Big|_{r=\bar{r}}$
"extrapolate dictionary for flat Space".
ω
Mellin
?
Δ
$\mathcal{O}^\epsilon_{\Delta,J}(z,\bar{z}) = -i\epsilon\,\Gamma(\Delta)\int_{-\infty}^{+\infty} du\, u^{-\Delta}\, \phi_J^\epsilon(u,z,\bar{z})$
celestial primary
carrollian primary


satisfy the correlation W.I
dictionary ensures: $\phi_J^\epsilon(u,z,\bar z)$
tion with celestial?
$= \lim_{\bar r \to \epsilon\infty} \overset{\text{Bulk}}{\Phi}_{(\Delta)} \times r^{1-\Delta}\Big|_{r=\bar r}$
"extrapolate dictionary for flat Space"
$k_1 = \frac{1-\epsilon i J_i}{2}$
Mellin
$\mathcal{O}^\epsilon_{\Delta,J}(z,\bar z) = -i\epsilon\,\Gamma(\Delta)\int_{-\infty}^{+\infty} du\, u^{-\Delta}\,\phi^\epsilon_J(u,z,\bar z)$
celestial primary
correlation primary
Example: 2-pt fct°: $\mathcal{A}_2 = K\,\frac{\delta(\omega_{12})}{\omega_1}\,\delta^2(z_{12})\,\delta_{J_1,J_2}$
$\langle\phi_1\phi_2\rangle = \frac{K}{4\pi}\int_0^{+\infty}\frac{d\omega}{\omega}\,e^{-i\epsilon u_{12}}\,\delta^2(z_{12})\,\delta_{J_1,J_2}$
$\langle\partial_{u_1}\phi_1\partial_{u_2}\phi_2\rangle = \frac{K}{4\pi^2}\,\frac{1}{u_{12}^2}\,\delta^2(z_{12})\,\delta_{J_1,J_2}$ ⇝ electric


* Relation with celestial?
Fourier
Mellin
Exemple: 2-pt fct°
Remark:
correlator

Relation with AdS/CFT

* Bondi coordinates:

$$ds^2_{AdS} = -\frac{r^2}{\ell^2} du^2 - 2\, du\, dr + 2r^2\, dz\, d\bar{z} \qquad (AdS_4)$$

* Flat limit $\ell \to \infty$: $\lim_{\ell\to\infty} ds^2_{AdS} = ds^2_{Mink}$

* $\lim_{\bar{r}\to\infty} \frac{1}{r^2} ds^2_{AdS}\big|_{r=\bar{r}} = \frac{1}{\ell^2} du^2 + 2\, du\, dz\, d\bar{z} \xrightarrow{\ell\to\infty} 0\, du^2 + 2\, du\, dz\, d\bar{z}$

$\ell^2 \to \epsilon^2$

$\pi_{\gamma,m}(\{\omega_i, z_i$

* Bondi coordinates

$$ds^2_{AdS} = -\frac{r^2}{\ell^2} du^2 - 2\, du\, dr + 2r^2 dz\, d\bar{z} \qquad (AdS_4)$$

* Flat limit $\ell \to \infty$: $\lim_{\ell\to\infty} ds^2_{AdS} = ds^2_{Mink}$

* $\lim_{\bar{r}\to\infty} \frac{1}{r^2} ds^2_{AdS}\big|_{r=\bar{r}} = -\frac{1}{\ell^2} du^2 + 2\, dz\, d\bar{z} \xrightarrow{\ell\to\infty} 0\, du^2 + 2\, dz\, d\bar{z}$

$\frac{1}{\ell^2} \to \epsilon^2$

Flat limit in the bulk
$\ell \to \infty$

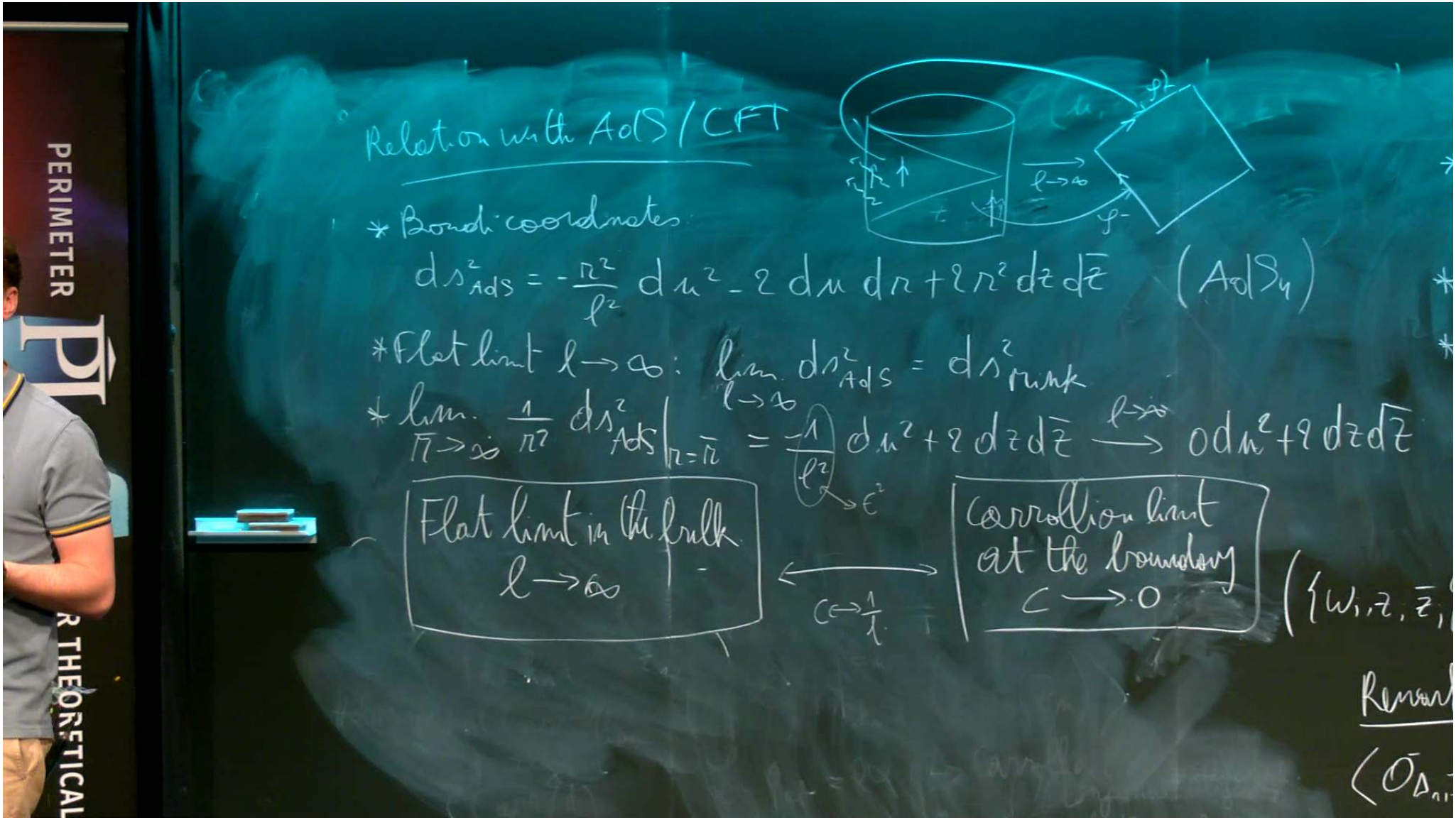
$\longleftrightarrow$

Carrollian limit at the boundary
$c \to 0$

$(\{\omega, z, \bar{z}, \xi$

Remark

$(\bar{O}_{\Delta,J}$


Relation with AdS / CFT
* Bondi coordinates
$ds^2_{AdS} = -\frac{r^2}{l^2} du^2 - 2 du\, dr + 2 r^2 dz d\bar{z}$ (AdS_4)
* Flat limit $l \to \infty$: $\lim_{l\to\infty} ds^2_{AdS} = ds^2_{Flat}$
* $\lim_{r\to\infty} \frac{1}{r^2} ds^2_{AdS}\big|_{r=\bar{r}} = \frac{-1}{l^2} du^2 + 2 dz d\bar{z} \xrightarrow{l\to\infty} 0\, du^2 + 2 dz d\bar{z}$
ϵ^2
Flat limit in the bulk
$l \to \infty$
$c \leftrightarrow \frac{1}{l}$
Carrollian limit at the boundary
$c \to 0$
$\{\omega, z, \bar{z},$
Rem
$\langle \mathcal{O}_{\Delta,$
PERIMETER
THEORETICAL