Title: Lecture - Carrollian Physics a

Speakers:

Collection: Celestial Holography Summer School 2024

Date: July 24, 2024 - 9:00 AM

URL: https://pirsa.org/24070012

Carrollian physics and holography

* Motivations
* Carrollian limit and symmetries
* Consequences of Carrollian invariance (W.I.)
* Carrollian and celestial amplitudes
* Flat limit of AdS/CFT

---

* Carrollian physics:
  - Null hypersurfaces
  - Cosmology
  - Condensed Matter
  - Fluids (Bjorken flow)
  - Holography.

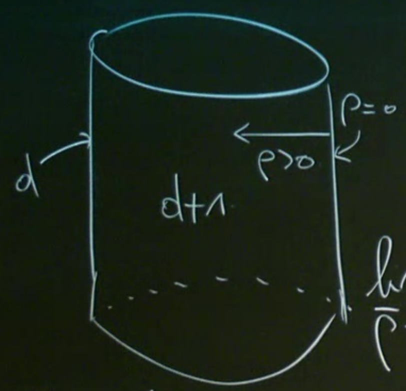
How to formulate flat Spac

How to formulate flat space holography?

(w.I.) $\longrightarrow$ Bottom-up

ws

AdS: $(d=3)$



(ρ, x^a) : Poincaré coordinates.

$$ds^2_{AdS} = \frac{\ell^2}{\rho^2}\Big(d\rho^2 \underbrace{-(dx^0)^2 + \sum_{a=1}^{d-1}(dx^a)^2}_{\eta_{ab}\,dx^a\,dx^b}\Big)$$

$$\lim_{\bar\rho\to 0}\ \underbrace{\left(\frac{\rho^2}{\ell^2}\right)}_{\Omega^2} ds^2_{AdS}\Big|_{\rho=\bar\rho} = \eta_{ab}\,dx^a dx^b = ds^2_{\partial AdS}.$$

* Symmetries:

$$\mathcal{L}_\xi \eta_{ab} = 2\alpha\,\eta_{ab} \qquad \Omega^2 \longrightarrow \Omega^2 \omega^2 \ ,\ \omega(x^a) \neq 0$$

$\hookrightarrow \xi \in SO(d,2) \Rightarrow$ CFT at ∂AdS.

flat Space holography?

Poincaré coordinates

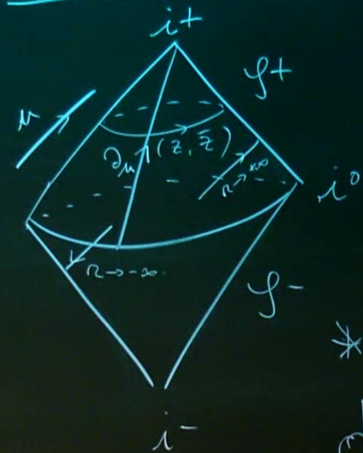
$$\frac{}{z}\Big(d\rho^2 - \underbrace{(dx^0)^2 + \sum_{a=1}^{d-1}(dx^a)^2}_{h_{ab}\,dx^a dx^b}\Big)$$

$$\rho=\bar\rho \;=\; h_{ab}\,dx^a dx^b = ds^2_{\partial AdS}$$

$\Omega^2\omega^2$, $\omega(x^a)\neq 0$

$\Rightarrow$ CFT at ∂AdS.

Flat (4d)



$\cdot\;(u,r,z,\bar z)$: Bondi coord.

$x^a=(u,z,\bar z)$

$$ds^2_{Mink} = -2\,du\,dr + 2r^2\,dz\,d\bar z$$

$$\left(ds^2 = -du^2 - 2\,du\,dr + \frac{4r^2\,dz\,d\bar z}{(1+z\bar z)^2}\right)$$

* Boundary structure?

$$ds^2_{\partial Mink} = \lim_{\bar r\to\infty}\underbrace{\left(\frac{1}{r^2}\right)}_{\Omega^2} ds^2_{Mink}\Big|_{r=\bar r}$$

$$= 0\,du^2 + 2\,dz\,d\bar z$$

$\equiv$ ↳ degenerate Metric $\equiv q_{ab}\,dx^a dx^b$

* $n^a\partial_a = \partial_u$ such that $q_{ab}n^b = 0$.

* Carrollian structure (q_{ab}, n^c) such that $q_{ab}n^b=0$

* $\Omega^2 \longrightarrow \Omega^2\omega^2 \Rightarrow (q_{ab}, n^c)$ defined up to rescaling "universal structure"

$\boxed{\text{...mulate flat Space holography?}}$

ρ :

(ρ, x^a) : Poincaré coordinates

$$ds^2_{AdS} = \frac{\ell^2}{\rho^2}\Big(d\rho^2 \underbrace{- (dx^0)^2 + \sum_{a=1}^{d-1} (dx^a)^2}_{h_{ab}\, dx^a dx^b} \Big)$$

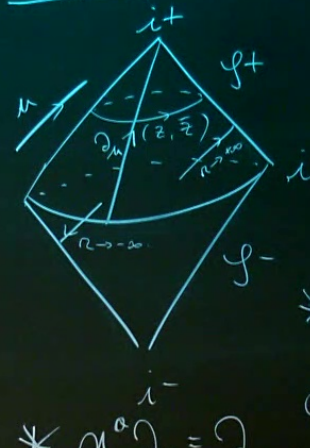
$$\underbrace{\left(\frac{\rho^2}{\ell^2}\right)}_{\Omega^2} ds^2_{AdS}\Big|_{\rho=\bar\rho} = h_{ab}\, dx^a dx^b = ds^2_{\partial AdS}$$

$$\Omega^2 \longrightarrow \Omega^2 \omega^2 \,, \quad \omega(x^a) \neq 0$$

$$O(d,2) \Rightarrow \text{CFT at } \partial AdS.$$

Flat (4d)

$\mathcal{L}_n q_{ab} = 0.$



$*\ n^a \partial_a = \partial_u$ such that $q_{ab} n^b = 0$

$*$ Carrollian structure

(q_{ab}, n^c) such that $q_{ab} n^b = 0$

$*\ \Omega^2 \longrightarrow \Omega^2 \omega^2 \Rightarrow (q_{ab}, n^c)$ defined up to rescaling

$$* \begin{cases} \mathcal{L}_\xi q_{ab} = 2\alpha\, q_{ab} \\ \mathcal{L}_\xi n^c = -\alpha\, n^c \end{cases}$$

$\cdot\ (u, r, z, \bar z)$: Bondi coord.

$x^a = (u, z, \bar z)$

$$ds^2_{Mink} = -2\, du\, dr + 2 r^2 dz\, d\bar z$$

$$\left(ds^2 = -du^2 - 2\, du\, dr + \frac{4 r^2 dz\, d\bar z}{(1+z\bar z)^2} \right)$$

$*$ Boundary structure? → "BMS symmetries as global spacetime symmetries"

$r = \bar r$

$$ds^2_{\partial Mink} = \lim_{\bar r \to \infty} \underbrace{\left(\frac{1}{r^2}\right)}_{\Omega^2} ds^2_{Mink}$$

$$= 0\, du^2 + 2\, dz\, d\bar z \equiv q_{ab}\, dx^a dx^b$$

↳ degenerate metric

"universal structure"

"conformal Carrollian symmetries"

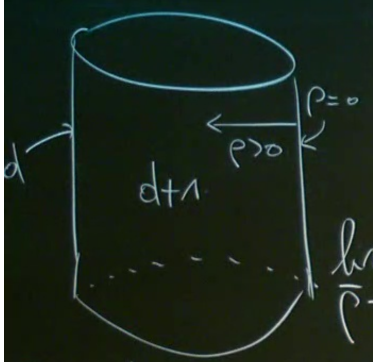
$\simeq BMS_4 \Rightarrow \boxed{\text{Carrollian CFT}}$

How to formulate flat space holography?

→ Bottom-up

Goal: $\boxed{\text{Study the properties of the Carrollian CFT}}$

AdS: $(d=3)$



(ρ, x^a) : Poincaré coordinates

$$ds^2_{AdS} = \frac{\ell^2}{\rho^2}\Big(d\rho^2 \underbrace{- (dx^0)^2 + \sum_{a=1}^{d-1} (dx^a)^2}_{\eta_{ab}\, dx^a dx^b}\Big)$$

$$\lim_{\bar\rho\to 0} \underbrace{\left(\frac{\rho^2}{\ell^2}\right)}_{\Omega^2} ds^2_{AdS}\Big|_{\rho=\bar\rho} = \eta_{ab}\, dx^a dx^b = ds^2_{\partial AdS}$$

Symmetries:

$\mathcal{L}_\xi \eta_{ab} = 2\alpha \eta_{ab}$ $\quad \Omega^2 \to \Omega^2 \omega^2$, $\omega(x^a) \neq 0$

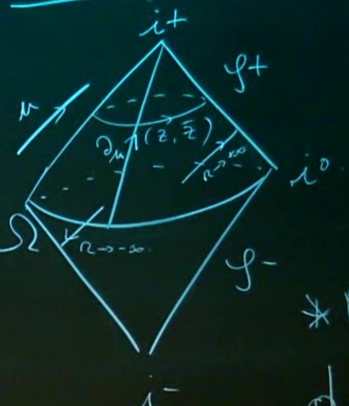
↳ $\xi \in SO(d,2) \Rightarrow$ CFT at ∂AdS.

Boundary diffeos

Flat (4d)

$\mathcal{L}_n q_{ab} = 0$.

$n^a = D^a \Omega$



$\cdot\ (u, r, z, \bar z)$: B

$x^a = (u, z, \bar z)$

$ds^2_{Mink} = -2\, du$

$(ds^2 = -du^2 - 2$

* Boundary structure

$ds^2_{\partial Mink} = \lim_{r\to\infty} \left(\frac{1}{r^2}\right.$ … Ω^2

$= 0\, du^2 + 2\, dz$

↳ degenera

* $n^a \partial_a = \partial_u$ such that $q_{ab} n^b = 0$

* Carrollian structure

(q_{ab}, n^c) such that $q_{ab} n^b = 0$ $\quad \equiv$

* $\Omega^2 \to \Omega^2 \omega^2 \Rightarrow$ (q_{ab}, n^c) defin

"universal stru

* $\begin{cases} \mathcal{L}_\xi q_{ab} = 2\alpha q_{ab} \\ \mathcal{L}_\xi n^c = -\alpha n^c \end{cases}$

"Conformal Ca

$\simeq BMS_4 \Rightarrow$

$(W.I.)$

...des

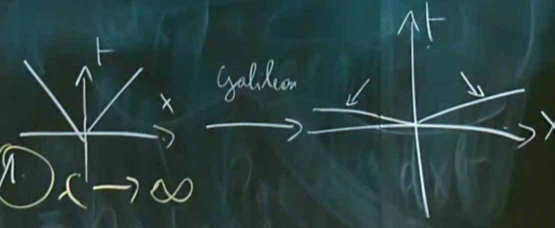
Carrollian limit

* Refers "$c \longrightarrow 0$" limit of Poincaré
* Lorentzian boost of parameter V (1+1 dim)

$$\begin{cases} \Delta x' = \dfrac{\Delta x + V\,\Delta t}{\sqrt{1-V^2}} \\[2ex] \Delta t' = \dfrac{\Delta t + V.\Delta x}{\sqrt{1-V^2}} \end{cases} \qquad \boxed{c=1}$$

1) Galilean limit: $V \ll 1$, $\dfrac{\Delta x}{\Delta t} \ll 1$ $\quad c \to \infty$

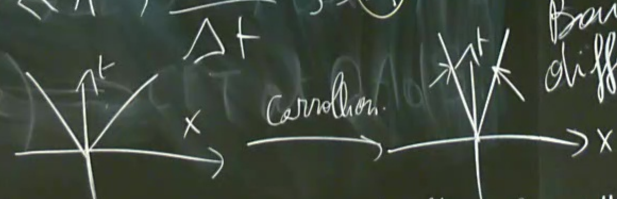
$$\begin{cases} \Delta x' = \Delta x + V.\Delta t \\ \Delta t' = \Delta t \end{cases}$$



2) Carrollian limit: $V \ll 1$, $\dfrac{\Delta x}{\Delta t} \gg 1$ $\quad c \longrightarrow 0$

$$\Delta x' = \Delta x$$

$$\Delta t' = \Delta t + V \Delta x$$

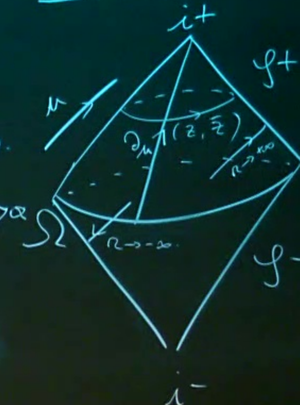


"ultra-local"

"Carroll" ~ Carroll Lewis.

Boundary diffeos

Flat (4d)



$\mathcal{L}_n q_{ab} = 0.$

$n^a = D^a \Omega$

* $n^a \partial_a = \partial_u$ such that $q_{ab} n^b = 0$
* Carrollian structure (q_{ab}, n^c) such that $q_{ab} n$...
* $\Omega^2 \longrightarrow \Omega^2 \omega^2 \Longrightarrow$ $(q_{ab}$...
* $\begin{cases} \mathcal{L}_\xi q_{ab} = 2\alpha\, q_{ab} \\ \mathcal{L}_\xi n^c = -\alpha\, n^c \end{cases}$

. $(u, r,$ ...

$x^a = (u$ ...

ds^2_{Mink}

$(ds^2 = -$ ...

* Boundary

$ds^2_{\partial Mink} =$ ...

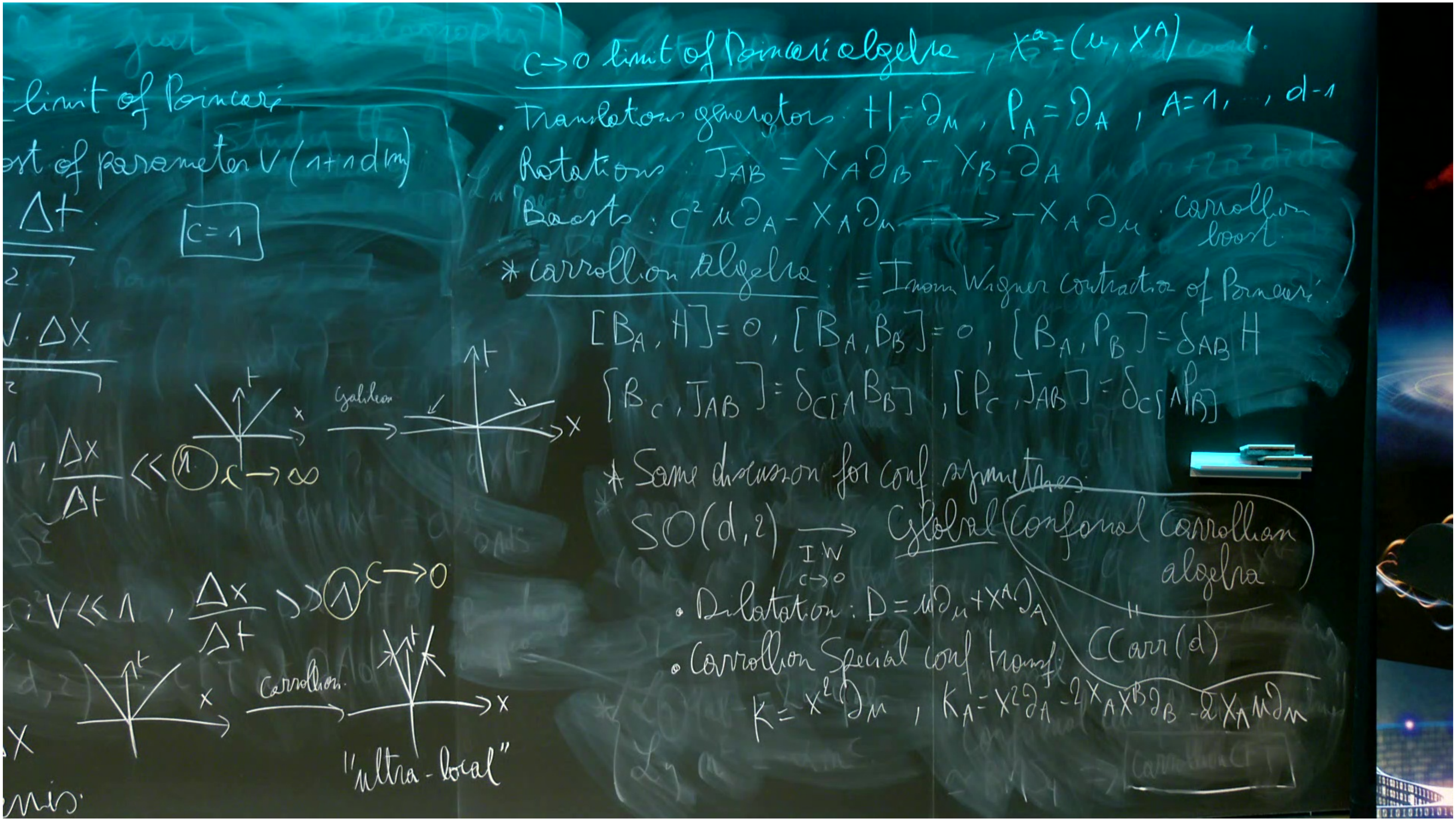
$= 0$

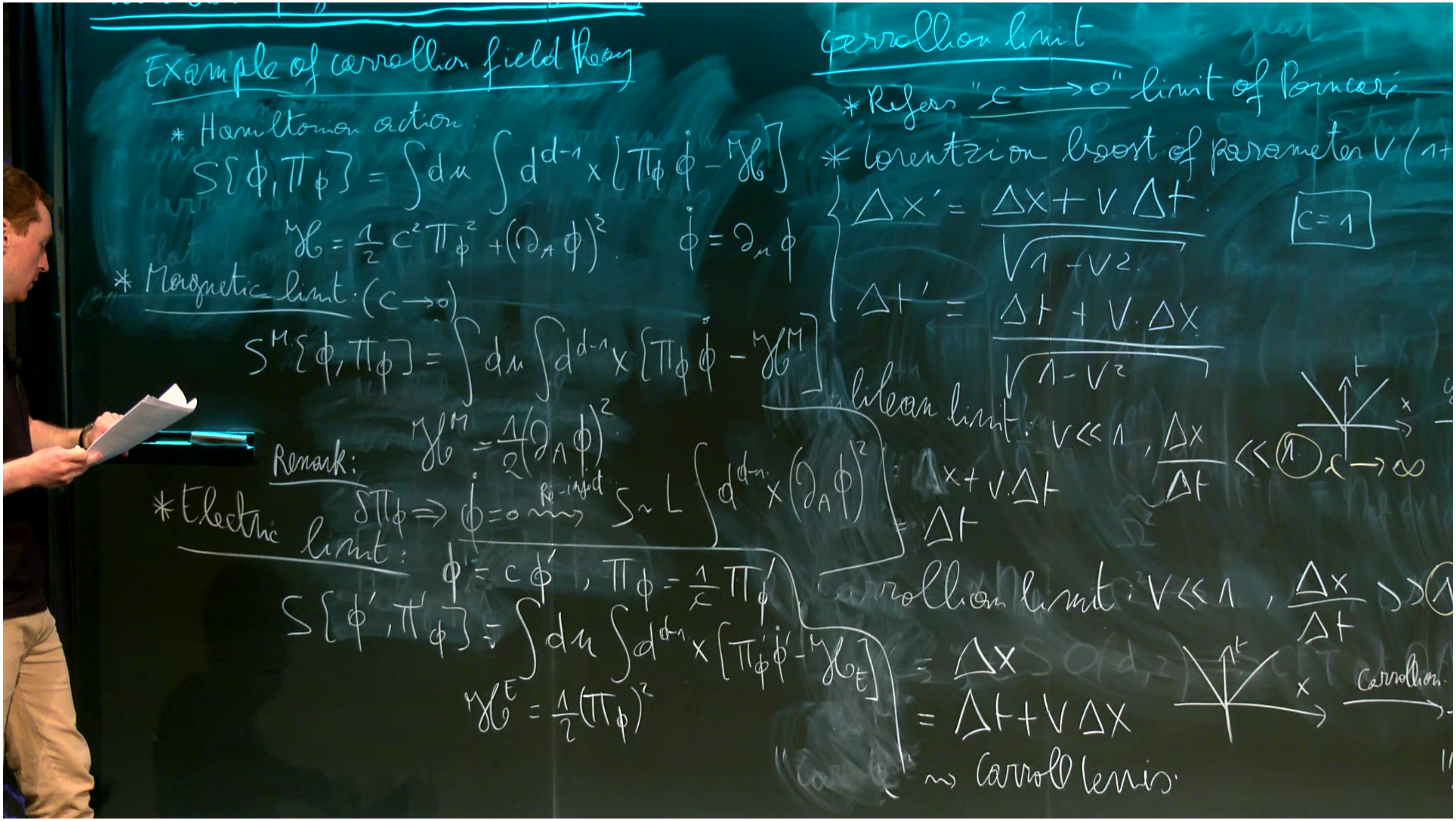
$\equiv$

"uni...

"Con...

$\simeq$ B...


c→0 limit of Poincaré algebra, $x^a = (u, x^A)$ coord.
Translation generators: $H = \partial_u$, $P_A = \partial_A$, $A = 1, \dots, d-1$
Rotations: $J_{AB} = x_A \partial_B - x_B \partial_A$
Boosts: $c^2 u \partial_A - x_A \partial_u \longrightarrow -x_A \partial_u$
Carrollian boost
* Carrollian algebra: = Inonu Wigner contraction of Poincaré
$[B_A, H] = 0,\ [B_A, B_B] = 0,\ [B_A, P_B] = \delta_{AB} H$
$[B_C, J_{AB}] = \delta_{C[A} B_{B]},\ [P_C, J_{AB}] = \delta_{C[A} P_{B]}$
* Same discussion for conf symmetries
$SO(d,2) \xrightarrow[c \to 0]{IW}$ Global Conformal Carrollian algebra
$= CCarr(d)$
• Dilatation: $D = u\partial_u + x^A \partial_A$
• Carrollian Special conf transf
$K = x^2 \partial_u$, $K_A = x^2 \partial_A - 2x_A x^B \partial_B - 2x_A u \partial_u$
limit of Poincaré
of parameter V (1+1 dim)
$c = 1$
Galilean
$c \to \infty$
$c \to 0$
Carrollian
"ultra-local"



Example of carrollian field theory
* Hamiltonian action:
$S[\phi,\Pi_\phi] = \int du \int d^{d-1}x \, [\Pi_\phi \dot\phi - \mathcal{H}]$
$\mathcal{H} = \frac{1}{2} c^2 \Pi_\phi^2 + (\partial_A \phi)^2$ $\dot\phi = \partial_u \phi$
* Magnetic limit: $(c \to 0)$
$S^M[\phi,\Pi_\phi] = \int du \int d^{d-1}x \, [\Pi_\phi \dot\phi - \mathcal{H}^M]$
$\mathcal{H}^M = \frac{1}{2}(\partial_A \phi)^2$
Remark: $\delta\Pi_\phi \Rightarrow \dot\phi = 0$ re-inject $S \sim L \int d^{d-1}x \, (\partial_A \phi)^2$
* Electric limit: $\phi = c\phi'$, $\Pi_\phi = \frac{1}{c}\Pi'_\phi$
$S[\phi',\Pi'_\phi] = \int du \int d^{d-1}x \, [\Pi'_\phi \dot\phi' - \mathcal{H}_E]$
$\mathcal{H}^E = \frac{1}{2}(\Pi_\phi)^2$
Carrollian limit
* Refers "$c \to 0$" limit of Poincaré
* Lorentzian boost of parameter v (1+
$\Delta x' = \frac{\Delta x + v\,\Delta t}{\sqrt{1-v^2}}$
$\Delta t' = \frac{\Delta t + v\,\Delta x}{\sqrt{1-v^2}}$
$c = 1$
Galilean limit: $v \ll 1$, $\frac{\Delta x}{\Delta t} \ll 1$, $c \to \infty$
$\Delta x + v\,\Delta t$
$= \Delta t$
Carrollian limit: $v \ll 1$, $\frac{\Delta x}{\Delta t} \gg 1$
$= \Delta x$
$= \Delta t + v\,\Delta x$
$\rightsquigarrow$ Carroll lenses
t
x
carrollian