

Title: Vision Talk

Speakers:

Collection: Celestial Holography Summer School 2024

Date: July 23, 2024 - 4:00 PM

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Taking Λ out of It:

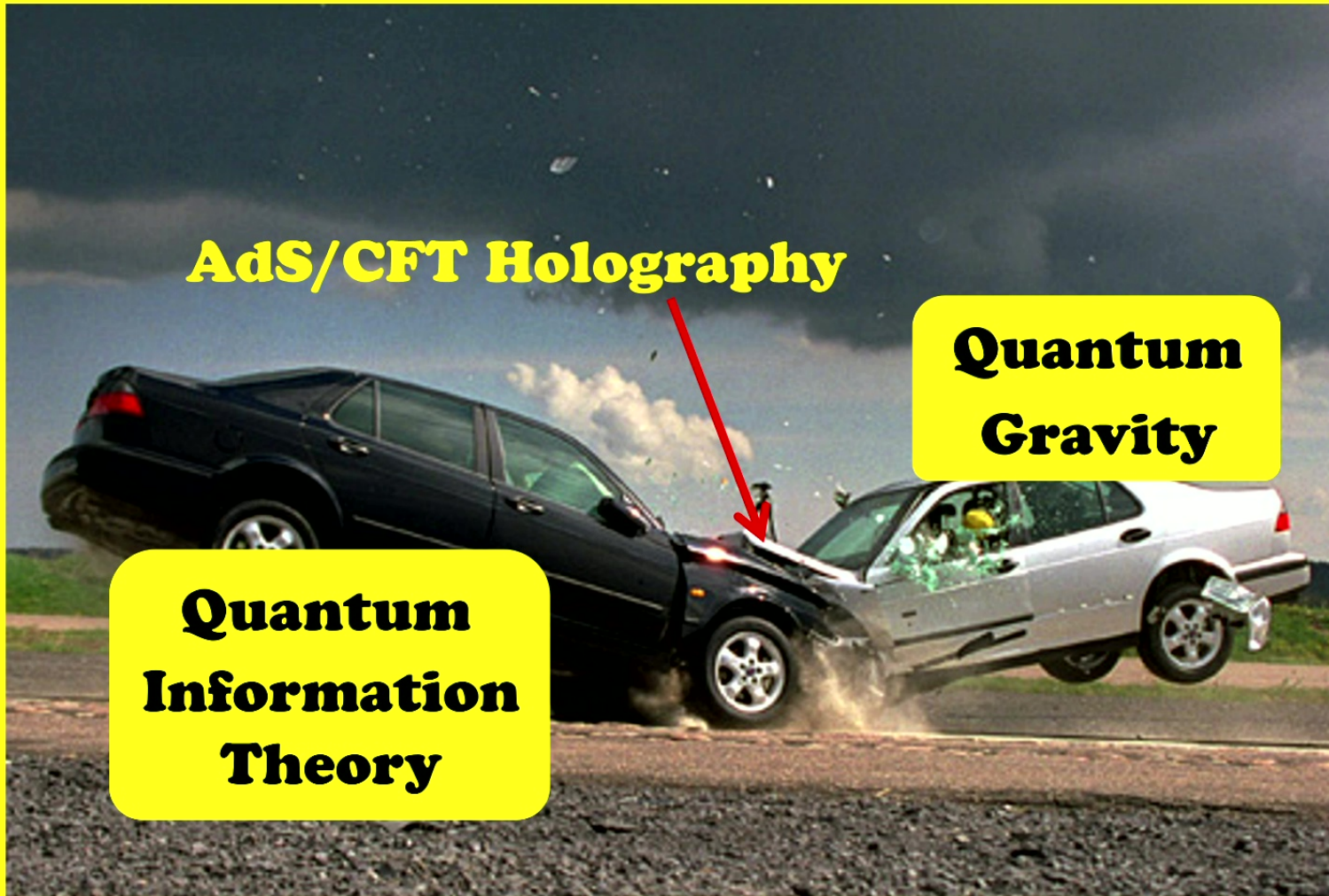
Quantum Information & Celestial Holography

Robert Myers,
Simons Celestial Holography
Summer School 2024

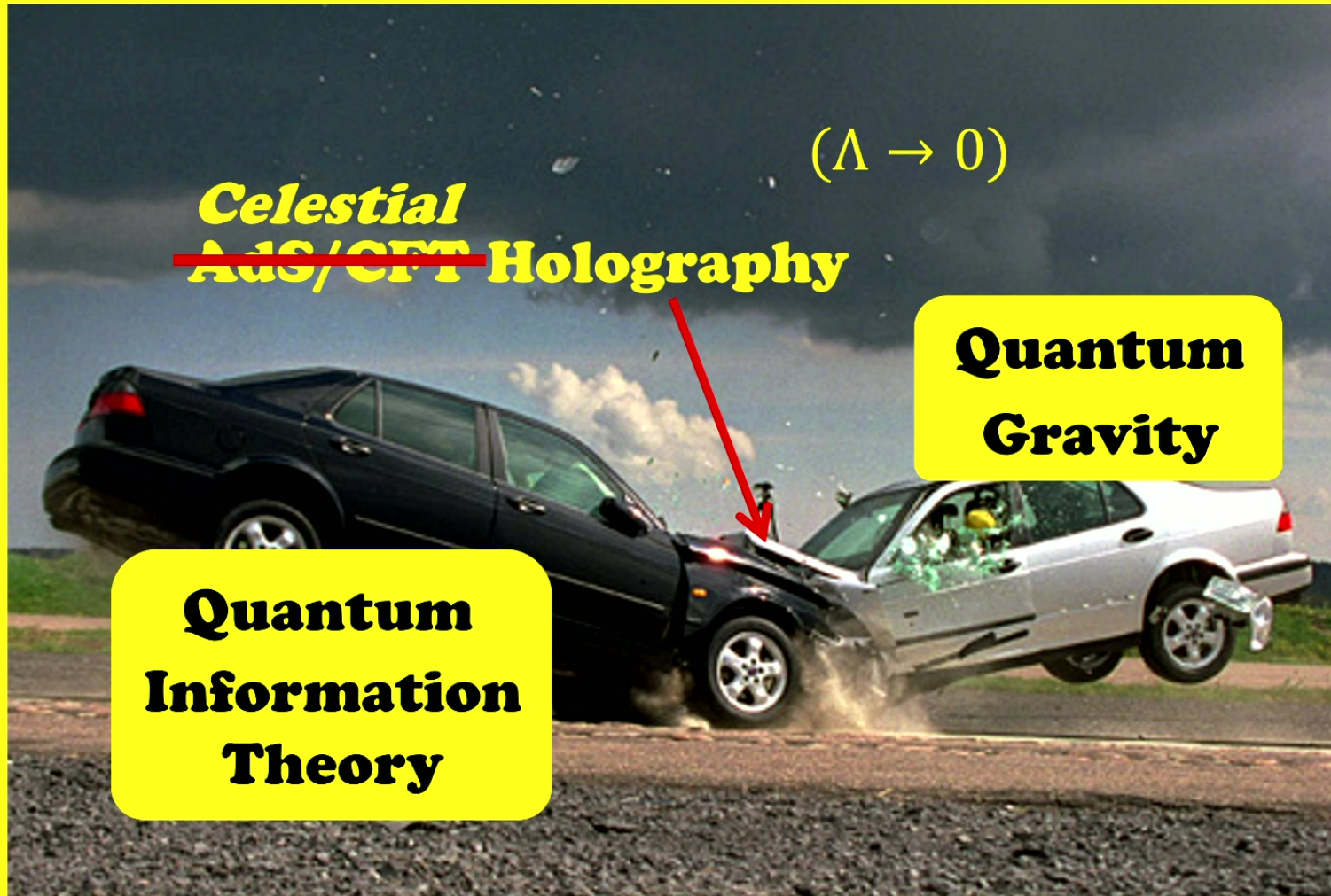
“It from Qubit”: New Collision of Ideas



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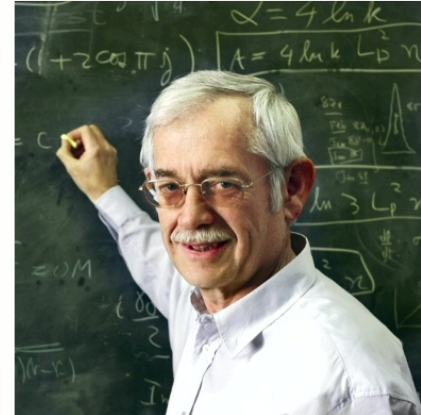
Overview:

- a) Preliminaries**
- b) Holographic EE – Take 1**
- c) Holographic EE – Take 2**
- d) Outlook**

Black Holes:

- Bekenstein: “black holes have entropy!”

$$S_{BH} = \eta \frac{k_B c^3}{\hbar} \frac{A_{horizon}}{G}$$



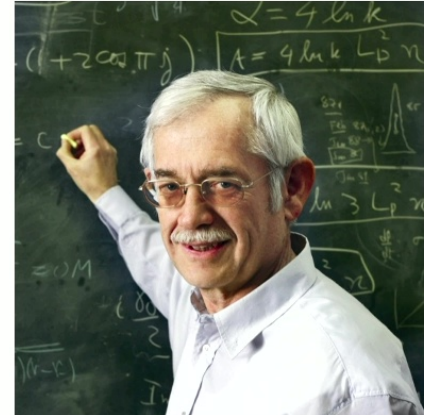
“we discuss black-hole physics from the point
of view of information theory”

“Black Holes and Entropy”, PRD, April 1973

Black Holes:

- Bekenstein and Hawking: “black holes have entropy!”

$$S_{BH} = \eta \frac{k_B c^3}{\hbar} \frac{A_{horizon}}{G}$$



- **Hawking radiation**: quantum fluctuations allow black holes to emit (almost pure) blackbody radiation with

$$T_H = \frac{\hbar}{k_B c} \frac{\kappa}{2\pi}$$

No Λ here!!



Black Hole Entropy:

- Bekenstein and Hawking: “black holes have entropy!”

microstates

geometry

$$S_{BH} = 2\pi \frac{A_{horizon}}{\ell_P^{d-2}}$$



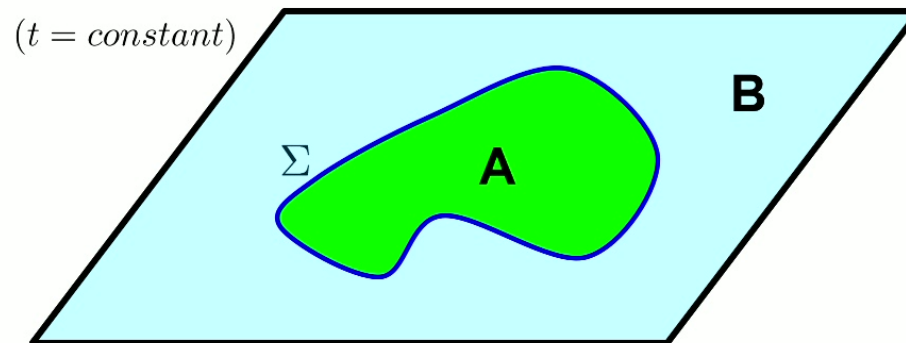
- area measured in terms of Planck scale: $\ell_P^{d-2} = 8\pi G \hbar/c^3$
- entropy is not extensive; grows with **area** rather than volume

→ Sorkin '84: black hole entropy \approx “**entanglement entropy**”

(Sorkin (1402.3589); Bombelli, Koul, Lee & Sorkin; Srednicki; Frolov & Novikov; . . .)

Entanglement Entropy in Quantum Field Theory:

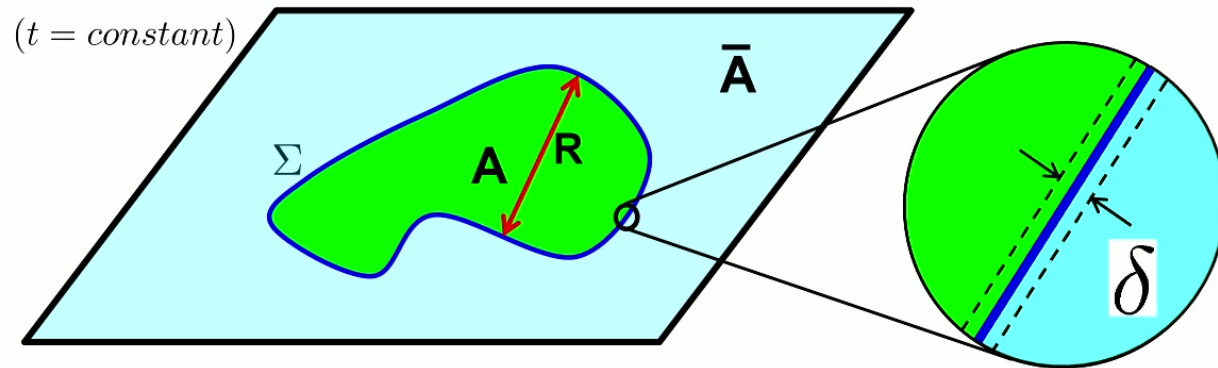
- EE is a quantitative measure of entanglement using **entropy** to detect correlations between two subsystems
 - in **QFT**, typically introduce a (smooth) boundary **or entangling surface** Σ which divides the space into two separate regions
 - integrate out degrees of freedom in “outside” region
 - remaining dof are described by a density matrix ρ_A
- calculate **von Neumann entropy**: $S_{EE} = -\text{Tr} [\rho_A \log \rho_A]$



Entanglement Entropy in Quantum Field Theory

- remaining dof are described by a density matrix ρ_A

→ calculate von Neumann entropy: $S_{EE} = -\text{Tr} [\rho_A \log \rho_A]$



- result is **UV divergent!** dominated by short-distance correlations
- must regulate calculation: $\delta = \text{short-distance cut-off}$

$$S = c_0 \frac{R^{d-2}}{\delta^{d-2}} + c_2 \frac{R^{d-4}}{\delta^{d-4}} + \dots \quad d = \text{spacetime dimension}$$

→ geometric structure, eg, $S = \tilde{c}_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \tilde{c}_2 \frac{\oint_\Sigma \text{“curvature”}}{\delta^{d-4}} + \dots$

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$$S_{BH} = 2\pi \frac{A_{horizon}}{\ell_P^{d-2}}$$



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$$S_{EE} = \tilde{c}_0 \frac{A_\Sigma}{\delta^{d-2}} + \dots \longrightarrow \text{“area law” suggestive of BH formula if } \delta^{d-2} \simeq \ell_P^{d-2}$$

(Sorkin (1402.3589); Bombelli, Koul, Lee & Sorkin; Srednicki; Frolov & Novikov; . . .)

Black Hole Entropy:

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- entropy is not extensive; grows with **area** rather than volume
 - Sorkin '84: black hole entropy \approx “**entanglement entropy**”
- entanglement entropy of QF's contributes to gravitational entropy
(Susskind & Uglum; ... Faulkner, Lewkowycz & Maldacena; ...)
- entanglement of microscopic gravitational dof also important

Holography: AdS/CFT correspondence

Bulk: quantum gravity
with negative Λ
in **d+1** dimensions

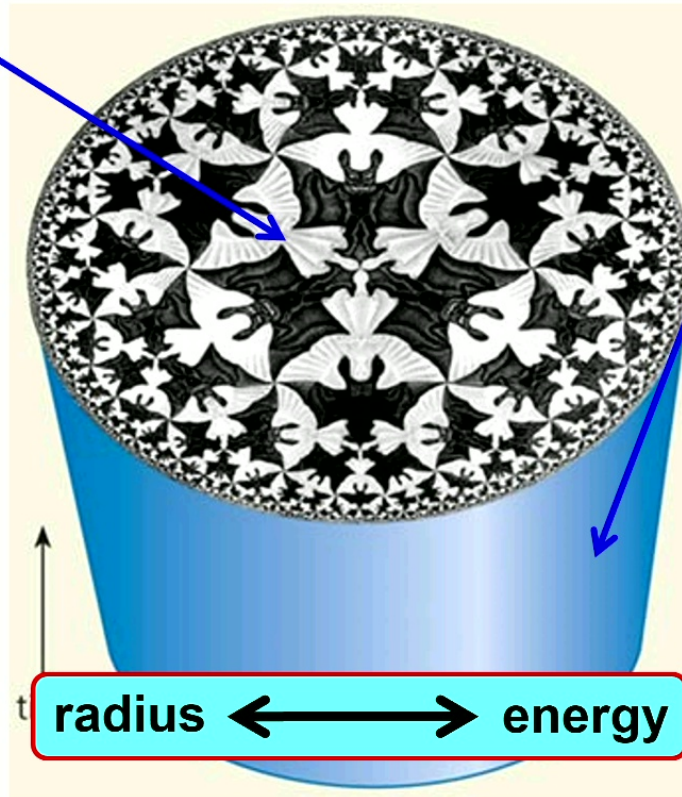
Boundary: quantum field theory
without intrinsic scales
in **d** dimensions

anti-de Sitter
space

↔
"holography"

conformal
field theory

$$\Lambda = -\frac{d(d-1)}{2L^2}$$



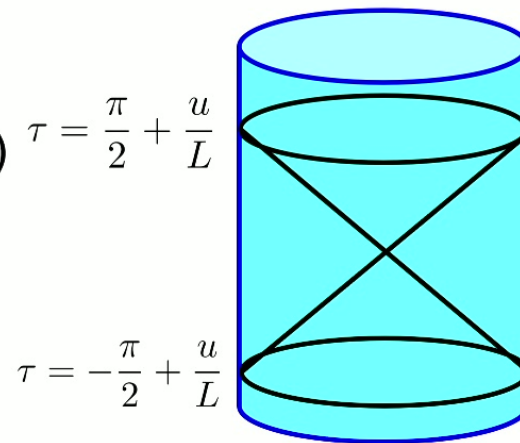
Flat space limit of AdS/CFT:

- extracting flat space S-matrix from AdS/CFT correlators has a long history

(Susskind, Polchinski, Giddings, Penedones, Fitzpatrick, Kaplan, Hijano,)

- celestial amplitudes arise from AdS Witten diagrams with particular kinematics

- celestial operators on \mathcal{I}^\pm mapped to CFT operators on narrow ($\sim 1/L$) band around $\tau = \pm \frac{\pi}{2} + \frac{u}{L}$



- celestial CCFT_{d-1} amplitudes can be obtained directly from correlation functions in a unitary Lorentzian CFT_d on $R \times S^{d-1}$
(de Gioia & Raclariu)

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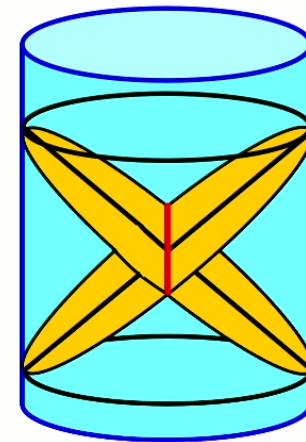
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- note Rindler horizons (or RT surfaces) near bulk point given by spacelike extremal surfaces anchored in intermediate zone



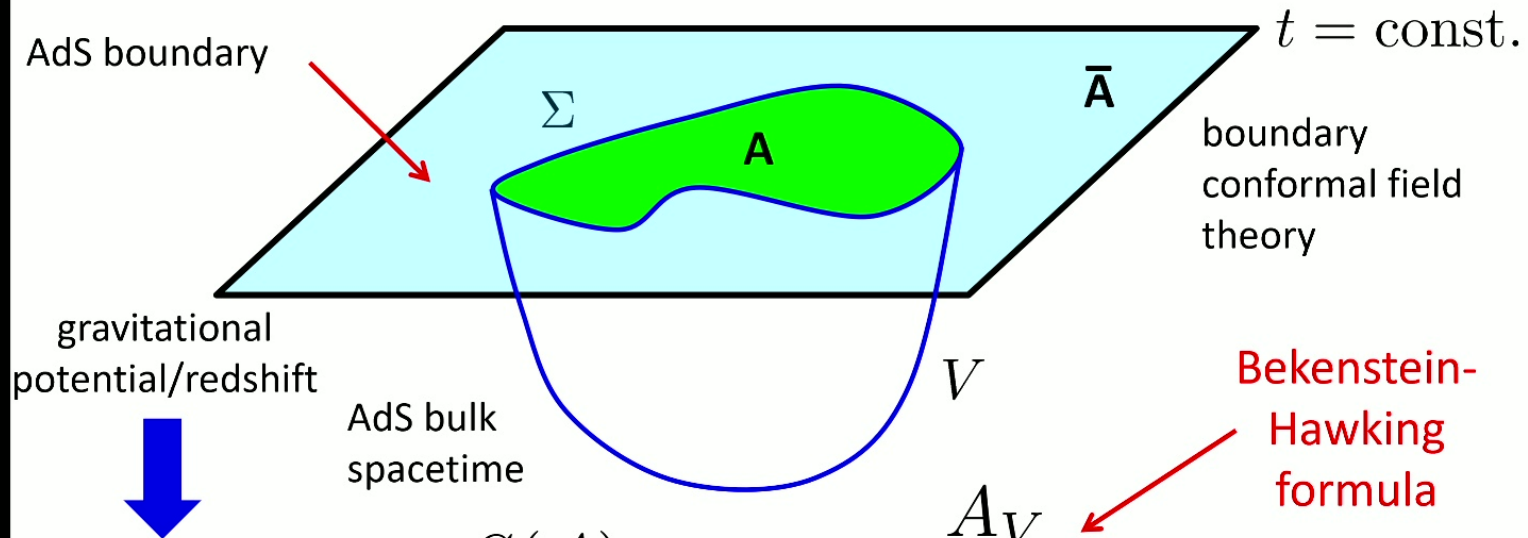
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(de Gioia & Raclariu)

(Ryu & Takayanagi '06)

Holographic Entanglement Entropy:

- CFT dof within **A** described by density matrix $\rho_A = \text{Tr}_{\bar{A}}(|\psi\rangle\langle\psi|)$

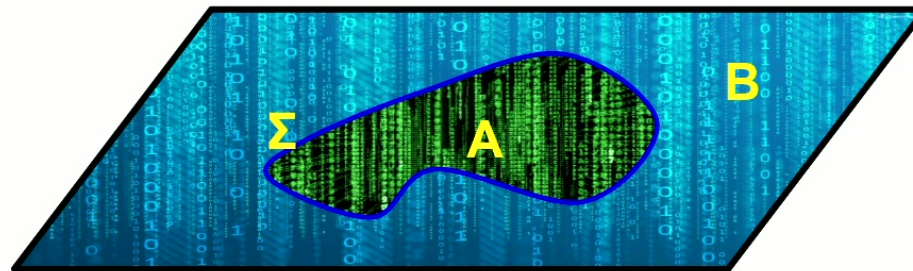
→ calculate von Neumann entropy: $S(A) = -\text{Tr}[\rho_A \log \rho_A]$



$$S(A) = \text{ext}_{V \sim A} \frac{A_V}{4G_N}$$

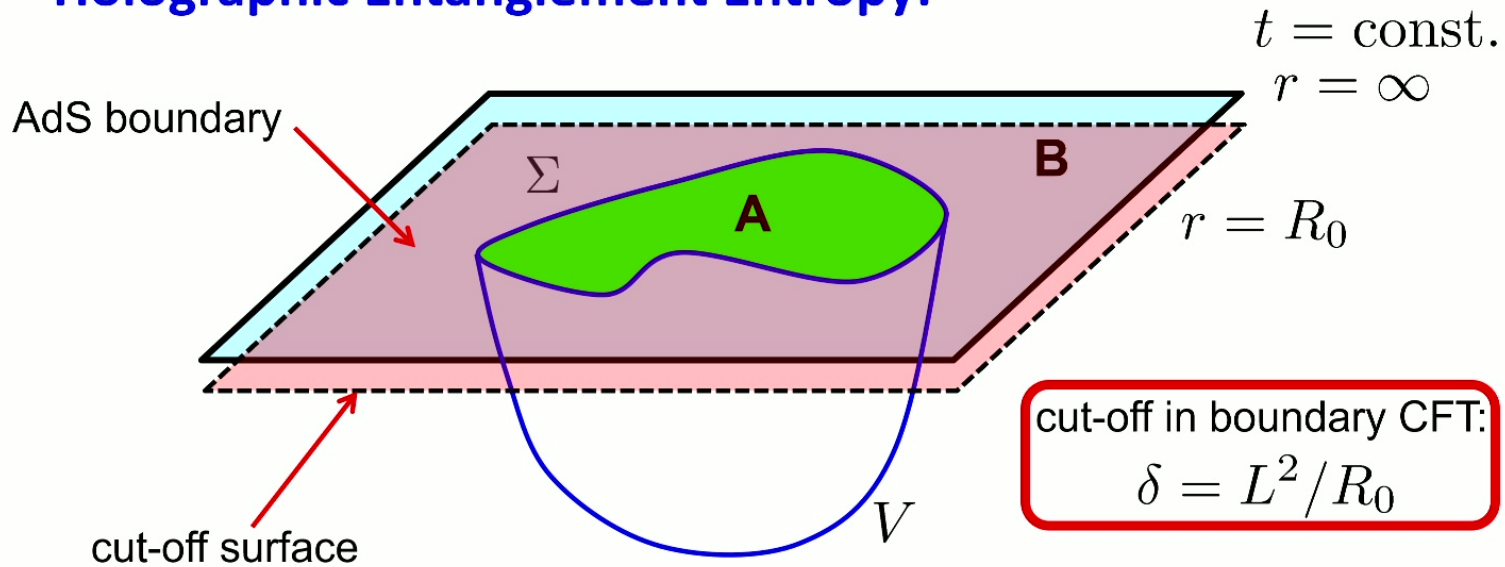
Spacetime Geometry = Entanglement

- connectivity of spacetime requires entanglement (van Raamsdonk)
- AdS spacetime as a tensor network (MERA) (Swingle, Vidal,)
- “ER = EPR” conjecture (Maldacena & Susskind)
- Einstein’s equations from entanglement (van Raamsdonk et al,)
- Spacetime as quantum error correcting code
(Almeirhi, Dong & Harlow,)
- Entanglement wedge reconstruction (Dong, Harlow & Wall,)
- Evaluation of Page Curve (Penington; Almeirhi et al,)



Holographic Entanglement Entropy:

(Ryu & Takayanagi '06)

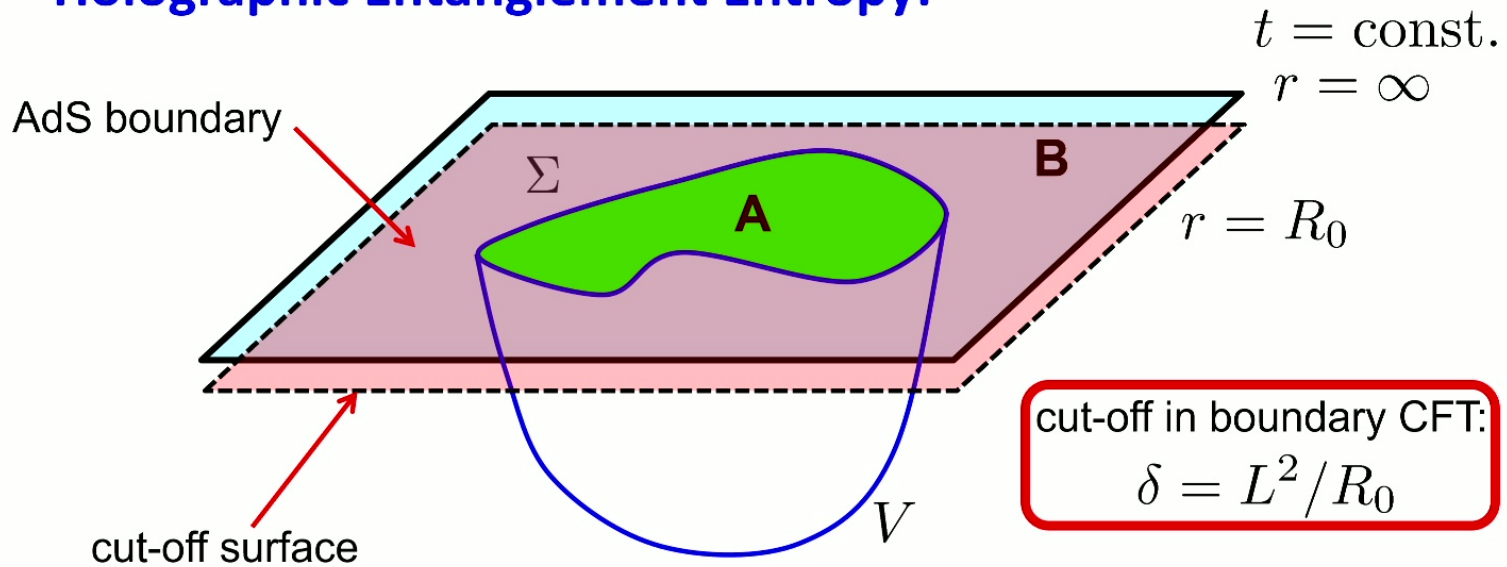


$$S(A) = \text{ext}_{V \sim A} \frac{A_V}{4G_N} \simeq \frac{L^{d-1}}{G_N} \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \dots$$

- “UV divergence” because area integral extends to $r = \infty$
- finite result by stopping radial integral at large radius: $r = R_0$
- short-distance cut-off in boundary theory: $\delta = L^2 / R_0$

Holographic Entanglement Entropy:

(Ryu & Takayanagi '06)



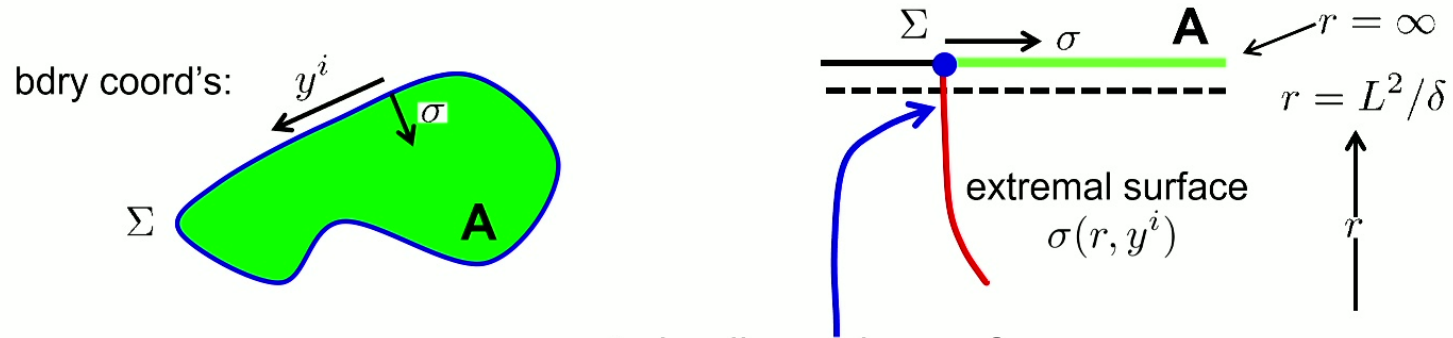
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central charge
 (counts dof) $(L/\ell_{Planck})^{d-1}$

“Area Law”

Area law contribution:

recall AdS metric: $ds^2 = \frac{r^2}{L^2} (-dt^2 + d\vec{x}^2) + \frac{L^2}{r^2} dr^2$



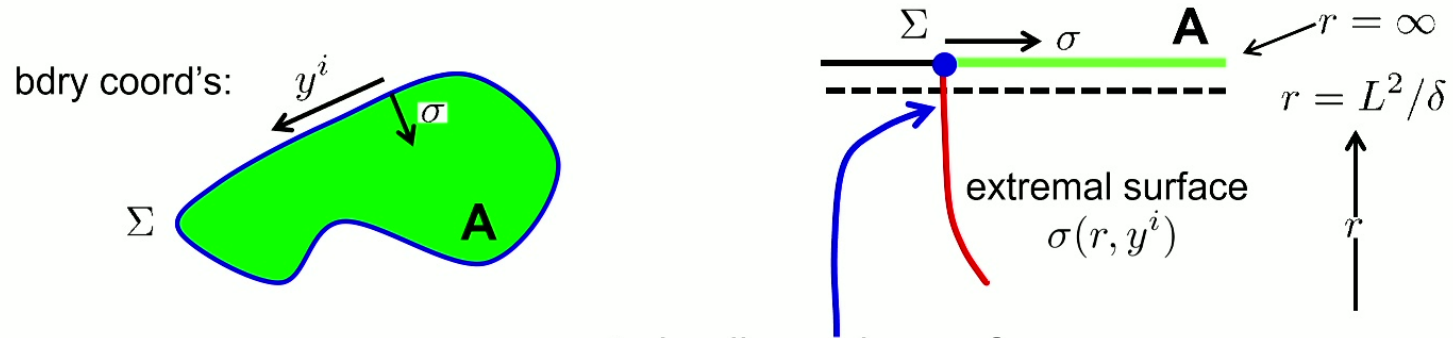
to leading order, surface falls straight down, ie, $\sigma \simeq 0$

$$S(A) = \frac{A_V}{4G_N} = \frac{1}{4G_N} \int d^{d-1} \sigma \sqrt{h}$$

$$= \frac{1}{4G_N} \int d^{d-2} y \sqrt{h_y} \left(\frac{r}{L} \right)^{d-2} \times \int^{R_0} dr \times \left(\frac{L}{r} \right)$$

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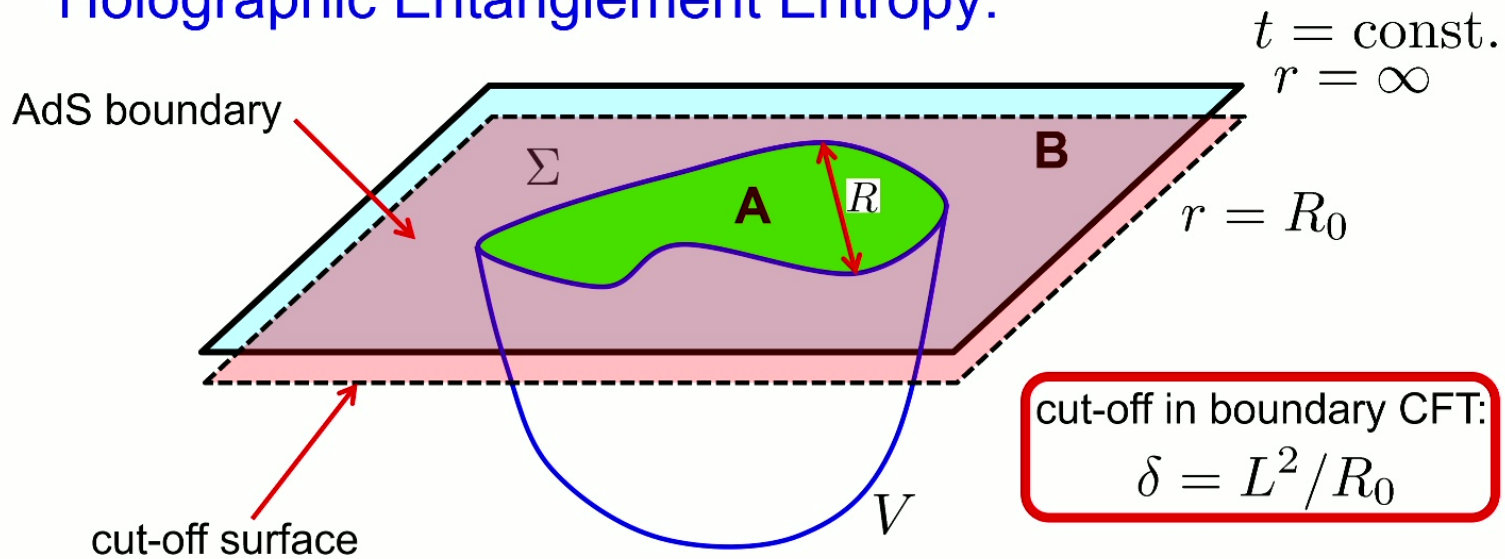
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$$S(A) = \frac{A_V}{4G_N} = \frac{1}{4G_N} \int d^{d-1}\sigma \sqrt{h}$$

$$= \frac{1}{4G_N} \mathcal{A}_\Sigma \int^{L^2/\delta} dr \left(\frac{r}{L}\right)^{d-3}$$

(Ryu & Takayanagi '06)

Holographic Entanglement Entropy:



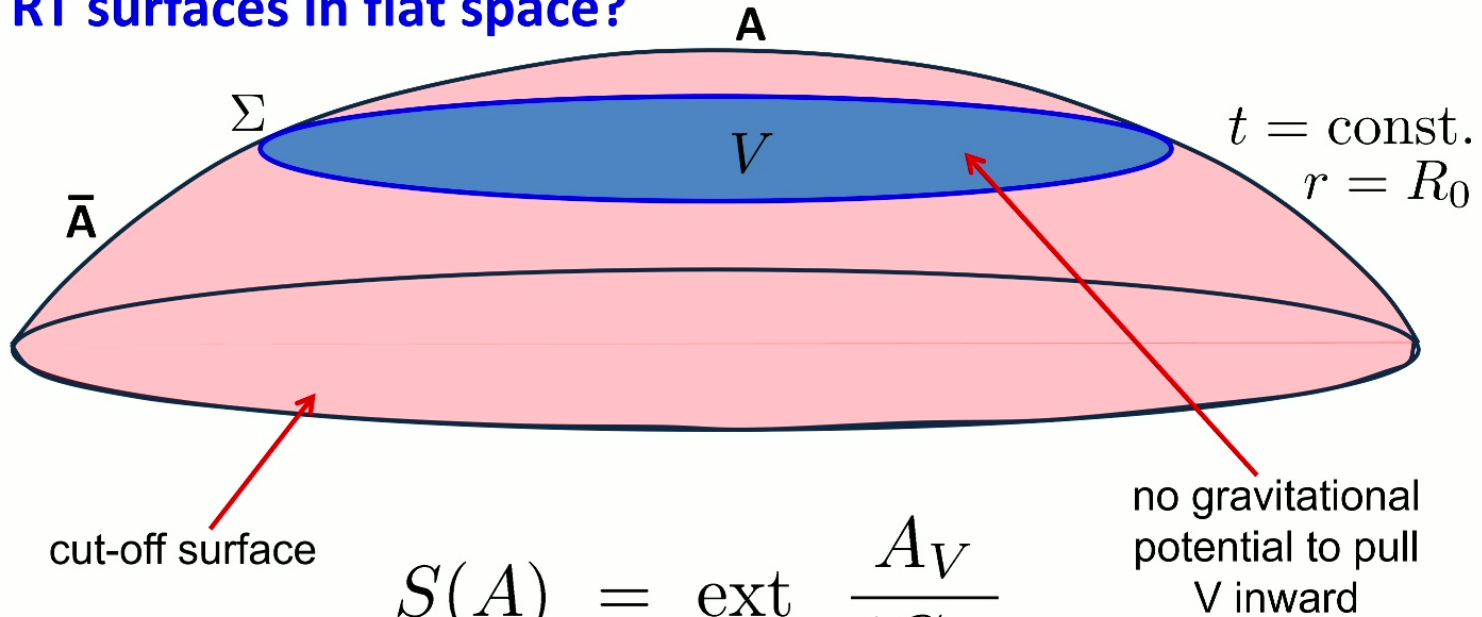
general expression (as desired):

$$S(A) \simeq c_0(R/\delta)^{d-2} + c_1(R/\delta)^{d-4} + \dots$$

$$\left[\begin{array}{l} +c_{d-2} \log(R/\delta) + \dots \quad (\text{d even}) \\ + \underbrace{c_{d-2} + \dots}_{\text{universal contributions}} \quad (\text{d odd}) \end{array} \right.$$

universal contributions

RT surfaces in flat space?



$$S(A) = \text{ext}_{V \sim A} \frac{A_V}{4G_N}$$

- let's try attaching extremal surfaces to boundary of flat space

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-1}^2$$

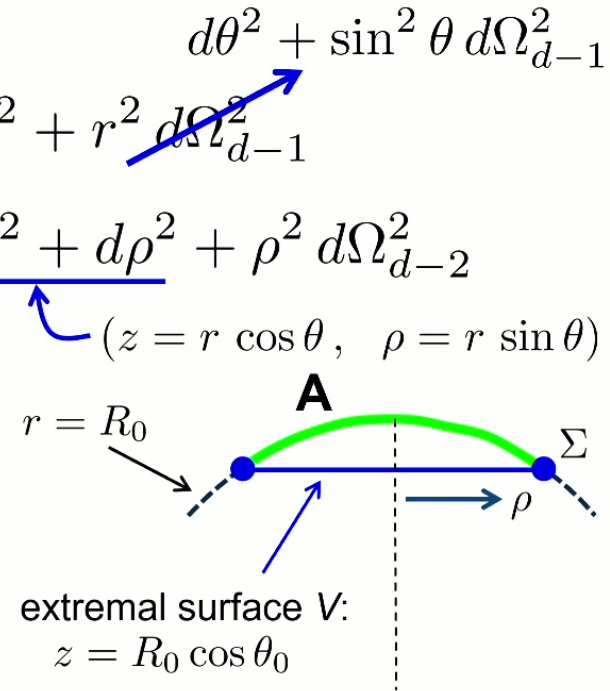
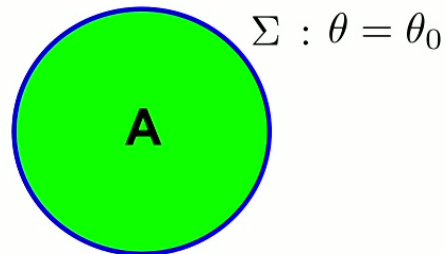
- regulate radial integral with cut-off surface at large radius: $r = R_0$

No area law contribution!!

flat space metric: $ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-1}^2$

$$= -dt^2 + \underline{dz^2 + d\rho^2} + \rho^2 d\Omega_{d-2}^2$$

Consider spherical cap:



$$S(A) = \frac{A_V}{4G_N} = \frac{1}{4G_N} \int d^{d-1} \sigma \sqrt{h}$$

$$= \frac{1}{4G_N} \int d^{d-2} \Omega \rho^{d-2} \times \int_0^{R_0 \sin \theta_0} d\rho \times \textcircled{1}$$

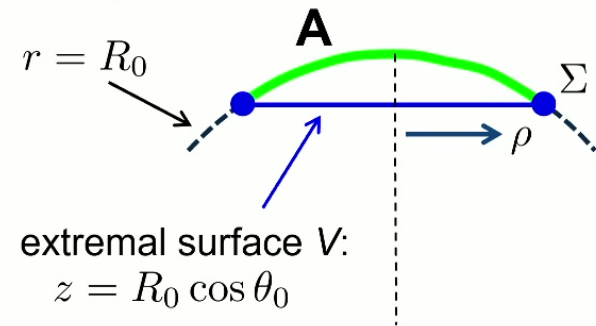
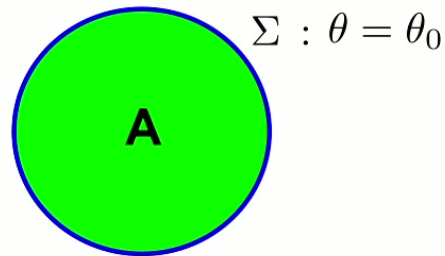
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flat space metric: $ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-1}^2$

$$= -dt^2 + \underline{dz^2 + d\rho^2} + \rho^2 d\Omega_{d-2}^2$$

$$d\theta^2 + \sin^2 \theta d\Omega_{d-1}^2$$

Consider spherical cap:



$$S(A) = \frac{A_V}{4G_N} = \frac{1}{4G_N} \int d^{d-1} \sigma \sqrt{h}$$

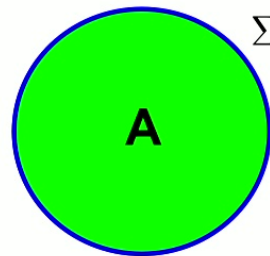
$$= \frac{\Omega_{d-2}}{4(d-1)G_N} (R_0 \sin \theta_0)^{d-1}$$

No area law contribution!!

flat space metric: $ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-1}^2$

$$= -dt^2 + \underline{dz^2 + d\rho^2} + \rho^2 d\Omega_{d-2}^2$$

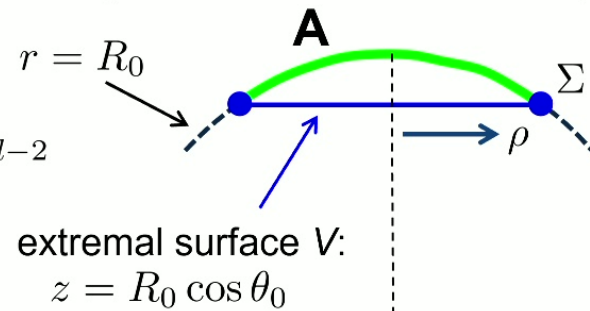
Consider spherical cap:



$$\Sigma : \theta = \theta_0$$

$$\mathcal{A}_\Sigma = \Omega_{d-2} (R_0 \sin \theta_0)^{d-2}$$

$$\text{Vol}_A = R_0^{d-1} f(\theta_0)$$

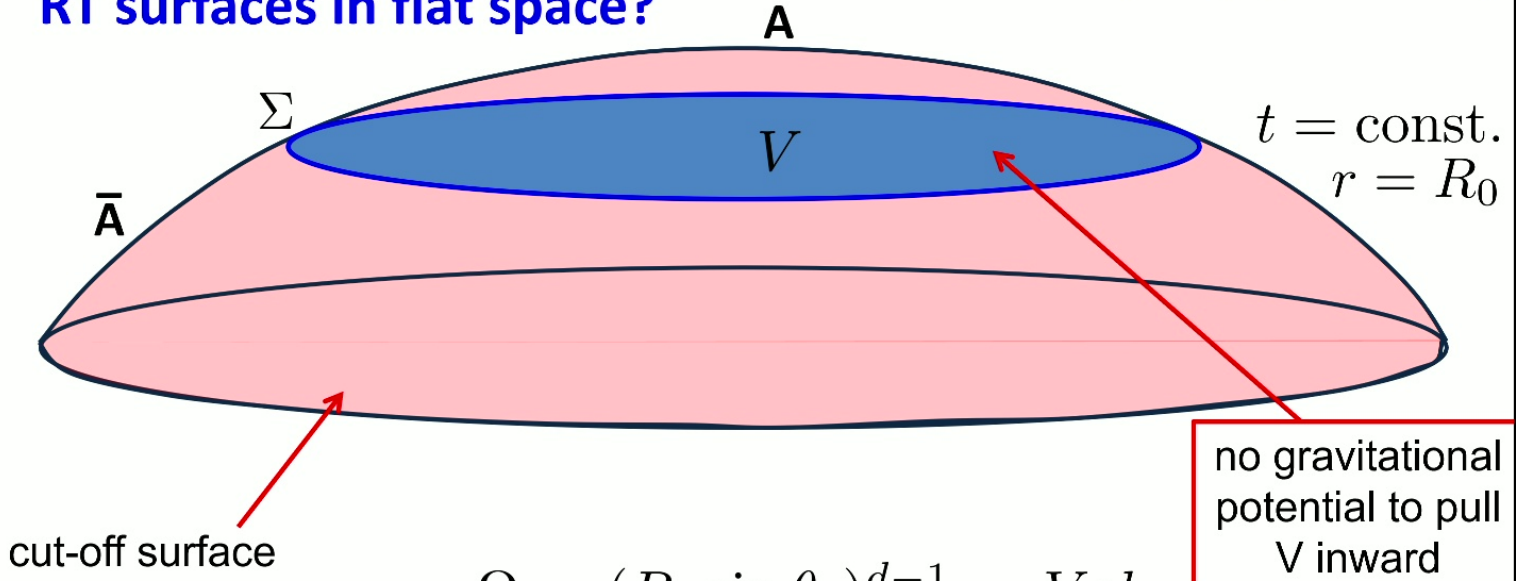


extremal surface V :
 $z = R_0 \cos \theta_0$

$$S(A) = \frac{A_V}{4G_N} = \frac{1}{4G_N} \int d^{d-1} \sigma \sqrt{h}$$

$$= \frac{\Omega_{d-2}}{4(d-1)G_N} (R_0 \sin \theta_0)^{d-1} \simeq \frac{\text{Vol}_A}{G_N} + \dots$$

RT surfaces in flat space?



$$S(A) = \frac{\Omega_{d-2} (R_0 \sin \theta_0)^{d-1}}{4(d-1)G_N} \simeq \frac{Vol_A}{G_N}$$

- leading contribution is a volume law, ie, “holo EE” is extensive
 - conjecture: holographic dual is nonlocal field theory (Li & Takayanagi)
 - perhaps holographic dual of flat space vacuum is highly excited state

RT surfaces in flat space?

$$S(A) = \frac{\Omega_{d-2} (R_0 \sin \theta_0)^{d-1}}{4(d-1)G_N} \simeq \frac{Vol_A}{G_N}$$

- what about a boundary cut-off? $R_0 = \frac{\ell^2}{\delta}$
 - ℓ^2 ← macroscopic distance
 - δ ← short distance cut-off

$$S(A) \simeq \underbrace{\frac{\ell^{d-1}}{G_N}}_{\text{"central charge" (depends on macroscopic scale)}} \frac{Vol'_A}{\delta^{d-1}} \quad \leftarrow \quad Vol'_A = \ell^{d-1} f(\theta_0)$$

- corrections to volume law? (consider $\theta_0 \ll 1$)

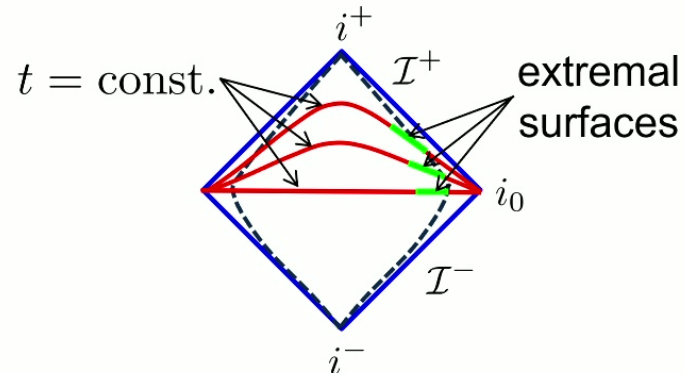
$$S(A) \simeq \frac{\ell^{d-1}}{G_N} \frac{Vol'_A}{\delta^{d-1}} \left(1 - \frac{\# \theta_0^2}{\dots} + \dots \right)$$

?? $\theta_0 \simeq (Vol'_A)^{1/d-1} / \ell$

RT surfaces in flat space?

$$S(A) = \frac{\Omega_{d-2} (R_0 \sin \theta_0)^{d-1}}{4(d-1)G_N} \simeq \frac{Vol_A}{G_N}$$

- implicitly surfaces are anchored at i_0 . What about \mathcal{I}^\pm ?

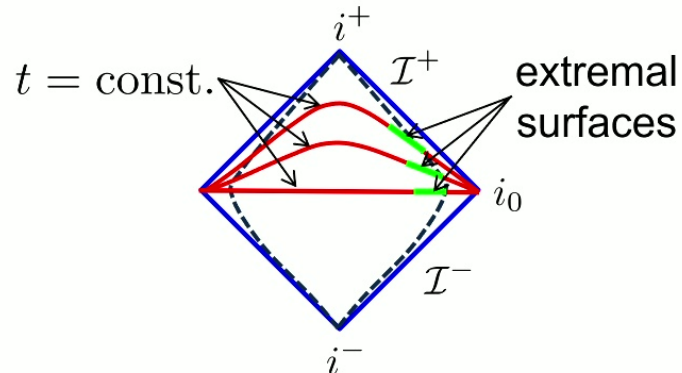


- straightforward to push cut-off surface out holding boundary region A at fixed $u = t - r$
- extremal surfaces are pushed towards cut-off surface/ \mathcal{I}^+ ??
- similarly for \mathcal{I}^- ??

RT surfaces in flat space?

$$S(A) = \frac{\Omega_{d-2}(R_0 \sin \theta_0)^{d-1}}{4(d-1)G_N} \simeq \frac{Vol_A}{G_N}$$

- implicitly surfaces are anchored at i_0 . What about \mathcal{I}^\pm ?



- straightforward to push cut-off surface out holding boundary region A at fixed $u = t - r$
- extremal surfaces are pushed towards cut-off surface/ \mathcal{I}^+ ??
- similarly for \mathcal{I}^- ??

- strong sub-additivity: $S(A) + S(B) \geq S(A \cup B) + S(A \cap B)$

- proof by pictures (Headrick & Takayanagi) still applies in flat space
- extended to dynamical settings (Wall) but subtleties arise for present flat space prescription (Grado-White, Marolf & Weinberg)



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- c) Holographic EE – Take 2
- d) Outlook

Holographic EE with ropes and swings?

- in three-dimensional AFS, the BMS group has following centrally extended Lie algebra:

$$\begin{aligned}
 [\mathcal{L}_n, \mathcal{L}_m] &= (n-m)\mathcal{L}_{n+m} + \frac{c_L}{12}n(n^2-1)\delta_{n+m,0} \\
 [\mathcal{L}_n, \mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_M}{12}n(n^2-1)\delta_{n+m,0} \\
 [\mathcal{M}_n, \mathcal{M}_m] &= 0
 \end{aligned}$$

↙ super-rotations
↘ super-translations

- with Einstein gravity in bulk: $c_M = \frac{3}{G_N} c_L = 0$ (Barnich & Compere)
- Wigner-Inönü contraction of bdy Vir X Vir in AdS₃/CFT₂:

$$\begin{aligned}
 [\mathcal{L}_n^\pm, \mathcal{L}_m^\pm] &= (n-m)\mathcal{L}_{n+m}^\pm + \frac{c^\pm}{12}n(n^2-1)\delta_{n+m,0} \\
 [\mathcal{L}_n^+, \mathcal{L}_m^-] &= 0
 \end{aligned}$$

Einstein gravity: $c^\pm = \frac{3L}{2G_N}$

$$L \rightarrow \infty \left\{ \begin{array}{l} \mathcal{L}_n = \mathcal{L}_n^+ - \mathcal{L}_{-n}^-, \quad \mathcal{M}_n = \frac{1}{L}(\mathcal{L}_n^+ + \mathcal{L}_{-n}^-) \\ c_M = \frac{1}{L}(c^+ + c^-), \quad c_L = c^+ - c^- \end{array} \right.$$

Holographic EE with ropes and swings?

- entanglement entropy for two-dimensional BMS field theory?
(Bagchi, Basu Grumiller & Riegler)

- consider EE in 2D CFT and take ultra-relativistic limit $c \rightarrow \infty$ *:

$$S(A) = - \lim_{n \rightarrow 1} \frac{\partial}{\partial n} \text{Tr} \rho_A^n \quad \text{where} \quad \text{Tr} \rho_A^n \propto \langle \Phi_{h_L, h_m}(x_1, y_1) \Phi_{h_L, h_M}(x_2, y_2) \rangle$$

- for interval with $(\Delta u, \Delta x)$ on a circle of circumference C :

$$S(A) = \frac{c_L}{6} \log \left(\frac{C}{\pi \varepsilon_x} \sin \frac{\pi \Delta x}{C} \right) + \frac{c_M}{6} \frac{C \Delta u}{\pi} \cot \frac{\pi \Delta x}{C}$$

- planar limit ($C \rightarrow \infty$): $S(A) = \frac{c_L}{6} \log \left(\frac{\Delta x}{\varepsilon_x} \right) + \frac{c_M}{6} \frac{\Delta u}{\Delta x}$
- similarly for thermal state on infinite line ($C \rightarrow i\beta$)
- Renyi entropies: $S_n(A) = \frac{1}{1-n} \log \text{Tr} \rho_A^n = \frac{1}{2} \left(1 + \frac{1}{n} \right) S(A)$

(* actually take non-relativistic limit $c \rightarrow 0$ and interchange $t \leftrightarrow x$)

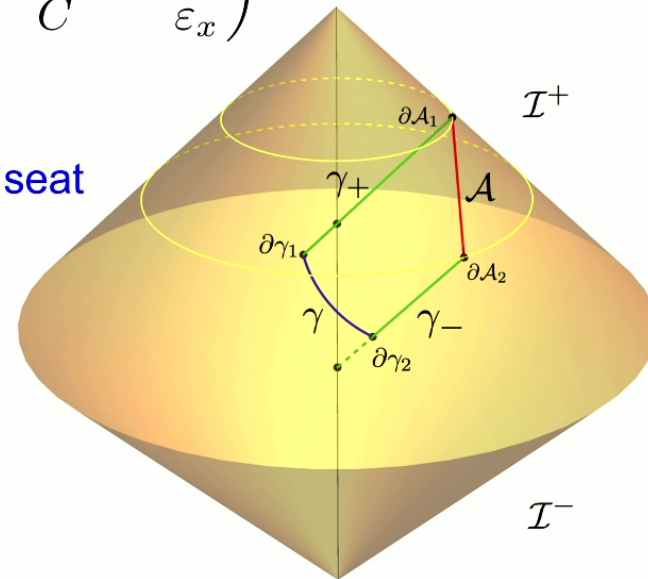
Holographic EE with ropes and swings?

- swing prescription:
 - 1) inscribe \mathcal{A} interval of interest on \mathcal{I}^+
 - 2) draw null geodesics γ_{\pm} from $\partial\mathcal{A}$ crossing origin (with $J = 0$)
 - 3) find minimal spacelike geodesic γ between γ_{\pm}
 - 4) evaluate BH formula on γ

$$S(A) = \frac{A_{\gamma}}{4G_N} = \frac{c_M}{6} \left(\frac{C \Delta u}{\pi} \cot \frac{\pi \Delta x}{C} - \frac{\varepsilon_u}{\varepsilon_x} \right)$$

γ : swing seat

γ_{\pm} : ropes



Holographic EE with ropes and swings?

- turn on c_L by considering topologically massive gravity

$$\mathcal{S}_{\text{TMG}} = \frac{1}{16\pi G_N} \int d^3x \sqrt{-g} \left[R + \frac{2}{L^2} + \frac{1}{2\mu} \varepsilon^{\alpha\beta\gamma} \left(\Gamma_{\alpha\sigma}^\rho \partial_\beta \Gamma_{\gamma\rho}^\sigma + \frac{2}{3} \Gamma_{\alpha\sigma}^\rho \Gamma_{\beta\eta}^\sigma \Gamma_{\gamma\rho}^\eta \right) \right]$$

$$c^\pm = \frac{3L}{2G_N} \left(1 \pm \frac{1}{\mu L} \right) \xrightarrow{L \rightarrow \infty} c_L = \frac{3}{\mu G_N}, \quad c_M = \frac{3}{G_N}$$

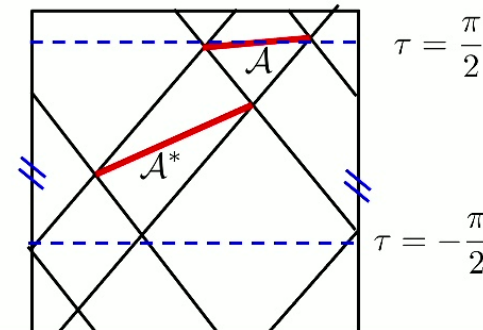
$$S(A) = \frac{c_L}{6} \log \left(\frac{C}{\pi \varepsilon_x} \sin \frac{\pi \Delta x}{C} \right) + \frac{c_M}{6} \left(\frac{C \Delta u}{\pi} \cot \frac{\pi \Delta x}{C} - \frac{\varepsilon_u}{\varepsilon_x} \right)$$

(see also: Castro, Detournay, Iqbal & Perlmutter)

Holographic EE with ropes and swings?

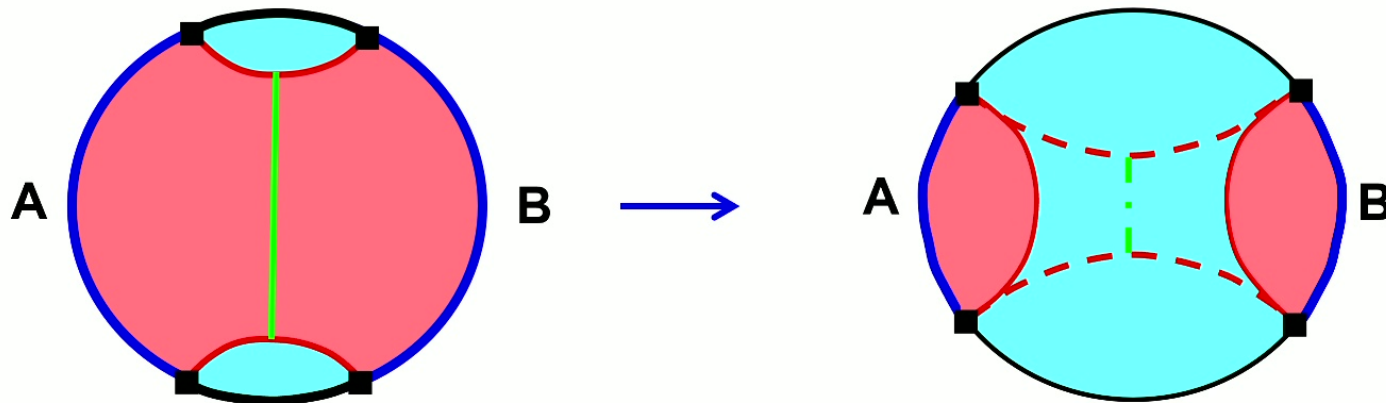
Comments/Personal confusions:

- derivation:
 - map entanglement entropy to thermal entropy (like CHM) but also add orbifold identifications(?)
- approach not covariant:
 - infinite family of timelike geodesics could define “origin”
 - correspond to changes in frame, absorbed by change of cutoffs (Grumiller, Parekh & Riegler)
- looking at finite portion of spacelike geodesic; why cutoffs?
- in lifting to AdS_3 , geodesic γ is not RT surface for \mathcal{A} but instead alternate \mathcal{A}^* (?)



Holographic EE with ropes and swings?

- could holographic BMS EE descend from alternative quantum information observable in $\text{AdS}_3/\text{CFT}_2$?
- consider reflected entropy: minimal cross-section of connected entanglement wedge, ie, minimal length of spacelike geodesic connecting to **spacelike** geodesics (in AdS_3)



- for cross-section to probe “bulk point” must shrink A and B
→ but then enter disconnected phase when $\ell_\gamma \sim L$!?

Other Ideas & Directions:

- Celestial Holography from qudits Guevara & Hu (2312.16298)
- Entanglement on the celestial sphere (& soft modes)
 Chen, Myers & Raclariu (2308.12341; 2403.13913)
- Black hole entropy using soft hair
 Hawking, Perry & Strominger (1601.00921; 1611.09175)
 Haco, Hawking, Perry & Strominger (1810.01847)
- Holographic entanglement entropy of general spacetimes
 Sanches & Weinberg (1603.05250)
 Nomura, Salzetta, Sanches & Weinberg (1611.02702)
 Grado-White, Marolf & Weinberg (2008.07022)
- Flat space with wedge holography
 Ogawa, Takayanagi, Tsuda & Waki (2207.06735)
- And more

Outlook/Questions:

- intersection of quantum information and quantum gravity led to significant advances in our understanding of holography and QG
→ holographic EE, role of QEC, **holographic complexity**, ...
- focus there was on AdS/CFT but **negative Λ was not essential**, eg, generalized gravitational entropy (Lewkowycz & Maldacena)
- QI in QG has great potential to provide important new insights for holography in flat space
- initial explorations point to unusual properties of dual bdy theory and/or holographic dictionary
- Many open questions:
 - role of the soft modes?
 - entanglement entropy in Carrollian theories?
 - role of Rindler horizons? black holes?

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Lots to explore!