Title: Lecture - Amplitudes b

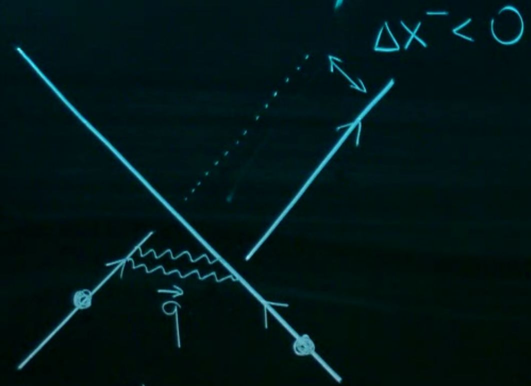
Speakers:

Collection: Celestial Holography Summer School 2024

Date: July 23, 2024 - 2:30 PM

URL: https://pirsa.org/24070010

YESTERDAY



$p_B \sim (p^+, 0, \vec{0})$ $\quad$ $p_A \sim (0, p^-, \vec{0})$

Exponentiation when $\underset{\overset{\parallel}{p^+p^-}}{s} \gg -\underset{\overset{\parallel}{\vec{q}^{\,2}}}{t} \sim 1/\vec{b}^{\,2}$

$$S(s,b) = e^{i\delta(s,b)} = e^{i}$$

Time delay: $\Delta x^- = \# \, \delta(s,b)$

Spin*: $J \leq 2$

$$\underset{\substack{\shortparallel\\ s}}{p^+p^-} \qquad \underset{\substack{\shortparallel\\ -t}}{\vec{q}^{\,2}}$$

Exponentiation when $s \gg -t \sim 1/\vec{b}^{\,2}$

$$S(s,b) = e^{i\delta(s,b)} = e^{i}$$

- Time delay: $\Delta x^- = \#\, \delta(s,b) \overset{?}{\lessgtr} 0$
- Spin*: $J \leq 2$

$\sim 1/\vec{b}^2$

Time delay is governed by tree-level exchanges:

$$\delta(s,\vec{b}) = \frac{\#}{s}\int d^2\vec{q}\; e^{i\vec{b}\cdot\vec{q}}\, \mathcal{M}^{\text{tree}}(s, t=-\vec{q}^{\,2})$$

$\vec{b} = (b,0)$, $\# > 0$

deform $q_1 \longmapsto q_1 + ik$

$e^{i\vec{b}\cdot\vec{q}} \mapsto e^{i\vec{b}\cdot\vec{q}}\, e^{-bk}$

$\overset{?}{\lessgtr} 0$

$1/\vec{b}^{2}$

$\overset{?}{\lessgtr} 0$

Time delay is governed by tree-level exchanges:

$$\delta(s,\vec{b}) = \frac{\#}{s}\int d^2\vec{q}\; e^{i\vec{b}\cdot\vec{q}}\,\mathcal{M}^{\text{tree}}(s, t=-\vec{q}^{\,2})$$

$\vec{b} = (b,0)$ $\qquad \# > 0$

deform $q_1 \longmapsto q_1 + ik$

$$e^{i\vec{b}\cdot\vec{q}} \mapsto e^{i\vec{b}\cdot\vec{q}}\, e^{-bk}$$

Roughly, $\delta(s,b)$

$\vec{q}^{\,2}$

$\|$

$-t \sim 1/\vec{b}^{\,2}$

$e^{i\,\cdots}$

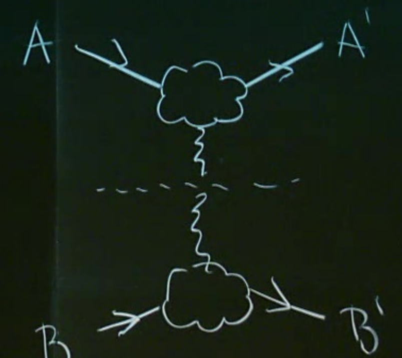
$\delta(s,b) \overset{?}{\lessgtr} 0$

Time delay is governed by tree-level exchanges:

$$\delta(s,\underset{\overset{\|}{(b,0)}}{\vec{b}}) = \underset{\# > 0}{\frac{\#}{s}} \int d^2\vec{q}\; e^{i\vec{b}\cdot\vec{q}}\, \mathcal{M}^{\text{tree}}(s, t=-\vec{q}^{\,2})$$

deform $q_1 \longmapsto q_1 + ik$

$$e^{i\vec{b}\cdot\vec{q}} \mapsto e^{i\vec{b}\cdot\vec{q}}\, e^{-bk}$$



$\mathcal{M}^A$

$\mathcal{M}^B$

Roughly, $\delta(s,b)$

$$\sim \sum_{\text{pol}} \mathcal{M}^A \mathcal{M}^B$$

s governed by tree-level exchanges:

$$\left(\frac{\#}{s}\int d^2\vec{q}\; e^{i\vec{b}\cdot\vec{q}}\;\mathcal{M}^{\text{tree}}(s, t=-\vec{q}^{\,2})\right)$$

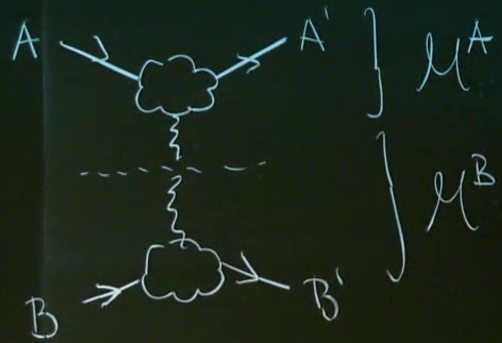
$\# > 0$

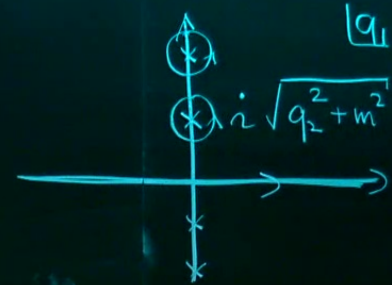
deform $q_1 \longmapsto q_1 + ik$

$$e^{i\vec{b}\cdot\vec{q}} \mapsto e^{i\vec{b}\cdot\vec{q}}\, e^{-bk}$$

$\delta(s,b)$

$\sum_{pol} \mathcal{M}^A \mathcal{M}^B$





- Strongest time delay/adv. from $m=0$.
- Governed by massless 3-pt amplitudes

Massless: $p_i^2 = 0$.

Mom. cons: $p_1^2 = (-p_2 - p_3)^2 = 2 p_2 \cdot p_3 = 0$.

$\Rightarrow p_i \cdot p_j = 0$.

$$\varepsilon_\lambda^{\mu\nu}, \quad q^\mu, \quad q_\mu \varepsilon^{\mu\nu} = 0$$

$$\mathcal{M}^A_{\phi\phi h} = \sqrt{G}\, \varepsilon^{\lambda}_{\mu\nu}\, p_A^\mu\, p_A^\nu$$

$$\mathcal{M}^B_{\phi\phi h} = \sqrt{G}\, \varepsilon^{\lambda *}_{\mu\nu}\, p_B^\mu\, p_B^\nu$$

$$\sum_{\text{pol.}\ \lambda=\pm 1} \varepsilon_\lambda^{\mu\nu}\, \varepsilon_\lambda^{\nu\rho *} \sim \frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right).$$

Time delay is governed by tree-level exchanges (in GR):

$$\delta(s, \vec{b}) = \frac{\#}{s} \int d^2\vec{q}\; e^{i\vec{b}\cdot\vec{q}} \left(- \frac{G s^2}{t} \right)$$

$\vec{b} = (b, 0)$, $\# > 0$

deform q_1

$$= \# G s \oint_{q_1 = +i|q_2|} dq_1\; e^{i b q_1} \int_{-\infty}^{\infty} dq_2 \frac{1}{q_1^2 + q_2^2}$$

compute res

$$= \# G s \int_0^{\infty} dq_2 \frac{e^{-b q_2}}{q_2} = \infty .$$

elay is governed by tree-level exchanges (in GR):

$$) = \frac{\#}{s}\int d^2\vec{q}\; e^{i\vec{b}\cdot\vec{q}}\left(-\frac{G s^2}{t}\right)$$

$,0)$ $\quad \# > 0$

deform q_1

$$= \# G s \oint_{q_1 = +i|q_2|} dq_1\, e^{i b q_1}\int_{-\infty}^{\infty} dq_2\, \frac{1}{q_1^2+q_2^2}$$

compute res.

$$= \# G s \int_{1/B}^{\infty} dq_2\, \frac{e^{-b q_2}}{q_2} = -\# G s \log(b/B)$$

Shapiro time delay.

IR cutoff $B \gg b$.

exchanges (in GR):

$$\left(- \frac{G s^2}{t} \right)$$

deform q_1

$$q_1 \int_{-\infty}^{\infty} dq_2 \, \frac{1}{q_1^2 + q_2^2}$$

compute res

$$-bq_2$$

$$q_2$$

$$= -\# G s \log(b/B)$$

Shapiro time delay.

utoff $B \gg b$.

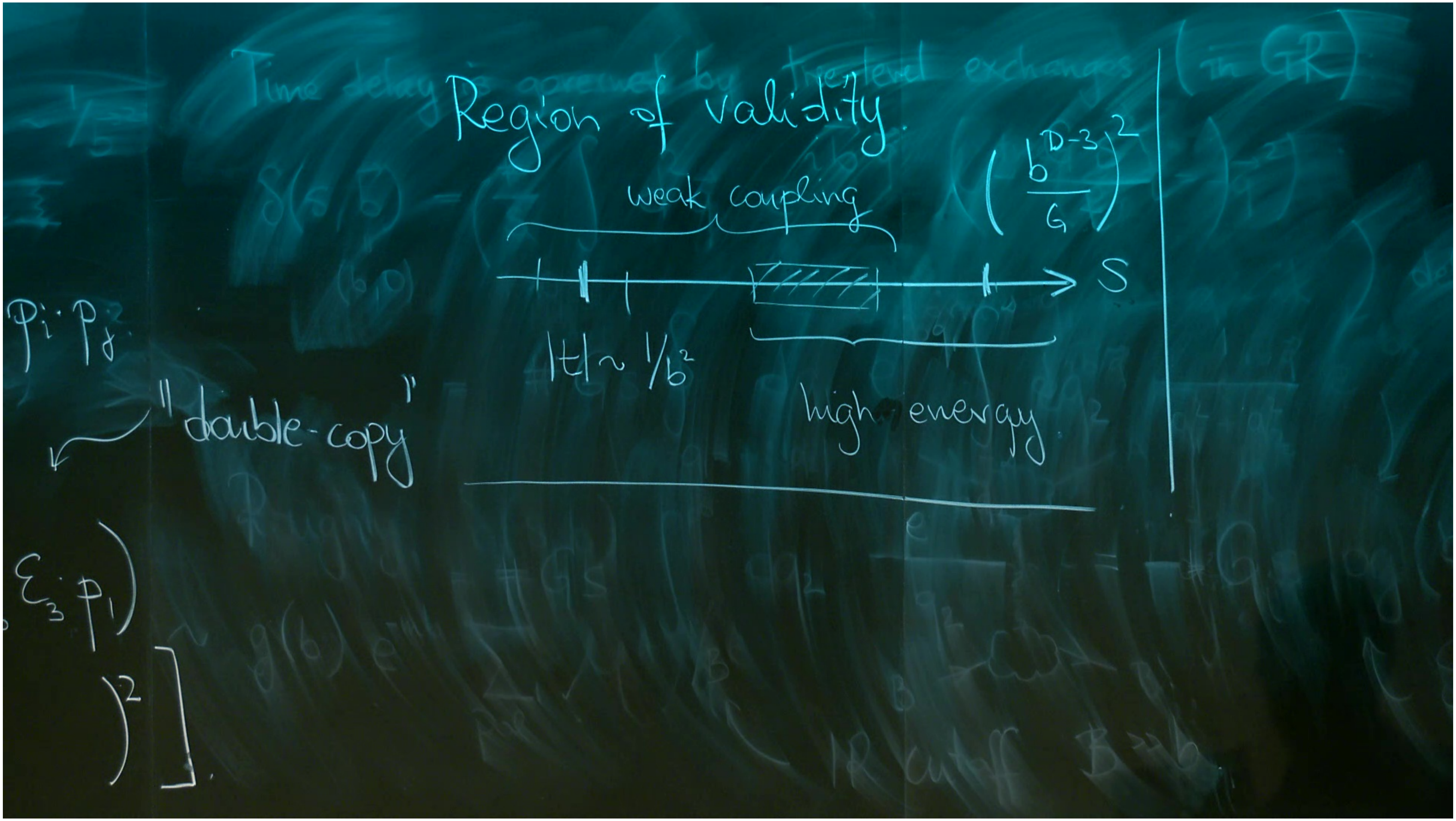
Bounds on modifications of GR $(D>4)$.

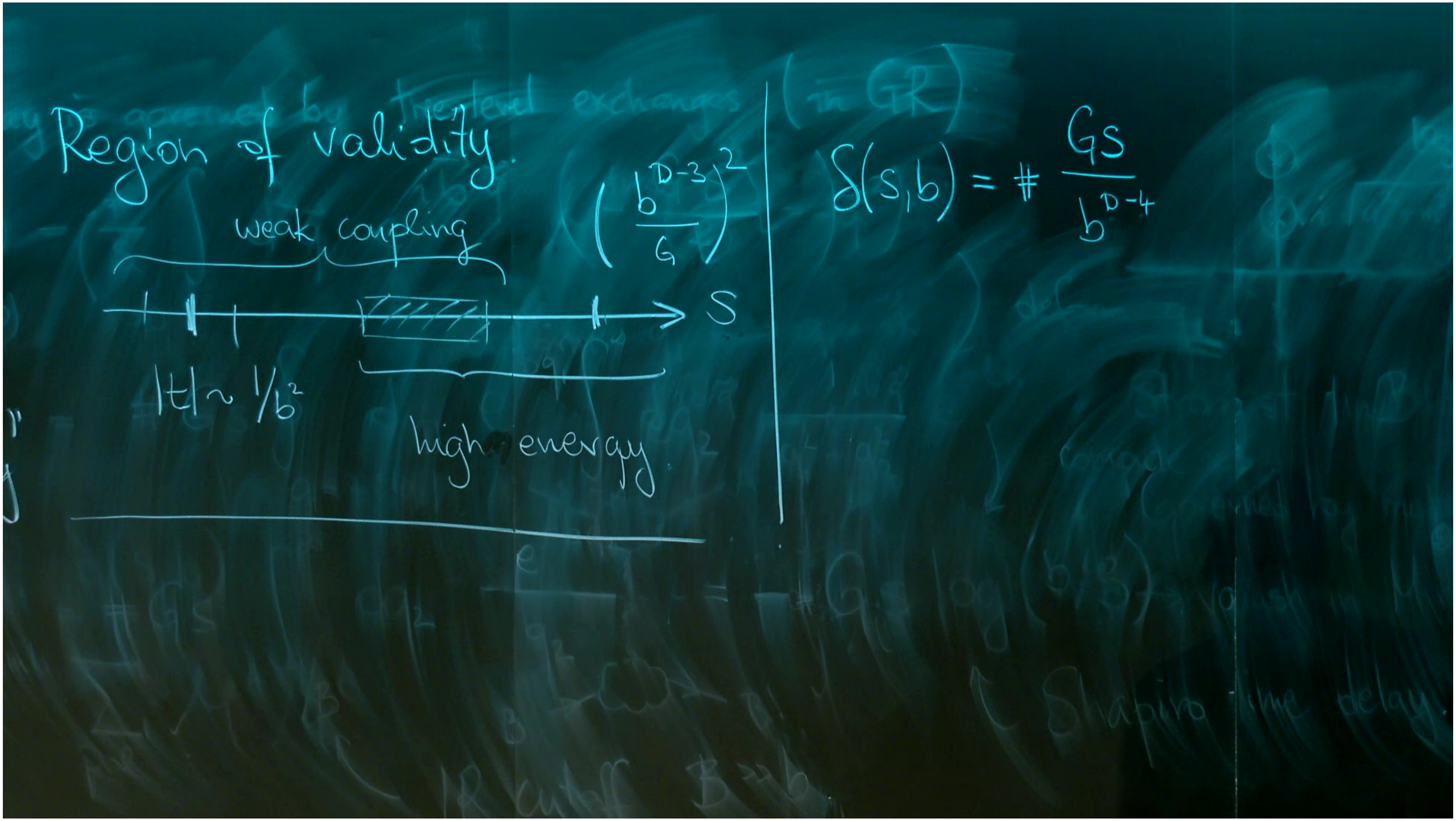
$p_i^\mu, \quad \varepsilon_i^{\mu\nu} = \varepsilon_i^\mu \varepsilon_i^\nu$

Constraints:

$$0 = p_i^2 = \varepsilon_i^2 = p_i \cdot \varepsilon_i = p_i \cdot p_j$$

$$\mathcal{M}_{hhh} = \pm\sqrt{G}\,\Big[\left(\varepsilon_1\cdot\varepsilon_2\, \varepsilon_3\cdot p_1 + \text{perms.} \right)^2 + \lambda_{GB} \left(\quad '' \quad \right)\left(\varepsilon_1\cdot p_2\, \varepsilon_2\cdot p_3\, \varepsilon_3\cdot p_1 \right) + \lambda_{R^3} \left(\quad '' \quad \right)^2 \Big].$$


Region of validity
weak coupling
(b^{D-3}/G)^2
S
|t| ~ 1/b^2
high energy
"double-copy"



Region of validity
weak coupling
$\left(\frac{b^{D-3}}{G}\right)^2$
S
$|t| \sim 1/b^2$
high energy
$\delta(s,b) = \# \frac{Gs}{b^{D-4}}$

$$\hat{\vec{b}} = \frac{\vec{b}}{|\vec{b}|}.$$

$$\delta(s,b) = \# \frac{Gs}{b^{D-4}} \left[1 + \left(\# \frac{\lambda_{GB}}{b^2} - \# \frac{\lambda_{R^3}}{b^4} \right) \left(\frac{(\vec{e}\cdot\hat{\vec{b}})^2}{\vec{e}\cdot\vec{e}} - \frac{1}{D-2} \right) + \# \frac{\lambda_{R^3}}{b^4} \left(\frac{(\vec{e}\cdot\hat{\vec{b}})^4}{(\vec{e}\cdot\vec{e})^2} - \frac{2}{D(D-2)} \right) \right].$$

$\varepsilon = (0, 0, \vec{e})$

dominate at small b

$$\hat{\vec b} = \frac{\vec b}{|\vec b|}.$$

(In GR)

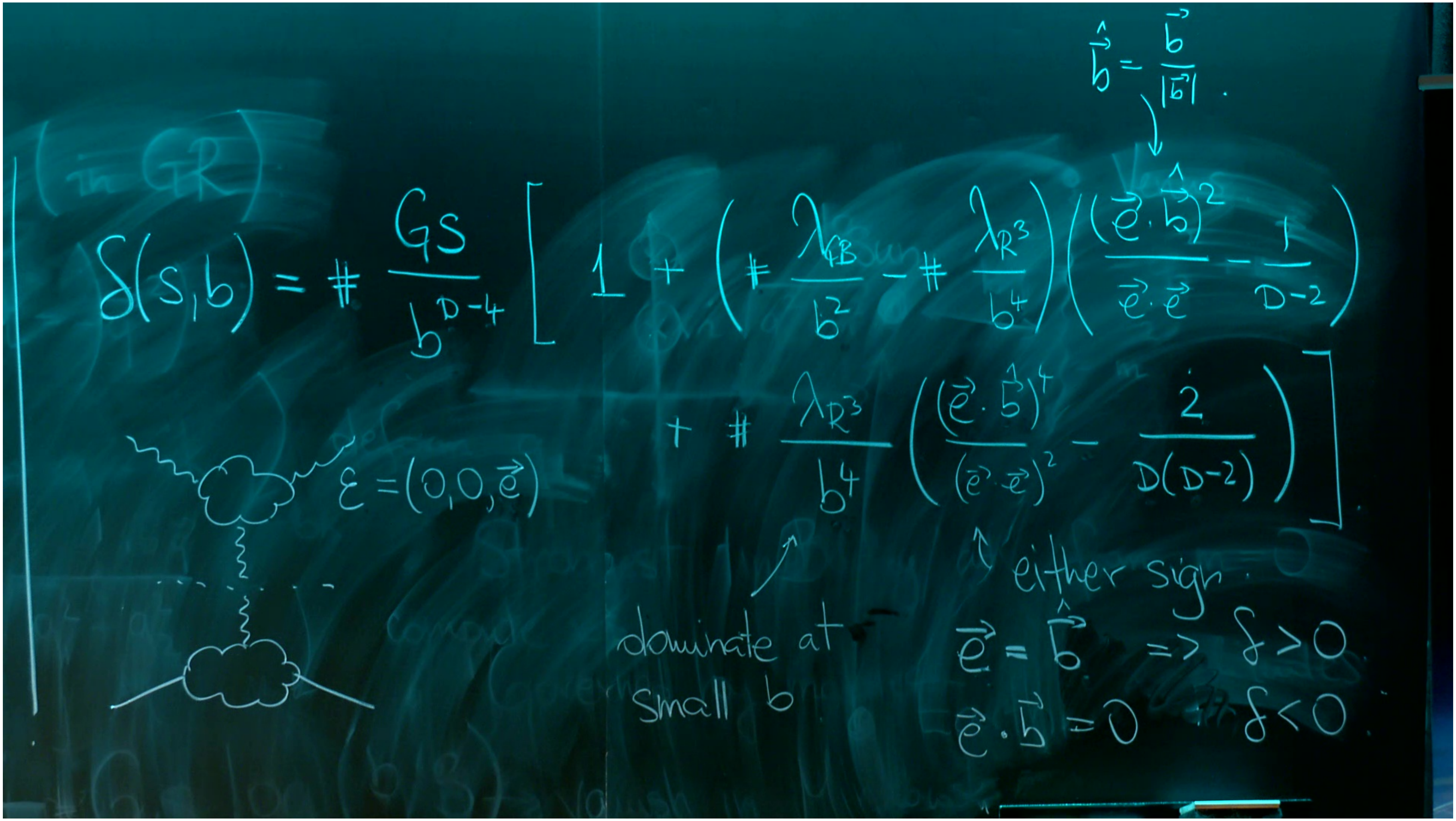
$$\delta(s,b) = \# \frac{Gs}{b^{D-4}}\left[1 + \left(\# \frac{\lambda_{GB}}{b^2} - \# \frac{\lambda_{R^3}}{b^4} \right)\left(\frac{(\vec e\cdot \hat{\vec b})^2}{\vec e \cdot \vec e} - \frac{1}{D-2} \right) + \# \frac{\lambda_{R^3}}{b^4}\left(\frac{(\vec e\cdot \hat{\vec b})^4}{(\vec e\cdot\vec e)^2} - \frac{2}{D(D-2)} \right) \right].$$

$\varepsilon = (0,0,\vec e)$

dominate at small b

either sign.

$\vec e = \hat{\vec b}$


$\hat{\vec{b}} = \frac{\vec{b}}{|\vec{b}|}$.
$\delta(s,b) = \# \frac{Gs}{b^{D-4}} \Big[1 + \left(\# \frac{\lambda_{GB}}{b^2} - \# \frac{\lambda_{R^3}}{b^4} \right) \left(\frac{(\vec{e}\cdot\hat{\vec{b}})^2}{\vec{e}\cdot\vec{e}} - \frac{1}{D-2} \right)$
$+ \# \frac{\lambda_{R^3}}{b^4} \left(\frac{(\vec{e}\cdot\hat{\vec{b}})^4}{(\vec{e}\cdot\vec{e})^2} - \frac{2}{D(D-2)} \right) \Big]$.
$\varepsilon = (0,0,\vec{e})$
dominate at small b
either sign.
$\vec{e} = \hat{\vec{b}} \Rightarrow \delta > 0$.
$\vec{e}\cdot\vec{b} = 0 \Rightarrow \delta < 0$.