Title: Lecture - Celestial Holography IIb

Speakers:

Collection: Celestial Holography Summer School 2024

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Recall:

$q^\mu = (1+z\bar{z}, z+\bar{z}, -i(z-\bar{z}), 1-z\bar{z})$

- Conformally primary WF

$\Phi^{\pm}_{\Delta,J}(X; z,\bar{z})$, $J = \pm s$

$\varphi_\Delta = \frac{1}{(-q\cdot X_\pm)^\Delta}$

$A_{\Delta,J=+1;\mu} = m_\mu \varphi_\Delta$ $\quad (\Delta, J) = (0,1)$

$h_{\Delta,J=+2;\mu\nu} = m_\mu m_\nu \varphi_\Delta$

- Conformal primary operators

$\mathcal{O}^{\pm}_{\Delta,J}(z,\bar{z}) = i\left(\hat{\mathcal{O}}^s(X), \Phi^{\pm}_{\Delta,J}(X; z,\bar{z})^*\right)$

inner product

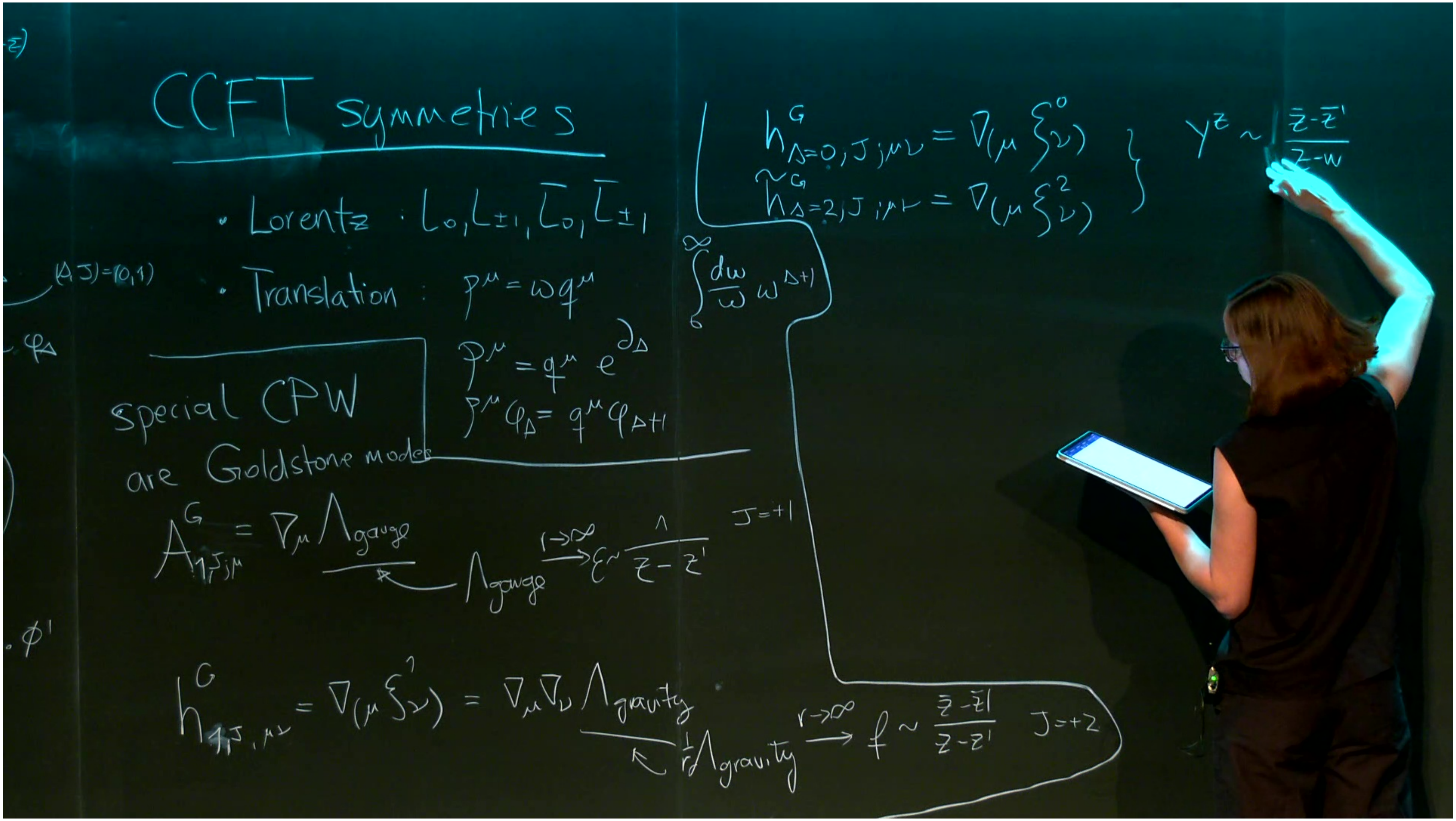
$s=0: \quad i(\phi, \phi'^*)_{KG} = \int_{\Sigma_0} d^3X^i \, \phi \overleftrightarrow{\partial}_{X^0} \phi'$

$= \Omega(\phi, \phi')$

CCFT symmetries

- Lorentz : $L_0, L_{\pm1}, \bar{L}_0, \bar{L}_{\pm1}$
- Translation : $p^\mu = \omega q^\mu$

$\int_0^\infty \frac{d\omega}{\omega} \omega^{\Delta+1}$


CCFT symmetries
• Lorentz : $L_0, L_{\pm 1}, \bar{L}_0, \bar{L}_{\pm 1}$
• Translation : $p^\mu = \omega q^\mu$
$p^\mu = q^\mu e^{\partial_\Delta}$
$p^\mu \varphi_\Delta = q^\mu \varphi_{\Delta+1}$
special CPW are Goldstone modes
$A^G_{1,J;\mu} = \nabla_\mu \Lambda_{gauge}$
$\Lambda_{gauge} \xrightarrow{r\to\infty} \varepsilon \sim \frac{1}{z-z'}$
$J=+1$
$h^G_{1,J,\mu\nu} = \nabla_{(\mu}\xi^1_{\nu)} = \nabla_\mu\nabla_\nu \Lambda_{gravity}$
$\frac{1}{r}\Lambda_{gravity} \xrightarrow{r\to\infty} f \sim \frac{\bar{z}-\bar{z}'}{z-z'}$
$J=+2$
$h^G_{\Delta=0,J;\mu\nu} = \nabla_{(\mu}\xi^0_{\nu)}$
$\tilde{h}^G_{\Delta=2,J;\mu\nu} = \nabla_{(\mu}\xi^2_{\nu)}$
$Y^z \sim \frac{\bar{z}-\bar{z}'}{z-w}$
$\int_0^\infty \frac{d\omega}{\omega}\omega^{\Delta+1}$
$(\Delta, J) = (0,1)$
φ_Δ
ϕ'

CCFT symmetries

- Lorentz : $L_0, L_{\pm 1}, \bar{L}_0, \bar{L}_{\pm 1}$
- Translation : $p^\mu = \omega q^\mu$

$$p^\mu = q^\mu e^{\partial_\Delta}$$

$$p^\mu \varphi_\Delta = q^\mu \varphi_{\Delta+1}$$

special CPW are Goldstone modes

$$A^G_{1,J;\mu} = \nabla_\mu \Lambda_{gauge} \qquad \Lambda_{gauge} \xrightarrow{r\to\infty} \varepsilon \sim \frac{1}{z-z'} \qquad J=+1$$

Superphaserotation

$$h^G_{1,J;\mu\nu} = \nabla_{(\mu}\xi^1_{\nu)} = \nabla_\mu \nabla_\nu \Lambda_{gravity} \qquad \frac{1}{r}\Lambda_{gravity} \xrightarrow{r\to\infty} f \sim \frac{\bar{z}-\bar{z}'}{z-z'} \qquad J=+2$$

BMS ST

$$\int_0^\infty \frac{d\omega}{\omega}\,\omega^{\Delta+1}$$

$$h^G_{\Delta=0,J;\mu\nu} = \nabla_{(\mu}\xi^0_{\nu)}$$

$$\tilde{h}^G_{\Delta=2,J;\mu\nu} = \nabla_{(\mu}\xi^2_{\nu)}$$

$$Y^z \sim \left(\frac{\bar{z}-\bar{z}'}{z-z'}\right. \qquad Y^z \sim \frac{1}{z-z'}$$

$$\frac{1}{(-q\cdot X_\pm)^\Delta}$$

$$\ldots_{;\mu} = m_\mu \varphi_\Delta \qquad (\Delta\,J)=(0,1)$$

$$\ldots_{;\mu\nu} = m_\mu m_\nu \varphi_\Delta$$

…ators

$$\ldots_{,J}(X; z,\bar{z})^*)$$

$$\int d^3X\, \phi \overleftrightarrow{\partial}_{X^0} \phi'$$

$$\Omega(\phi, \phi')$$

$$h^G_{\Delta=0,J;\mu\nu} = \nabla_{(\mu}\xi^0_{\nu)} \qquad Y^z \sim \frac{(\bar z-\bar z')^2}{z-z'} \quad J=-2$$

$$\tilde h^G_{\Delta=2,J;\mu\nu} = \nabla_{(\mu}\xi^2_{\nu)} \qquad Y^z \sim \frac{1}{z-z'} \quad J=+2$$

Diff(S^2)

BMS SR

Vir SR

$$\int_0^\infty \frac{d\omega}{\omega}\,\omega^{\Delta+1}$$

e^{∂_Δ}

$\varphi_{\Delta+1}$

± 1

$$\boxed{\text{Special CPW are Memory modes}}$$

$$\lim_{\Delta\to 1}\partial_\Delta\left(A^\pm_{\Delta,J,\mu} + \tilde A^\pm_{2-\Delta,J,\mu}\right) = A^{\log\pm}_{\Delta=1,J,\mu}$$

$$\frac{1}{2\pi i}\left(A^{\log,+}_{\Delta=1,J,\mu} - A^{\log,-}_{\Delta=1,J,\mu}\right) = A^M_{\Delta=1,J,\mu} = A^G_{\Delta=1,J,\mu}\left[(q\cdot X)\underbrace{\log(X^2)\,\delta(q\cdot X)}_{\text{shockwave}} + \mathcal{O}(X^2)\right]$$

$$\frac{\Lambda}{z-z'} \quad J=+1$$

$\Lambda_{gravity}$

$$\frac{1}{r}\Lambda_{gravity} \xrightarrow{r\to\infty} f \sim \frac{\bar z-\bar z'}{z-z'} \quad J=+2$$

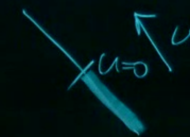
BMS ST

$$= \nabla_{(\mu}\xi^{0}_{\nu)} \qquad Y^{z} \sim \frac{(\bar z-\bar z')^{2}}{z-z'} \qquad J=-2 \quad \xleftarrow{} \ \mathrm{Diff}(S^2)$$

$$= \nabla_{(\mu}\xi^{2}_{\nu)} \qquad Y^{z} \sim \frac{1}{z-z'} \qquad J=+2 \quad \xleftarrow{} \ \text{Vir SR}$$

BMS SR

$$\boxed{\text{Special CPW are Memory modes}}$$



$$\partial_{\Delta}\left(A^{\pm}_{\Delta,J,\mu} + \tilde{A}^{\pm}_{2-\Delta,J,\mu}\right) = A^{\log\pm}_{\Delta=1,J,\mu}$$

$$\frac{1}{2\pi i}\left(A^{\log,+}_{\Delta=1,J,\mu} - A^{\log,-}_{\Delta=1,J,\mu}\right) = A^{M}_{\Delta=1,J,\mu} = A^{G}_{\Delta=1,J,\mu}\left[(q\cdot X)\underbrace{\log(X^{2})\,\delta(q\cdot X)}_{\text{shockwave}} + \underbrace{\theta(X^{2})}_{\text{shift}}\right]$$

$$i\left(A^{G}_{\Delta=1,J}(z),\ A^{M}_{2-\Delta,J}(z')\right) = \#\,\delta^{(2)}(z-z')$$

$$\frac{\bar z-\bar z'}{z-z'} \qquad J=+2$$

BMS ST

CCFT symmetries

Conformal multiplets

↳ organize CCFT data in reps of $SL(2,\mathbb{C})$

$$(\Delta, J) = \tfrac{1}{2}(h+\bar{h},\ h-\bar{h})$$

- $|h,\bar{h}\rangle$ is primary if $L_1|h,\bar{h}\rangle = 0 = \bar{L}_1|h,\bar{h}\rangle$

  construct descendants: $\underbrace{(L_{-1})}_{\partial \equiv \partial_z}{}^k$ or $\underbrace{(\bar{L}_{-1})}_{\bar{\partial} \equiv \partial_{\bar{z}}}{}^{\bar{k}}$

- $|h',\bar{h}'\rangle \equiv (L_{-1})^k |h,\bar{h}\rangle$ is primary descendant

  if $L_1 (L_{-1})^k |h,\bar{h}\rangle = 0 \iff h = \frac{1-k}{2}$, $k \in \mathbb{Z}_{>0}$

  $h' = \frac{1+k}{2} = 1-h$

CCFT symmetries

multiplets

CCFT data in reps of $SL(2,\mathbb{C})$

$= \frac{1}{2}(h+\bar{h},\, h-\bar{h})$

is primary if $L_1|h,\bar{h}\rangle = 0 = \bar{L}_1|h,\bar{h}\rangle$

ruct descendants: $\underbrace{(L_{-1})^k}_{\partial \equiv \partial_z}$ or $\underbrace{(\bar{L}_{-1})^{\bar{k}}}_{\bar{\partial} \equiv \partial_{\bar{z}}}$

$\equiv (L_{-1})^k |h,\bar{h}\rangle$ is primary descendant

$$L_1 (L_{-1})^k |h,\bar{h}\rangle = 0 \iff \begin{cases} h = \frac{1-k}{2} & k \in \mathbb{Z}_{>0} \\ h' = \frac{1+k}{2} = 1-h \end{cases}$$

$$\begin{tikzcd} & \mathcal{O}_{h,\bar{h}} \arrow[dl, "\bar{\partial}^k"'] \arrow[dr, "\partial^k"] & \\ \bar{\partial}^{\bar{k}} \mathcal{O}_{h,\bar{h}} \arrow[dr] & & \partial^k \mathcal{O}_{h,\bar{h}} \arrow[dl] \\ & \partial^k \bar{\partial}^{\bar{k}} \mathcal{O}_{h,\bar{h}} = 0 & \end{tikzcd}$$

conservation equation

$\downarrow$

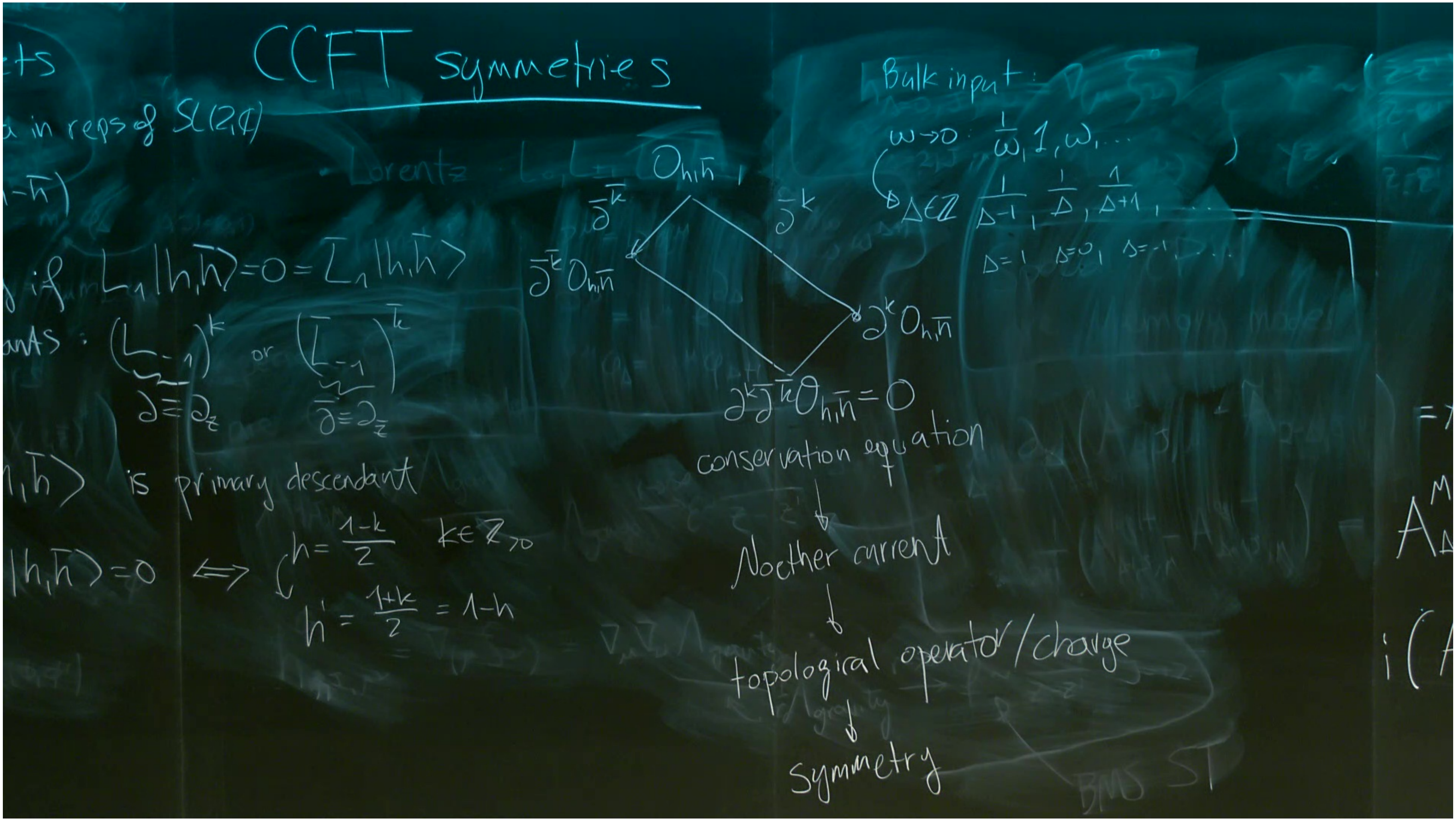
Noether current

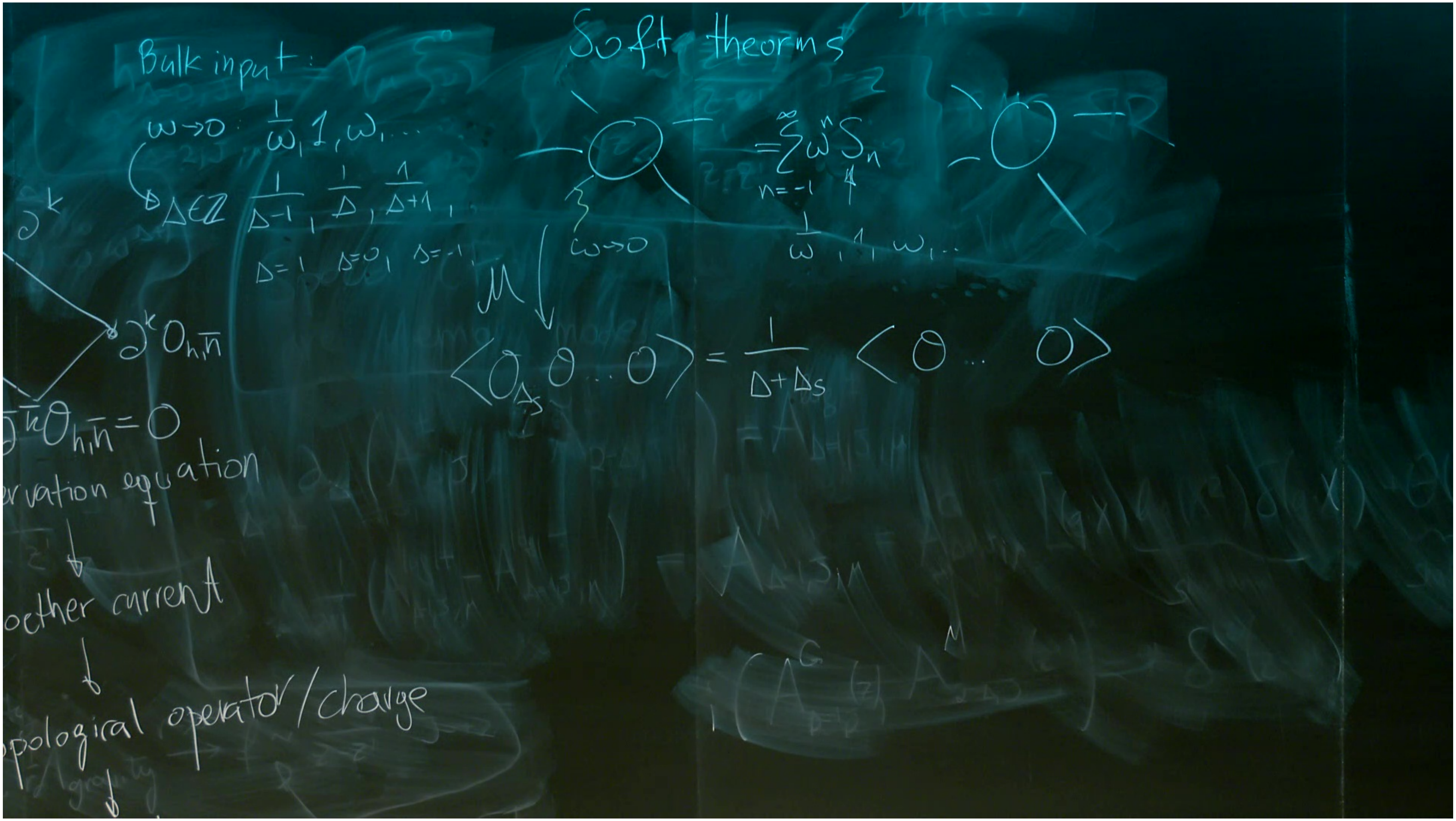
$\downarrow$

topological operator/charge

$\downarrow$

symmetry


CCFT symmetries
Bulk input:
$\omega \to 0: \frac{1}{\omega}, 1, \omega, \ldots$
$\Delta \in \mathbb{Z}: \frac{1}{\Delta-1}, \frac{1}{\Delta}, \frac{1}{\Delta+1}, \ldots$
$\Delta = 1, \Delta = 0, \Delta = -1, \ldots$
in reps of $SL(2,\mathbb{C})$
Lorentz
$\mathcal{O}_{h,\bar h}$
$\bar\partial^{\bar k}$
∂^{k}
$\bar\partial^{\bar k}\mathcal{O}_{h,\bar h}$
$\partial^{k}\mathcal{O}_{h,\bar h}$
$\partial^k\bar\partial^{\bar k}\mathcal{O}_{h,\bar h} = 0$
conservation equation
Noether current
topological operator/charge
symmetry
if $L_1|h,\bar h\rangle = 0 = \bar L_1|h,\bar h\rangle$
$(L_{-1})^k$ or $(\bar L_{-1})^{\bar k}$
$\partial \equiv \partial_z$
$\bar\partial \equiv \partial_{\bar z}$
$|h,\bar h\rangle$ is primary descendant
$|h,\bar h\rangle = 0 \iff h = \frac{1-k}{2}$, $k \in \mathbb{Z}_{>0}$
$h' = \frac{1+k}{2} = 1-h$
BMS ST


Bulk input:
ω→0: 1/ω, 1, ω, ...
Δ∈Z 1/(Δ-1), 1/Δ, 1/(Δ+1), ...
Δ=1, Δ=0, Δ=-1
Soft theorms
= Σ_{n=-1}^∞ ω^n S_n
1/ω, 1, ω, ...
ω→0
M
⟨O_{Δs} O ... O⟩ = 1/(Δ+Δs) ⟨O ... O⟩
∂^k O_{h,h̄}
∂̄^k O_{h,h̄} = 0
ervation equation
oether current
pological operator/charge

$$\lim_{\Delta\to 1}(\Delta-1)\,\mathcal{M}_{n+1}(\Delta, J=+1; \dots) = \sum_i \frac{Q_i}{z-z_i}\,\mathcal{M}_n(\dots)$$

$$\langle J \dots \mathcal{O}_{\Delta_i, J_i} \dots \rangle = \sum_i \frac{Q_i}{z-z_i}\langle \dots \mathcal{O}_{\Delta_i J_i} \dots \rangle$$

$(\Delta, J) = (1,1)$ $(h, \bar{h}) = (1,0)$

J is conserved, $\bar\partial J = 0$

$$\begin{tikzcd}[column sep=small] & \phi \arrow[dl] \arrow[dr] & \\ \bar\partial\phi = \bar{J} \arrow[dr] & & J = \partial\phi \arrow[dl] \\ & \partial\bar\partial\phi = 0 & \end{tikzcd}$$

Noether: $\mathcal{J} = \epsilon J$

$\bar\partial \mathcal{J} = 0 \to \bar\partial \epsilon = 0$ $\epsilon = \epsilon(z) = \sum_{n\in\mathbb{Z}} \epsilon_n z^n$

$$Q_\Sigma = \oint_\Sigma dz\, \mathcal{J}(z) = \oint dz\, \epsilon J(z)$$

$\epsilon = 1$ global charge

$\epsilon(z)$ superphaserotation

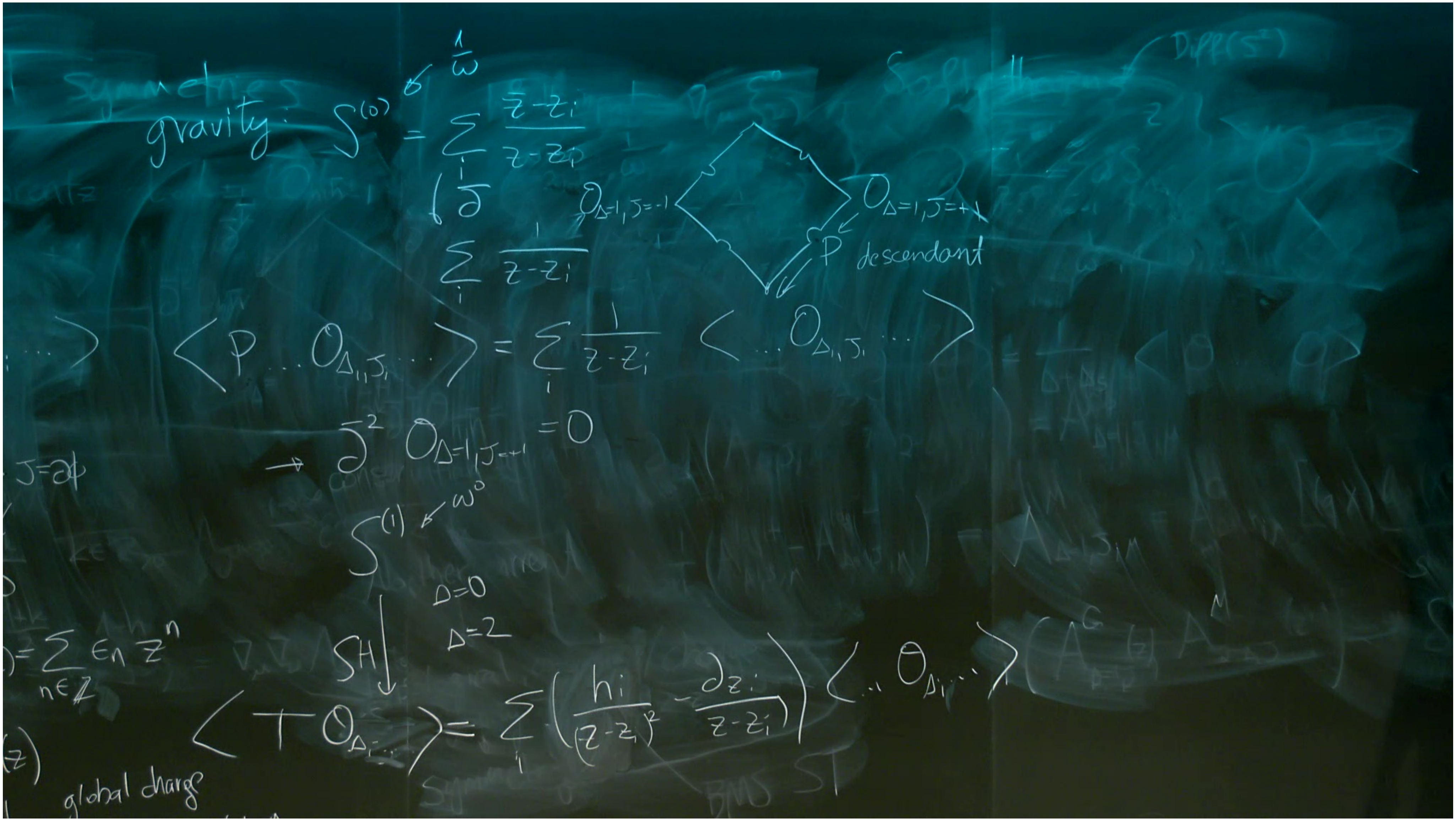
Noether curr

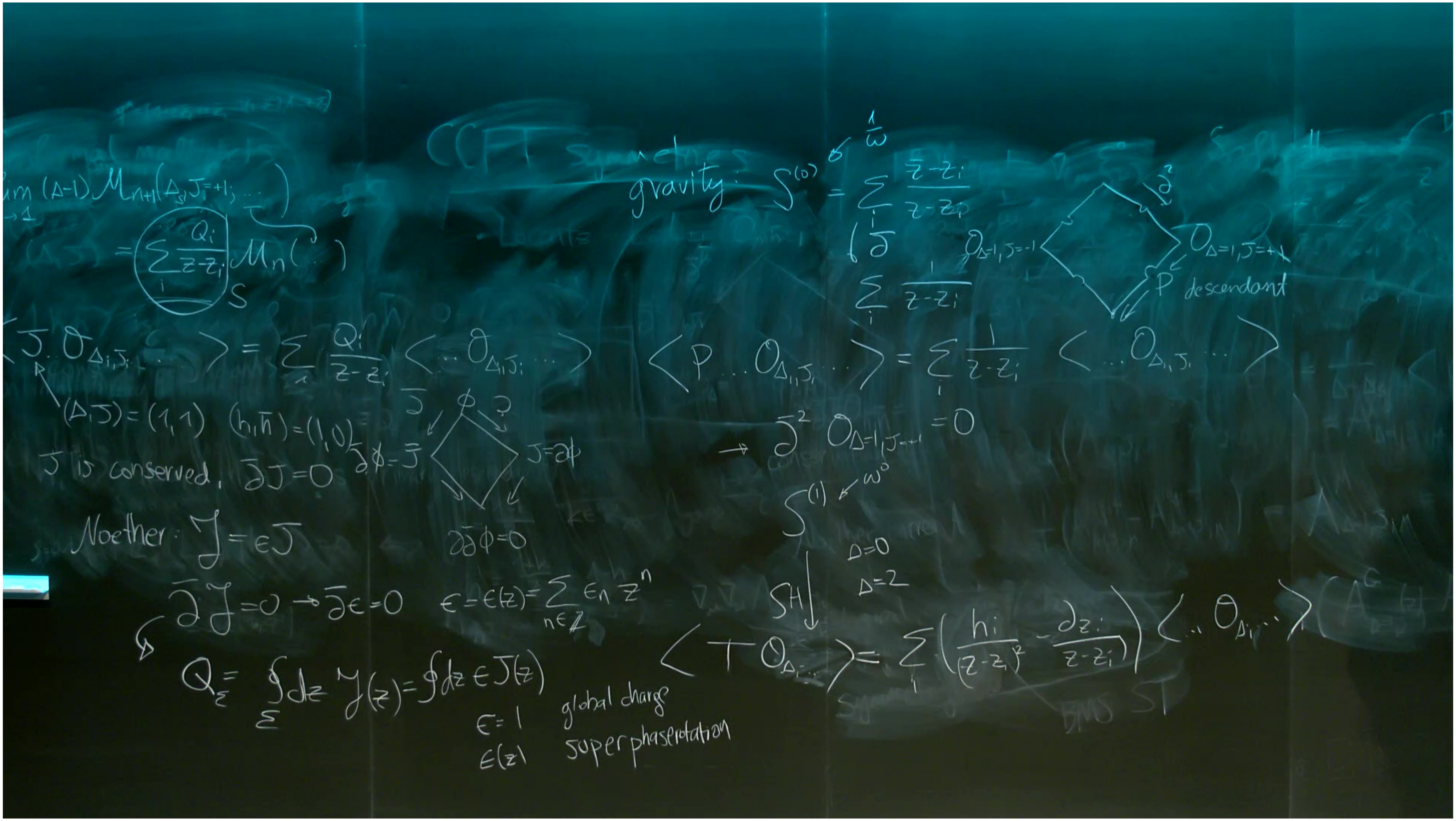
$\downarrow$

topological

$\downarrow$

symmet


gravity: $S^{(0)} = \sum_i \frac{\bar{z}-\bar{z}_i}{z-z_i}$
$\frac{1}{\omega}$
$\sum_i \frac{1}{z-z_i}$
$\mathcal{O}_{\Delta=1,J=-1}$
$\mathcal{O}_{\Delta=1,J=+1}$
P descendant
$\langle P \ldots \mathcal{O}_{\Delta_i,J_i} \ldots \rangle = \sum_i \frac{1}{z-z_i} \langle \ldots \mathcal{O}_{\Delta_i,J_i} \ldots \rangle$
$\bar{\partial}^2 \mathcal{O}_{\Delta=1,J=+1} = 0$
$J = \partial\phi$
$S^{(1)}$ ω^0
$\Delta=0$
$\Delta=2$
SH
$\langle T \mathcal{O}_{\Delta_i} \ldots \rangle = \sum_i \left(\frac{h_i}{(z-z_i)^2} - \frac{\partial z_i}{z-z_i} \right) \langle \ldots \mathcal{O}_{\Delta_i} \ldots \rangle$
$= \sum_{n \in \mathbb{Z}} \epsilon_n z^n$
global charge
Diff(S^2)


$\lim_{\Delta\to 1} (\Delta-1)\mathcal{M}_{n+1}(\Delta, J=+1; \dots) = \left(\sum_i \frac{Q_i}{z-z_i}\right)\mathcal{M}_n(\dots)$
S
$\langle J \dots O_{\Delta_i,J_i} \dots \rangle = \sum_i \frac{Q_i}{z-z_i} \langle \dots O_{\Delta_i,J_i} \dots \rangle$
$(\Delta, J) = (1,1) \quad (h,\bar h) = (1,0)$
J is conserved, $\bar\partial J = 0$
$\partial\phi = J \quad J = \partial\phi \quad \partial\bar\partial\phi = 0$
Noether: $\mathcal{J} = \epsilon J$
$\bar\partial \mathcal{J} = 0 \to \bar\partial\epsilon = 0 \quad \epsilon = \epsilon(z) = \sum_{n\in\mathbb{Z}} \epsilon_n z^n$
$Q_\varepsilon = \oint_\Sigma dz\, \mathcal{J}(z) = \oint dz\, \epsilon J(z)$
$\epsilon = 1$ global charge
$\epsilon(z)$ superphaserotation
gravity: $S^{(0)} = \sum_i \frac{\bar z - \bar z_i}{z - z_i}$
$\frac{1}{\omega}$
$\bar\partial$
$\sum_i \frac{1}{z-z_i}$
$O_{\Delta=1,J=-1}$
$O_{\Delta=1,J=+1}$
$\bar\partial^2$
P descendant
$\langle P \dots O_{\Delta_i,J_i} \dots \rangle = \sum_i \frac{1}{z-z_i} \langle \dots O_{\Delta_i,J_i} \dots \rangle$
$\to \bar\partial^2 O_{\Delta=1,J=-1} = 0$
$S^{(1)} \swarrow \omega^0$
$\Delta=0$
$\Delta=2$
SH
$\langle T\, O_{\Delta_i} \dots \rangle = \sum_i \left(\frac{h_i}{(z-z_i)^2} - \frac{\partial_{z_i}}{z-z_i} \right) \langle \dots O_{\Delta_i} \dots \rangle$