Title: Lecture - Celestial Holography IIa

Speakers:

Collection: Celestial Holography Summer School 2024

Date: July 23, 2024 - 9:00 AM

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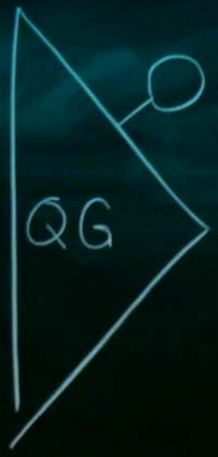
Celestial Holography



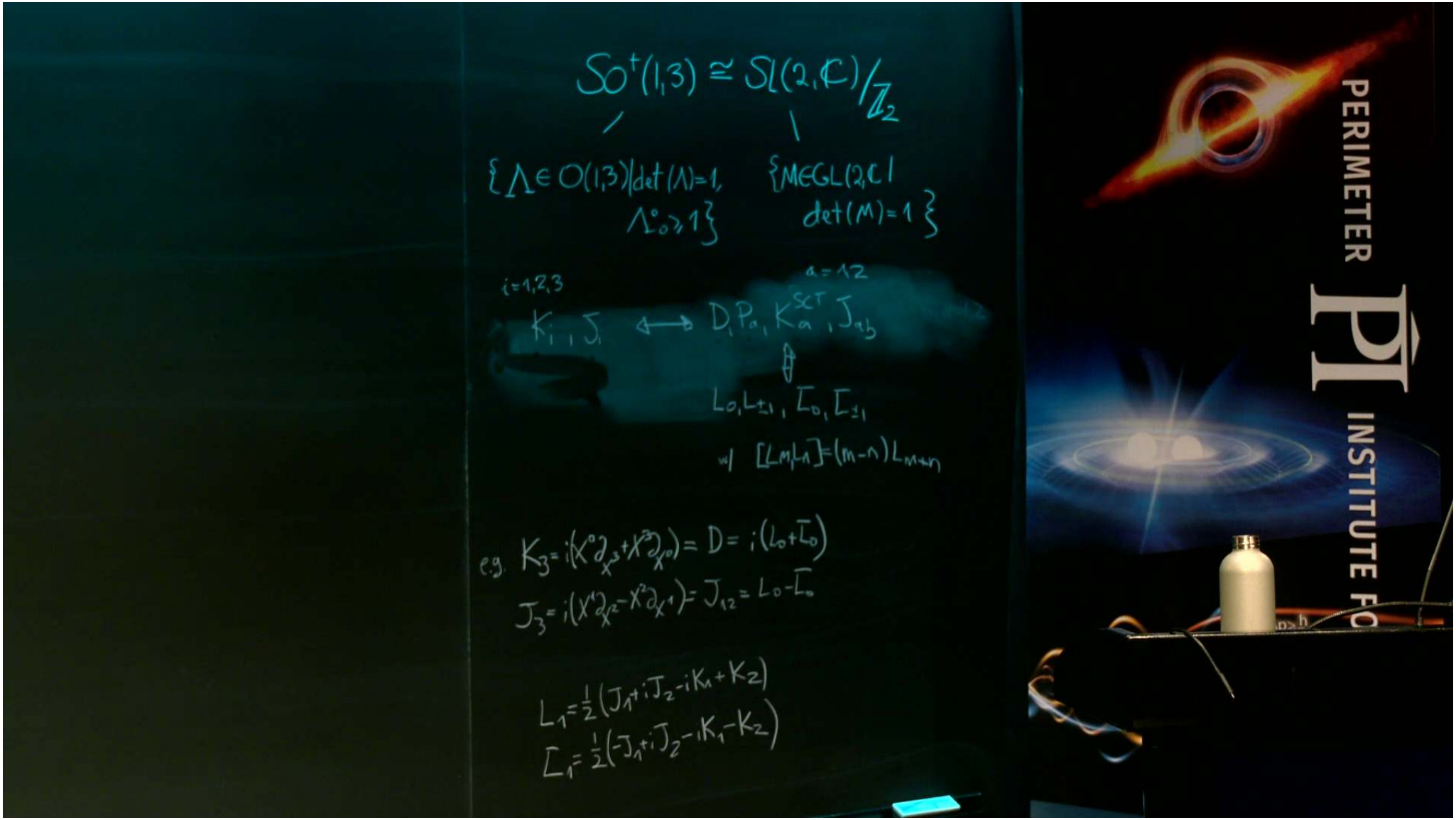
QG in 4D ASF = "Celestial" CFT on S^2

2 basic hints

1) Symmetry:

Lorentz group in $D = d+2$ dim $\cong$ Euclidean conf group in d dim

D: 4 ; d: 2


$SO^+(1,3) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$
$\{\Lambda \in O(1,3) | \det(\Lambda)=1, \Lambda^0_0 \geq 1\}$
$\{M \in GL(2,\mathbb{C}) | \det(M)=1\}$
$i=1,2,3$
$a=1,2$
$K_i, J_i \longleftrightarrow D, P_a, K^{SCT}_a, J_{ab}$
$L_0, L_{\pm 1}, \bar{L}_0, \bar{L}_{\pm 1}$
w/ $[L_m, L_n] = (m-n)L_{m+n}$
e.g. $K_3 = i(x^0\partial_{x^3} + x^3\partial_{x^0}) = D = i(L_0 + \bar{L}_0)$
$J_3 = i(x^1\partial_{x^2} - x^2\partial_{x^1}) = J_{12} = L_0 - \bar{L}_0$
$L_1 = \frac{1}{2}(J_1 + iJ_2 - iK_1 + K_2)$
$\bar{L}_1 = \frac{1}{2}(-J_1 + iJ_2 - iK_1 - K_2)$
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2) Observables: S-matrix

$|p_i\rangle = |\omega_i, x^i\rangle$

energy-momentum basis

standard amplitudes

$\langle p_n^{out} | S | p_n^{in} \rangle$

translation symmetry

Mellin

$\int_0^\infty d\omega\, \omega^{\Delta-1}$

$|\Delta_i, x^i\rangle$

boost weight basis

celestial amplitudes

$\langle \mathcal{O}^-_{\Delta_1}(x_1) \dots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$

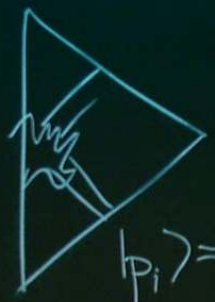
ean conf

d dim

↑

2

2) Observables: S-matrix



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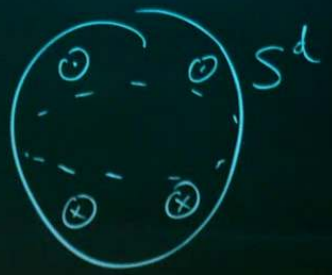


$|\Delta_i, x^i\rangle$

boost weight basis

celestial amplitudes

$\langle \mathcal{O}^-_{\Delta_1}(x_1) \dots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$ *

* kinematic Dirac δ from $\delta^{(4)}(\sum_i p_i^n)$

$\{\Lambda \in O($

$i=1,2,3$

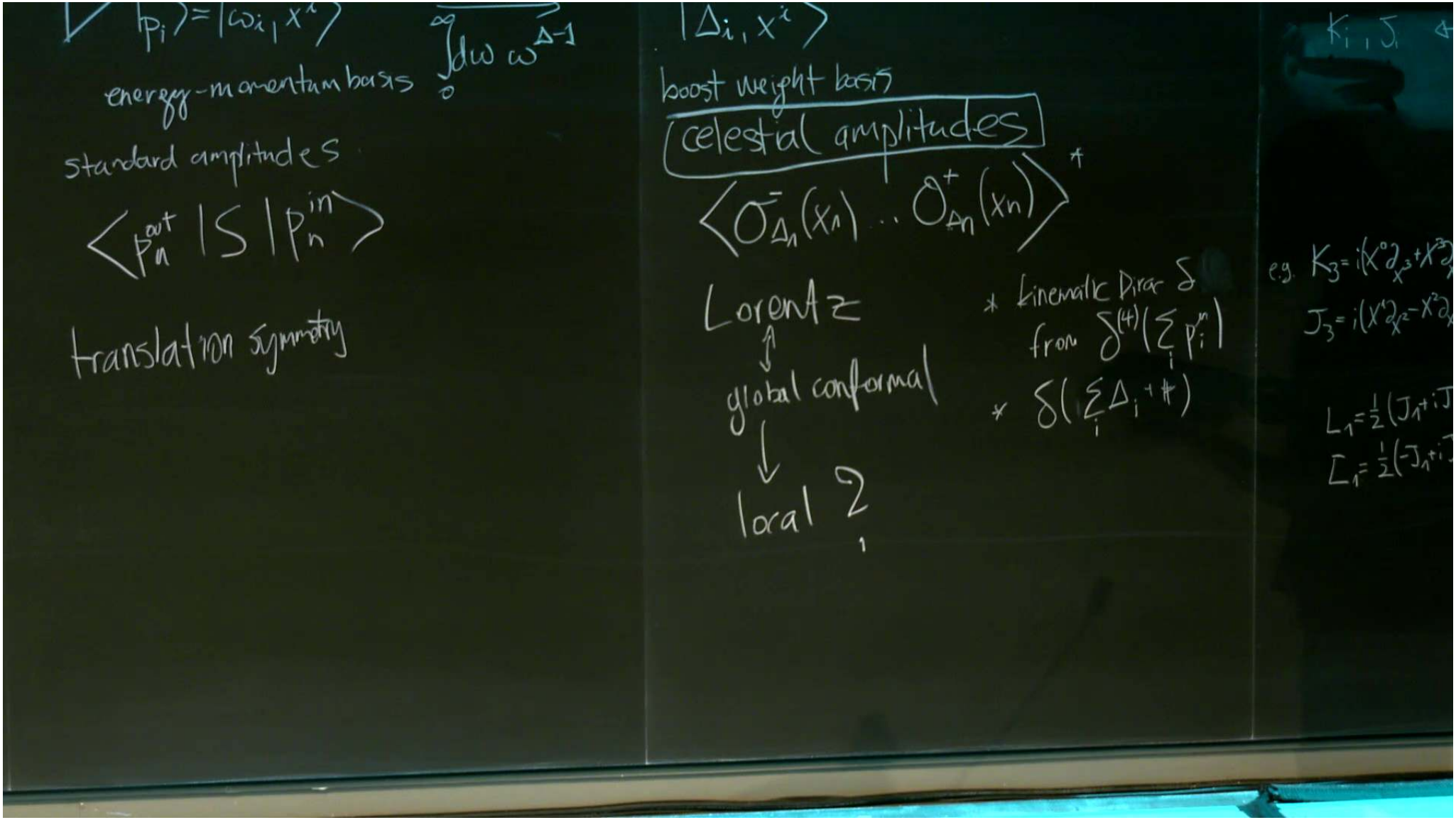
K_i, J

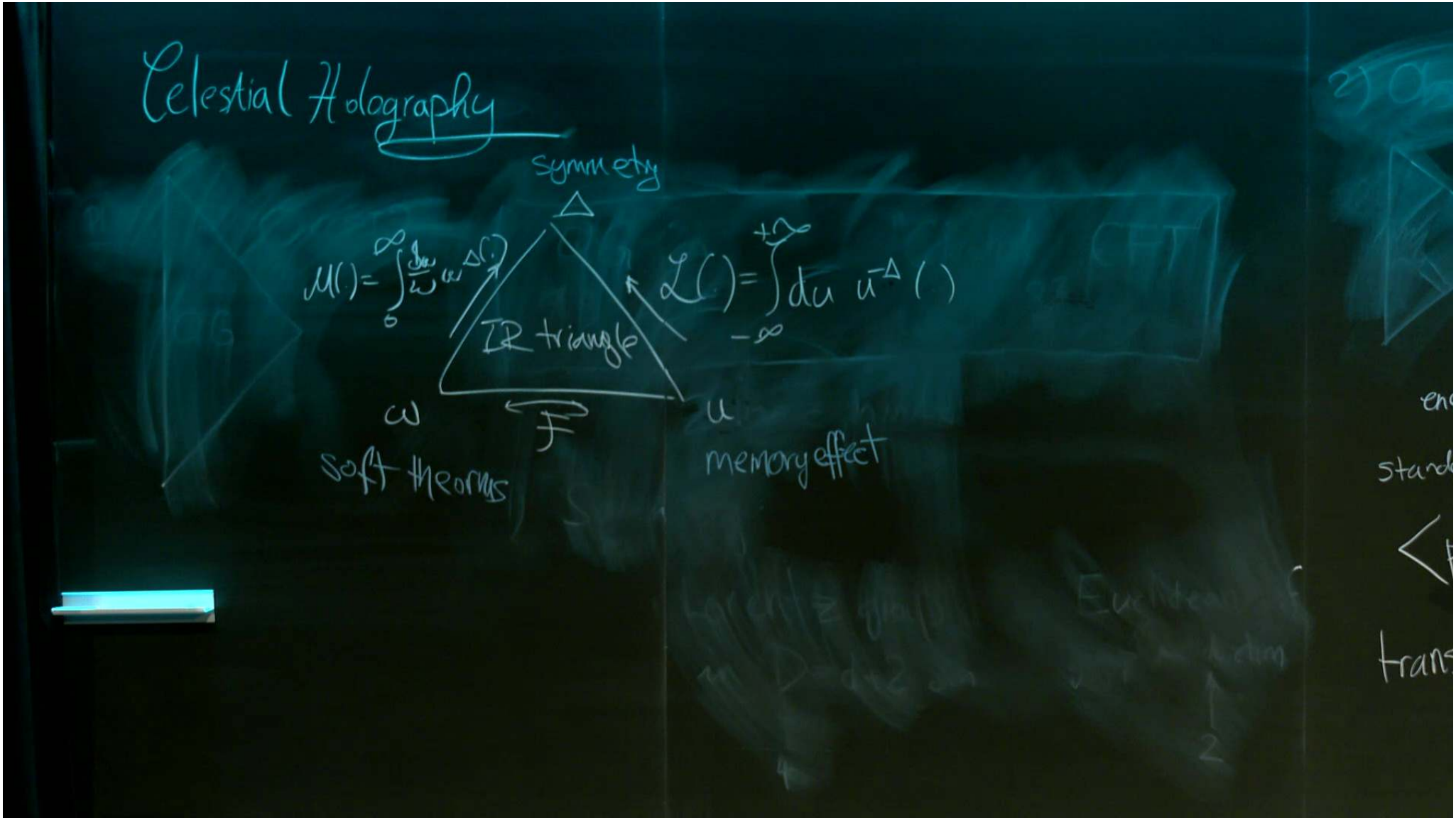
e.g. $K_3 = i(x^0$

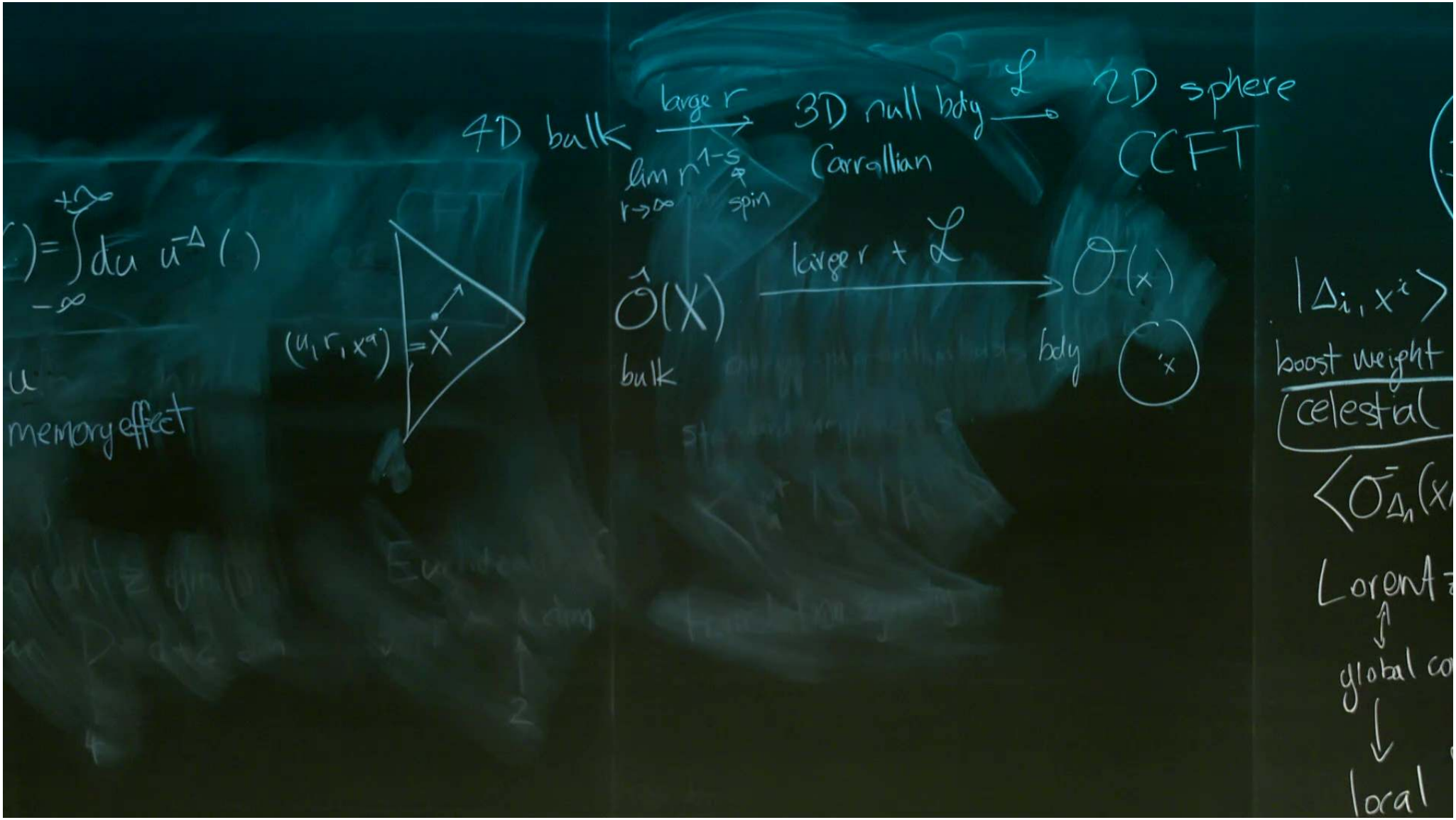
$J_3 = i(x^2$

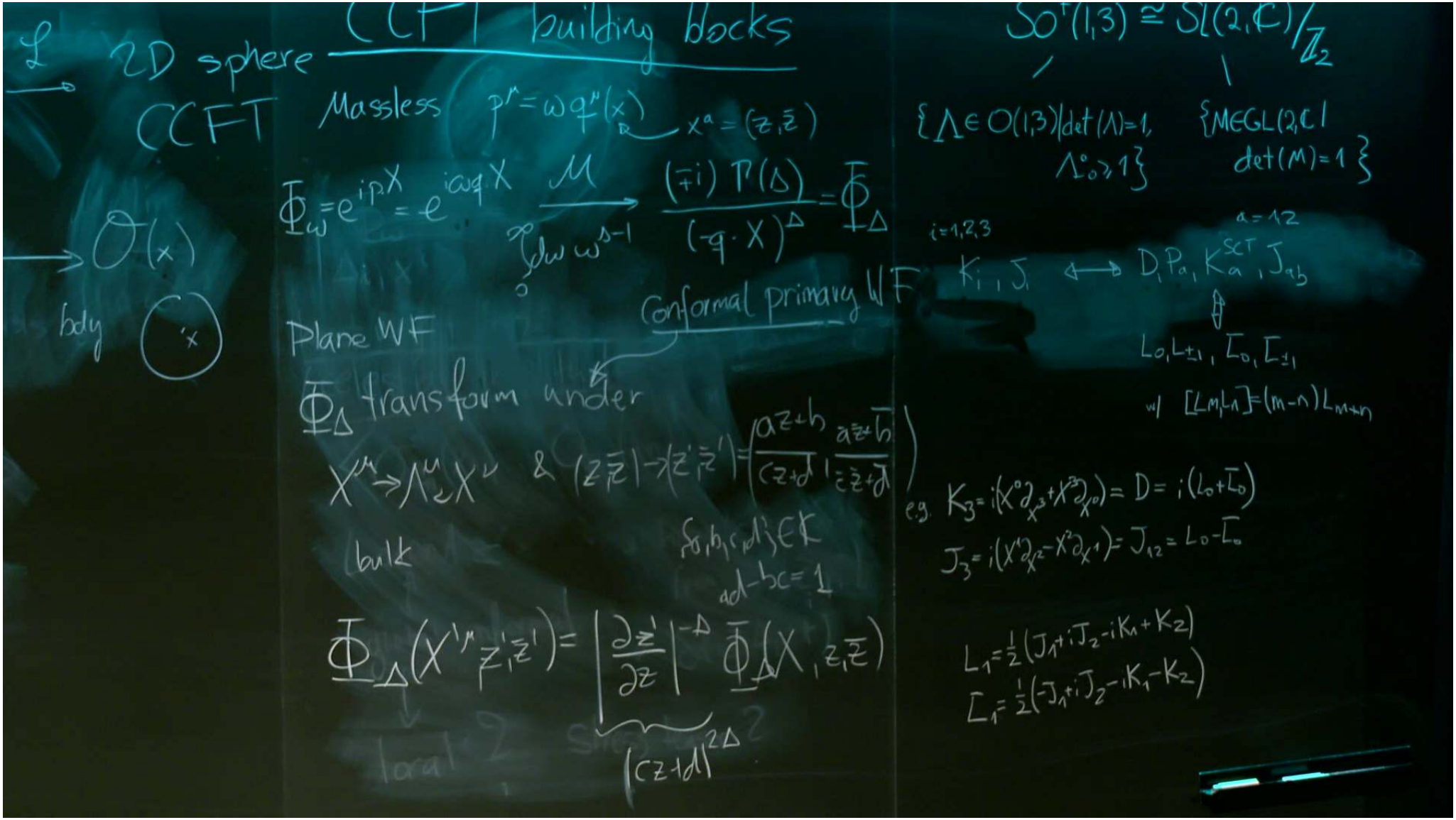
$L_1 = \frac{1}{2}($

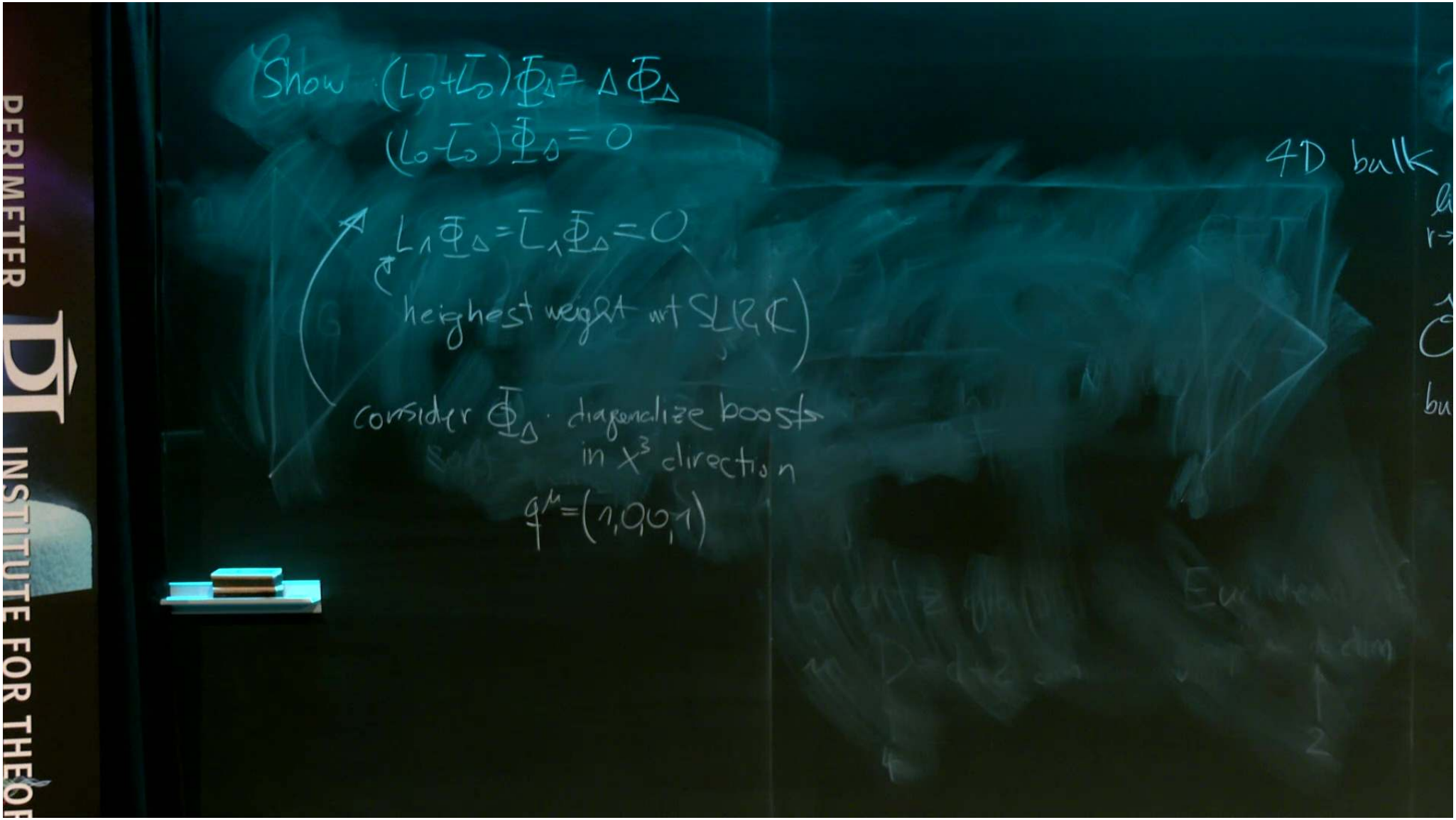
$\bar{L}_1 = \frac{1}{2}$


$|p_i\rangle = |\omega_i, x^i\rangle$
$\int_0^\infty d\omega\, \omega^{\Delta-1}$
energy-momentum basis
standard amplitudes
$\langle p_n^{out} | S | p_n^{in} \rangle$
translation symmetry
$|\Delta_i, x^i\rangle$
boost weight basis
celestial amplitudes
$\langle \mathcal{O}^-_{\Delta_1}(x_1) \dots \mathcal{O}^+_{\Delta_n}(x_n) \rangle$
Lorentz
global conformal
local ?
* kinematic Dirac δ from $\delta^{(4)}(\sum_i p_i^\mu)$
* $\delta(\sum_i \Delta_i + \#)$
K_i, J_i
e.g. $K_3 = i(x^0\partial_{x^3} + x^3\partial$
$J_3 = i(x^1\partial_{x^2} - x^2\partial$
$L_1 = \frac{1}{2}(J_1 + iJ$
$\bar{L}_1 = \frac{1}{2}(-J_1 + i$


Celestial Holography
symmetry
$\mathcal{M}(\omega) = \int_0^\infty \frac{du}{\omega} e^{i\omega u} \Delta(\omega)$
$\mathcal{L}(u) = \int_{-\infty}^{+\infty} du\, e^{-i\omega u} \Delta(\omega)$
IR triangle
ω
soft theorems
F
u
memory effect


4D bulk
larger r
3D null bdy
2D sphere
CCFT
Carrollian
spin
larger + $\mathcal{L}$
$\hat{O}(X)$
bulk
$O(x)$
bdy
memory effect
boost weight
celestial
Lorent
global co
local


CCFT building blocks
2D sphere
CCFT
Massless $p^\mu = \omega q^\mu(x)$
$x^a = (z, \bar{z})$
$\Phi_\omega = e^{ip\cdot X} = e^{i\omega q\cdot X}$
$\mathcal{M}$
$\int_0^\infty d\omega\, \omega^{s-1}$
$\frac{(\mp i)\,\Gamma(\Delta)}{(-q\cdot X)^\Delta} = \Phi_\Delta$
$\mathcal{O}(x)$
bdy
Plane WF
Conformal primary WF
Φ_Δ transform under
$X^\mu \to \Lambda^\mu{}_\nu X^\nu$ & $(z,\bar{z}) \to (z',\bar{z}') = \left(\frac{az+b}{cz+d}, \frac{a\bar{z}+\bar{b}}{c\bar{z}+\bar{d}}\right)$
bulk
$a,b,c,d \in \mathbb{C}$
$ad-bc=1$
$\Phi_\Delta(X', z', \bar{z}') = \left|\frac{\partial z'}{\partial z}\right|^{-\Delta} \Phi_\Delta(X, z, \bar{z})$
$|cz+d|^{2\Delta}$
$SO^+(1,3) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$
$\{\Lambda \in O(1,3) / \det(\Lambda)=1, \Lambda^0{}_0 \geq 1\}$
$\{M \in GL(2,\mathbb{C}) / \det(M)=1\}$
$i=1,2,3$
$a=1,2$
$K_i, J_i \longleftrightarrow D, P_a, K^{SCT}_a, J_{ab}$
$L_0, L_{\pm 1}, \bar{L}_0, \bar{L}_{\pm 1}$
w/ $[L_m, L_n] = (m-n)L_{m+n}$
e.g. $K_3 = i(x^0\partial_{x^3} + x^3\partial_{x^0}) = D = i(L_0+\bar{L}_0)$
$J_3 = i(x^1\partial_{x^2} - x^2\partial_{x^1}) = J_{12} = L_0 - \bar{L}_0$
$L_1 = \frac{1}{2}(J_1 + iJ_2 - iK_1 + K_2)$
$\bar{L}_1 = \frac{1}{2}(-J_1 + iJ_2 - iK_1 - K_2)$


Show $(L_0+\bar{L}_0)\Phi_\Delta = \Delta\Phi_\Delta$
$(L_0-\bar{L}_0)\Phi_\Delta = 0$
$L_1\Phi_\Delta = \bar{L}_1\Phi_\Delta = 0$
highest weight wrt SL(2,R)
consider Φ_Δ · diagonalize boosts in x^3 direction
$q^\mu = (1,0,0,1)$
4D bulk
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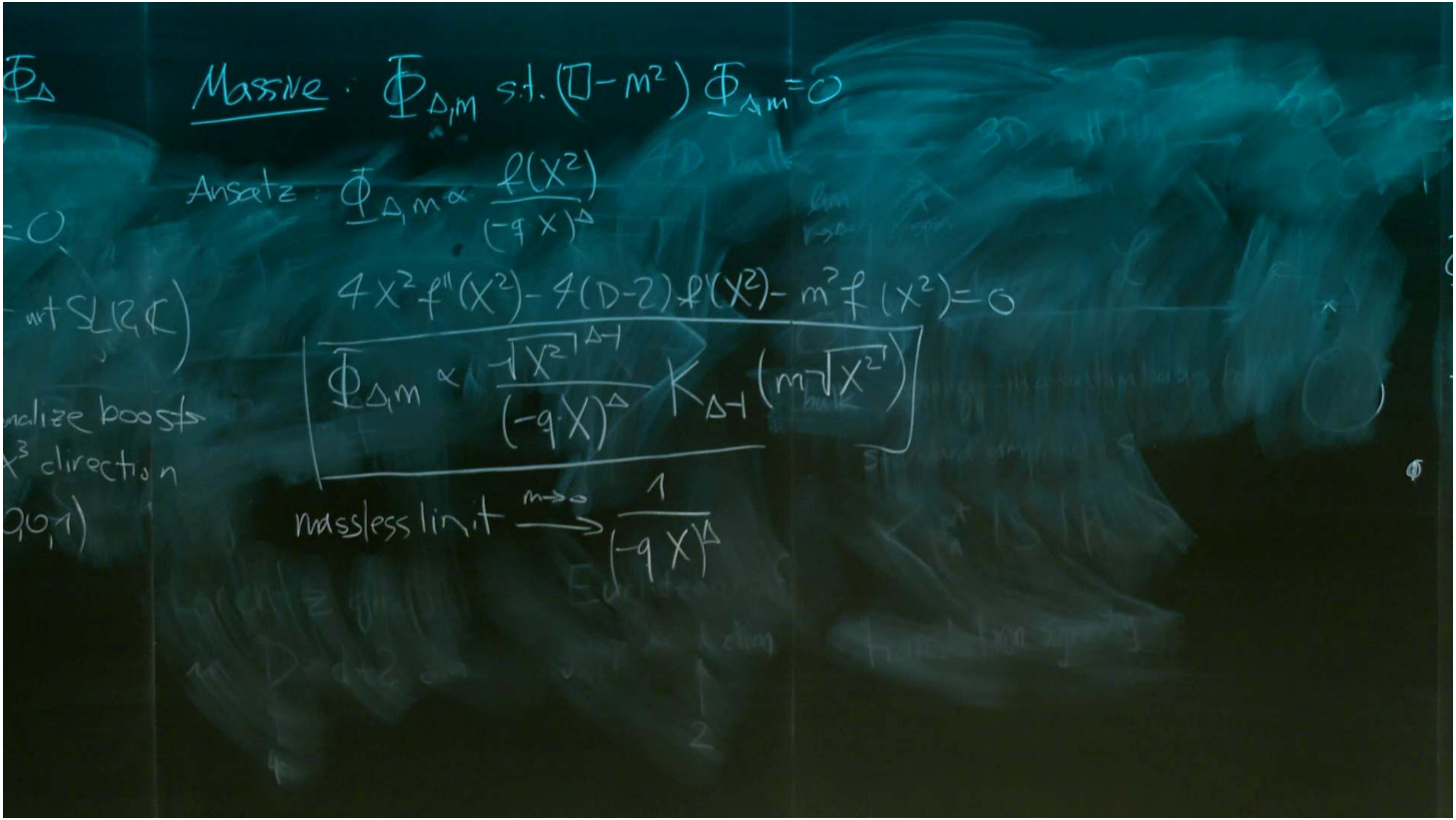
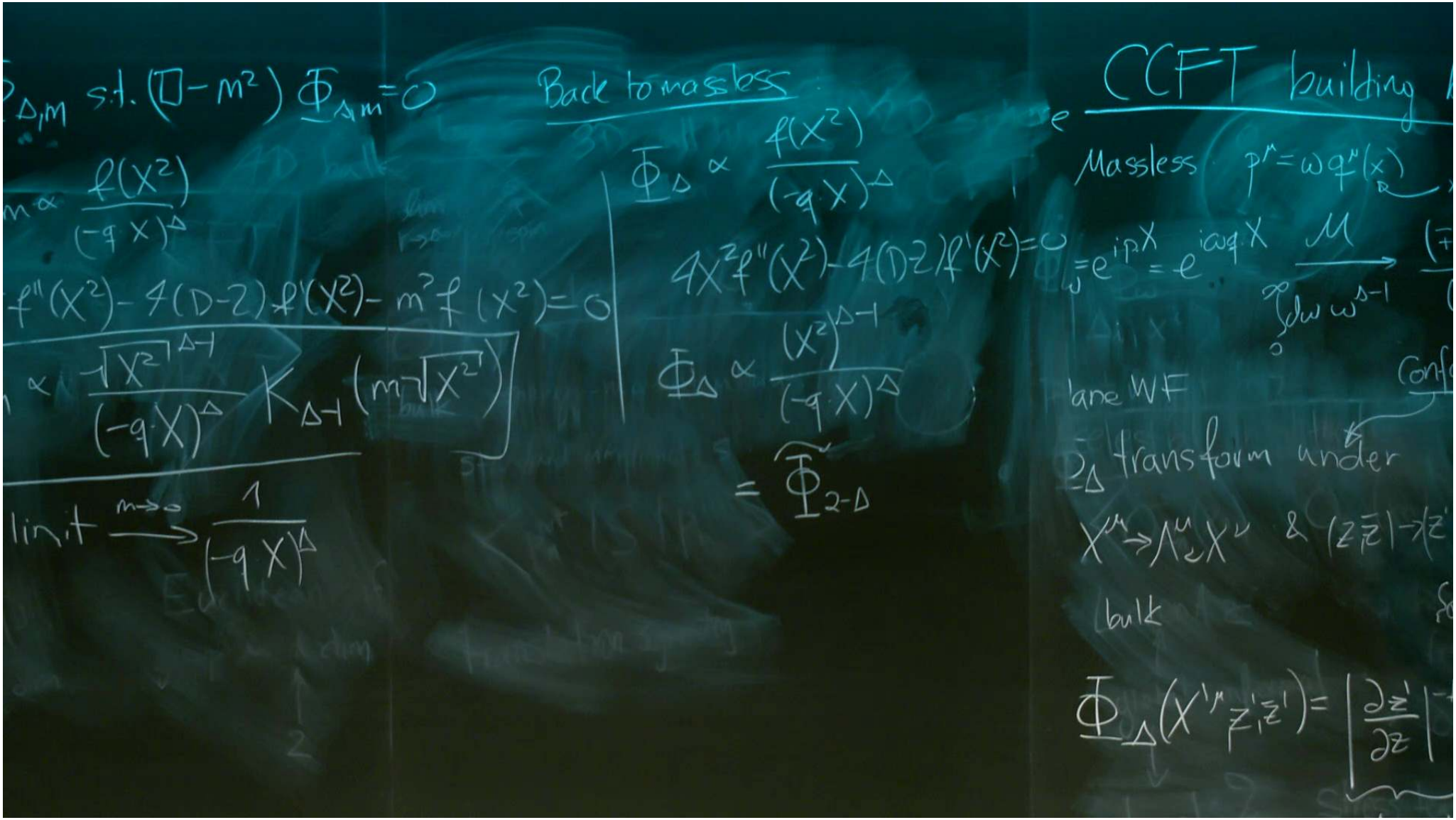
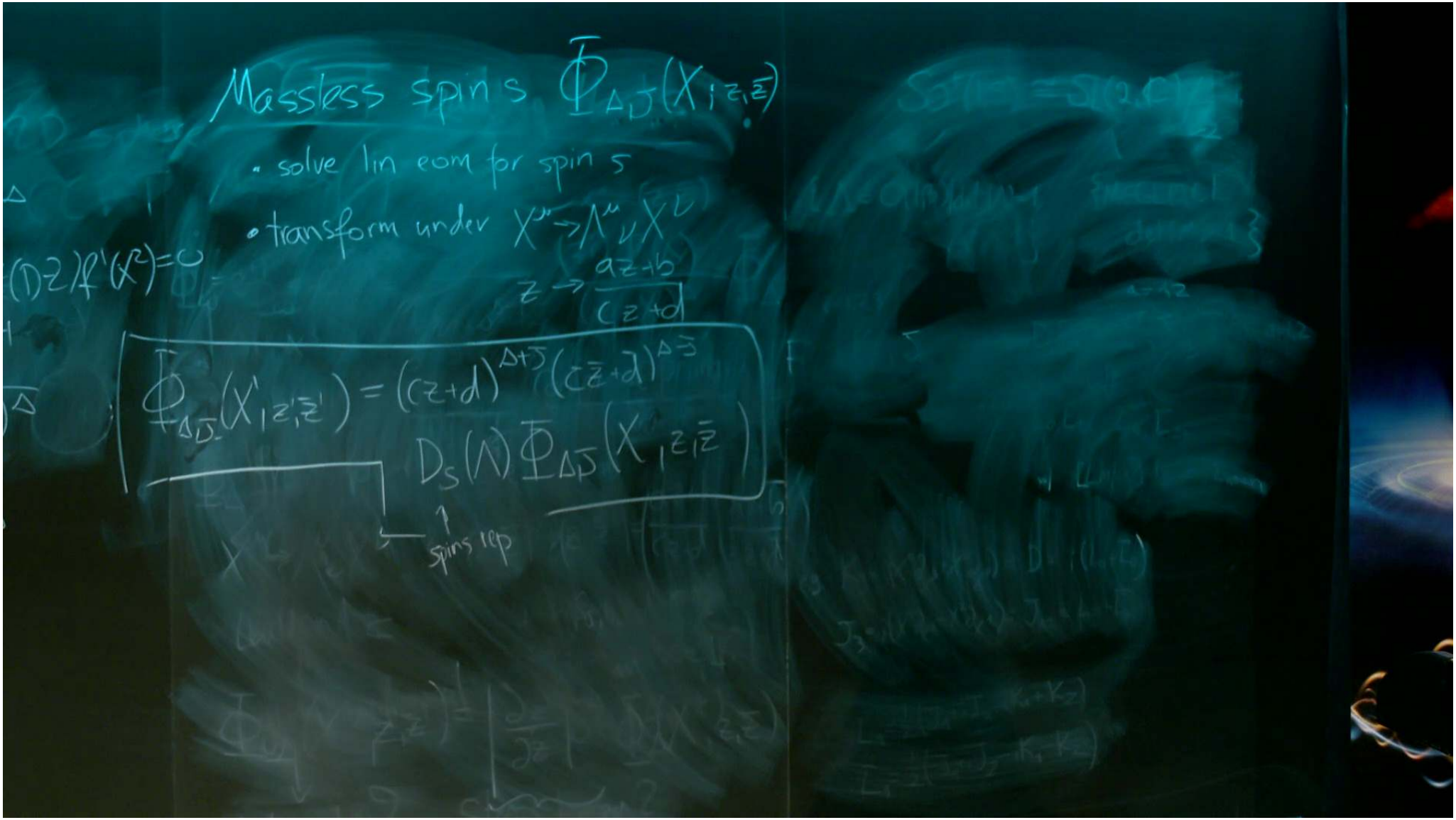
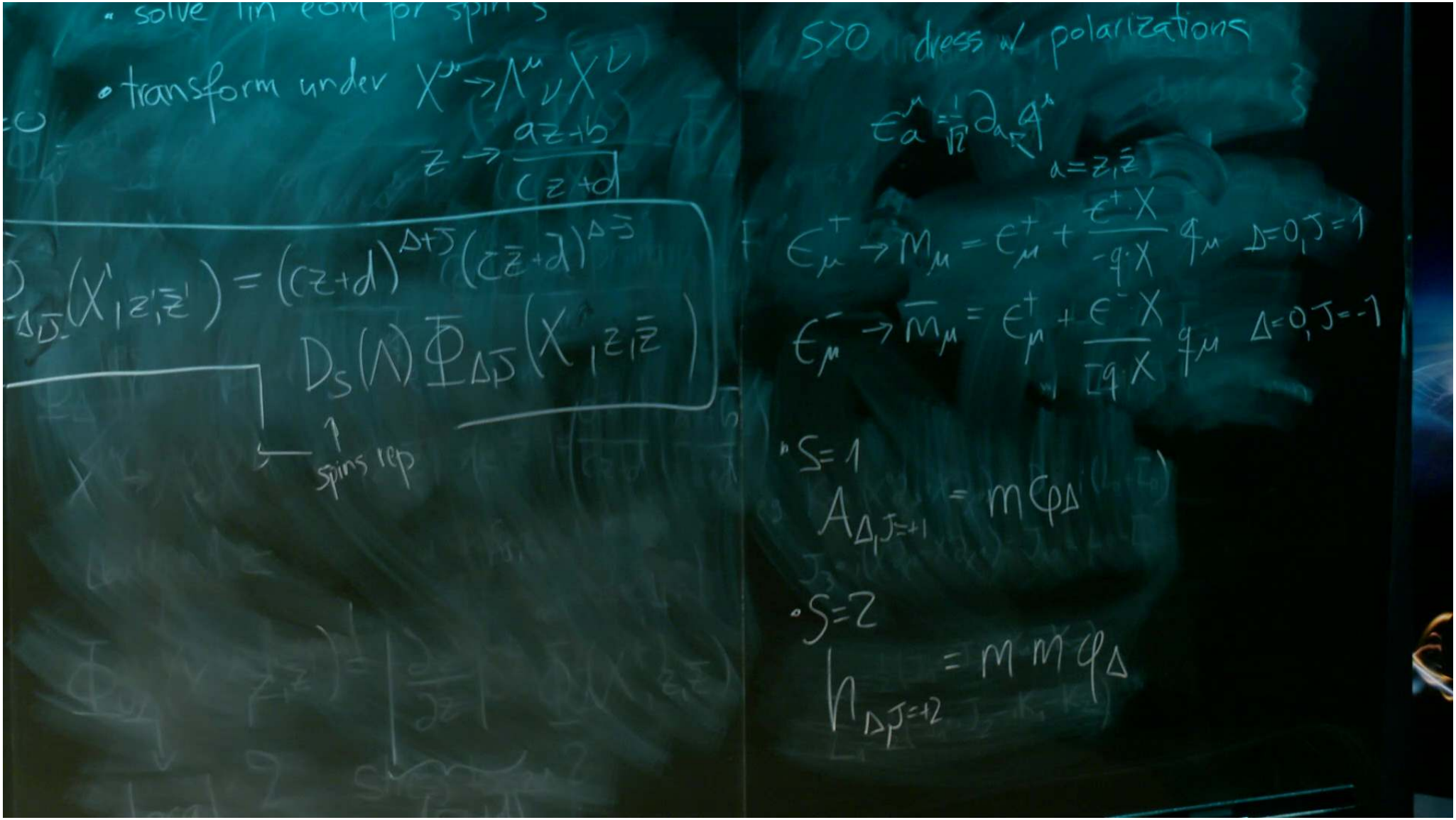
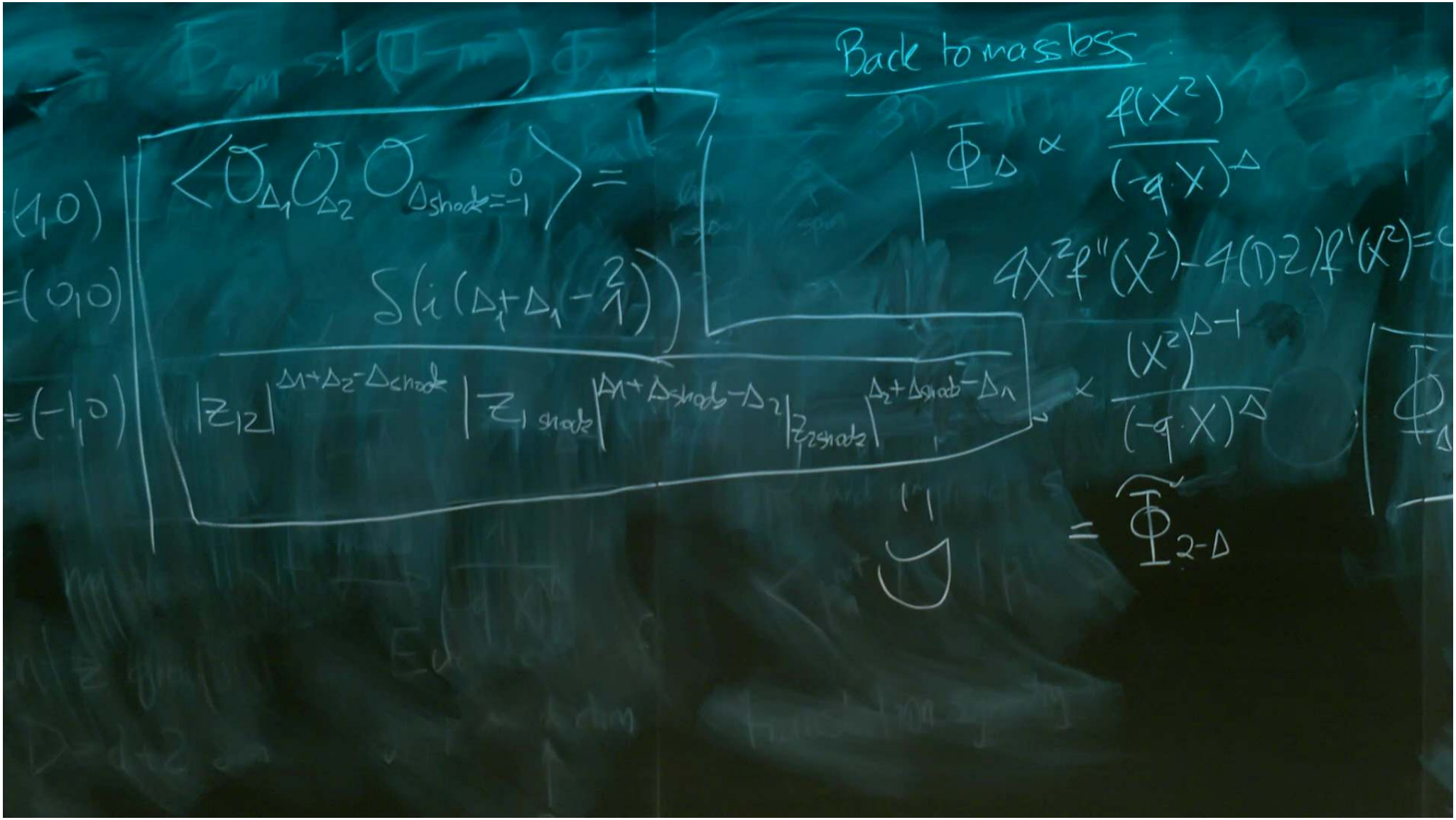
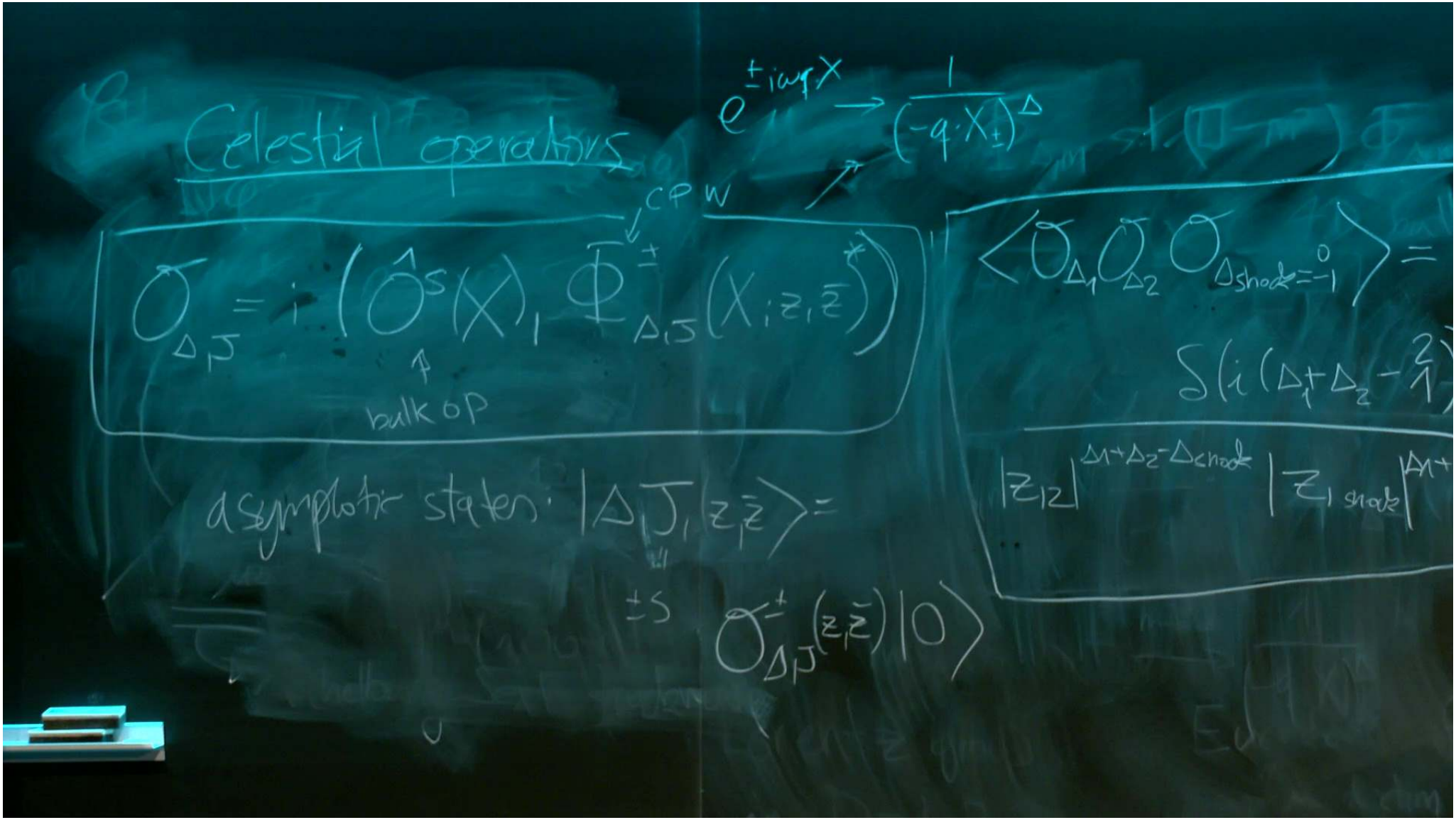

Massive: $\Phi_{\Delta,m}$ s.t. $(\Box - m^2)\Phi_{\Delta,m} = 0$
Ansatz: $\Phi_{\Delta,m} \propto \frac{f(X^2)}{(-q\cdot X)^\Delta}$
$4X^2 f''(X^2) - 4(D-2) f'(X^2) - m^2 f(X^2) = 0$
$\Phi_{\Delta,m} \propto \frac{\sqrt{X^2}^{\Delta-1}}{(-q\cdot X)^\Delta} K_{\Delta-1}(m\sqrt{X^2})$
massless limit $\xrightarrow{m\to 0}$ $\frac{1}{(-q\cdot X)^\Delta}$

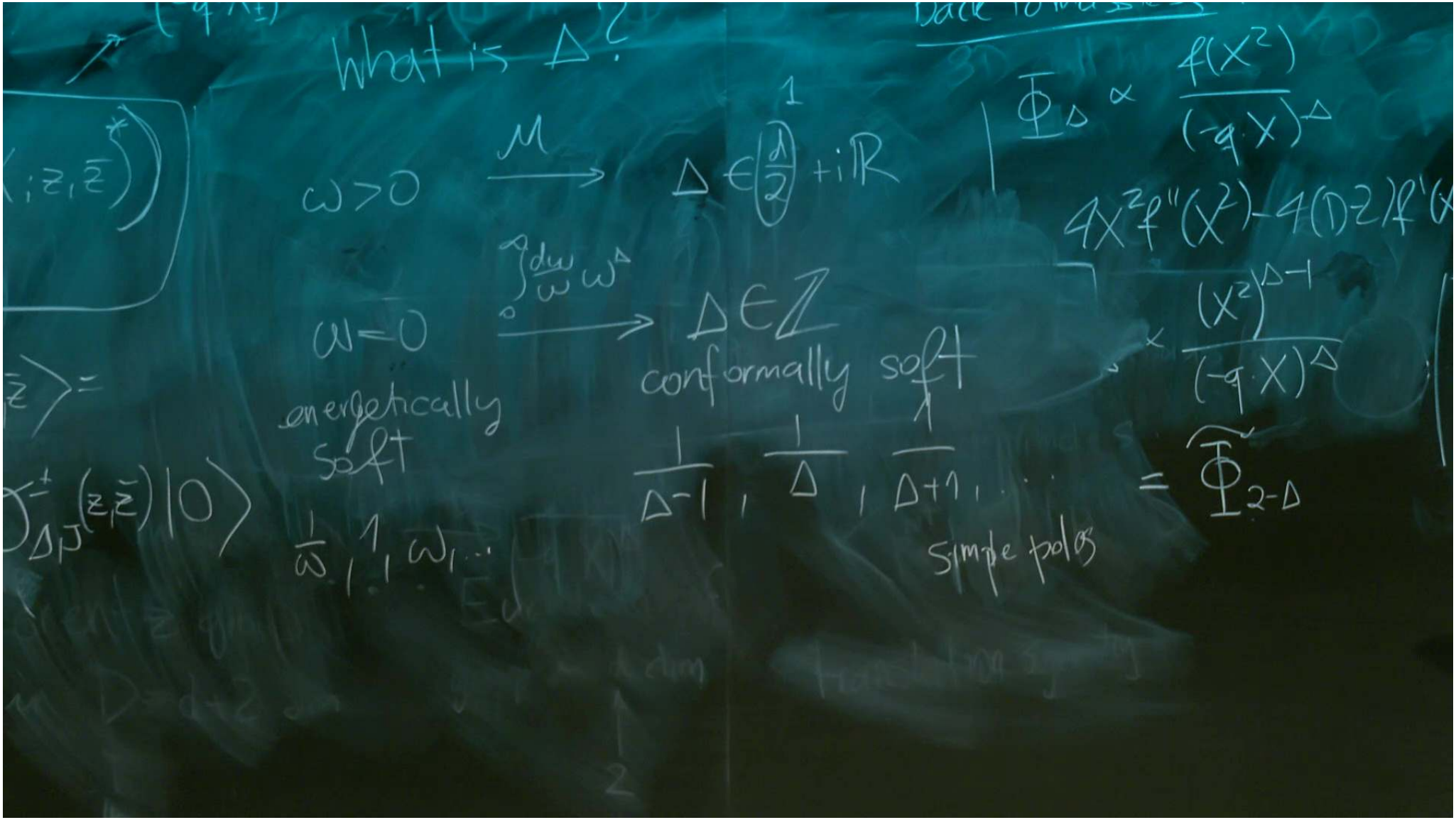

$\Phi_{\Delta,m}$ s.t. $(\Box - m^2)\,\Phi_{\Delta m} = 0$
$\propto \frac{f(X^2)}{(-q\cdot X)^\Delta}$
$f''(X^2) - 4(D-2)f'(X^2) - m^2 f(X^2) = 0$
$\propto \frac{\sqrt{X^2}^{\Delta-1}}{(-q\cdot X)^\Delta} K_{\Delta-1}(m\sqrt{X^2})$
limit $\xrightarrow{m\to\infty}$ $\frac{1}{(-q\cdot X)^\Delta}$
Back to massless
$\Phi_\Delta \propto \frac{f(X^2)}{(-q\cdot X)^\Delta}$
$4X^2 f''(X^2) - 4(D-2)f'(X^2) = 0$
$\Phi_\Delta \propto \frac{(X^2)^{\Delta-1}}{(-q\cdot X)^\Delta}$
$= \widetilde{\Phi}_{2-\Delta}$
CCFT building
Massless $p^\mu = \omega q^\mu(x)$
$= e^{ip\cdot X} = e^{i\omega q\cdot X}$ $\xrightarrow{\mathcal{M}}$ $\int_0^\infty d\omega\, \omega^{s-1}$
bare WF
Φ_Δ transform under
$X^\mu \to \Lambda^\mu{}_\nu X^\nu$ & $(z\bar{z}) \to (z'$
bulk
$\Phi_\Delta(X'^\mu, z', \bar{z}') = \left|\frac{\partial z'}{\partial z}\right|$


Massless spins $\bar{\Phi}_{\Delta,J}(X;z,\bar{z})$
• solve lin eom for spins
• transform under $X^\mu \to \Lambda^\mu{}_\nu X^\nu$
$z \to \frac{az+b}{cz+d}$
$\bar{\Phi}_{\Delta,J}(X';z',\bar{z}') = (cz+d)^{\Delta+J}(c\bar{z}+d)^{\Delta-J}$
$D_S(\Lambda)\bar{\Phi}_{\Delta,J}(X;z,\bar{z})$
spins rep


• solve lin eom for spins
• transform under $X^\mu \to \Lambda^\mu{}_\nu X^\nu$
$z \to \frac{az+b}{cz+d}$
$D_{\Delta,J}(X', z', \bar{z}') = (cz+d)^{\Delta+J}(c\bar{z}+d)^{\Delta-J}$
$D_S(\Lambda)\,\Phi_{\Delta,J}(X, z, \bar{z})$
spins rep
S20 dress w/ polarizations
$\epsilon_a^\mu = \frac{1}{\sqrt{2}}\partial_{a_\perp} q^\mu$
$a = z, \bar{z}$
$\epsilon_\mu^+ \to M_\mu = \epsilon_\mu^+ + \frac{\epsilon^+ \cdot X}{-q \cdot X} q_\mu \quad \Delta=0, J=1$
$\epsilon_\mu^- \to \bar{M}_\mu = \epsilon_\mu^+ + \frac{\epsilon^- \cdot X}{-q \cdot X} q_\mu \quad \Delta=0, J=-1$
• S=1
$A_{\Delta, J=\pm 1} = m \varphi_\Delta$
• S=2
$h_{\Delta, J=\pm 2} = m\, m\, \varphi_\Delta$


Back to massless
$\langle O_{\Delta_1} O_{\Delta_2} O_{\Delta_{shadow}=-1} \rangle =$
$\delta(i(\Delta_1+\Delta_1-2))$
$|z_{12}|^{\Delta_1+\Delta_2-\Delta_{shadow}} |z_{1\,shadow}|^{\Delta_1+\Delta_{shadow}-\Delta_2} |z_{2\,shadow}|^{\Delta_2+\Delta_{shadow}-\Delta_1}$
$\Phi_\Delta \propto \frac{f(X^2)}{(-q\cdot X)^\Delta}$
$4X^2 f''(X^2) - 4(D-2) f'(X^2) =$
$\propto \frac{(X^2)^{\Delta-1}}{(-q\cdot X)^\Delta}$
$= \tilde{\Phi}_{2-\Delta}$


Celestial operators
$e^{\pm i w_q \cdot X} \to \frac{1}{(-q \cdot X_\pm)^\Delta}$
CPW
$\mathcal{O}_{\Delta,J} = i\left(\hat{\mathcal{O}}^S(X), \Phi^{\pm}_{\Delta,J}(X;z,\bar{z})\right)$
bulk OP
$\langle \mathcal{O}_{\Delta_1} \mathcal{O}_{\Delta_2} \mathcal{O}_{\Delta_{shock}=-1} \rangle =$
$\delta(i(\Delta_1+\Delta_2-2$
$|z_{12}|^{\Delta_1+\Delta_2-\Delta_{shock}} |\bar{z}_{1\,shock}|^{\Delta_1+}$
asymptotic states: $|\Delta, J, z, \bar{z}\rangle =$
$\pm S$
$\mathcal{O}^{\pm}_{\Delta,J}(z,\bar{z})|0\rangle$


What is Δ?
Back to basics
$\omega > 0 \xrightarrow{M} \Delta \in \frac{1}{2} + i\mathbb{R}$
$\omega = 0 \xrightarrow{\int_0^\infty \frac{d\omega}{\omega}\omega^\Delta} \Delta \in \mathbb{Z}$
energetically soft
conformally soft
$\frac{1}{\omega}, 1, \omega, \ldots$
$\frac{1}{\Delta-1}, \frac{1}{\Delta}, \Delta+1, \ldots$
simple poles
$\Phi_\Delta \propto \frac{f(X^2)}{(-q\cdot X)^\Delta}$
$4X^2 f''(X^2) - 4(D-2) f'(X$
$\propto \frac{(X^2)^{\Delta-1}}{(-q\cdot X)^\Delta}$
$= \widetilde{\Phi}_{2-\Delta}$