Title: Lecture - Celestial Holography IIa

Speakers:

Collection: Celestial Holography Summer School 2024

Date: July 23, 2024 - 9:00 AM

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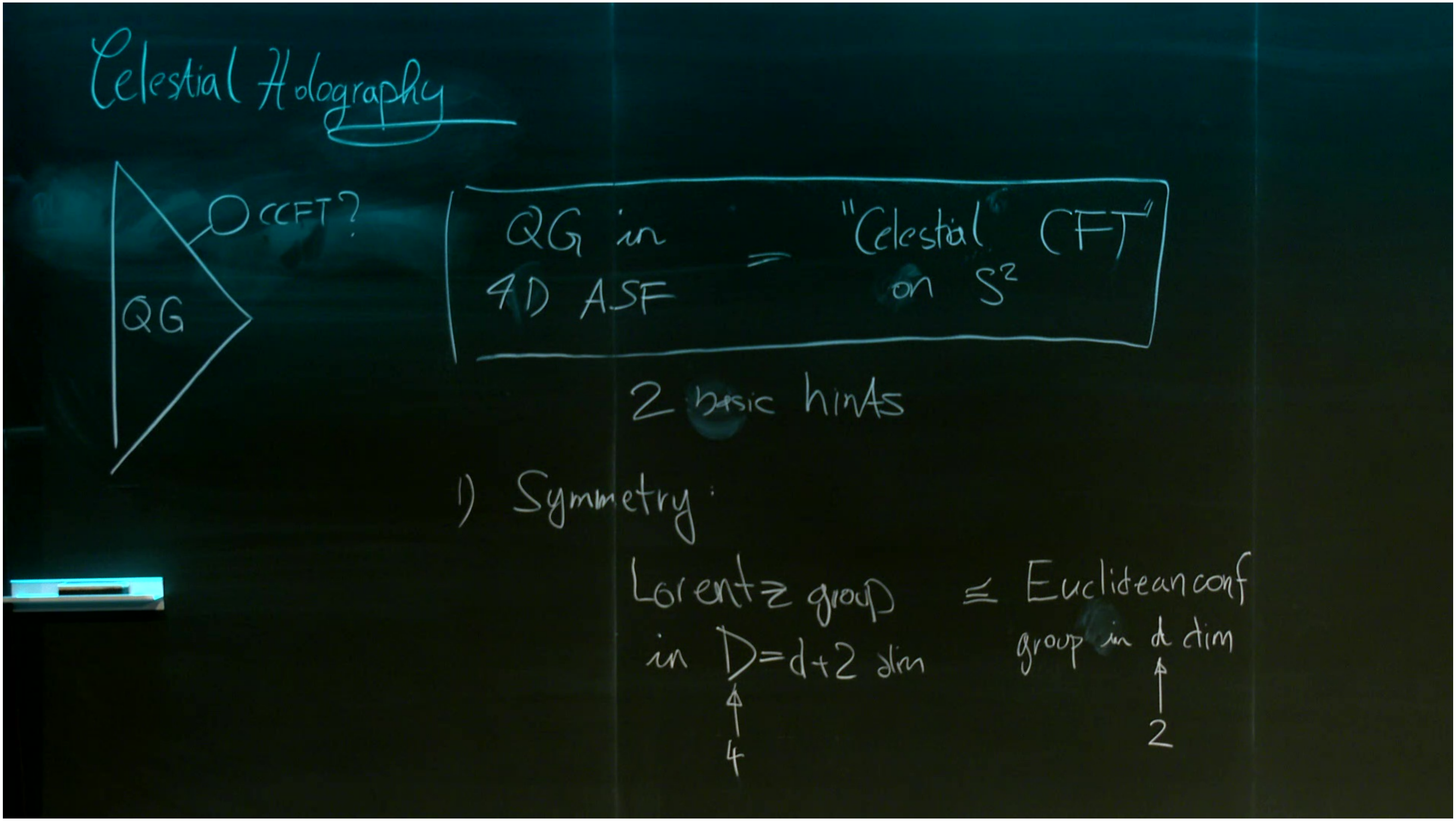
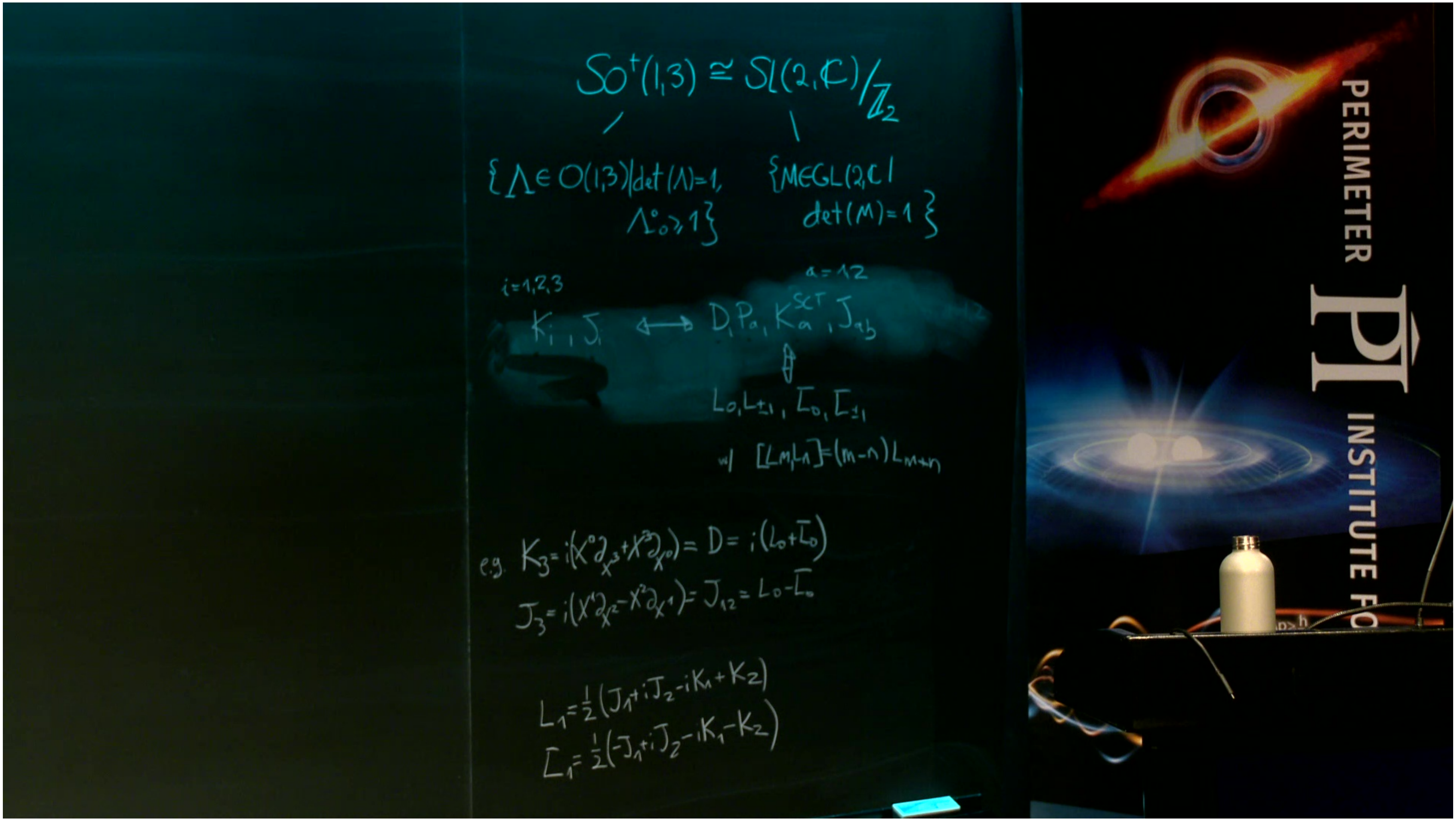


Celestial Holography
QG
CCFT?
QG in 4D ASF = "Celestial CFT" on S^2
2 basic hints
1) Symmetry:
Lorentz group in D=d+2 dim ≅ Euclidean conf group in d dim
4
2


$SO^{+}(1,3) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$
$\{\Lambda \in O(1,3) \mid \det(\Lambda)=1, \Lambda^0{}_0 \geq 1\}$
$\{M \in GL(2,\mathbb{C}) \mid \det(M)=1\}$
$i=1,2,3$
$a=1,2$
$K_i, J_i \longleftrightarrow D, P_a, K_a^{SCT}, J_{ab}$
$L_0, L_{\pm 1}, \bar{L}_0, \bar{L}_{\pm 1}$
w/ $[L_m, L_n] = (m-n) L_{m+n}$
e.g. $K_3 = i(x^0 \partial_{x^3} + x^3 \partial_{x^0}) = D = i(L_0 + \bar{L}_0)$
$J_3 = i(x^1 \partial_{x^2} - x^2 \partial_{x^1}) = J_{12} = L_0 - \bar{L}_0$
$L_1 = \frac{1}{2}(J_1 + iJ_2 - iK_1 + K_2)$
$\bar{L}_1 = \frac{1}{2}(-J_1 + iJ_2 - iK_1 - K_2)$
PERIMETER
PI
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ean conf
d dim
$\uparrow$
2

2) Observables: S-matrix

$|p_i\rangle = |\omega_i, x^i\rangle$

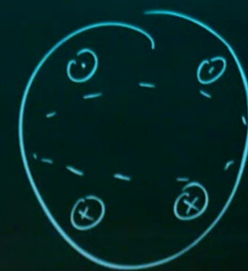
energy-momentum basis

standard amplitudes

$\langle p_n^{out} | S | p_n^{in} \rangle$

translation symmetry

$$\xrightarrow{\text{Mellin}} \quad \int_0^\infty d\omega\, \omega^{\Delta-1}$$



S^d

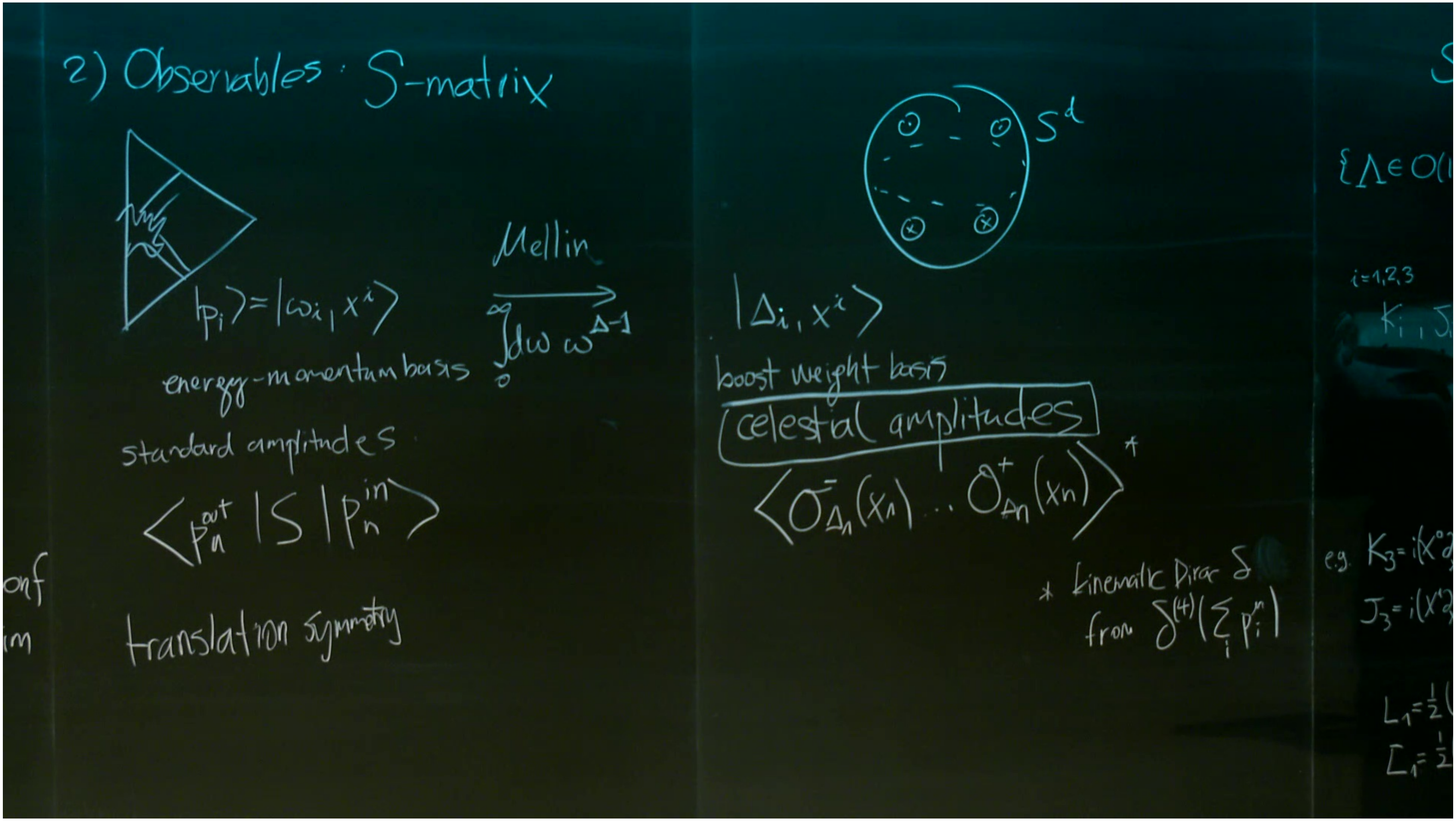
$|\Delta_i, x^i\rangle$

boost weight basis

$\boxed{\text{celestial amplitudes}}$

$\langle O^-_{\Delta_1}(x_1) \dots O^+_{\Delta_n}(x_n) \rangle$

e.g.


2) Observables: S-matrix
$|p_i\rangle = |\omega_i, x^i\rangle$
energy-momentum basis
standard amplitudes
$\langle p^{out}_n | S | p^{in}_n \rangle$
translation symmetry
Mellin
$\int_0^\infty d\omega\, \omega^{\Delta-1}$
S^d
$|\Delta_i, x^i\rangle$
boost weight basis
celestial amplitudes
$\langle O^-_{\Delta_1}(x_1) \dots O^+_{\Delta_n}(x_n) \rangle^*$
* kinematic Dirac δ
from $\delta^{(4)}(\sum_i p^n_i)$
conf
im
$\{\Lambda \in O($
$i=1,2,3$
K_i, J
e.g. $K_3 = i(x^0\partial$
$J_3 = i(x^1\partial$
$L_1 = \frac{1}{2}($
$\bar{L}_1 = \frac{1}{2}$

$|p_i\rangle = |\omega_i, x^i\rangle$ $\quad \int_0^\infty d\omega\, \omega^{\Delta-1}$ $\quad |\Delta_i, x^i\rangle$ $\quad K_i, J_i$

energy-momentum basis

standard amplitudes

$\langle p_n^{out} | S | p_n^{in} \rangle$

translation symmetry

boost weight basis

$\boxed{\text{celestial amplitudes}}$

$\langle \mathcal{O}^-_{\Delta_1}(x_1) \ldots \mathcal{O}^+_{\Delta_n}(x_n) \rangle^*$

Lorentz

$\updownarrow$

global conformal

$\downarrow$

local ?

* kinematic Dirac δ from $\delta^{(4)}(\sum_i p_i^{in})$

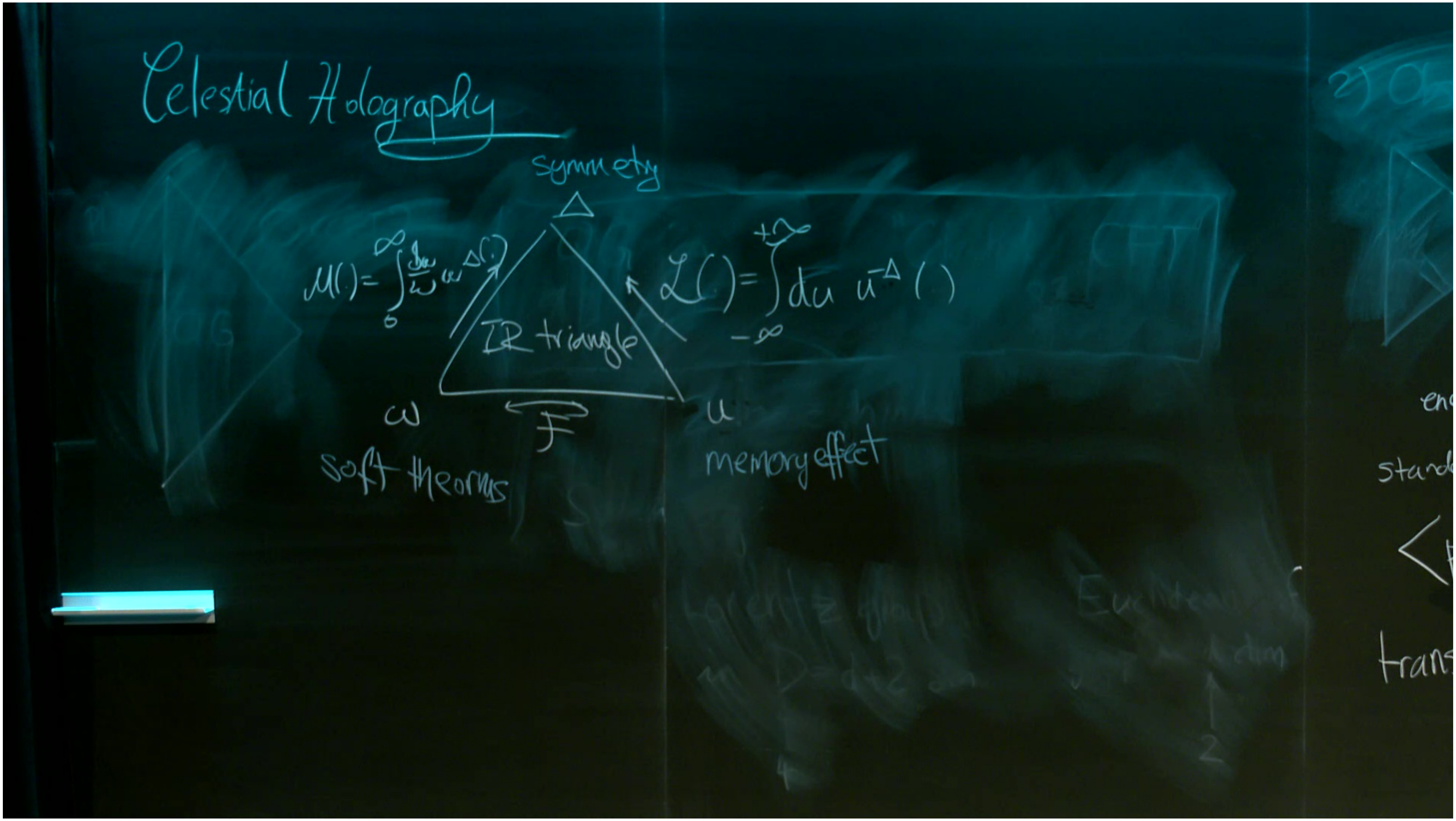
* $\delta(\sum_i \Delta_i + \#)$

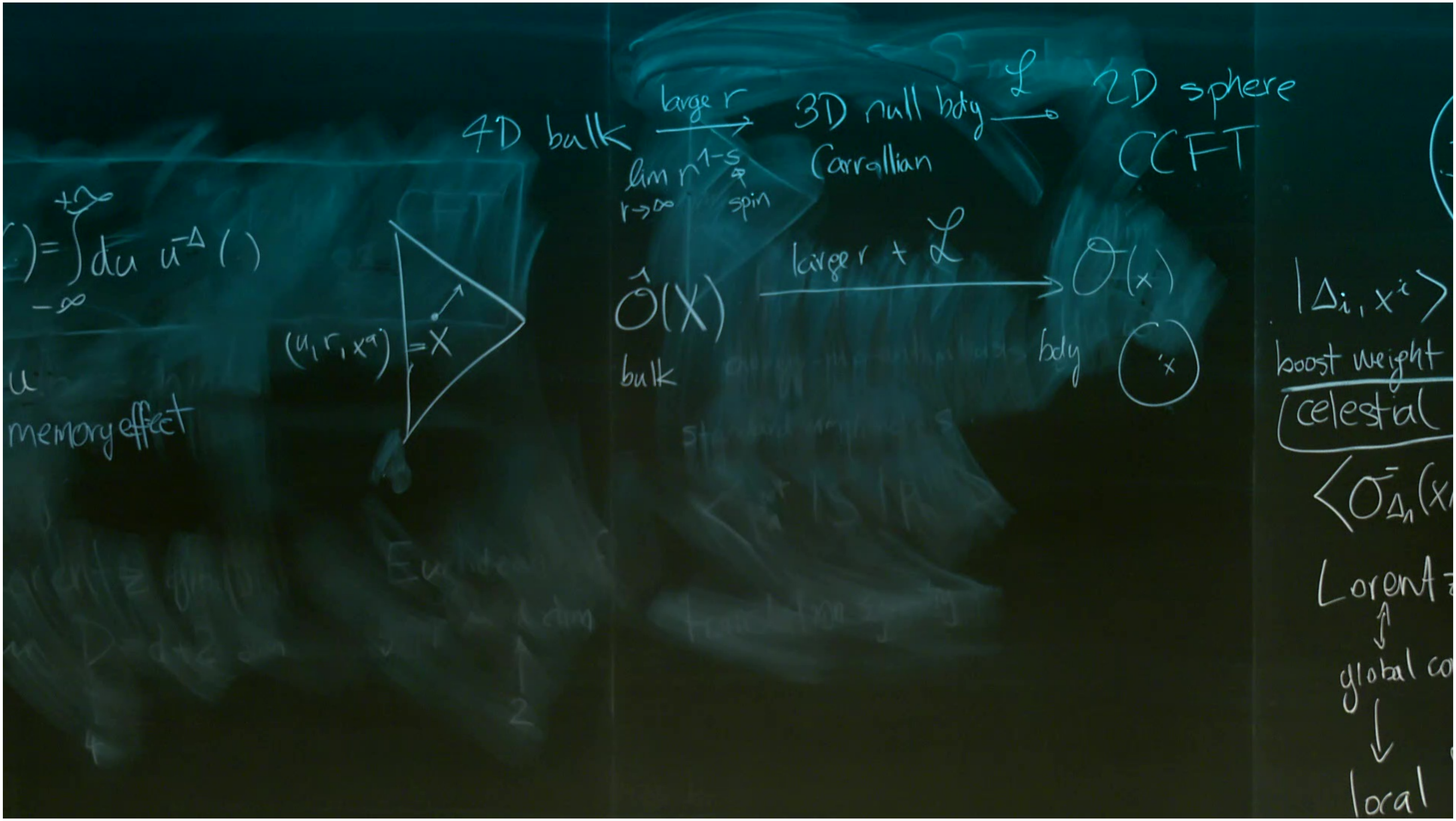
e.g. $K_3 = i(x^0 \partial_{x^3} + x^3 \partial_{x^0})$

$J_3 = i(x^1 \partial_{x^2} - x^2 \partial_{x^1})$

$L_1 = \frac{1}{2}(J_1 + iJ$

$\bar{L}_1 = \frac{1}{2}(-J_1 + iJ$


Celestial Holography
symmetry
Δ
$\mathcal{M}(\,) = \int_0^\infty \frac{d\omega}{\omega}\,\omega^{\Delta}(\,)$
$\mathcal{L}(\,) = \int_{-\infty}^{+\infty} du\, u^{-\Delta}(\,)$
IR triangle
ω
$\mathcal{F}$
u
soft theorems
memory effect


4D bulk
large r
3D null bdy
Carrollian
2D sphere
CCFT
lim r^(1-s)
r→∞
spin
larger r + ℒ
Ô(X)
bulk
O(x)
bdy
|Δi, x^i⟩
boost weight
celestial
Lorentz
global co
local
memory effect
(u, r, x^a) = X

$\mathcal{L}$ 2D sphere

$\to$ CCFT

$\to \mathcal{O}(x)$

bdy

CCFT building blocks

Massless $p^\mu = \omega q^\mu(x)$, $x^a = (z, \bar{z})$

$$\Phi_\omega = e^{ip\cdot X} = e^{i\omega q\cdot X} \xrightarrow[\int_0^\infty d\omega\, \omega^{\Delta-1}]{\mathcal{M}} \frac{(\mp i)\,\Gamma(\Delta)}{(-q\cdot X)^\Delta} = \Phi_\Delta$$

Plane WF $\qquad$ Conformal primary WF

Φ_Δ transform under

$$X^\mu \to \Lambda^\mu{}_\nu X^\nu \quad \& \quad (z,\bar{z}) \to (z',\bar{z}') = \left(\frac{az+b}{cz+d}, \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}\right)$$

bulk $\qquad \{a,b,c,d\} \in \mathbb{C}$, $ad-bc=1$

$$\Phi_\Delta(X'^\mu, z', \bar{z}') = \underbrace{\left|\frac{\partial z'}{\partial z}\right|^{-\Delta}}_{|cz+d|^{2\Delta}} \Phi_\Delta(X, z, \bar{z})$$

$$SO^+(1,3) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$$

$\{\Lambda \in O(1,3) \mid \det(\Lambda)=1, \Lambda^0{}_0 \geq 1\}$ $\qquad$ $\{M \in GL(2,\mathbb{C}) \mid \det(M)=1\}$

$i=1,2,3$ $\qquad$ $a=1,2$

$$K_i, J_i \longleftrightarrow D, P_a, K_a^{SCT}, J_{ab}$$

$$\Updownarrow$$

$$L_0, L_{\pm1}, \bar{L}_0, \bar{L}_{\pm1}$$

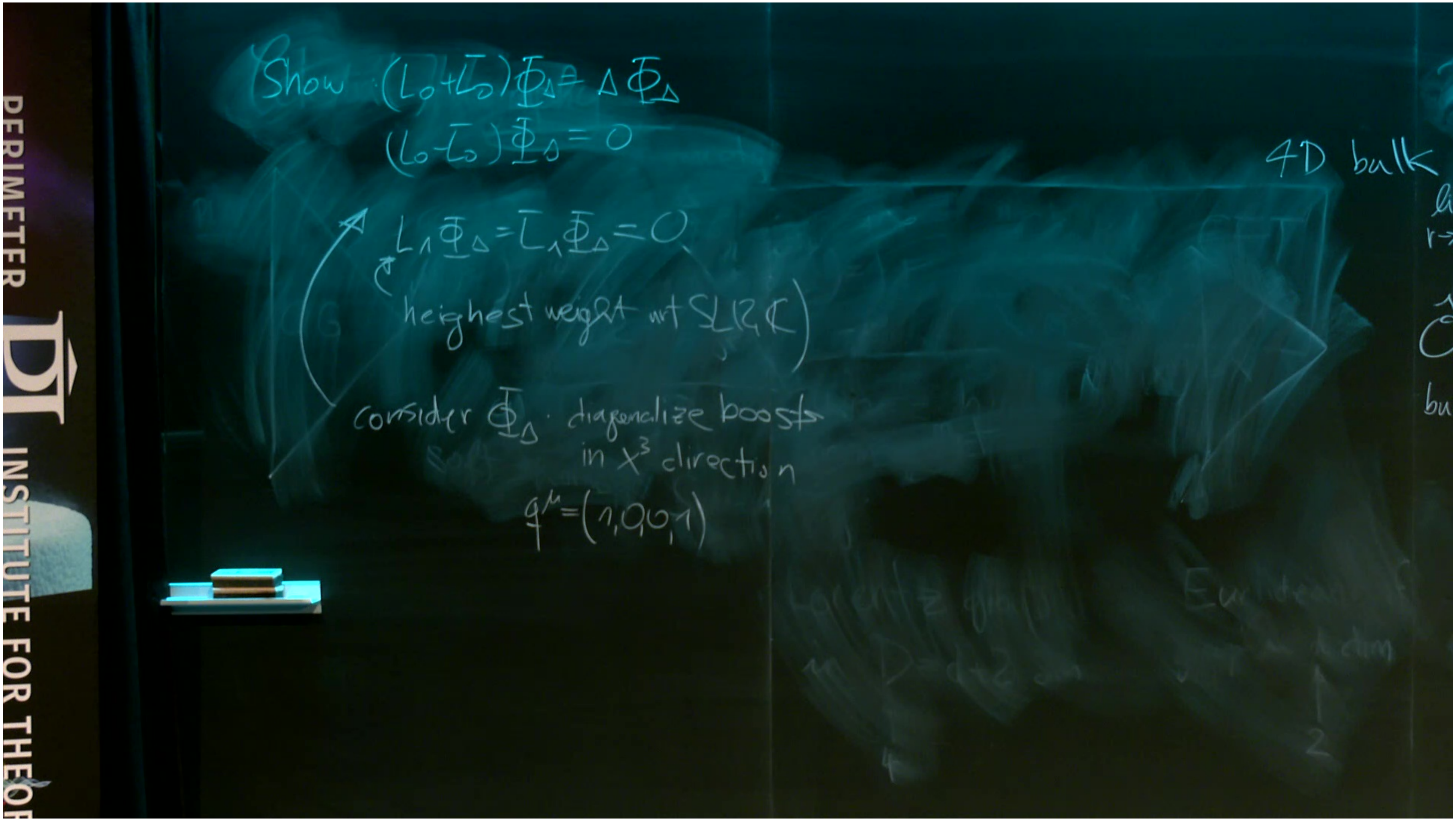
w/ $[L_m, L_n] = (m-n)L_{m+n}$

e.g. $K_3 = i(X^0\partial_{X^3} + X^3\partial_{X^0}) = D = i(L_0 + \bar{L}_0)$

$J_3 = i(X^1\partial_{X^2} - X^2\partial_{X^1}) = J_{12} = L_0 - \bar{L}_0$

$L_1 = \frac{1}{2}(J_1 + iJ_2 - iK_1 + K_2)$

$\bar{L}_1 = \frac{1}{2}(-J_1 + iJ_2 - iK_1 - K_2)$



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Show $(L_0+\bar{L}_0)\Phi_\Delta = \Delta\Phi_\Delta$
$(L_0-\bar{L}_0)\Phi_\Delta = 0$
$L_1\Phi_\Delta = \bar{L}_1\Phi_\Delta = 0$
heighest weight wrt $SL(2,\mathbb{C})$
consider Φ_Δ: diagonalize boosts
in x^3 direction
$q^\mu = (1,0,0,1)$
4D bulk

Φ_Δ

$=0$

wrt $SL(2,\mathbb{C})$

nalize boosts

x^3 direction

$(0,0,1)$

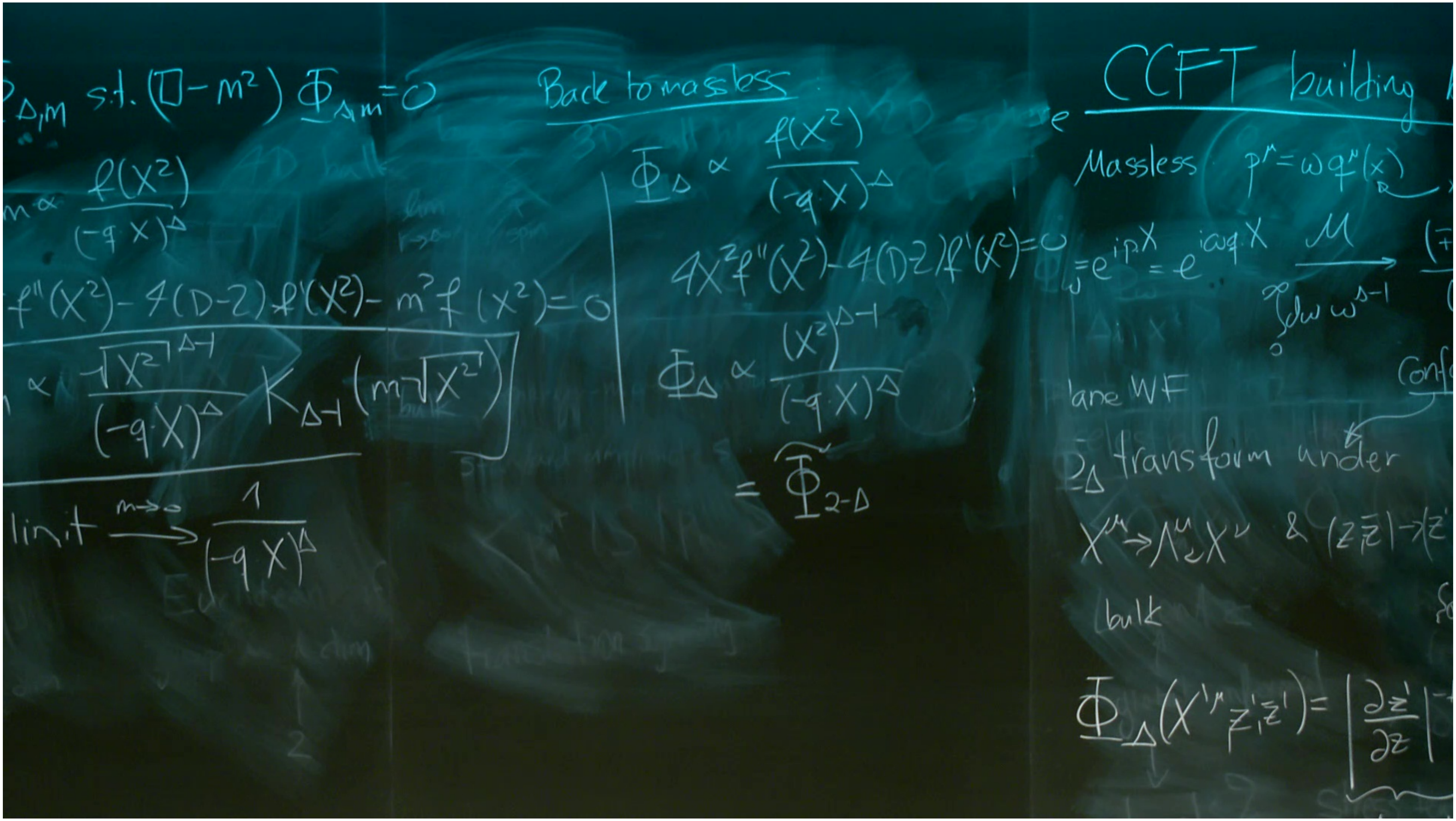
Massive: $\Phi_{\Delta,m}$ s.t. $(\Box - m^2)\,\Phi_{\Delta,m} = 0$

Ansatz: $\Phi_{\Delta,m} \propto \frac{f(X^2)}{(-q\cdot X)^\Delta}$

$$4X^2 f''(X^2) - 4(D-2) f'(X^2) - m^2 f(X^2) = 0$$

$$\boxed{\Phi_{\Delta,m} \propto \frac{\sqrt{X^2}^{\,\Delta-1}}{(-q\cdot X)^\Delta} K_{\Delta-1}\left(m\sqrt{X^2}\right)}$$

massless limit $\xrightarrow{m\to 0} \frac{1}{(-q\cdot X)^\Delta}$


$\Phi_{\Delta,m}$ s.t. $(\Box - m^2)\,\Phi_{\Delta m} = 0$
$\propto \frac{f(X^2)}{(-q\cdot X)^\Delta}$
$f''(X^2) - 4(D-2) f'(X^2) - m^2 f(X^2) = 0$
$\propto \frac{\sqrt{X^2}^{\Delta-1}}{(-q\cdot X)^\Delta} K_{\Delta-1}(m\sqrt{X^2})$
limit $\xrightarrow{m\to 0} \frac{1}{(-q\cdot X)^\Delta}$
Back to massless
$\Phi_\Delta \propto \frac{f(X^2)}{(-q\cdot X)^\Delta}$
$4X^2 f''(X^2) - 4(D-2) f'(X^2) = 0$
$\Phi_\Delta \propto \frac{(X^2)^{\Delta-1}}{(-q\cdot X)^\Delta}$
$= \widetilde{\Phi}_{2-\Delta}$
CCFT building
Massless $p^\mu = \omega q^\mu(x)$
$= e^{ip\cdot X} = e^{i\omega q\cdot X}$
$\mathcal{M}$
$\int_0^\infty d\omega\, \omega^{\Delta-1}$
lane WF
Φ_Δ transform under
$X^\mu \to \Lambda^\mu{}_\nu X^\nu$ & $(z,\bar z) \to$
bulk
$\Phi_\Delta(X'^\mu, z', \bar z') = \left|\frac{\partial z'}{\partial z}\right|$

Massless spins $\Phi_{\Delta,J}(X; z,\bar{z})$

- solve lin eom for spin s
- transform under $X^\mu \to \Lambda^\mu{}_\nu X^\nu$

$$z \to \frac{az+b}{cz+d}$$

$$\boxed{\Phi'_{\Delta,J}(X', z', \bar{z}') = (cz+d)^{\Delta+J}(c\bar{z}+d)^{\Delta-J}\, D_s(\Lambda)\, \Phi_{\Delta J}(X, z, \bar{z})}$$

spin s rep

• solve lin eom for spin s

• transform under $X^\mu \to \Lambda^\mu{}_\nu X^\nu$

$$z \to \frac{az+b}{cz+d}$$

$$\boxed{\Phi'_{\Delta,J}(X', z', \bar z') = (cz+d)^{\Delta+J}(c\bar z+d)^{\Delta-J}\, D_s(\Lambda)\, \Phi_{\Delta J}(X, z, \bar z)}$$

spins rep

$S>0$ dress w/ polarizations

$$\epsilon^\mu_a = \frac{1}{\sqrt{2}}\partial_a q^\mu \qquad a = z, \bar z$$

$$\epsilon^+_\mu \to m_\mu = \epsilon^+_\mu + \frac{\epsilon^+\cdot X}{-q\cdot X} q_\mu \qquad \Delta=0, J=1$$

$$\epsilon^-_\mu \to \bar m_\mu = \epsilon^+_\mu + \frac{\epsilon^-\cdot X}{-q\cdot X} q_\mu \qquad \Delta=0, J=-1$$

• $S=1$

$$A_{\Delta, J=+1} = m\varphi_\Delta$$

• $S=2$

$$h_{\Delta, J=+2} = m\, m\, \varphi_\Delta$$

$(1,0)$

$=(0,0)$

$=(-1,0)$

$$\langle O_{\Delta_1} O_{\Delta_2} O_{\Delta_{\text{shadow}}=\,0,\,-1} \rangle = \delta\left(i\left(\Delta_1+\Delta_1-\begin{smallmatrix}2\\1\end{smallmatrix}\right)\right)$$

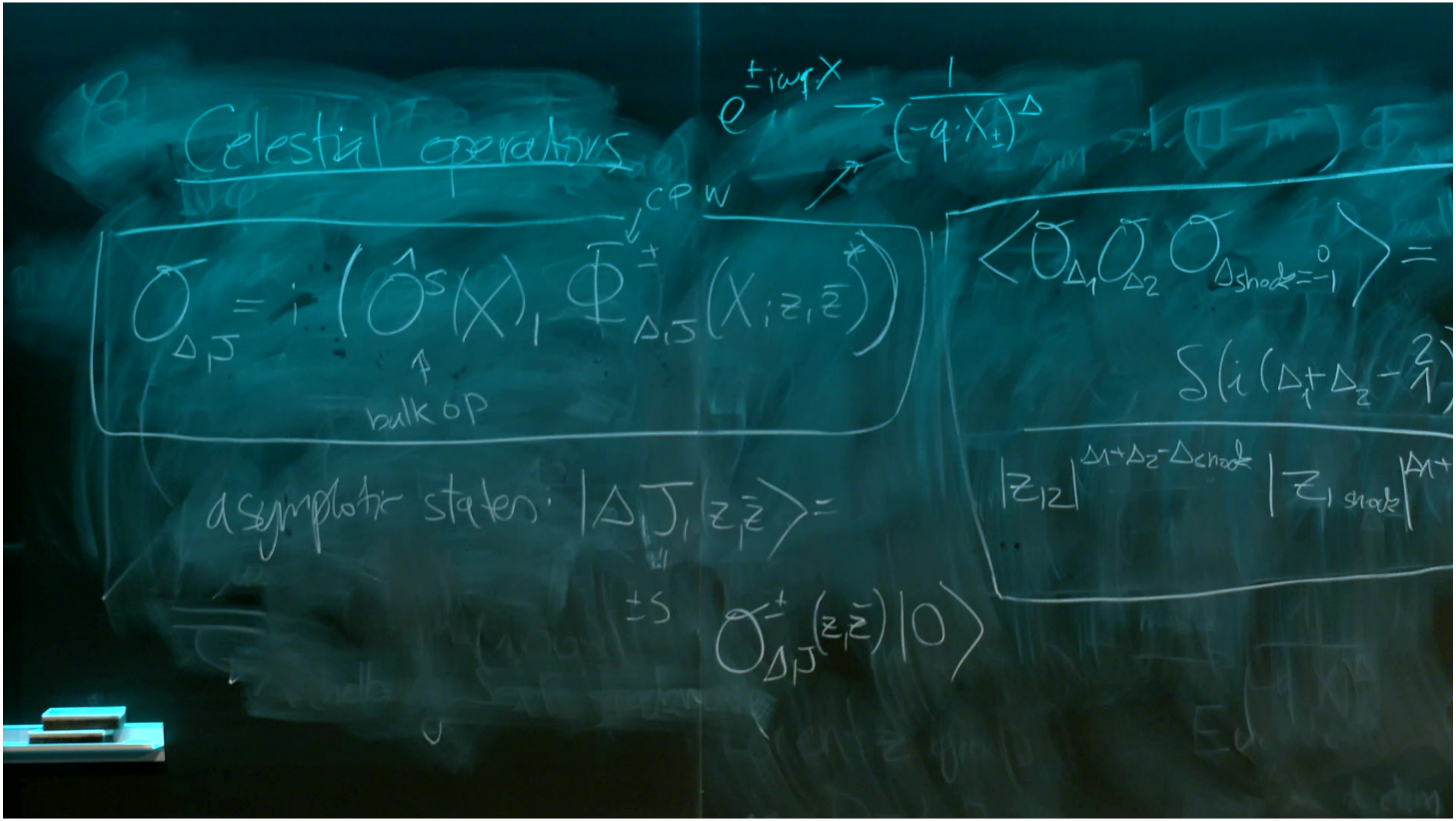
$$|z_{12}|^{\Delta_1+\Delta_2-\Delta_{\text{shadow}}}\,|z_{1\,\text{shadow}}|^{\Delta_1+\Delta_{\text{shadow}}-\Delta_2}\,|z_{2\,\text{shadow}}|^{\Delta_2+\Delta_{\text{shadow}}-\Delta_1}$$

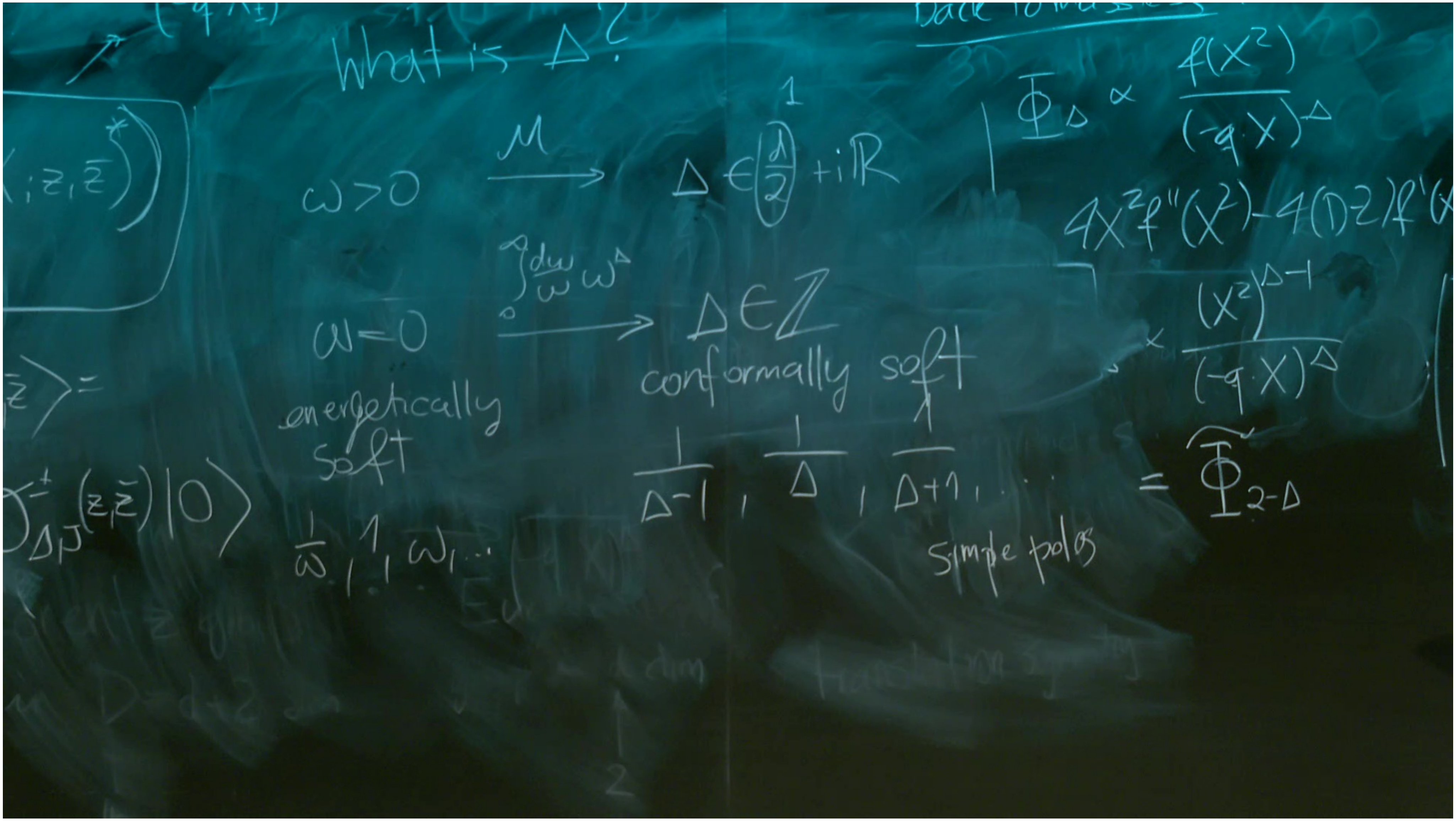
Back to massless:

$$\Phi_\Delta \propto \frac{f(X^2)}{(-q\cdot X)^{\Delta}}$$

$$4X^2 f''(X^2) - 4(D-2)f'(X^2) = 0$$

$$\times \frac{(X^2)^{\Delta-1}}{(-q\cdot X)^{\Delta}} = \tilde{\Phi}_{2-\Delta}$$


Celestial operators
$e^{\pm i\omega q\cdot X} \to \frac{1}{(-q\cdot X_\pm)^\Delta}$
CPW
$\tilde{O}_{\Delta,J} = i\left(\hat{O}^s(X), \Phi^{\pm}_{\Delta,J}(X;z,\bar{z})^*\right)$
bulk op
$\langle \tilde{O}_{\Delta_1} \tilde{O}_{\Delta_2} \tilde{O}_{\Delta_{shock}=0,-1} \rangle =$
$|z_{12}|^{\Delta_1+\Delta_2-\Delta_{shock}}$ $|z_{1\,shock}|$
asymptotic states: $|\Delta, J, (z,\bar{z})\rangle =$
$\tilde{O}^{\pm}_{\Delta,J}(z,\bar{z})|0\rangle$


What is Δ?
ω>0
ω=0
Δ∈Z
conformally soft
energetically soft
simple poles