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Speakers:

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Celestial symmetries from Twistor space

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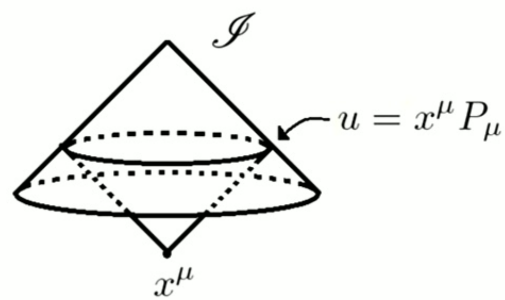
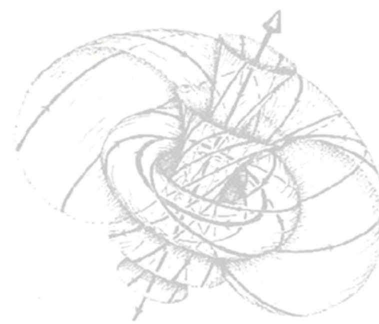
On: 2407.04028 with Adam Kmec, Romain Ruzziconi & Akshay Yelleshpur.
& older work with: Adamo & Sharma 2103.16984, 2110.06066, 2203.02238, Dunajski, 00s.
Cf also: 2403.18011 [Bittleston, Bogna, Kmec, Heuveline, & Skinner]

Holography from null infinity, via light-cones

- Newman '70's: tries to rebuild space-time from 'cuts' of $\mathscr{I}$.
- Yields instead '$\mathcal{H}$-space' a complex self-dual space-time.
- Penrose: $\rightsquigarrow$ asymptotic Twistor space $P\mathcal{T}$ & *nonlinear graviton*.



- Applies beyond asymptotically flat sector.
- Embodies integrability of SD sector.
- $\rightsquigarrow$ hierarchies of hidden symmetries.
- gives Noether charges via twistor actions.
- $\leftrightarrow$ Celestial Holography construction from soft OPEs of S-matrix?



Celestial symmetries and their deformations

[Strominger, Guevara,Himwich, Pate,]

- Postulate amplitudes are correlators in a *celestial CFT*.

$$\mathcal{M}(k_1, k_2, \ldots) = \langle \mathcal{O}(k_1)\mathcal{O}(k_2)\ldots\rangle$$

- Set $k_i = \omega_i(1 + |z_i|^2, \Re z_i, \Im z_i, 1 - |z_i|^2)$ soft expansion

$$\mathcal{O}(k) = \sum_p \omega^p H^p(z, \tilde{z})$$

- Collinear limits in amplitudes lead to 'splitting functions'

$$\mathcal{M}(k_1, k_2, k_3, \ldots) \xrightarrow{k_1 \| k_2} \mathrm{Split}(k_1, k_2, k_1 + k_2)\mathcal{M}(k_1 + k_2, k_3, \ldots)$$

- Splitting functions define OPE coefficients

$$H^{p_1}(z, \tilde{z})H^{p_2}(0, \tilde{z}) \xrightarrow{z \to 0} \frac{C^{p_1 p_2}_{p_3}}{z} H^{p_3}(0, \tilde{z}) + \ldots$$

- Mode expansion $\rightsquigarrow Lw_{1+\infty} = L\mathfrak{ham}(\mathbb{C}^2)$:

$$[h^p_{m,a}, h^q_{n,b}] = (m(q-1) - n(p-1))h^{p+q-2}_{m+n,a+b}\,.$$

Realization as asymptotic charges

Friedel, Pranzetti & Raclariu, 2112.15573, Geiller 2403..

On $\mathscr{I} = \mathbb{R} \times S^2$ with coordinates $(u, z, \bar{z})$, introduce

- asymptotic shear (gravity data) $\leftrightarrow (\sigma(u, z, \bar{z}), \bar{\sigma}(u, z, \bar{z}))$,
- Weyl tensor NP components Ψ_n^0, $n = 0, \dots, 4$

$$\Psi_4^0 = \ddot{\sigma}, \qquad \Psi_3^0 = \bar{\eth}\dot{\sigma},$$

- Asymptotic Bianchi identities

$$\partial_u \Psi_n^0 = \bar{\eth}\Psi_{n+1}^0 + (3-n)\bar{\sigma}\Psi_{n+2}^0, \qquad n = 0, 1, 2, 3\,.$$

- Generalise to recursion $\mathcal{Q}_s = \Psi_{2-s}^0$

$$\partial_u \mathcal{Q}_s = \bar{\eth}\mathcal{Q}_{s-1} + (s+1)\bar{\sigma}\mathcal{Q}_{s-2}, \qquad s = -1, 0, 1, 2 \dots$$

- Obtain charge integrals for each $\tau_s(z, \bar{z})$ spin s on S^2:

$$H_s = \int_{u=u_o} d^2 z\, \tau_s \mathcal{Q}_s + \dots$$

- But guesswork, ... complicated:

The full formulae at $\mathscr{I}$

Friedel,Pranzetti, Raclariu][Geiller][Kmec,M., Ruzziconi, Yelleshpur]

$$H_{-1} = \int_{\mathcal{S}} d^2 z\, \tau_{-1}\mathcal{Q}_{-1},$$

$$H_0 = \int_{\mathcal{S}} d^2 z\, \tau_0(\mathcal{Q}_0 - u\bar{\eth}\mathcal{Q}_{-1}),$$

$$H_1 = \int_{\mathcal{S}} d^2 z\, \tau_1\left(\mathcal{Q}_1 - u\bar{\eth}\mathcal{Q}_0 + \frac{u^2}{2}\bar{\eth}^2\mathcal{Q}_{-1} - 2\mathcal{Q}_{-1}\partial_u^{-1}\bar{\sigma}\right),$$

$$H_2 = \int_{\mathcal{S}} d^2 z\, \tau_2\left(\mathcal{Q}_2 - u\bar{\eth}\mathcal{Q}_1 + \frac{u^2}{2}\bar{\eth}\mathcal{Q}_0 - \frac{u^3}{6}\bar{\eth}^3\mathcal{Q}_{-1} - 3\mathcal{Q}_0\partial_u^{-1}\bar{\sigma} + 3\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-2}\bar{\sigma} + 2\bar{\eth}\left(\mathcal{Q}_{-1}\partial_u^{-1}(u\bar{\sigma})\right)\right),$$

$$\begin{aligned}H_3 = \int_{\mathcal{S}} d^2 z\, \tau_3\Big(&\mathcal{Q}_3 - u\bar{\eth}\mathcal{Q}_2 + \frac{u^2}{2}\bar{\eth}^2\mathcal{Q}_1 - \frac{u^3}{6}\bar{\eth}^3\mathcal{Q}_0 + \frac{u^4}{24}\bar{\eth}^4\mathcal{Q}_{-1}\\ &- 4\mathcal{Q}_1\partial_u^{-1}\bar{\sigma} + 3\bar{\eth}\left(\mathcal{Q}_0\partial_u^{-1}(u\bar{\sigma})\right) + 4\bar{\eth}\mathcal{Q}_0\partial_u^{-2}\bar{\sigma} + 8\mathcal{Q}_{-1}\partial_u^{-1}(\bar{\sigma}\partial_u^{-1}\bar{\sigma})\\ &- 3\bar{\eth}\left(\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-2}(u\bar{\sigma})\right) - \bar{\eth}^2\left(\mathcal{Q}_{-1}\partial_u^{-1}(u^2\bar{\sigma})\right) - 4\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-3}\bar{\sigma}\Big).\end{aligned}$$

We will derive these from twistor space.

Outline

1. Symmetries in twistor space and their hierarchy.
2. Higher symmetries $\leftrightarrow$ gravitons via Penrose's nonlinear graviton.
3. Twistor action for (SD) Einstein gravity.
4. Noether charges, and Hamiltonians.
5. Recursion operators and hierarchies.

Poisson diffeos of plane & w_∞

W_N = higher spin symmetries in 2d CFT [Zamolodchikov 1980s].

- For $N \to \infty$ classical limit w_∞ = Poisson diffeos of plane.
- Plane has coords $\mu^{\dot\alpha}$, $\dot\alpha = \dot 0, \dot 1$ with Poisson bracket

$$\{f, g\} := \varepsilon^{\dot\alpha\dot\beta} \frac{\partial f}{\partial \mu^{\dot\alpha}} \frac{\partial g}{\partial \mu^{\dot\alpha}} , \qquad \varepsilon^{\dot\alpha\dot\beta} = \varepsilon^{[\dot\alpha\dot\beta]}, \quad \varepsilon^{\dot 0 \dot 1} = 1 .$$

- Basis of $w_{1+\infty} \leftrightarrow$ polynomial hamiltonians

$$w^p_m = (\mu^{\dot 0})^{p-m-1} (\mu^{\dot 1})^{p+m-1} , \qquad |m| \leq p - 1, \quad 2p - 2 \in \mathbb{N}$$

- Poisson brackets $\leftrightarrow$ commutation relations of $w_{1+\infty}$:

$$\{w^p_m, w^q_n\} = (2(p-1)n - 2(q-1)m) w^{p+q-2}_{m+n} .$$

- Loop algebra $Lw_{1+\infty}$, loop coord $z = \mathrm{e}^{i\theta}$, and generators

$$g^p_{m,r} = w^p_m / z^r , \qquad r \in \mathbb{Z}.$$

- Poisson brackets now

$$\{g^p_{m,r}, g^q_{n,s}\} = (2(p-1)n - 2(q-1)m)\, g^{p+q-2}_{m+n,r+s} .$$

Twistor construction for self-dual space-times

Flat twistor space: $\mathbb{T} = \mathbb{C}^4$, projectively $\mathbb{PT}$ with hgs coords:

$$W = (\lambda_\alpha, \mu^{\dot\alpha}) \in \mathbb{T}, \qquad W \sim aW\,, a \neq 0\,.$$

We have

- Projection $\mathbb{PT} - \mathbb{CP}^1_{\{\lambda_\alpha=0\}}$ onto celestial sphere

$$p : \mathbb{PT} \to \mathbb{CP}^1\,, \qquad p(W) = \lambda_\alpha\,, \quad z = \lambda_1/\lambda_0\,.$$

- Poisson bracket:

$$\{f, g\} = \varepsilon^{\dot\alpha\dot\beta}\frac{\partial f}{\partial \mu^{\dot\alpha}}\frac{\partial g}{\partial \mu^{\dot\beta}} = \left[\frac{\partial f}{\partial \mu}\,\frac{\partial g}{\partial \mu}\right].$$

Theorem (Penrose 1976)

There is a 1 : 1 *correspondence between:*

- *SD Ricci flat holomorphic metrics on regions in* $\mathbb{C}^4$*, and*
- *deformations* $P\mathcal{T}$ *of twistor space* $\mathbb{PT}$ *preserving fibration* $p : \mathcal{T} \to \mathbb{CP}^1$ *and Poisson structure on fibres of* p*.*

Patching functions for deformations

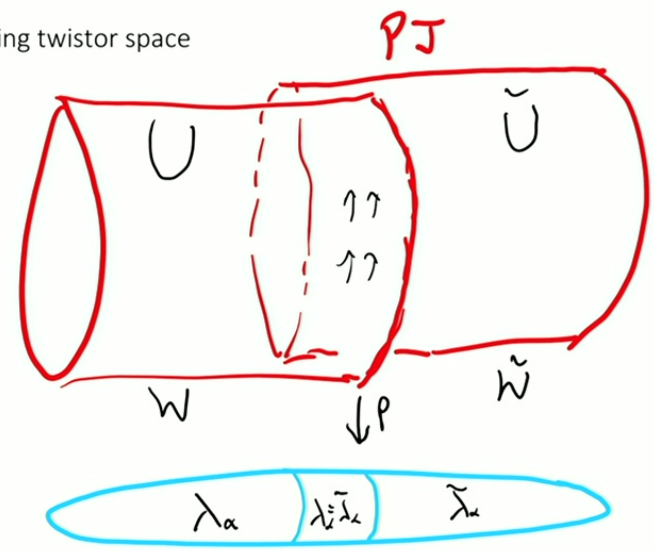
SD metrics $\leftrightarrow$ deformations $\mathbb{PT} \to P\mathcal{T} \leftrightarrow$ Patching functions

- Cover $\mathbb{PT} - \mathbb{CP}^1$ by patches

$$U = \{\lambda_0 \neq 0\}\,, \qquad \tilde{U} = \{\lambda_1 \neq 0\}$$

  coords $W = (\lambda_\alpha, \mu^{\dot\alpha})$ and $\tilde{W} = (\lambda_\alpha, \tilde{\mu}^{\dot\alpha})$ resp..
- 'Patch' $\tilde{W} = \tilde{W}(W)$ on $U \cap \tilde{U}$;
- $\lambda_\alpha = \tilde{\lambda}_\alpha$ to preserve $p : P\mathcal{T} \to \mathbb{CP}^1_{\lambda_\alpha}$.

Patching twistor space



Patching functions and $Lw_{1+\infty}$

Patching function $\tilde{\mu}^{\dot\alpha} = \tilde{\mu}^{\dot\alpha}(\lambda_\alpha, \mu^{\dot\alpha})$ must preserve $\{,\} \Rightarrow$

Proposition

Space of SD metrics $\simeq$ Loop group of Poisson diffeos.

- Coord transform preserves $\{,\} \leftrightarrow$ generating function: $G(\lambda_\alpha, \mu^{\dot 0}, \tilde{\mu}^{\dot 1})$, homogeneity degree 2 determines:

$$\mu^{\dot 1} = \frac{\partial G}{\partial \mu^{\dot 0}}, \qquad \tilde{\mu}^{\dot 0} = \frac{\partial G}{\partial \tilde{\mu}^{\dot 1}}.$$

- Infinitesimally $h = \delta G \in Lw_{1+\infty}$ can be expanded in

$$h^p_{m,r} = \frac{(\mu^{\dot 0})^{p-m-1}(\mu^{\dot 1})^{p+m-1}}{\lambda_0^{2p-4-r}\lambda_1^r}, \qquad p-2 \in \mathbb{N}, \quad r \in \mathbb{Z}.$$

- When g is quadratic $\rightsquigarrow$ global symmetry $\rightsquigarrow$ Poincaré$^+$.

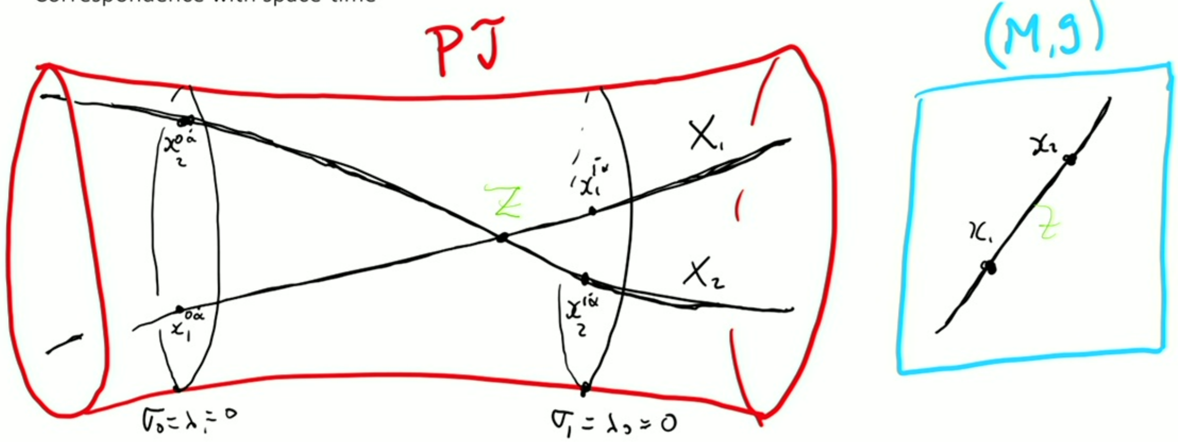
The self-dual space-time from holomorphic curves

To reconstruct self-dual space-time

$$(M^4, g) = \{ \text{Holomorphic degree-1 } \mathbb{CP}^1\text{s in } P\mathcal{T} \}$$

- Parametrize the $\mathbb{CP}^1_{x,\sigma} \subset \mathbb{PT}$, with coords σ, $Z = Z(x, \sigma)$

Correspondence with space-time



Dolbeault version: deform $\bar{\partial}$-operator with Poisson compatible Beltrami differential $\rightsquigarrow$ d-bar eq for $\mathbb{C}$-curves in deformed $\mathbb{PT}$:

$$\bar{\partial}_\sigma Z^A = \{H, Z^A\}_\Lambda\,, \qquad H \in \Omega^{0,1}(2)\,.$$

Null infinity $\mathscr{I}$

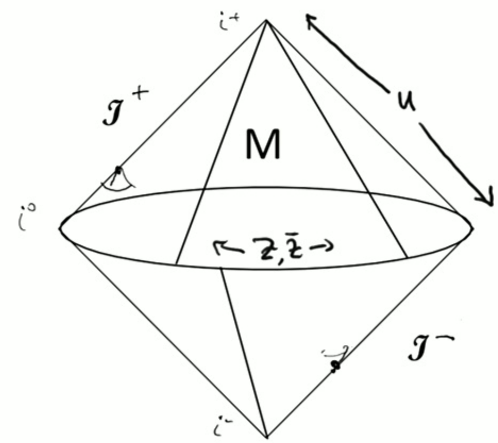
Asymptotically simple (M, g) has $\mathscr{I}^{\pm}$ and light rays meet both.

- Bondi coordinates $(u = t - r, z, \bar{z})$.
- Flat space conformal to

$$2dudR - dzd\bar{z} + O(R^0), \qquad R = \frac{1 + |z|^2}{r}.$$

- Use spinor coordinates on on $S^2 = \mathbb{CP}^1$

$$\lambda_\alpha = (1, z), \quad \alpha = 0, 1.$$



  - Rescale curved g near $\mathscr{I}$ by $\tilde{g} = R^2 g$ as $R \to 0$ to give

$$2dudR - dzd\bar{z} + R(\boldsymbol{\sigma}^0 d\bar{z}^2 + c.c.) + O(1),$$

  - $\boldsymbol{\sigma}^0(u, z, \bar{z})$ is *asymptotic shear*; SD gravitational data at $\mathscr{I}$.

Asymptotic Twistor space

Penrose's nonlinear graviton at $\mathscr{I}$ with deformed $\bar\partial$ operator.

Twistor space $\mathcal{T} = \mathbb{C}^4$ or projective $\mathbb{P}\mathcal{T}$, homogeneous coords:

$$W = (\lambda_\alpha, \mu^{\dot\alpha}) \in \mathbb{T}, \qquad W \sim aW\,, a \neq 0\,.$$

Poisson bracket $\{f, g\} = \varepsilon^{\dot\alpha\dot\beta} \frac{\partial f}{\partial \mu^{\dot\alpha}} \frac{\partial g}{\partial \mu^{\dot\beta}} = \left[\frac{\partial f}{\partial \mu} \frac{\partial g}{\partial \mu}\right]$ as before.

- $q : \mathcal{T} \to \mathscr{I}$ given in Bondi coordinates

$$z = \frac{\lambda_1}{\lambda_0}\,, \qquad u = \frac{\mu^{\dot\alpha}\bar\lambda_{\dot\alpha}}{|\lambda_0|^2 + |\lambda_1|^2}\,.$$

- Asymptotic shear $\boldsymbol{\sigma}^0 d\bar z$ pulls back to $\mathcal{T}$ to define

$$\mathbf{h} = \int^u du' \boldsymbol{\sigma}^0(u', z, \bar z) d\bar z = h(\mu^{\dot\alpha}\bar\lambda_{\dot\alpha}, \lambda, \bar\lambda)[\bar\lambda d\bar\lambda] \in \Omega^{0,1}(2)\,.$$

- $\bar\partial$-operator on $\mathcal{T}$ is deformed by 'Hamiltonian' h to

$$\bar\partial_h f := \bar\partial_0 f + \{h, f\}\,.$$

SD Twistor action

Holomorphic Poisson Chern-Simons action [M. & Wolf 2007]

To go off-shell, use Dolbeault framework:

- Gravitational data, SD: $\mathbf{h} \in \Omega^{0,1}(2)$, ASD: $\mathbf{g} \in \Omega^{0,1}(-6)$,

$$\mathbf{h}(aZ) = a^2\mathbf{h}(Z), \qquad \mathbf{g}(aZ) = a^{-6}\mathbf{g}(Z)\,, \qquad \mathbf{h} = \mathbf{h}_{\bar{A}} d\bar{Z}^{\bar{A}} \text{ etc..}$$

- To guarantee compatibility with $\{,\}$, deform $\bar{\partial}$-operator by

$$\bar{\partial}_{\mathbf{h}} f(Z, \bar{Z}) := \bar{\partial}_0 f + \{\mathbf{h}, f\}$$

- Need integrability $\bar{\partial}\mathbf{h}^2 = 0 \rightsquigarrow$ SD field equations

$$\bar{\partial}_0 \mathbf{h} + \frac{1}{2}\{\mathbf{h}, \mathbf{h}\} = 0\,, \qquad \bar{\partial}_{\mathbf{h}} G = 0\,.$$

- Follow from action, with $D^3 Z = \frac{1}{24}\varepsilon_{ABCD} Z^A dZ^B \wedge \ldots \wedge dZ^D$,

$$S[\mathbf{g}, \mathbf{h}] := \int_{\mathbb{PT}} \mathbf{g} \wedge (\bar{\partial}\mathbf{h} + \frac{1}{2}\{\mathbf{h}, \mathbf{h}\}) \wedge D^3 Z\,.$$

Hamiltonians for symmetries

So $[\mathbf{g}] \in H^1(\mathcal{PT}, \mathcal{O}(-6))$ and $\mathbf{h}$ a nonlinear $H^1(\mathcal{PT}, \mathcal{O}(2))$.

- local 'gauge' symmetries $= (\chi, \xi) \in (\mathcal{O}(-6), \mathcal{O}(2))$.

$$\delta(\mathbf{g}, \mathbf{h}) = (\bar{\partial}_{\mathbf{h}}\chi + \{\xi, \mathbf{g}\}, \bar{\partial}_{\mathbf{h}}\xi)$$

- $\xi \leftrightarrow$ generators of smooth local Poisson diffeos.
- $\chi \leftrightarrow$ gauge for $\mathbf{g} \in$ cohomology class.
- Action $\rightsquigarrow$ symplectic form

$$\Omega(\delta_1, \delta_2) = \int_{\Sigma^5} \delta_1 \mathbf{g} \wedge \delta_2 \mathbf{h} \wedge D^3 Z + (1 \leftrightarrow 2)\,.$$

- Gauge generated by Hamiltonians

$$\mathcal{H}_{\chi,\xi} = \int_{\Sigma^5} (\bar{\partial}_{\mathbf{h}}\xi \wedge \mathbf{g} + (\bar{\partial}_0 \chi + \frac{1}{2}\{\mathbf{h}, \chi\}) \wedge \mathbf{h}) \wedge D^3 Z$$

- Compute Poisson brackets in gravitational phase space:

$$\{\mathcal{H}_{\chi_1,\xi_1}, \mathcal{H}_{\chi_2,\xi_2}\}_\Omega = \mathcal{H}_{\{\xi_1,\chi_2\}-\{\xi_2,\chi_1\},\{\xi_1,\xi_2\}}\,.$$

Algebra of charges

- 'Large' gauge transformations are only symmetries if holomorphic at $\partial\Sigma^5 \subset \infty$', $\bar{\partial}_{\mathbf{h}}\xi = 0 = \bar{\partial}_{\mathbf{h}}\chi$.
- On-shell, Hamiltonians given by boundary charges

$$Q_{\chi,\xi} = \int_{\partial\Sigma^5} \xi \mathbf{g} + \chi \mathbf{h}\, D^3 Z\,.$$

  Here $\partial\Sigma^5 = \mathbb{CP}^1 \times \mathcal{S}$ for $M \supset \mathcal{S}$, some 2-surface at ∞.
- As above obtain gauge algebra

$$\{Q_{\chi_1,\xi_1}, Q_{\chi_2,\xi_2}\}_\Omega = Q_{\{\xi_1,\chi_2\}-\{\xi_2,\chi_1\}_\Lambda,\{\xi_1,\xi_2\}}\,.$$

- Here to be symmetries, (χ, ξ) are holomorphic near $\partial\Sigma^5$.
- If global on $\partial\Sigma^5$, then $\xi =$ quadratic $\leftrightarrow$ Poincaré$^+$ symmetries, and $\xi = 0$.
- For $Lw_{1+\infty}$, need $(\mathbf{g}, \mathbf{h}) = 0$ near $\lambda_0 = 0$ & $\lambda_1 = 0$.

Bramson-Tod formula for transform to $\mathscr{I}$ and recursion

Integrable hierarchies [M. & Woodhouse 1996], [Dunajski, M. 2002,3]

Penrose transform charge formulae to $\mathscr{I}$ needs transform for g:

- Point $(u, z, \bar{z}) \in \mathscr{I} \leftrightarrow \mathbb{CP}^1_{(u,z,\bar{z})} \ni q$ given by

$$z = \frac{\lambda_1}{\lambda_0}, \qquad \mu^{\dot{\alpha}} = uT^{\alpha\dot{\alpha}}\lambda_\alpha + q\bar{\lambda}^{\dot{\alpha}}, \qquad T^{\alpha\dot{\alpha}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

- Bramson-Tod formula gives $\Psi^0_n = Q_{2-n}$ for $n = 0, .., 3$:

$$\mathcal{Q}_s = \int_{\mathbb{CP}^1_{(u,z,\bar{z})}} q^{s+2} g \wedge \mathrm{d}q$$

- Solves asymptotic Bianchi identities & recurrence relation

$$\partial_u \mathcal{Q}_s = \bar{\eth}\mathcal{Q}_{s-1} + (s+1)\bar{\sigma}\mathcal{Q}_{s-2}, \qquad s = -1, 0, 1, 2$$

- Extension to larger $s \leftrightarrow$ how $Lw_{1+\infty}$ arises from Poincaré$^+$ symmetries by multiplication by $\frac{\mu_1}{\lambda_0}$ etc., here $q = \frac{\langle\lambda\, T\, \mu]}{\langle\lambda T \bar{\lambda}]}$.

Gauge fixing the Poisson diffeos

Our ξ generates Poisson diffeos of $P\mathcal{T}$ that destroys gauge fixing of $\mathbf{h}$ that relates $\partial_u \mathbf{h}$ to $\sigma(u, z, \bar{z})$. We fix this as follows:

- We need the gauge fixing $\partial_q \mathbf{h} = 0$ which gives

$$\partial_q(q\partial_u - \bar{\eth} + \bar{\sigma}\partial_q)\xi = 0\,.$$

- Solve by ansatze $\xi_s = \sum_{n=0}^{s+1} \xi_{s,n} q^n$ and recurrence

$$\partial_u \xi_{s,n-1} - \bar{\eth}\xi_{s,n} + (n+1)\bar{\sigma}\xi_{s,n+1} = 0\,, \qquad \xi_{s,s+1} := \tau_s(\lambda, \bar{\lambda})\,.$$

- Start with $\tilde{\xi}_s \leftrightarrow \tau_s(\lambda, \bar{\lambda})$ spin s on S^2, poly degree $s+1$ in q

$$\tilde{\xi}_s = \frac{\mu^{\dot{\alpha}_1} \cdots \mu^{\dot{\alpha}_{s+1}}}{(s+1)!} \frac{\partial^{s+1}\tau_s}{\partial\bar{\lambda}^{\dot{\alpha}_1} \cdots \partial\bar{\lambda}^{\dot{\alpha}_{s+1}}} = \sum_{n=-1}^{s} \frac{u^{s-n} q^{n+1}}{(s-n)!} \bar{\eth}^{s-n}\tau_s\,,$$

- 'Wedge' condition $\Leftrightarrow \bar{\eth}^{s+2}\tau_s = 0 \Leftrightarrow \tilde{\xi}_s$ holomorphic.
- Then find gauge fixed $\xi_s = \tilde{\xi}_s + f_s$ solving recursively for f_s.

But, ξ_s now *field dependent* $\rightsquigarrow$ nonintegrable (non-Hamiltonian)!

The charges from twistor space to $\mathscr{I}$

Define surface charge integrals by $H_s := \int_{\partial\Sigma^5} \xi_s \, \mathbf{g} \wedge D^3 Z$ to give:

$$H_{-1} = \int_{\mathcal{S}} d^2z\, \tau_{-1} \mathcal{Q}_{-1},$$

$$H_0 = \int_{\mathcal{S}} d^2z\, \tau_0 (\mathcal{Q}_0 - u\bar{\eth}\mathcal{Q}_{-1}),$$

$$H_1 = \int_{\mathcal{S}} d^2z\, \tau_1 \left(\mathcal{Q}_1 - u\bar{\eth}\mathcal{Q}_0 + \frac{u^2}{2}\bar{\eth}^2\mathcal{Q}_{-1} - 2\mathcal{Q}_{-1}\partial_u^{-1}\bar{\sigma} \right),$$

$$H_2 = \int_{\mathcal{S}} d^2z\, \tau_2 \left(\mathcal{Q}_2 - u\bar{\eth}\mathcal{Q}_1 + \frac{u^2}{2}\bar{\eth}\mathcal{Q}_0 - \frac{u^3}{6}\bar{\eth}^3\mathcal{Q}_{-1} - 3\mathcal{Q}_0\partial_u^{-1}\bar{\sigma} + 3\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-2}\bar{\sigma} + 2\bar{\eth}\left(\mathcal{Q}_{-1}\partial_u^{-1}(u\bar{\sigma}) \right) \right),$$

$$H_3 = \int_{\mathcal{S}} d^2z\, \tau_3 \left(\mathcal{Q}_3 - u\bar{\eth}\mathcal{Q}_2 + \frac{u^2}{2}\bar{\eth}^2\mathcal{Q}_1 - \frac{u^3}{6}\bar{\eth}^3\mathcal{Q}_0 + \frac{u^4}{24}\bar{\eth}^4\mathcal{Q}_{-1} - 4\mathcal{Q}_1\partial_u^{-1}\bar{\sigma} + 3\bar{\eth}\left(\mathcal{Q}_0\partial_u^{-1}(u\bar{\sigma}) \right) + 4\bar{\eth}\mathcal{Q}_0\partial_u^{-2}\bar{\sigma} + 8\mathcal{Q}_{-1}\partial_u^{-1}(\bar{\sigma}\partial_u^{-1}\bar{\sigma}) - 3\bar{\eth}\left(\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-2}(u\bar{\sigma}) \right) - \bar{\eth}^2\left(\mathcal{Q}_{-1}\partial_u^{-1}(u^2\bar{\sigma}) \right) - 4\bar{\eth}\mathcal{Q}_{-1}\partial_u^{-3}\bar{\sigma} \right).$$

Conclusions & discussion

Conclusions:

- Penrose nonlinear graviton $\rightsquigarrow$:
  $\{\text{SD gravitons}\} = \{Lw_{1+\infty}\}$.
- Twistor action yields charge & charge algebra.
- Integrability $\rightsquigarrow$ recursion operator and hierarchies for SD sector. SDYM [M. & Woodhouse 1996] and SD gravity [Dunajski & M. 2001/2]
- Asymptotic twistor space's Penrose transform gives charges at $\mathscr{I}$ $\leftrightarrow$ improving [Friedel, Pranzetti, Raclariu][Geiller].

Where does take us?

- Celestial symmetries are symmetries of SD sector.
- Acts locally on twistor space.
- $\leftrightarrow$ 3d analogue of Virasoro for integrable celestial CFT_3s.
- Are these symmetries of S-matrix?
- Connections with worldsheet amplitude formulae?

Further questions

- Ward's Λ extension of Penrose's nonlinear graviton underpins Taylor-Zhu Λ-extension of $L\mathfrak{ham}(\mathbb{C}^2)$ at $\Lambda = 0$.
  [Bittleston, Bogna, Heuveline, Kmec,M., Skinner, 2403.18011].
- For $\Lambda \neq 0$ use 'Heaven on earth' construction [Lebrun 1983].
- Analogue of Virasoro for honest integrable CFT_3s!
- Use full twistor actions beyond SD sector cf. [M.,Boels, Sharma, Skinner].
- Connections with conventional $\mathrm{AdS}_4/\mathrm{CFT}_3$ e.g., ABJM?

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