

Title: Lecture - Amplitudes a

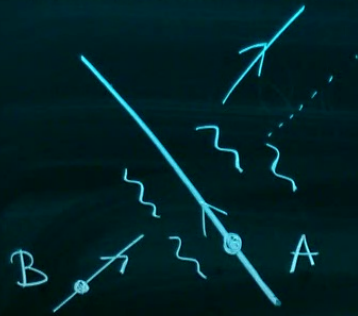
Speakers:

Collection: Celestial Holography Summer School 2024

Date: July 22, 2024 - 2:30 PM

URL: <https://pirsa.org/24070005>

SCATTERING AMPLITUDES AND TIME MACHINES



time delay/advance



high-energy scattering



fixed by symmetry

REFERENCES:

- Camanho, Edelstein, Maddacena, Zhiboedov, "Causality Constraints ..."
[1407.5597]
- Giddings, "The gravitational S-matrix"
[1105.2036]
- Di Vecchia, Heissenberg, Russo, Veneziano, "The gravitational eikonal"
[2306.16488]
- SM, "Physics of the Analytic S-Matrix"
[2306.05395]



Lightcone coords. $ds^2 = dx^+ dx^- - d\vec{x}^2$



Momenta: $p_A \sim (0, p^-, \vec{0})$

$$p_B \sim (p^+, 0, \vec{0})$$

$$p_A - p_{A'} = (0, 0, \vec{q})$$

Kinematic invariants: $p_A^2 = p_B^2 = 0$

$$s = (p_A + p_B)^2 = p^+ p^- > 0$$

$$t = (p_A - p_{A'})^2 = -\vec{q}^2 < 0$$

$$-d\vec{x}^2$$

High-energy limit (Regge/
eikonal/classical)

$$p^+ \gg |\vec{q}| \Leftrightarrow s \gg -t$$

$$p_B^2 = 0$$

$$(p_A + p_B)^2 = p^+ p^- \rightarrow 0$$

$$(p_A - p_B)^2 = -\vec{q}^2 < 0$$

REFERENCES:

- Camanho, Edelstein, Maldacena, Zhiboedov, "Causality Constraints on Gravitational Scattering" [140]
- Giddings, "The graviton" [110]
- Di Vecchia, Heissenberg, Russo, Veneziano, "The graviton" [23]
- SM, "Physics of the Anomalous" [22]

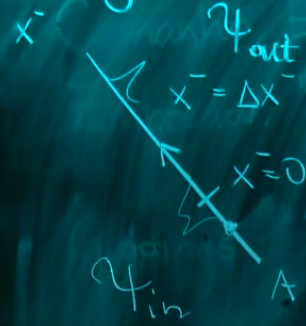
High-energy limit (Regge/
eikonal/classical)

$$p^+ \gg |\vec{q}| \Leftrightarrow s \gg -t$$

$$p^- > 0$$

$$q^2 < 0$$

REFERENCES
Signal model in 1D:



Erstein, Mottaiana, "Causality constraints" [1007.5577]
The gravitational field [1208.2281]
Russo
The gravitational field [1007.5577]

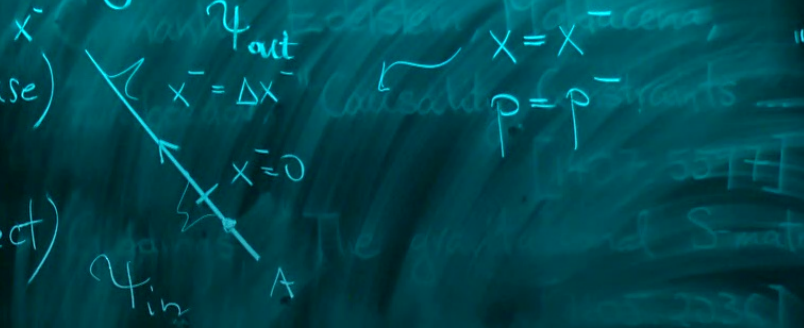
Causality

If $\tilde{\Psi}_{in}(x) = 0 \quad x < 0$ (cause)

then $\tilde{\Psi}_{out}(x) = 0 \quad x < 0$ (effect)

REFERENCES

Signal model in 1D:



$$\Psi_{out}(p) = S(p) \Psi_{in}(p)$$

scattering ampe as a fn of p
(p+, q fixed)

$$\begin{aligned}
 \tilde{\Psi}_{\text{out}}(x) &= \# \int dp e^{-ipx} S(p) \tilde{\Psi}_{\text{in}}(p) \\
 &= \# \int dp dx' dx'' e^{-ip(x-x'-x'')} \\
 &\quad \times \tilde{S}(x'') \tilde{\Psi}_{\text{in}}(x') \\
 &= \# \int dx' \tilde{S}(x-x') \tilde{\Psi}_{\text{in}}(x) \\
 &= 0 \text{ if } x-x' = \Delta x < 0.
 \end{aligned}$$

Causality
 If $\tilde{\Psi}_{\text{in}}(x) = 0$
 then $\tilde{\Psi}_{\text{out}}(x) = 0$

$$S(p) \tilde{\Psi}_{in}(p)$$

$$= e^{-ip(x-x'-x'')} \tilde{S}(x'') \tilde{\Psi}_{in}(x')$$

$$\tilde{\Psi}_{in}(x)$$

if $x-x' = \Delta x < 0$

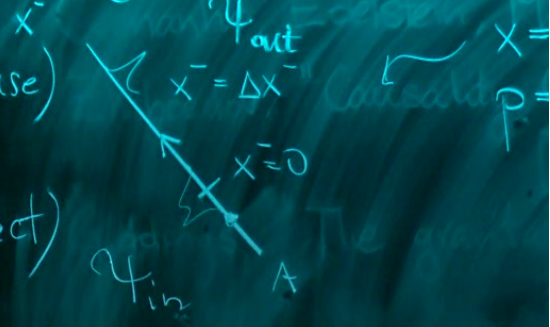
Causality

If $\tilde{\Psi}_{in}(x) = 0 \quad x < 0$ (cause)
 then $\tilde{\Psi}_{out}(x) = 0 \quad x < 0$ (effect)

$$S(p) = \int_0^{\infty} d(\Delta x) e^{ip\Delta x} \tilde{S}(\Delta x)$$

↑ causality

REFERENCES
 Signal model in 1D

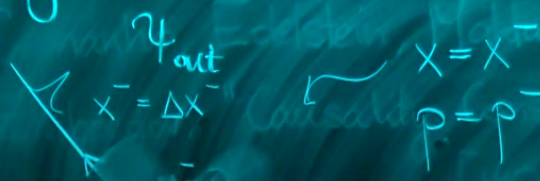


$$\tilde{\Psi}_{out}(p) = S(p) \tilde{\Psi}_{in}(p)$$

scattering amp^{le} as $\tilde{S}(p)$

causality

If $\tilde{\Psi}_{in}(x) = 0 \quad x < 0$ (cause)
 then $\tilde{\Psi}_{out}(x) = 0 \quad x < 0$ (effect)



$$S(p) = \# \int_0^{\infty} d(\Delta x) e^{ip\Delta x} \tilde{S}(\Delta x)$$

(x')
 (x)
 $= \Delta x < 0$

$\Rightarrow S(p)$ is analytic in UHP

causality

change

$p \mapsto p + ik$
 $e^{ip\Delta x} \mapsto e^{ip\Delta x} e^{-k\Delta x}$
 exponen. suppr. if $k > 0$

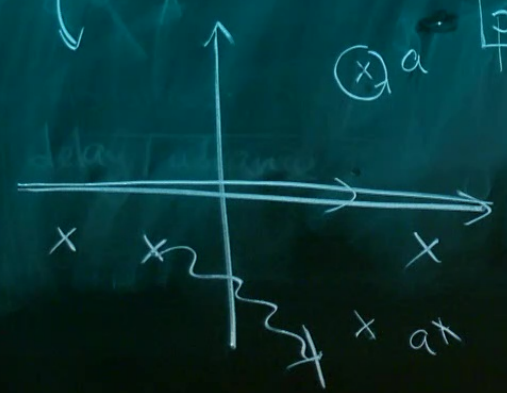
SCATTERING AMPLITUDES IN TIME-MONOMES

Unitarity $|S(p)| = 1$ for real p .

analytic

$$S(p) = e^{2\delta(p)}$$

↖ phase shift



$$S(a) = \frac{2i \operatorname{Im} a}{2\pi i} \int_{p=a}^{\infty} \frac{S(p) dp}{(p-a)(p-a^*)}$$

$$= \frac{\operatorname{Im} a}{\pi} \int_{-\infty}^{\infty} \frac{S(p) dp}{(p-a)(p-a^*)}$$

$\tilde{\psi}_{out}(x) =$

$S(p) = e^{2\delta(p)}$ ← phase shift.

$S(a) = \frac{2i \operatorname{Im} a}{2\pi i} \oint_{p=a} \frac{S(p) dp}{(p-a)(p-a^*)}$

$= \frac{\operatorname{Im} a}{\pi} \int_{-\infty}^{\infty} \frac{S(p) dp}{(p-a)(p-a^*)}$

$|S(a)| \leq \frac{\operatorname{Im} a}{\pi} \int_{-\infty}^{\infty} \frac{|S(p)| dp}{|(p-a)(p-a^*)|}$

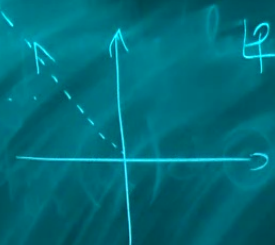
Additional notes on the right side of the board:

$\psi_{\text{out}}(x) = \# \int dp dx' d$
 $= \# \int dx'$

On the left side, there are some diagrams and labels:

$\otimes a$ $\frac{p}{p}$
 x
 x a^*

Ex: $S(p) = e^{ip\#}, e^{-ip\#}, e^{ip^2\#}$



$S(p) = e^{i c_1 p + c_2 p^2}$

causal if

$c_1 \geq 0$

$c_2 \in [-1, 1]$

causality

$\Rightarrow S(p)$ is analytic in U^+

1

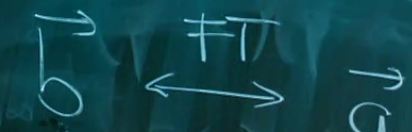
Back to 4D

Ex.

$$iM(s,t) = \text{diagrams}$$

The equation shows the imaginary part of the scattering amplitude $iM(s,t)$ as a sum of four diagrams. The first diagram consists of two parallel horizontal lines with arrows pointing to the right. The second diagram shows two horizontal lines connected by a vertical wavy line, with a downward-pointing arrow labeled \vec{q} next to it. The third diagram shows two horizontal lines connected by two vertical wavy lines. The fourth diagram shows two horizontal lines connected by a vertical wavy line that has a small loop at its top and bottom vertices.

Easiest in the impact param space



$$S(s,b) = \int d^2\vec{q} e^{i\vec{b}\cdot\vec{q}} iM(s,t) = \mathcal{O}(s)$$

$P_A = p^+$

$$= \frac{\#}{S} \int d^2 \vec{q} e^{i \vec{b} \cdot \vec{q}} \left(\frac{p^+}{p^-} \delta^2(\vec{q}) \right) = 1$$

$P_B = p^-$



$$\mathcal{E}^{\mu_1 \mu_2 \dots \mu_J} = \mathcal{E}^{\mu_1} \mathcal{E}^{\mu_2} \dots \mathcal{E}^{\mu_J} \left(\frac{S^J}{-q^2 - m^2} \right) \sim i S^{J-1} f(b)$$

P_B

$$2 \# \sum_{\text{pol.}} \frac{(\mathcal{E} \cdot p_A)^J (\mathcal{E} \cdot p_B)^J}{t - m^2} \sim \pm 2 \# \frac{S^J}{-q^2 - m^2}$$

Back to 4D

$$iM(s,t) = \begin{array}{c} \text{---} \rightarrow \\ \text{---} \rightarrow \\ \vdots \end{array} + \begin{array}{c} \text{---} \rightarrow \\ \text{---} \rightarrow \\ \vdots \end{array} + \begin{array}{c} \text{---} \rightarrow \\ \text{---} \rightarrow \\ \vdots \end{array} + \begin{array}{c} \text{---} \rightarrow \\ \text{---} \rightarrow \\ \vdots \end{array} + \begin{array}{c} \text{---} \rightarrow \\ \text{---} \rightarrow \\ \vdots \end{array} + \dots$$

$$\begin{array}{c} \text{---} \rightarrow \\ \text{---} \rightarrow \\ \vdots \end{array} + \begin{array}{c} \text{---} \rightarrow \\ \text{---} \rightarrow \\ \vdots \end{array} = \frac{1}{2} \left[i s^{J-1} f(b) \right]^2$$

Erkonal exponentiation:

$$S(s,b) = 1 + \sum_{L=0}^{\infty} \frac{1}{(L+1)!} \left(\dots \right)$$

Ex

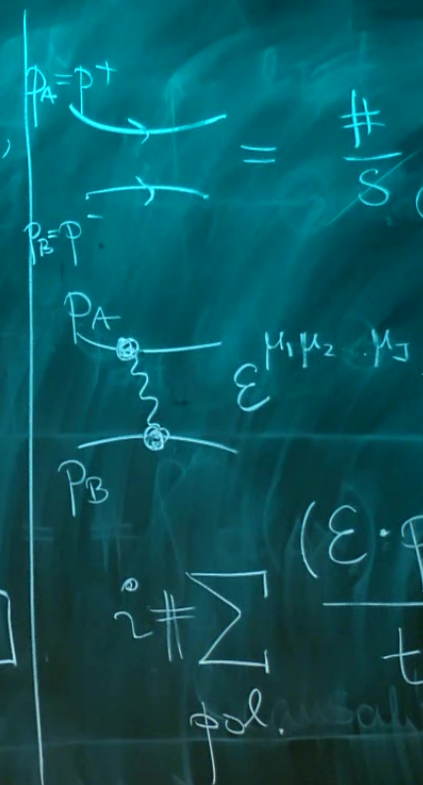
Ex: $S(p) = e^{ip\#}, e^{-ip\#}, e^{ip^2\#}$

$S(p) = e^{i c_1 p + c_2 p^2}$

causal if

$c_1 \geq 0$

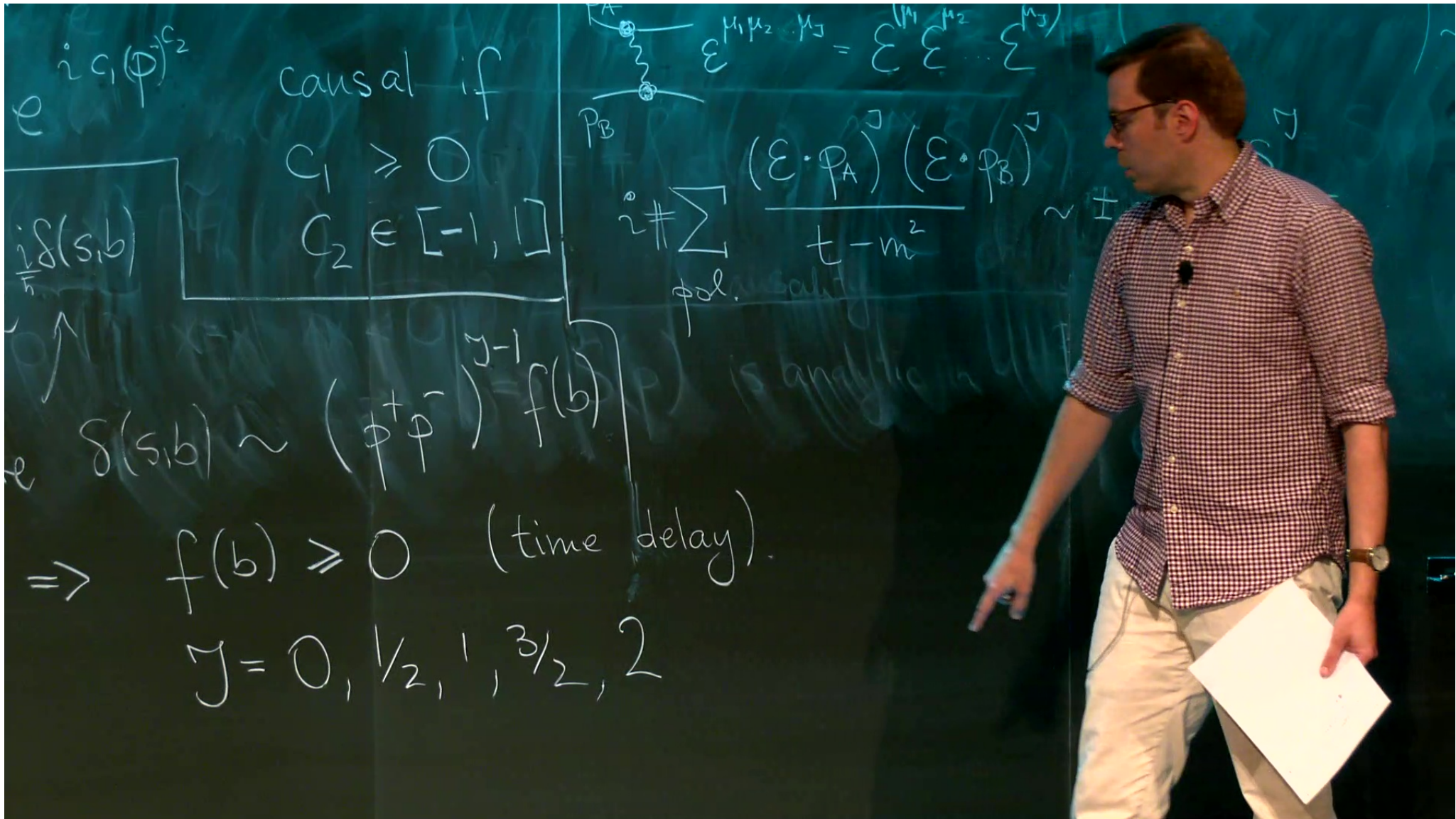
$c_2 \in [-1, 1]$



$\sum_{l=0}^{\infty}$

$\frac{1}{(L+1)!} \left(\frac{1}{h} s^{L-1} f(b) \right)^{L+1} = e^{i\delta(s,b)}$

where $\delta(s,b)$



$$e^{i\alpha(\phi)^{s_2}}$$

causal if

$$C_1 \geq 0$$

$$C_2 \in [-1, 1]$$

$$\frac{1}{i} \delta(s, b)$$

$$e^{\delta(s, b)}$$

$$\sim \left(\phi^+ \phi^- \right)^{\gamma-1} f(b)$$

$$\Rightarrow f(b) \geq 0 \quad (\text{time delay})$$

$$\gamma = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$$

$$\epsilon^{\mu_1 \mu_2 \dots \mu_J} = \epsilon^{\mu_1} \epsilon^{\mu_2} \dots \epsilon^{\mu_J}$$

$$i \# \sum_{\text{pol.}} \frac{(\epsilon \cdot p_A)^J (\epsilon \cdot p_B)^J}{t - m^2} \sim \pm$$