Title: Lecture - Amplitudes a

Speakers:

Collection: Celestial Holography Summer School 2024

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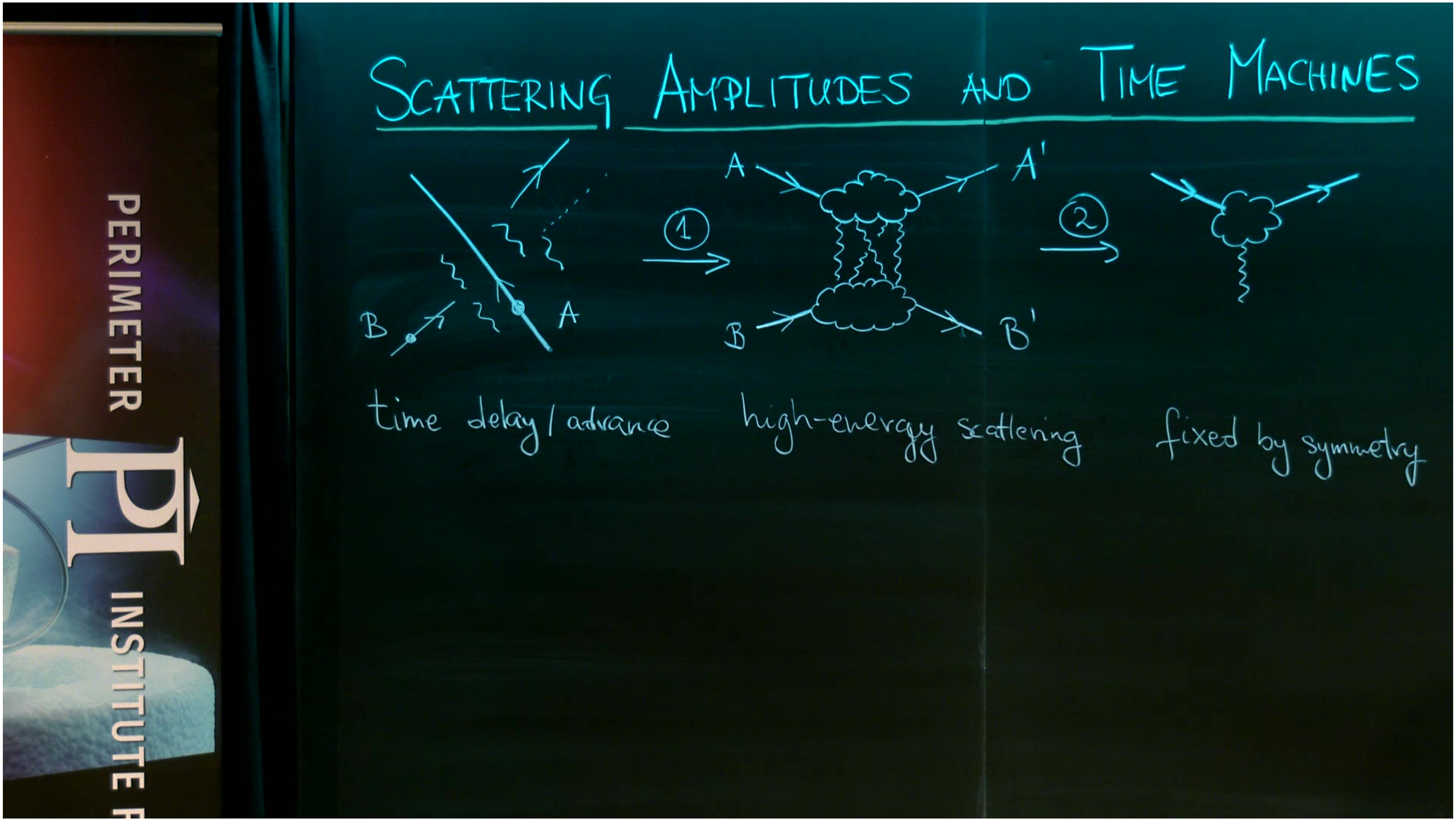

PERIMETER
INSTITUTE
SCATTERING AMPLITUDES AND TIME MACHINES
B
A
1
A
A'
B
B'
2
time delay / advance
high-energy scattering
fixed by symmetry

REFERENCES:

- Camanho, Edelstein, Maldacena, Zhiboedov, "Causality Constraints..." [1407.5597]
- Giddings, "The gravitational S-matrix" [1105.2036]
- Di Vecchia, Heissenberg, Russo, Veneziano, "The gravitational eikonal" [2306.16488]
- SM, "Physics of the Analytic S-Matrix" [2306.05395]

Lightcone coords. $ds^2 = dx^+ dx^- - d\vec{x}^2$.

Momenta: $p_A \sim (0, p^-, \vec{0})$

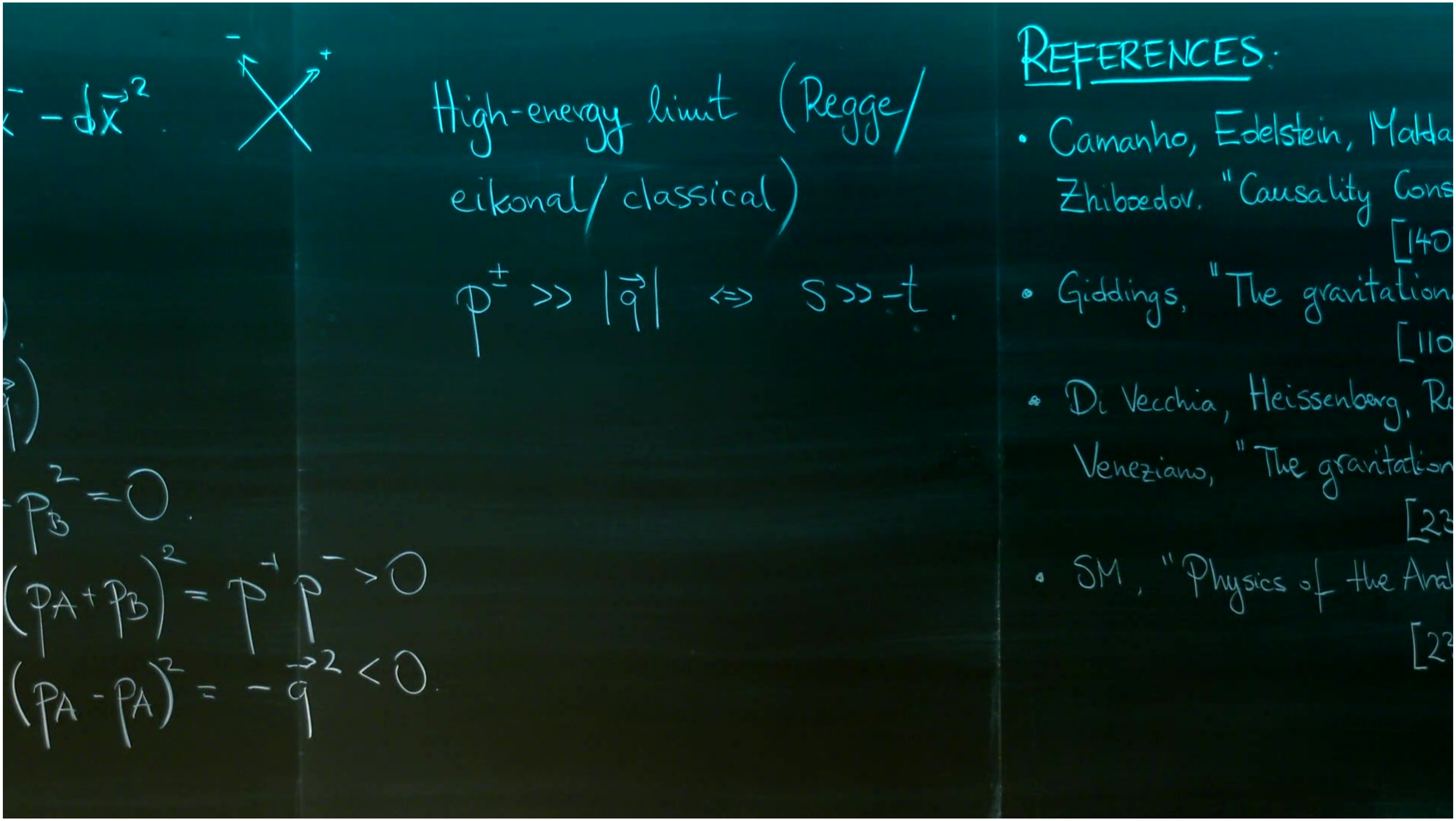
$p_B \sim (p^+, 0, \vec{0})$.

$p_A - p_{A'} = (0, 0, \vec{q})$

Kinematic invariants: $p_A^2 = p_B^2 = 0$.

$s = (p_A + p_B)^2 = p^+ p^- > 0$

$t = (p_A - p_A)^2 = -\vec{q}^2 < 0$.


High-energy limit (Regge/
eikonal/ classical)
p^± >> |q⃗| <=> s >> -t.
REFERENCES:
• Camanho, Edelstein, Malda
Zhiboedov, "Causality Cons
[140
• Giddings, "The gravitation
[110
• Di Vecchia, Heissenberg, R
Veneziano, "The gravitation
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• SM, "Physics of the Anal
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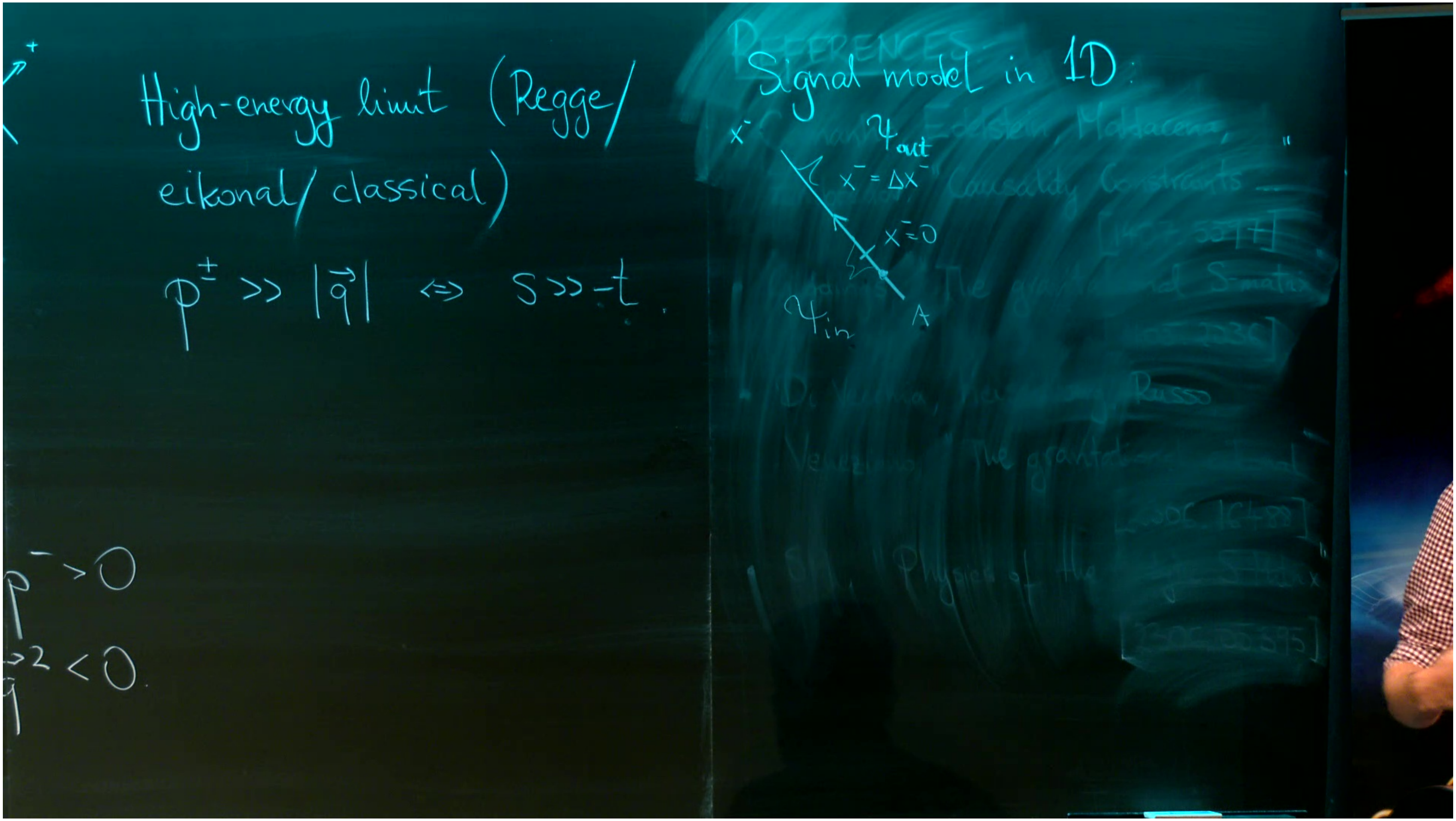
(pA + pB)^2 = p^+ p^- > 0
(pA - pA)^2 = - q⃗^2 < 0.

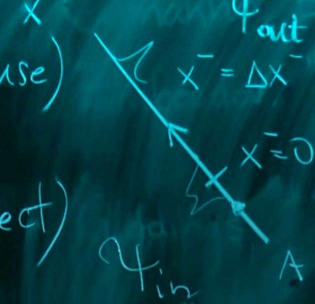

High-energy limit (Regge/ eikonal/ classical)
$p^{\pm} \gg |\vec{q}| \Leftrightarrow s \gg -t$.
$p^{-} > 0$
$\vec{q}^{\,2} < 0$.
Signal model in 1D:
x^-
ψ_{out}
$x^- = \Delta x^-$
$x^- = 0$
ψ_{in}
A

Causality

If $\tilde{\psi}_{in}(x) = 0 \quad x < 0$ (cause)

then $\tilde{\psi}_{out}(x) = 0 \quad x < 0$ (effect)

Signal model in 1D:



$x = x^-$

$p = p^-$

$$\psi_{out}(p) = S(p)\, \psi_{in}(p)$$

scattering ampl. as a fn of p^-

(p^+, $\vec{q}$ fixed)

$$\# > 0$$

$$\tilde{\psi}_{out}(x) = \# \int dp \, e^{-ipx} \, S(p) \, \psi_{in}(p)$$

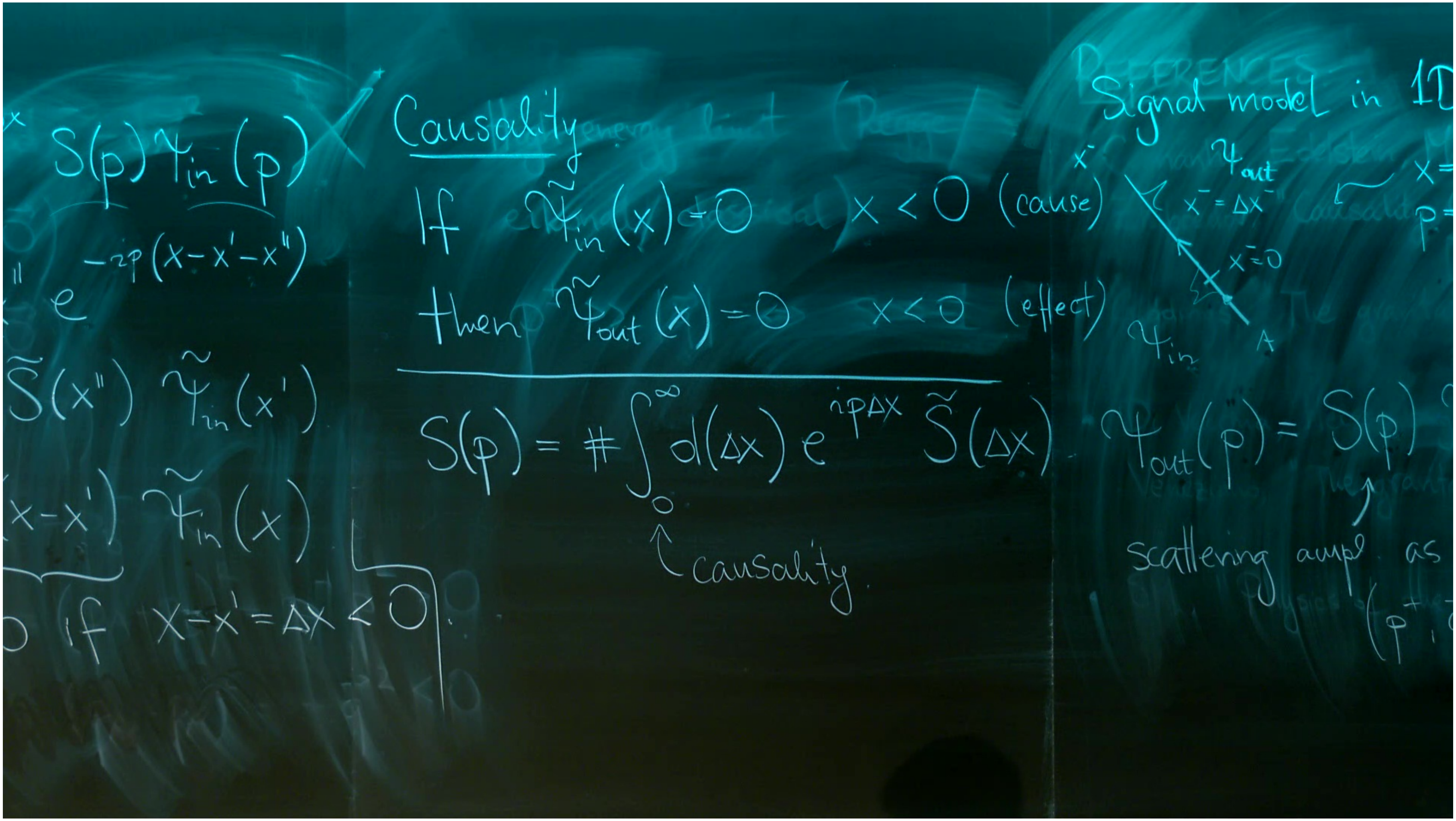
$$= \# \int dp \, dx' \, dx'' \, e^{-ip(x - x' - x'')} \times \tilde{S}(x'') \, \tilde{\psi}_{in}(x')$$

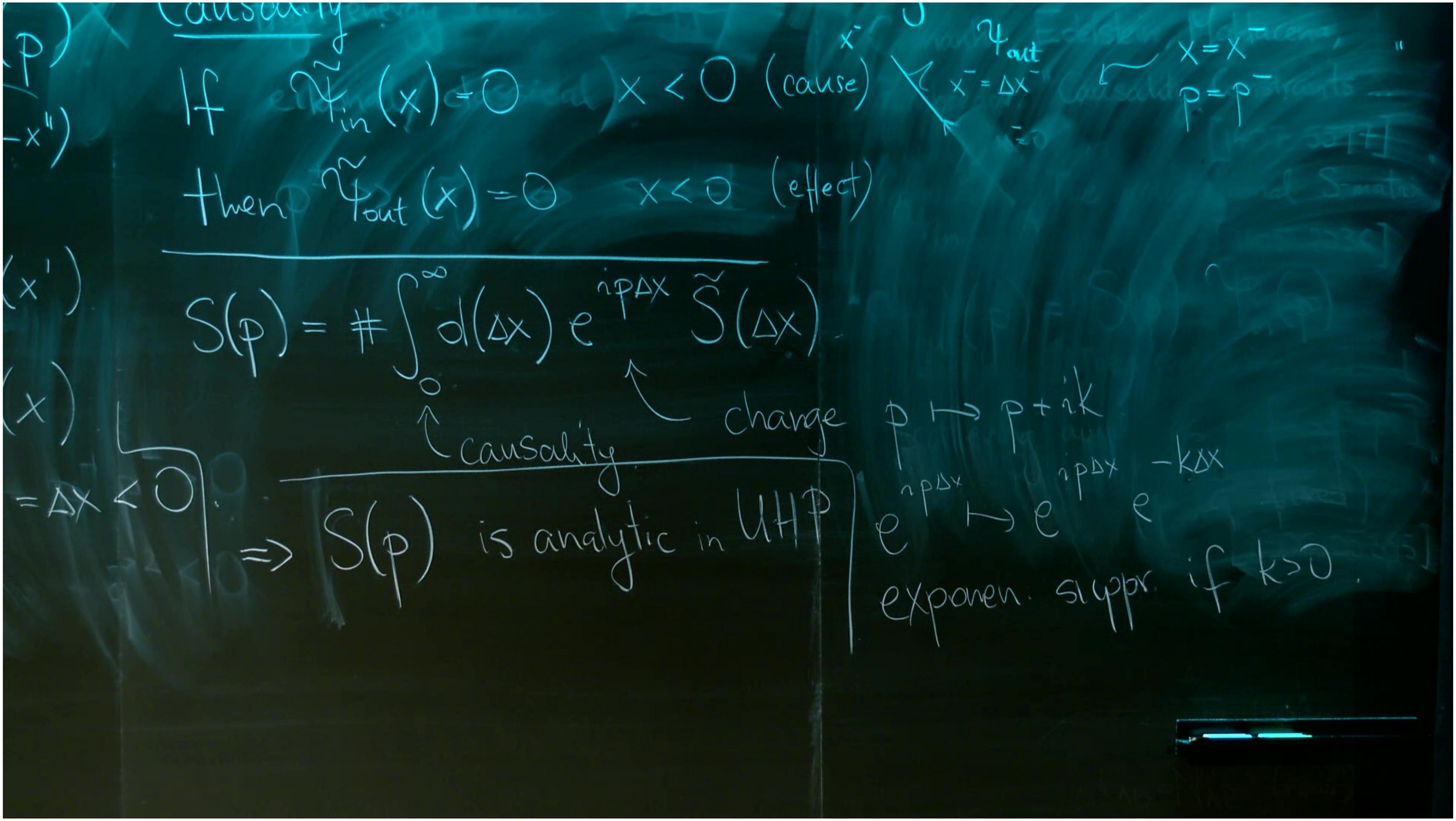
$$= \# \int dx' \, \underbrace{\tilde{S}(x - x')}_{=0 \text{ if } x - x' = \Delta x < 0} \, \tilde{\psi}_{in}(x)$$

Causality

If $\tilde{\psi}_{in}(x) = 0$

then $\tilde{\psi}_{out}(x) = 0$


Causality
If $\tilde{\psi}_{in}(x) = 0$ $x < 0$ (cause)
then $\tilde{\psi}_{out}(x) = 0$ $x < 0$ (effect)
$S(p) = \# \int_0^\infty d(\Delta x)\, e^{ip\Delta x}\, \tilde{S}(\Delta x)$
causality
$S(p)\,\psi_{in}(p)$
$e^{-ip(x-x'-x'')}$
$\tilde{S}(x'')\,\tilde{\psi}_{in}(x')$
$\tilde{\psi}_{in}(x)$
if $x - x' = \Delta x < 0$
Signal model in 1D
ψ_{out}
$x^- = \Delta x^-$
$x^- = 0$
ψ_{in}
$\psi_{out}(p) = S(p)$
scattering ampl. as

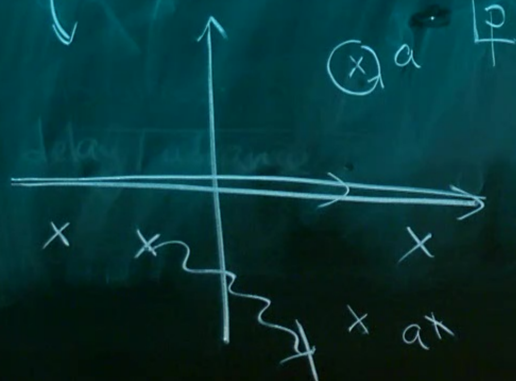

Causality
If $\tilde{\psi}_{in}(x) = 0$ $x < 0$ (cause)
then $\tilde{\psi}_{out}(x) = 0$ $x < 0$ (effect)
ψ_{out}
$x^- = \Delta x^-$
$x = x^-$
$p = p^-$
$S(p) = \# \int_0^\infty d(\Delta x)\, e^{ip\Delta x}\, \tilde{S}(\Delta x)$
causality
change $p \mapsto p + ik$
$= \Delta x < 0$
$\Rightarrow S(p)$ is analytic in UHP
$e^{ip\Delta x} \mapsto e^{ip\Delta x} e^{-k\Delta x}$
exponen. suppr. if $k > 0$.

#

Unitarity. $|S(p)| = 1$ for real p.

analytic

$$S(p) = e^{2i\delta(p)} \leftarrow \text{phase shift.}$$



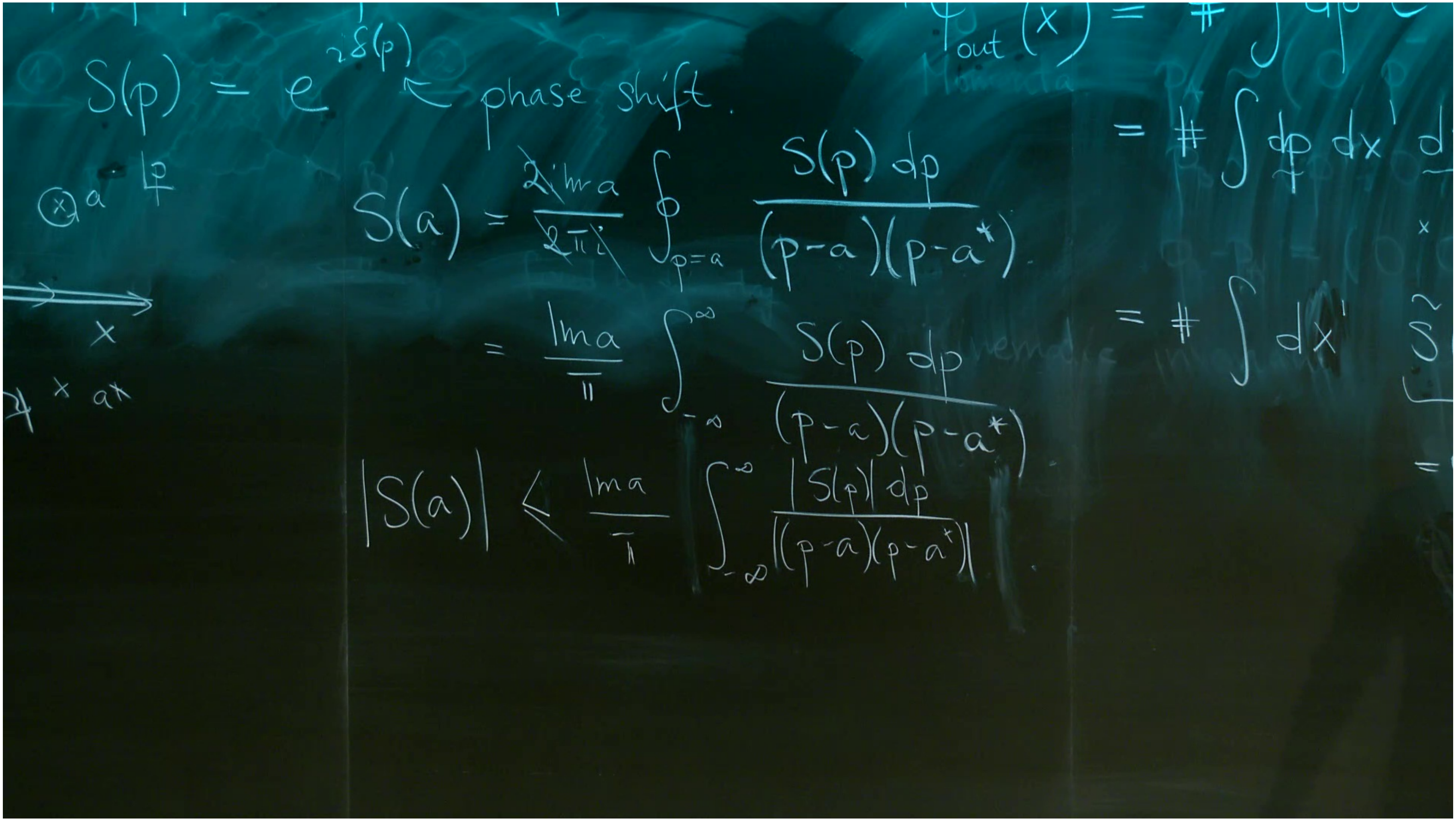
$$S(a) = \frac{\cancel{2i}\,\mathrm{Im}\,a}{\cancel{2\pi i}} \oint_{p=a} \frac{S(p)\,dp}{(p-a)(p-a^*)}.$$

$$= \frac{\mathrm{Im}\,a}{\pi} \int_{-\infty}^{\infty} \frac{S(p)\,dp}{(p-a)(p-a^*)}.$$

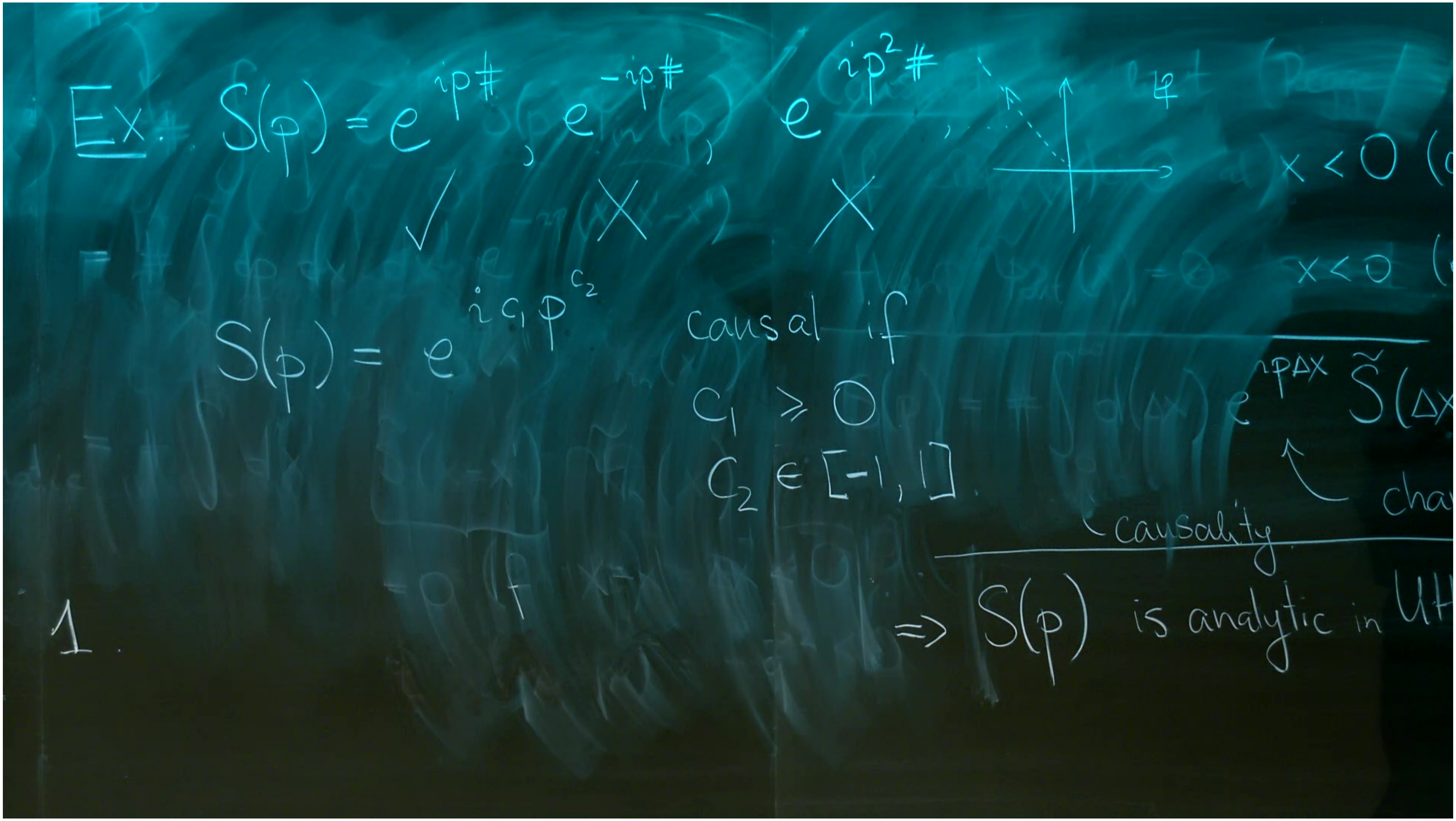
$$\tilde{\psi}_{out}(x) =$$

$$=$$

$$=$$



$S(p) = e^{2\delta(p)}$ phase shift
$\psi_{out}(x) = \# \int dp$
$S(a) = \frac{\cancel{2i}\,\mathrm{Im}\,a}{\cancel{2\pi i}} \oint_{p=a} \frac{S(p)\,dp}{(p-a)(p-a^*)}$
$= \# \int dp\, dx'$
$= \frac{\mathrm{Im}\,a}{\pi} \int_{-\infty}^{\infty} \frac{S(p)\,dp}{(p-a)(p-a^*)}$
$= \# \int dx'\, \tilde{S}$
$|S(a)| \leq \frac{\mathrm{Im}\,a}{\pi} \int_{-\infty}^{\infty} \frac{|S(p)|\,dp}{|(p-a)(p-a^*)|}$


Ex. $S(p) = e^{ip\#}, e^{-ip\#}, e^{ip^2\#}$
$x < 0$
$S(p) = e^{ic_1 p^{c_2}}$
causal if
$c_1 \geq 0$
$c_2 \in [-1, 1]$
$e^{ip\Delta x} \tilde{S}(\Delta x)$
causality
1.
$\Rightarrow S(p)$ is analytic in

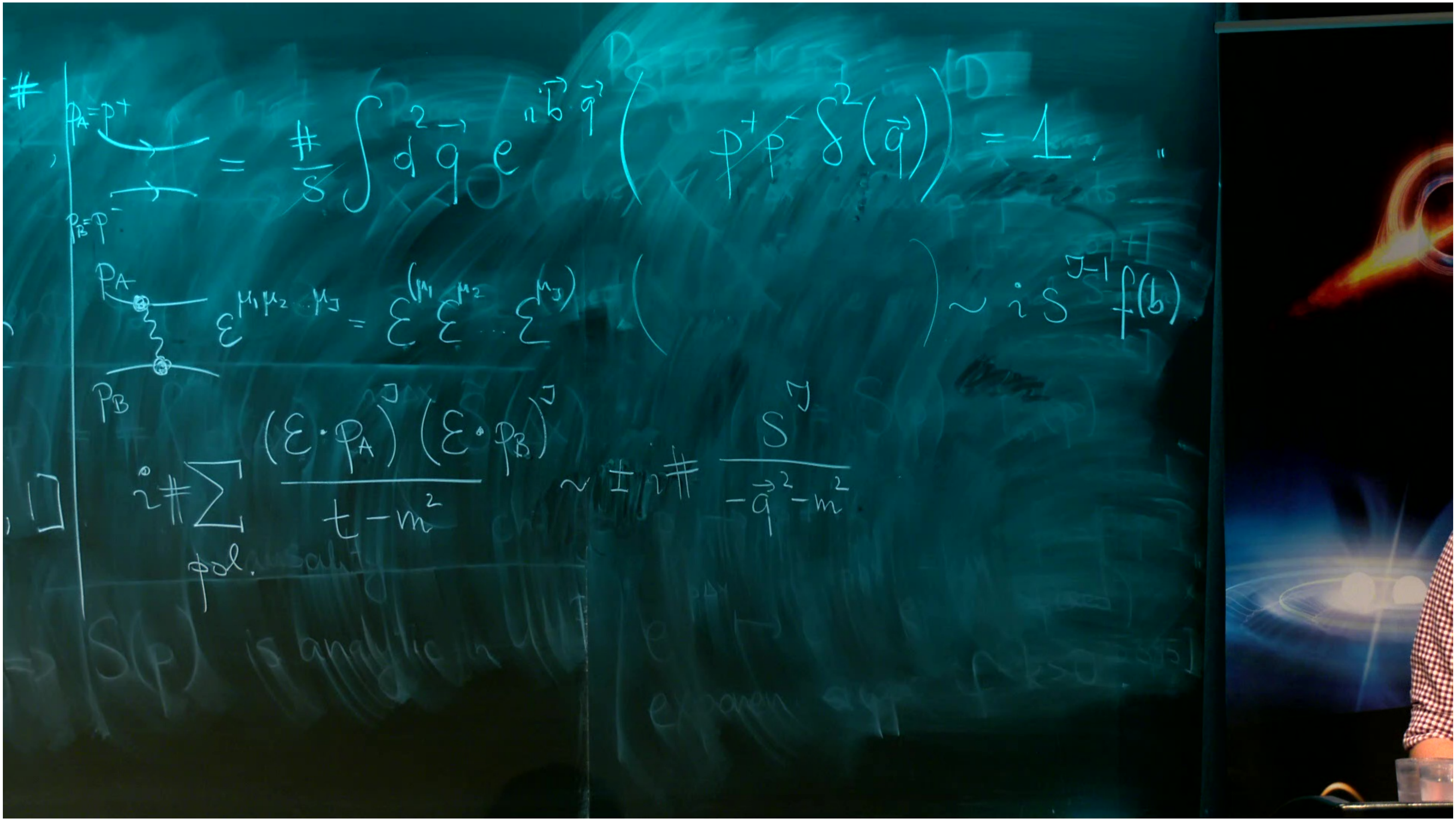
Back to 4D

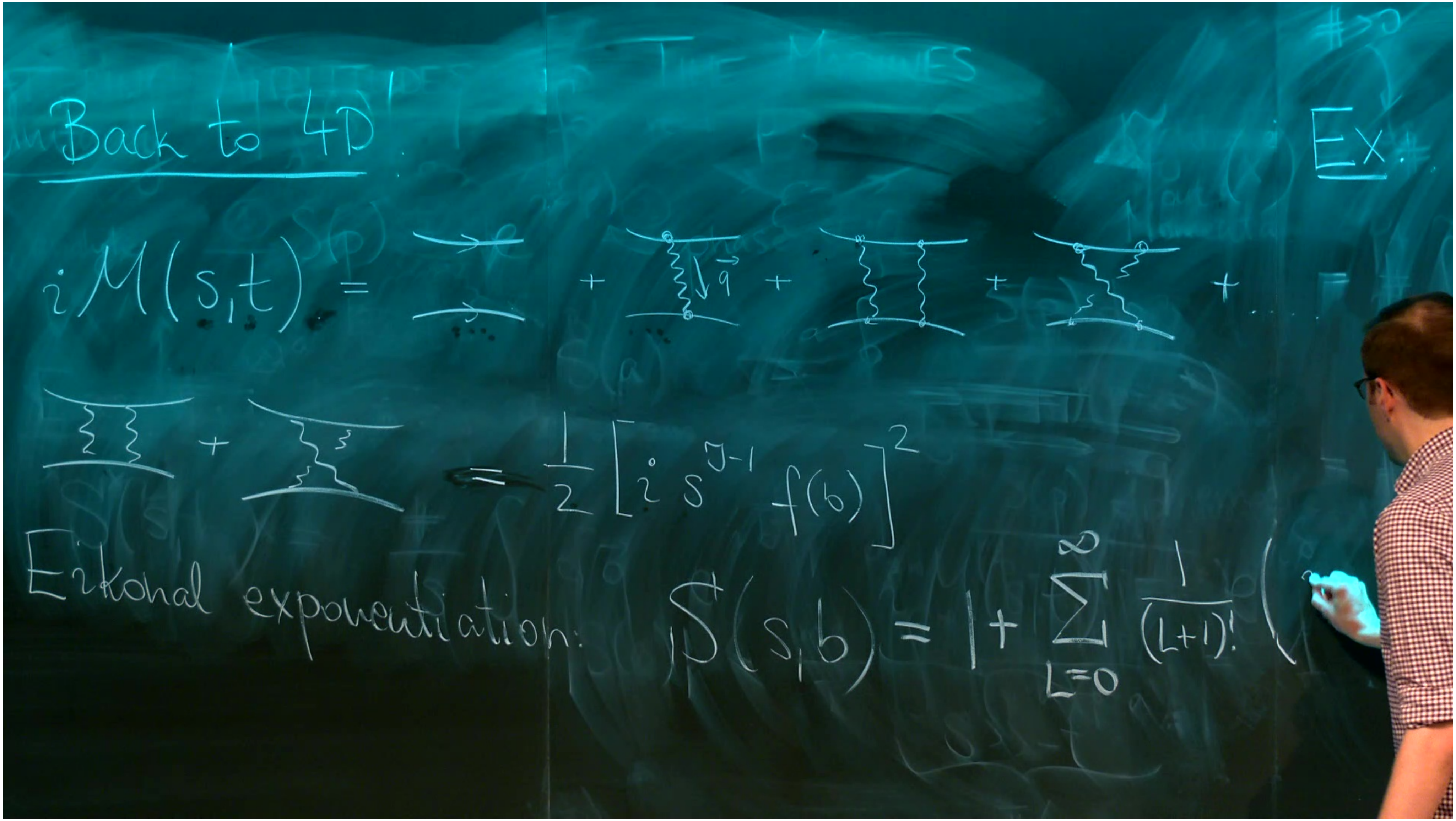
Ex.

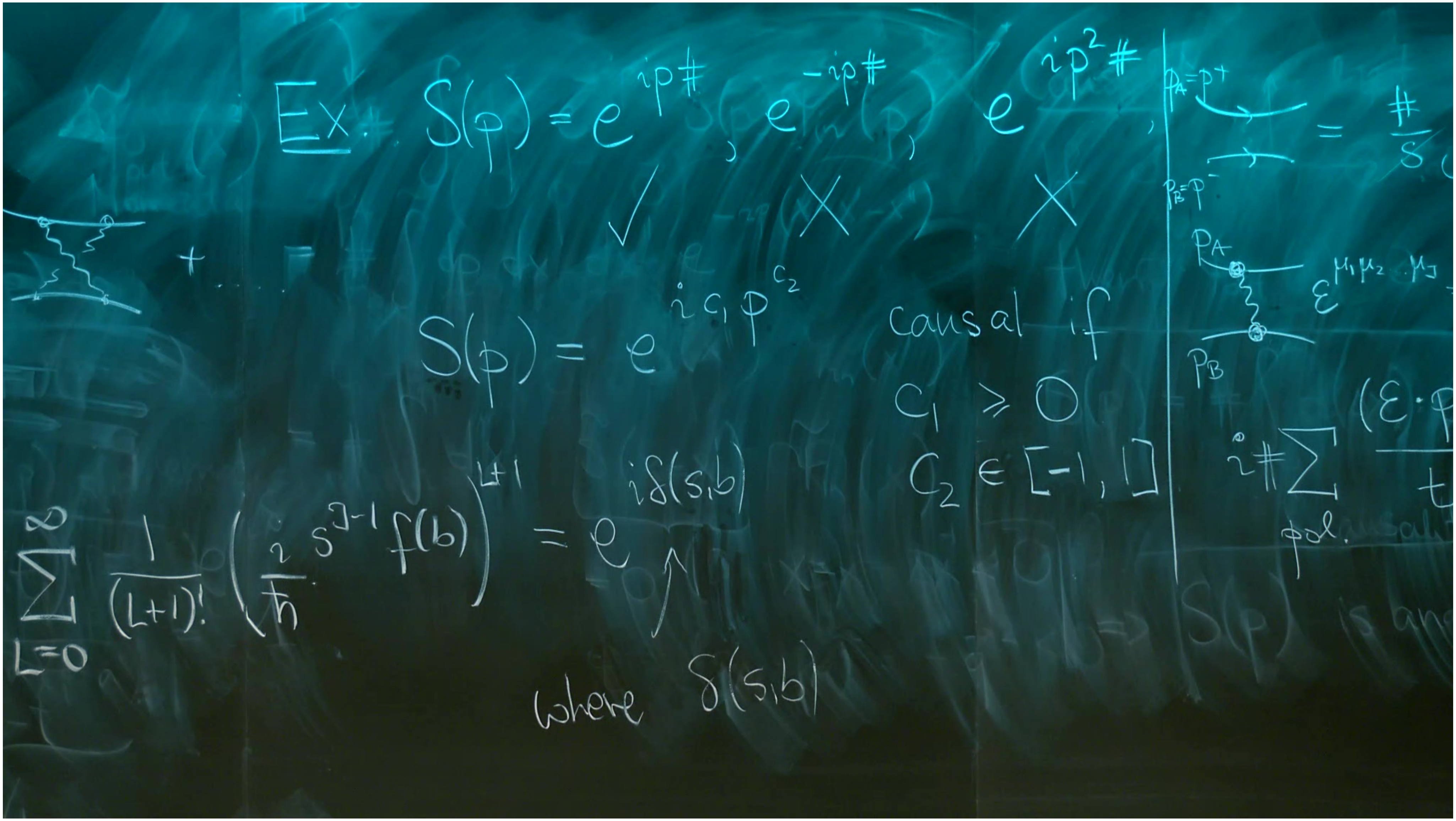
$i\mathcal{M}(s,t) =$  $+ \dots$

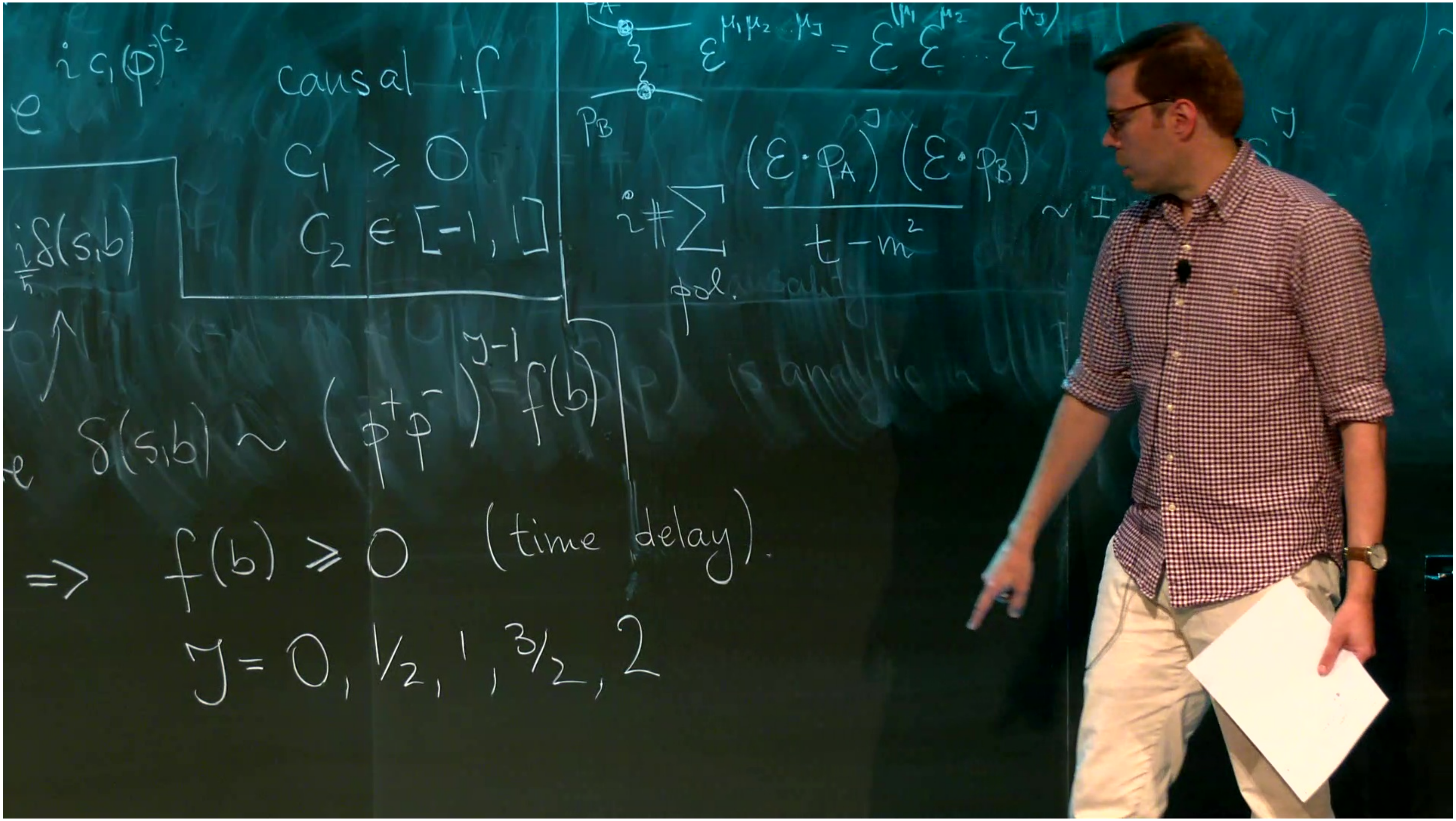
Easiest in the impact param. space $\vec{b} \xleftrightarrow{FT} \vec{q}$

$$S(s,b) = \int d^2\vec{q}\; e^{i\vec{b}\cdot\vec{q}}\; i\mathcal{M}(s,t) = e^{i\delta}$$


$p_A = p^+$
$= \frac{\#}{s}\int d^2\vec{q}\, e^{i\vec{b}\cdot\vec{q}}\left(p^+p^-\delta^2(\vec{q})\right) = 1$
$p_B = p^-$
p_A
$\varepsilon^{\mu_1\mu_2\ldots\mu_J} = \varepsilon^{(\mu_1}\varepsilon^{\mu_2}\ldots\varepsilon^{\mu_J)}$
$\left(\quad\right) \sim i s^{J-1} f(b)$
p_B
$i\#\sum_{pol.} \frac{(\varepsilon\cdot p_A)^J(\varepsilon\cdot p_B)^J}{t-m^2} \sim i\# \frac{s^J}{-\vec{q}^2-m^2}$


Back to 4D
Ex.
iM(s,t) =
q
= 1/2 [i s^{J-1} f(b)]^2
Eikonal exponentiation:
S(s,b) = 1 + Σ_{L=0}^{∞} 1/(L+1)! (


Ex: $S(p) = e^{ip\#}$, $e^{-ip\#}$, $e^{ip^2\#}$,
$S(p) = e^{i c_1 p^{c_2}}$
causal if
$c_1 \geq 0$
$c_2 \in [-1, 1]$
$\sum_{L=0}^{\infty} \frac{1}{(L+1)!} \left(\frac{i}{\hbar} s^{J-1} f(b) \right)^{L+1} = e^{i\delta(s,b)}$
where $\delta(s,b)$
$p_A = p^+$
$p_B = p^-$
$= \frac{\#}{s}$
p_A
p_B
$\varepsilon^{\mu_1 \mu_2 \ldots \mu_J}$
$i\# \sum_{pol}$
$(\varepsilon \cdot p$
t
$S(p)$


causal if
$c_1 \geq 0$
$c_2 \in [-1, 1]$
$\varepsilon^{\mu_1 \mu_2 \ldots \mu_J} = \varepsilon^{(\mu_1}\varepsilon^{\mu_2}\ldots\varepsilon^{\mu_J)}$
$i \sum_{pol} \frac{(\varepsilon \cdot p_A)^J (\varepsilon \cdot p_B)^J}{t - m^2}$
$\delta(s,b) \sim (p^+ p^-)^{J-1} f(b)$
$\Rightarrow f(b) \geq 0$ (time delay).
$J = 0, 1/2, 1, 3/2, 2$