Title: Lecture - Amplitudes a

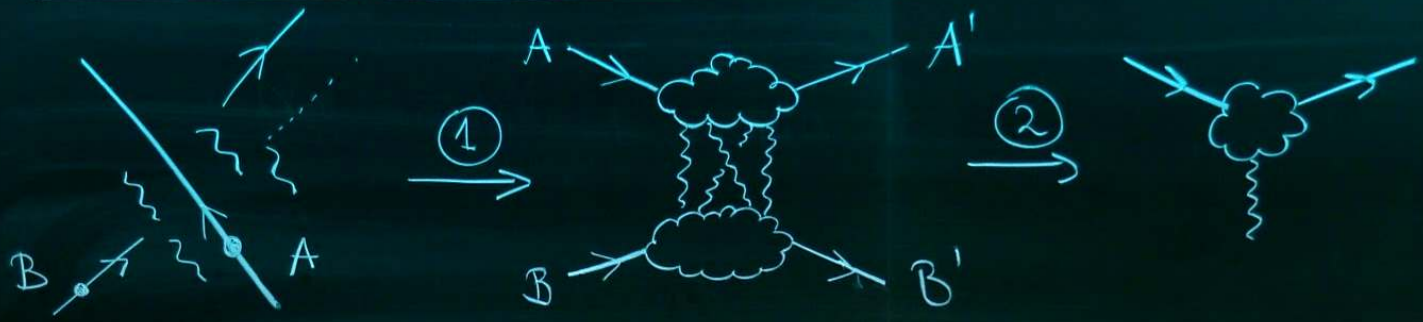
Speakers:

Collection: Celestial Holography Summer School 2024

Date: July 22, 2024 - 2:30 PM

URL: https://pirsa.org/24070005

Scattering Amplitudes and Time Machines



time delay / advance high-energy scattering fixed by symmetry

PERIMETER INSTITUTE

REFERENCES:

- Camanho, Edelstein, Maldacena, Zhiboedov. "Causality Constraints ..." [1407.5597]
- Giddings, "The gravitational S-matrix" [1105.2036]
- Di Vecchia, Heissenberg, Russo, Veneziano, "The gravitational eikonal" [2306.16488]
- SM, "Physics of the Analytic S-Matrix" [2306.05395]

Lightcone coords. $ds^2 = dx^+ dx^- - d\vec{x}^2$

Momenta: $p_A \sim (0, p^-, \vec{0})$

$p_B \sim (p^+, 0, \vec{0})$

$p_A - p_{A'} = (0, 0, \vec{q})$

Kinematic invariants: $p_A^2 = p_B^2 = 0$.

$s = (p_A + p_B)^2 = p^+ p^- > 0$

$t = (p_A - p_{A'})^2 = -\vec{q}^2 < 0$.

$-d\vec{x}^2$

High-energy limit (Regge/ eikonal/ classical)

$p^{\pm} \gg |\vec{q}| \iff s \gg -t$

$\vec{q})$

$p_B^2 = 0$

$(p_A + p_B)^2 = p^+ p^- > 0$

$(p_A - p_A)^2 = -\vec{q}^2 < 0$

REFERENCES:

- Camanho, Edelstein, Malda Zhiboedov. "Causality Cons [140
- Giddings, "The gravitation [110
- Di Vecchia, Heissenberg, R Veneziano, "The gravitation [23
- SM, "Physics of the Ana [2

High-energy limit (Regge/ eikonal/ classical)

$p^{\pm} \gg |\vec{q}| \Leftrightarrow s \gg -t$

$p^- > 0$

$q^2 < 0$

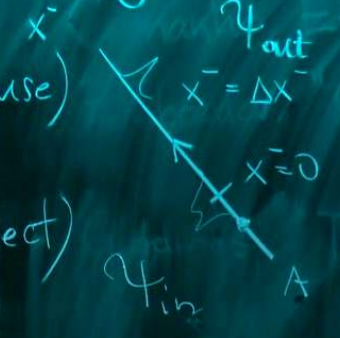
Signal model in 1D:

Causality

If $\tilde{\psi}_{in}(x) = 0 \quad x < 0$ (cause)

then $\tilde{\psi}_{out}(x) = 0 \quad x < 0$ (effect)

Signal model in 1D



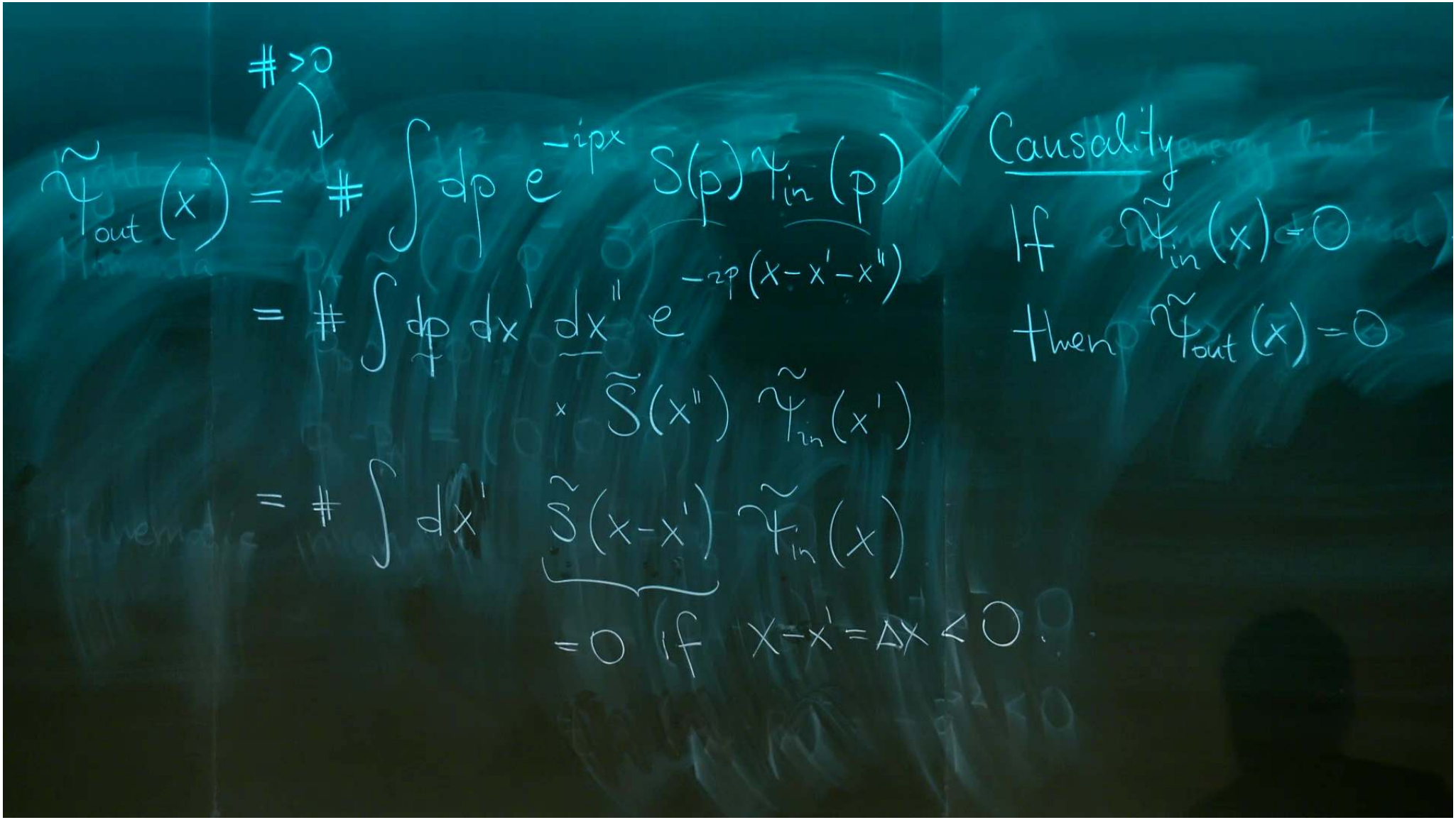
$x = x^-$

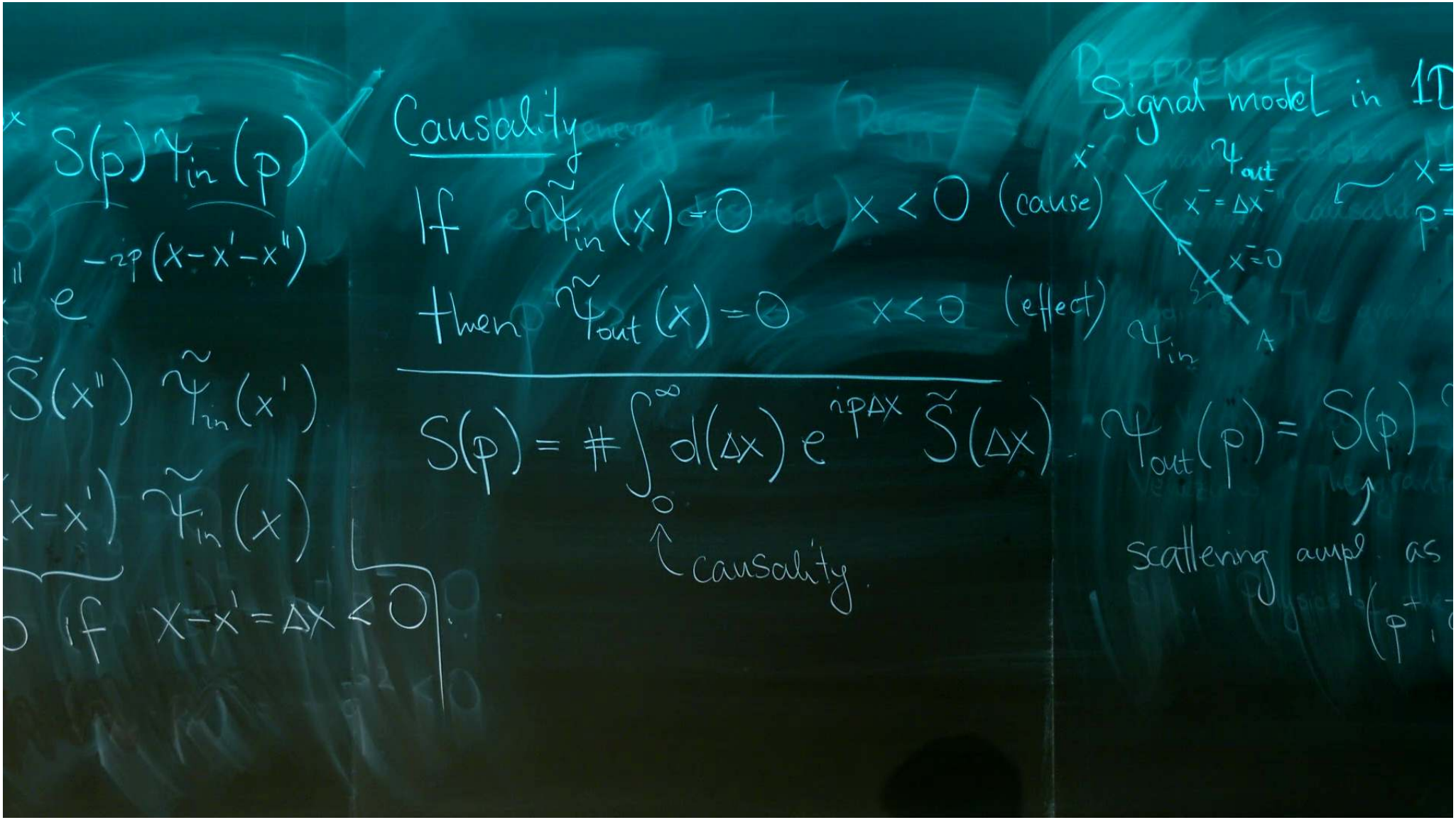
$p = p^-$

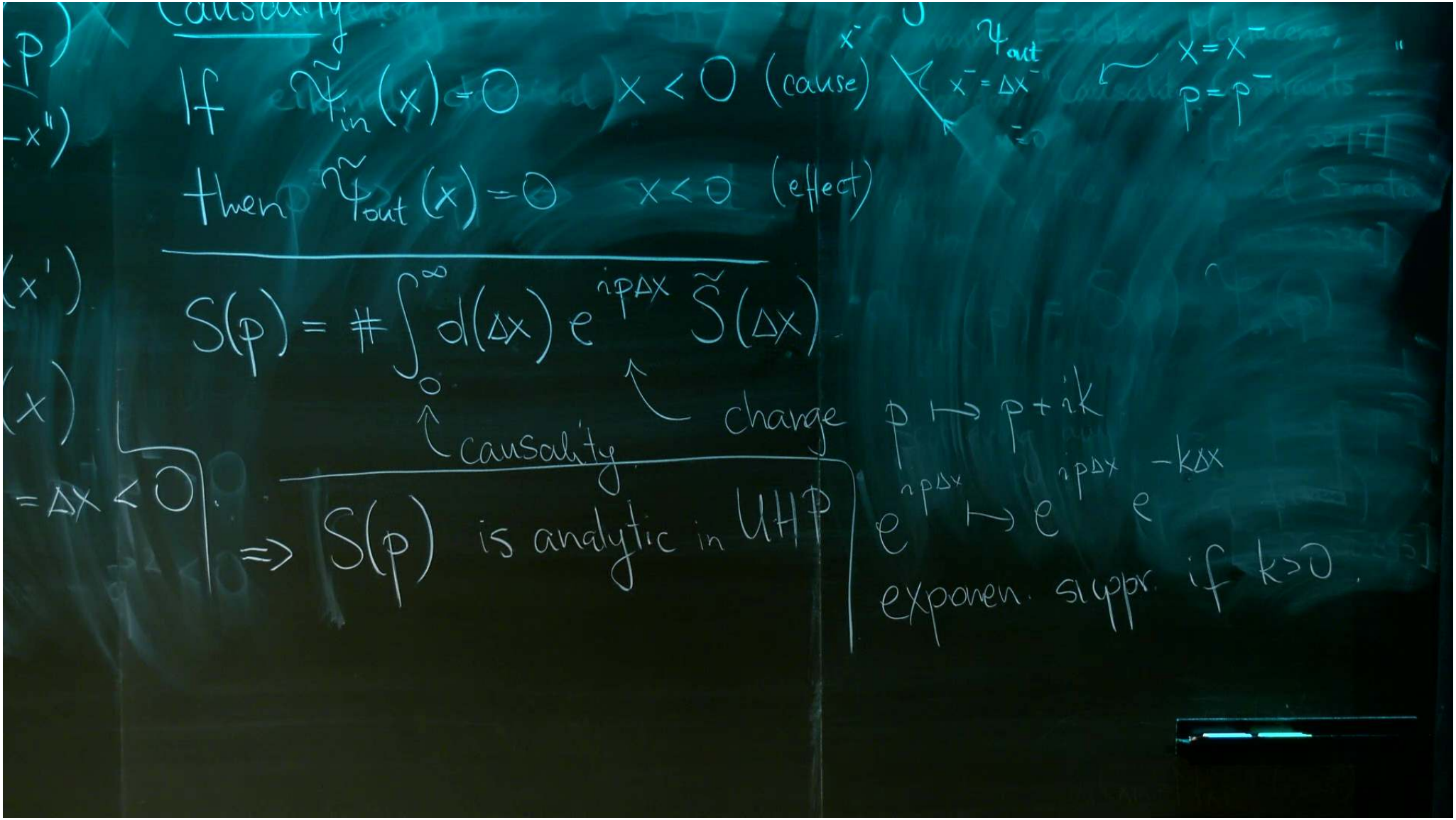
$$\psi_{out}(p) = S(p)\,\psi_{in}(p)$$

scattering ampl. as a fn of p^-

($p^+, \vec{q}$ fixed)

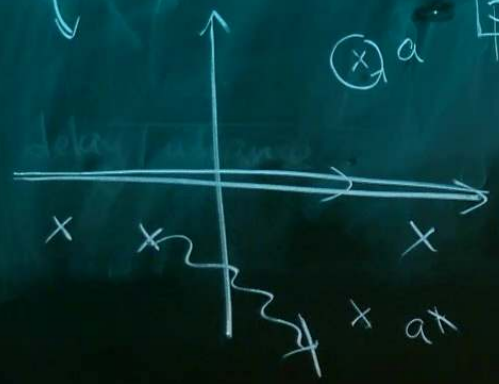

$\# > 0$
$\tilde{\psi}_{out}(x) = \# \int dp\, e^{-ipx}\, S(p)\, \tilde{\psi}_{in}(p)$
$= \# \int dp\, dx'\, dx''\, e^{-ip(x-x'-x'')} \times \tilde{S}(x'')\, \tilde{\psi}_{in}(x')$
$= \# \int dx'\, \tilde{S}(x-x')\, \tilde{\psi}_{in}(x)$
$= 0$ if $x - x' = \Delta x < 0$
Causality
If $\tilde{\psi}_{in}(x) = 0$
then $\tilde{\psi}_{out}(x) = 0$


$S(p)\,\tilde{\psi}_{in}(p)$
$e^{-2p(x-x'-x'')}$
$\tilde{S}(x'')\;\tilde{\psi}_{in}(x')$
$(x-x')\;\tilde{\psi}_{in}(x)$
0 if $x-x'=\Delta x<0$
Causality
If $\tilde{\psi}_{in}(x)=0$ $x<0$ (cause)
then $\tilde{\psi}_{out}(x)=0$ $x<0$ (effect)
$S(p)=\#\int_0^\infty d(\Delta x)\,e^{ip\Delta x}\,\tilde{S}(\Delta x)$
causality
Signal model in 1D
ψ_{out}
$x^-=\Delta x^-$
$x^-=0$
ψ_{in}
A
$\psi_{out}(p)=S(p)$
scattering ampl as


Causality
If $\tilde{\psi}_{in}(x) = 0$ $x < 0$ (cause)
then $\tilde{\psi}_{out}(x) = 0$ $x < 0$ (effect)
ψ_{out}
$x^- = \Delta x^-$
$x = x^-$
$p = p^-$
$S(p) = \# \int_0^\infty d(\Delta x)\, e^{ip\Delta x}\, \tilde{S}(\Delta x)$
causality
change
$p \mapsto p + ik$
$e^{ip\Delta x} \mapsto e^{ip\Delta x} e^{-k\Delta x}$
exponen. suppr. if $k > 0$
$\Rightarrow S(p)$ is analytic in UHP

Unitarity $|S(p)| = 1$ for real p.

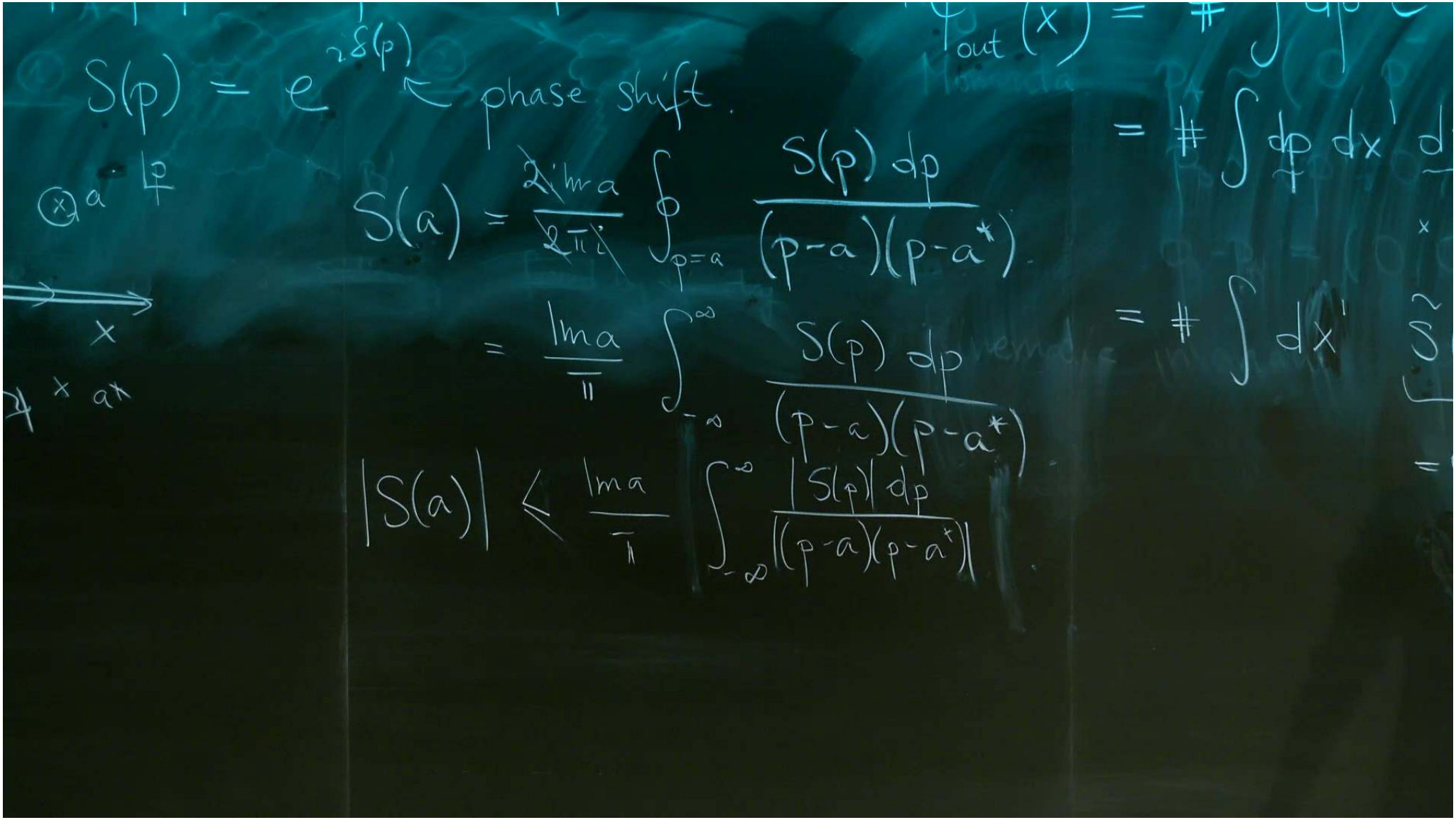
analytic $S(p) = e^{2\delta(p)}$ ⟷ phase shift.



$$S(a) = \frac{2\pi i \operatorname{Im} a}{2\pi i} \oint_{p=a} \frac{S(p)\,dp}{(p-a)(p-a^*)}$$

$$= \frac{\operatorname{Im} a}{\pi} \int_{-\infty}^{\infty} \frac{S(p)\,dp}{(p-a)(p-a^*)}$$

$$\tilde{\psi}_{out}(x) =$$


$S(p) = e^{2i\delta(p)}$ ← phase shift
$S(a) = \frac{2\pi i \cdot \operatorname{Im} a}{2\pi i} \oint_{p=a} \frac{S(p)\,dp}{(p-a)(p-a^*)}$
$= \frac{\operatorname{Im} a}{\pi} \int_{-\infty}^{\infty} \frac{S(p)\,dp}{(p-a)(p-a^*)}$
$|S(a)| \leq \frac{\operatorname{Im} a}{\pi} \int_{-\infty}^{\infty} \frac{|S(p)|\,dp}{|(p-a)(p-a^*)|}$
$\psi_{out}(x) = \# \int dp$
$= \# \int dp\, dx'$
$= \# \int dx'$

Ex. $S(p) = e^{ip\#}$ ✓ ; $e^{-ip\#}$ ✗ ; $e^{ip^2\#}$ ✗

$x < 0$ (

$x < 0$ (

$S(p) = e^{ic_1 p^{c_2}}$

causal if

$c_1 \geq 0$

$c_2 \in [-1, 1]$

$e^{ip\Delta x}\ \tilde{S}(\Delta x$

cha

causality

1.

$\Rightarrow S(p)$ is analytic in UH

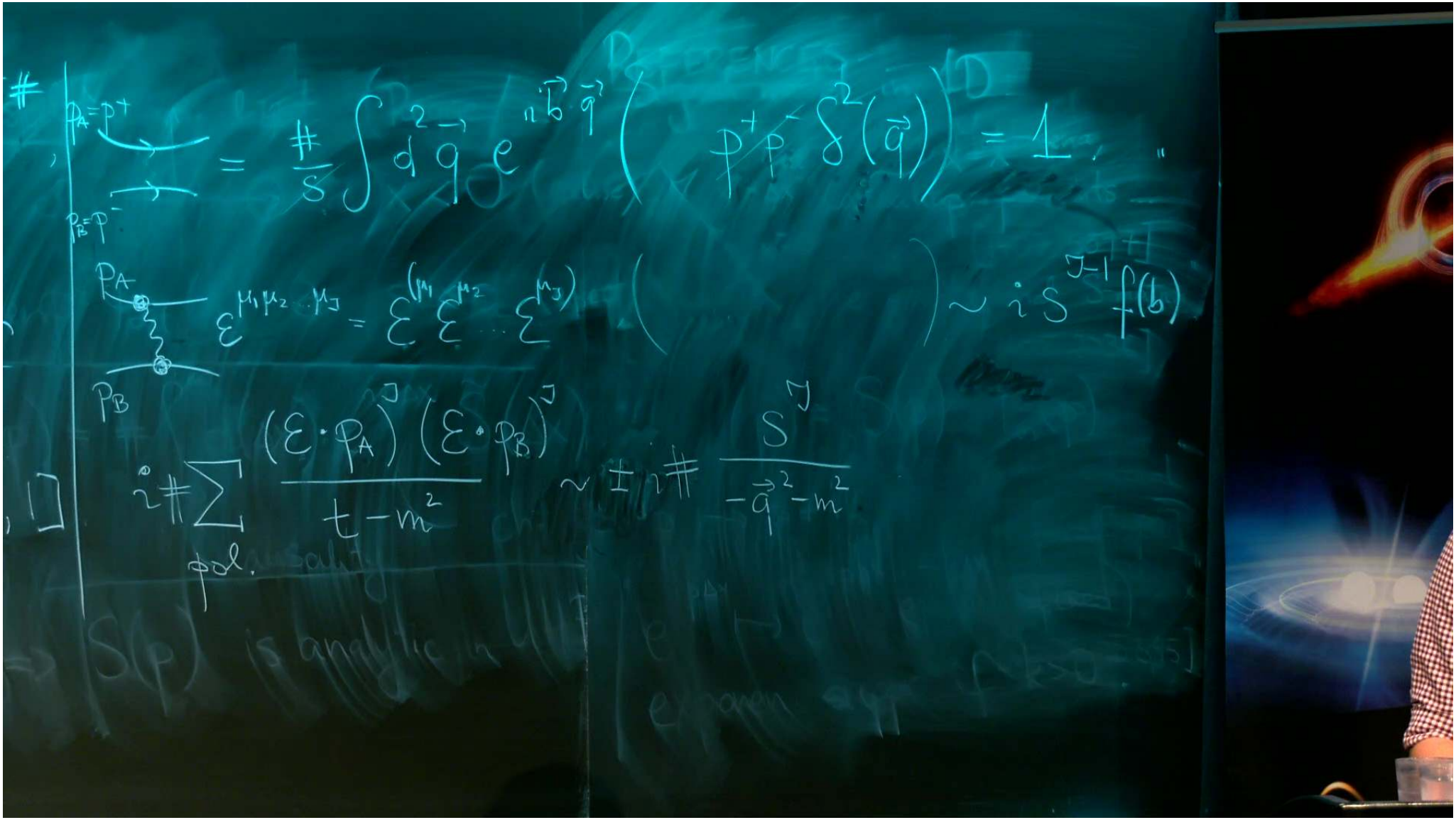
Back to 4D

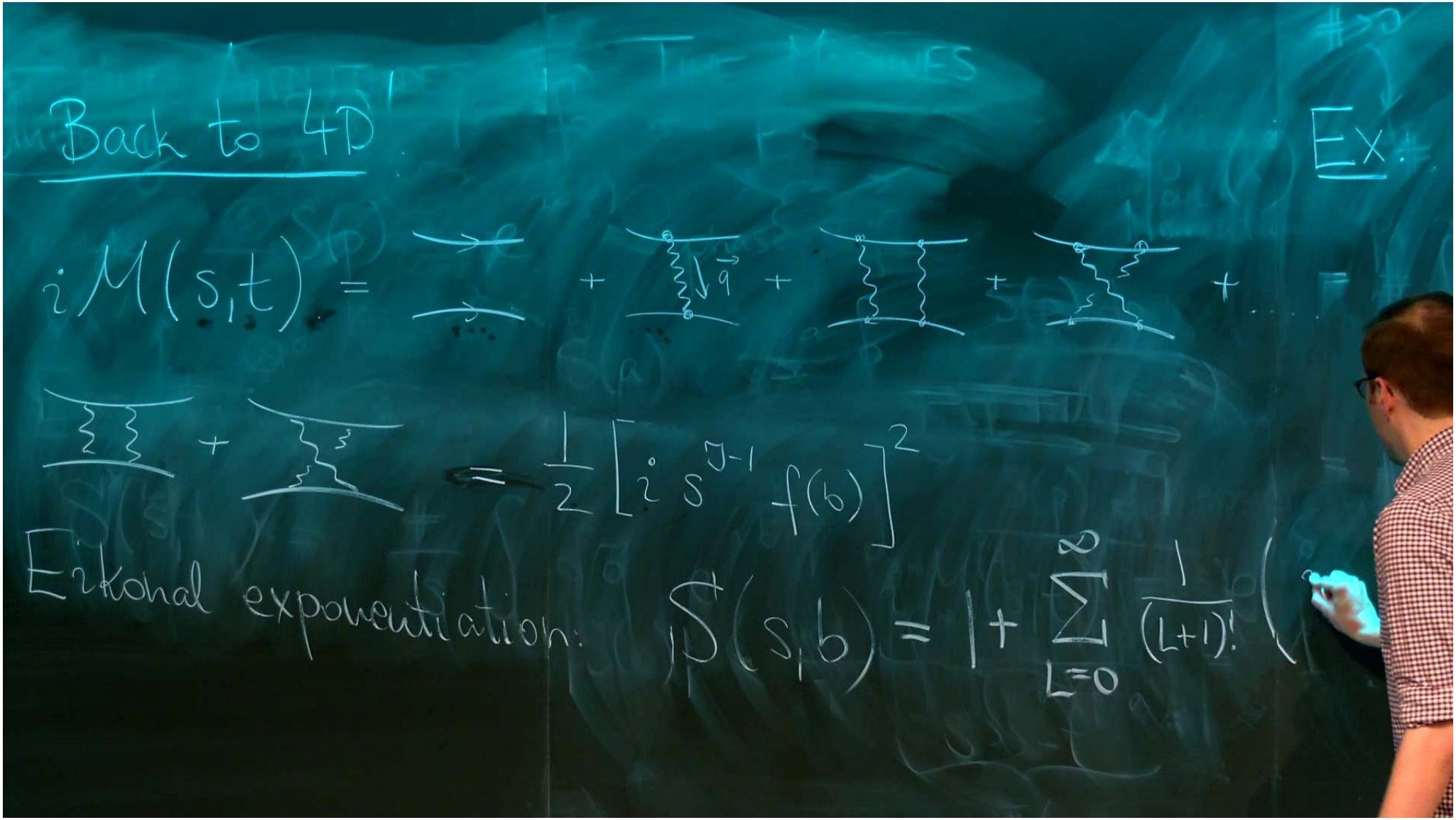
Ex.

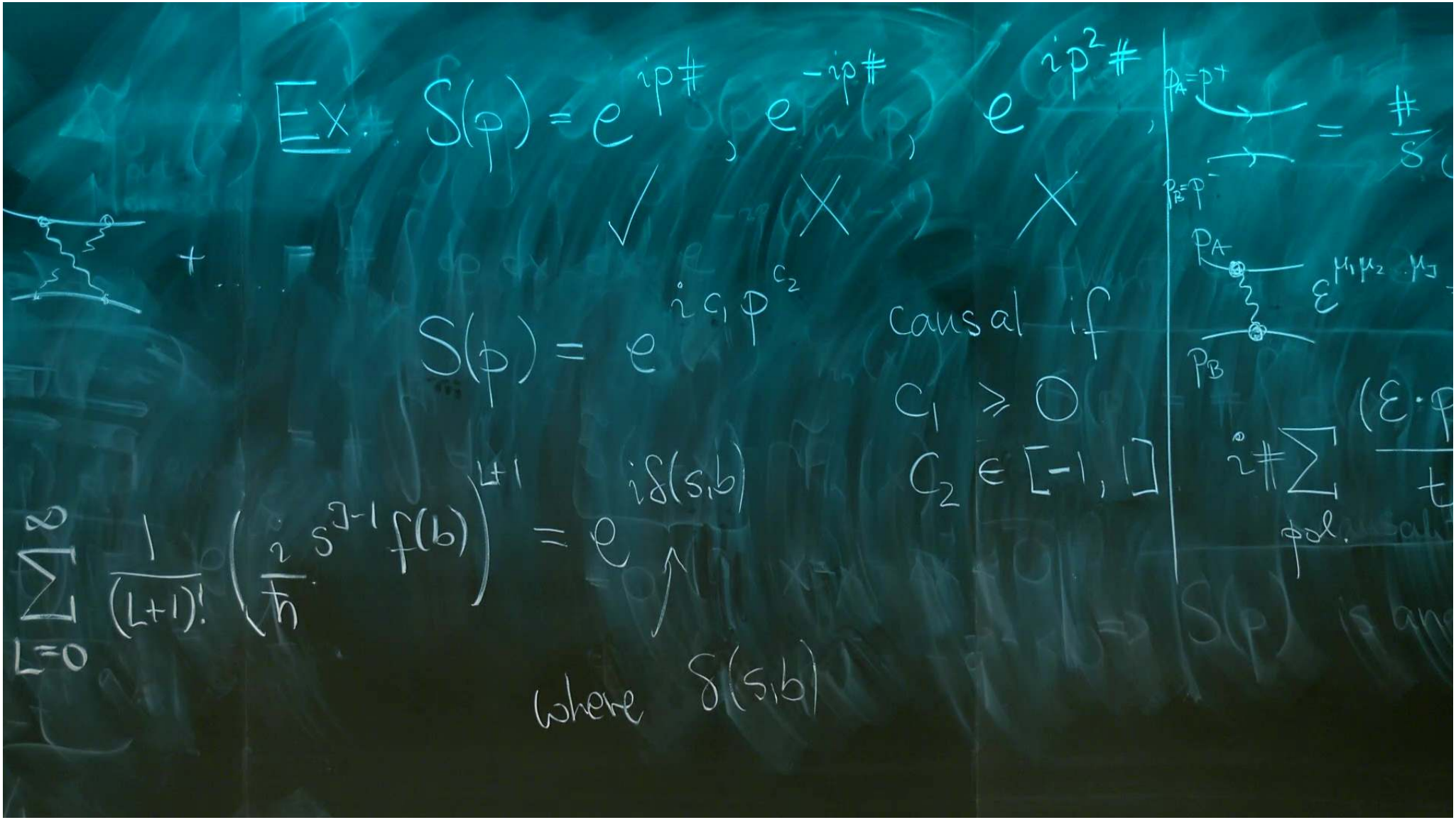
$i\mathcal{M}(s,t) =$ $+ \cdots$

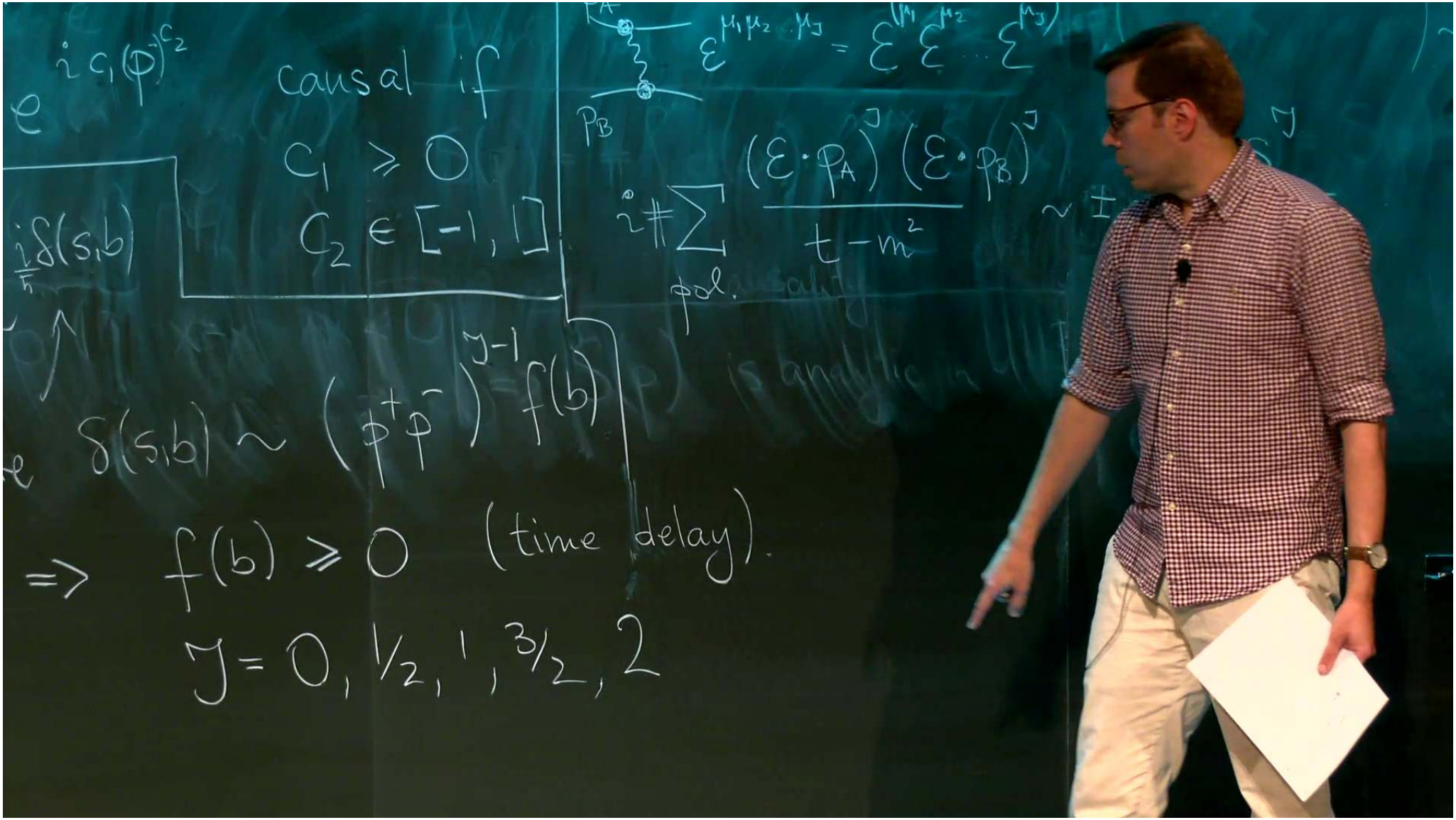
Easiest in the impact param. space $\vec{b} \overset{FT}{\longleftrightarrow} \vec{q}$

$$S(s,b) = \int d^2\vec{q}\, e^{i\vec{b}\cdot\vec{q}}\, i\mathcal{M}(s,t) = e^{i\delta}$$


$p_A = p^+$
$p_B = p^-$
$= \frac{\#}{s}\int d^2\vec{q}\, e^{i\vec{b}\cdot\vec{q}} \left(p^+ p^- \delta^2(\vec{q}) \right)^D = 1$
p_A
p_B
$\varepsilon^{\mu_1\mu_2\ldots\mu_J} = \varepsilon^{(\mu_1}\varepsilon^{\mu_2}\ldots\varepsilon^{\mu_J)}$
$\left(\quad \right) \sim i s^{J-1} f(b)$
$i\# \sum_{\text{pol.}} \frac{(\varepsilon\cdot p_A)^J (\varepsilon\cdot p_B)^J}{t - m^2} \sim \pm i\# \frac{s^J}{-\vec{q}^2 - m^2}$


Back to 4D
Ex.
$i\mathcal{M}(s,t) =$
$\vec{q}$
$= \frac{1}{2}\left[i\, s^{J-1} f(b)\right]^2$
Eikonal exponentiation: $S(s,b) = 1 + \sum_{L=0}^{\infty} \frac{1}{(L+1)!}$


Ex. $S(p) = e^{ip\#}, e^{-ip\#}, e^{ip^2\#}$
$S(p) = e^{ic_1 p^{c_2}}$
causal if
$c_1 \geq 0$
$c_2 \in [-1, 1]$
$\sum_{L=0}^{\infty} \frac{1}{(L+1)!} \left(\frac{i}{\hbar} s^{J-1} f(b) \right)^{L+1} = e^{i\delta(s,b)}$
where $\delta(s,b)$
$p_A = p^+$
$p_B = p^-$
p_A
p_B
$\varepsilon^{\mu_1 \mu_2 \ldots \mu_J}$
$i\# \sum_{pol.}$


causal if
$c_1 \geq 0$
$c_2 \in [-1, 1]$
$\delta(s,b) \sim (p^+p^-)^{J-1} f(b)$
$\Rightarrow f(b) \geq 0$ (time delay)
$J = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$
$\varepsilon^{\mu_1\mu_2\ldots\mu_J} = \varepsilon^{(\mu_1}\varepsilon^{\mu_2}\cdots\varepsilon^{\mu_J)}$
p_A
p_B
$\sum_{\text{pol.}} \frac{(\varepsilon\cdot p_A)^J (\varepsilon\cdot p_B)^J}{t - m^2}$