Title: Lecture - Celestial Holography Ib

Speakers:

Collection: Celestial Holography Summer School 2024

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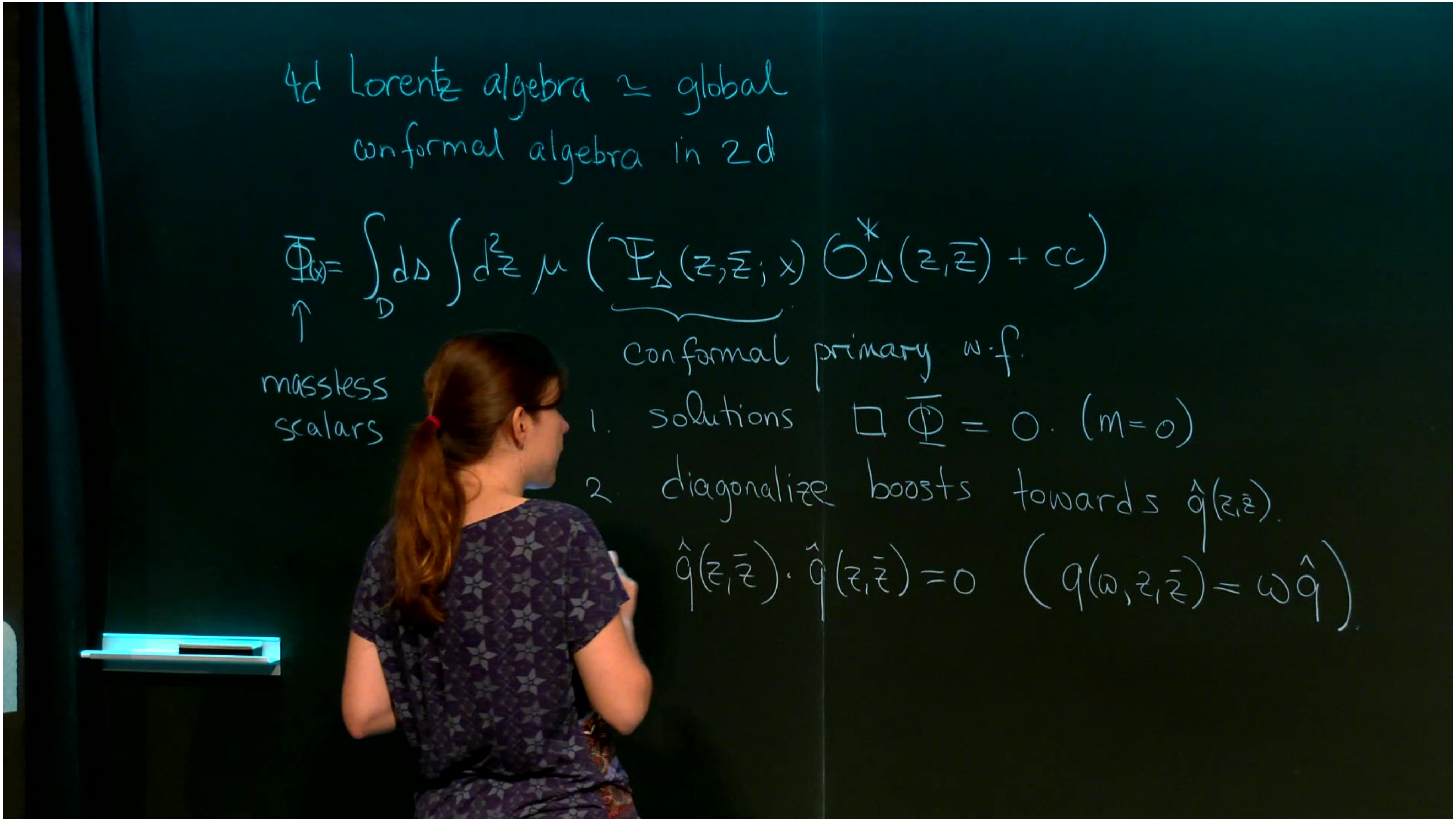

4d Lorentz algebra ≃ global conformal algebra in 2d
$\Phi(x) = \int_D d\Delta \int d^2z\, \mu \left(\underbrace{\Psi_\Delta(z,\bar{z};x)}_{\text{conformal primary w.f.}} \mathcal{O}^*_\Delta(z,\bar{z}) + cc \right)$
massless scalars
1. solutions $\Box \Phi = 0$. $(m=0)$
2. diagonalize boosts towards $\hat{q}(z,\bar{z})$.
$\hat{q}(z,\bar{z}) \cdot \hat{q}(z,\bar{z}) = 0 \quad \left(q(\omega,z,\bar{z}) = \omega \hat{q} \right)$

$$\Phi(x) = \int_D d\Delta \int d^2z \, \mu \left(\underbrace{\Psi_\Delta(z,\bar z; x)}_{\text{conformal primary w.f.}} \mathcal{O}^*_\Delta(z,\bar z) + cc \right)$$

$\uparrow$

massless scalars

1. solutions $\Box \Phi = 0$. $(m=0)$

2. diagonalize boosts towards $\hat q(z,\bar z)$.

$$\hat q(z,\bar z)\cdot \hat q(z,\bar z) = 0 \quad \left(q(\omega, z, \bar z) = \omega \hat q \right)$$

with eigenvalue Δ. $\left[(L_0 + \bar L_0) \Psi_\Delta = \Delta \Psi_\Delta \right]$

3. highest weight wrt Lorentz:

$$L_1 \Psi_\Delta = \bar L_1 \Psi_\Delta = 0$$

$)$

$m=0)$

rds $\hat{q}(z,\bar{z})$.

$(\omega,z,\bar{z})=\omega\hat{q})$

$$\left[(L_0+\bar{L}_0)\Psi_\Delta=\Delta\Psi_\Delta\right]$$

orentz:

generators of Lorentz $\{J^i, K^i\}$, $i=1,2,3$

rotations   boosts

$\updownarrow$

$m,n=-1,0,1 \leftarrow L_m, \bar{L}_m$ – $SL(2,\mathbb{C})$ gens.

$$[L_m,L_n]=(m-n)L_{m+n}$$

– " – for $\bar{L}_m$

* modes $O_\Delta(z,\bar{z}) \longleftrightarrow$ primary operators in 2d CFT.

rotations $\updownarrow$ boosts.

$m,n=-1,0,1 \leftarrow L_m, \bar{L}_m$ — $SL(2,\mathbb{C})$ gens.

$$[L_m, L_n] = (m-n)L_{m+n}$$

— " — for $\bar{L}_m$

* modes $O_\Delta(z,\bar{z}) \longleftrightarrow$ primary operators in 2d CFT.

$\ldots\Psi_\Delta = \Delta\psi_\Delta]$

* $$\Psi^{\pm}_{\Delta}(z,\bar{z};x) = \frac{(\mp i)^{\Delta}\,\Gamma(\Delta)}{(-\hat{q}\cdot x)^{\Delta}} = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i\omega\hat{q}\cdot x}$$

c) gens.

L_{m+n}

rators

$\int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i\omega \hat{q}\cdot x}$

Examples :

Gravitons (spin-2 fields) are the quanta of C_{AB} in a mom. estate basis

$$C_{zz} = \frac{1}{2\pi}$$

$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du\,dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

curvature wrt γ_{AB} — Bondi mass aspect

$$\frac{V}{r} = -\frac{R}{2} + \frac{2M}{r} + \mathcal{O}(r^{-2})$$

$$\beta = \frac{1}{r^2}\left(-\frac{1}{32}C_{AB}C^{AB}\right) + \mathcal{O}(r^{-3})$$

angular mom a

$$U^A = -\frac{1}{2r^2}D_B C^{BA} - \frac{2}{3}\frac{1}{r^3}\left[N^A - \frac{1}{2}C^{AB}D^C C_{BC}\right] + \mathcal{O}(r^{-4})$$

V, g_{AB}, U^A are fct^{n}s of (u, r, x^A)

$$g_{AB} = r^2\gamma_{AB} + r\boxed{C_{AB}(u,x^A)} + \dots + \frac{T_{AB}(u,x^A)}{r}$$

rotations $\updownarrow$ boosts

$\leftarrow L_m, \bar{L}_m$ — $SL(2,\mathbb{C})$ gens.

$$[L_m, L_n] = (m-n) L_{m+n}$$

— " — for $\bar{L}_m$

$z,\bar{z})$ $\longleftrightarrow$ primary operators

$$;x) = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-\hat{q}\cdot x)^\Delta} = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i\omega\hat{q}\cdot x}$$

EX

• Gravitons (spin-2 fields) are the quanta of C_{AB} in a mom. estate basis

$$C_{zz}^{(u,z,\bar{z})} = \frac{1}{2\pi}\int_0^\infty d\omega \left[e^{-i\omega u} \underbrace{\varepsilon^{+}_{zz}\, \overset{a_+(\omega\hat{q})}{\overset{\text{|||}}{a_+}}}_{\overset{\text{|||}}{\tilde{C}_+(\omega,z,\bar{z})}} + h.c. \right]$$

creates an outgoing graviton of +-ve helicity

$$\langle 0 | \tilde{C}_+(\omega,z,\bar{z})$$

- Conformal primary gravitons are boost e-state quanta of C_{AB}:

$$\hat{C}_{zz}(\Delta, z, \bar{z}) \equiv \int_0^\infty d\omega\, \omega^{\Delta-1}\, \tilde{C}_+(\omega)$$

$$= i^\Delta\, \Gamma(\Delta) \int_{-\infty}^{\infty} du\, \frac{C_{zz}(u)}{(u+i\epsilon)^\Delta}$$

- Conformally soft gravitons

Example

- Grav

are

in a

$C^{(\omega,}_{zz}$

$\hat{q})$.

$)\Psi_\Delta = \Delta\Psi_\Delta]$

cc)

. $(m=0)$

owards $\hat{q}(z,\bar{z})$

$q(\omega,z,\bar{z}) = \omega\hat{q}$)

Δ $\left[(L_0 + \bar{L}_0)\Psi_\Delta = \Delta \Psi_\Delta \right]$

Lorentz:

$\bar{L}_1 \Psi_\Delta = 0$

$$\hat{C}_{zz}(\Delta, z, \bar{z}) \equiv \int_0^\infty d\omega\, \omega^{\Delta-1} \tilde{C}_+(\omega)$$

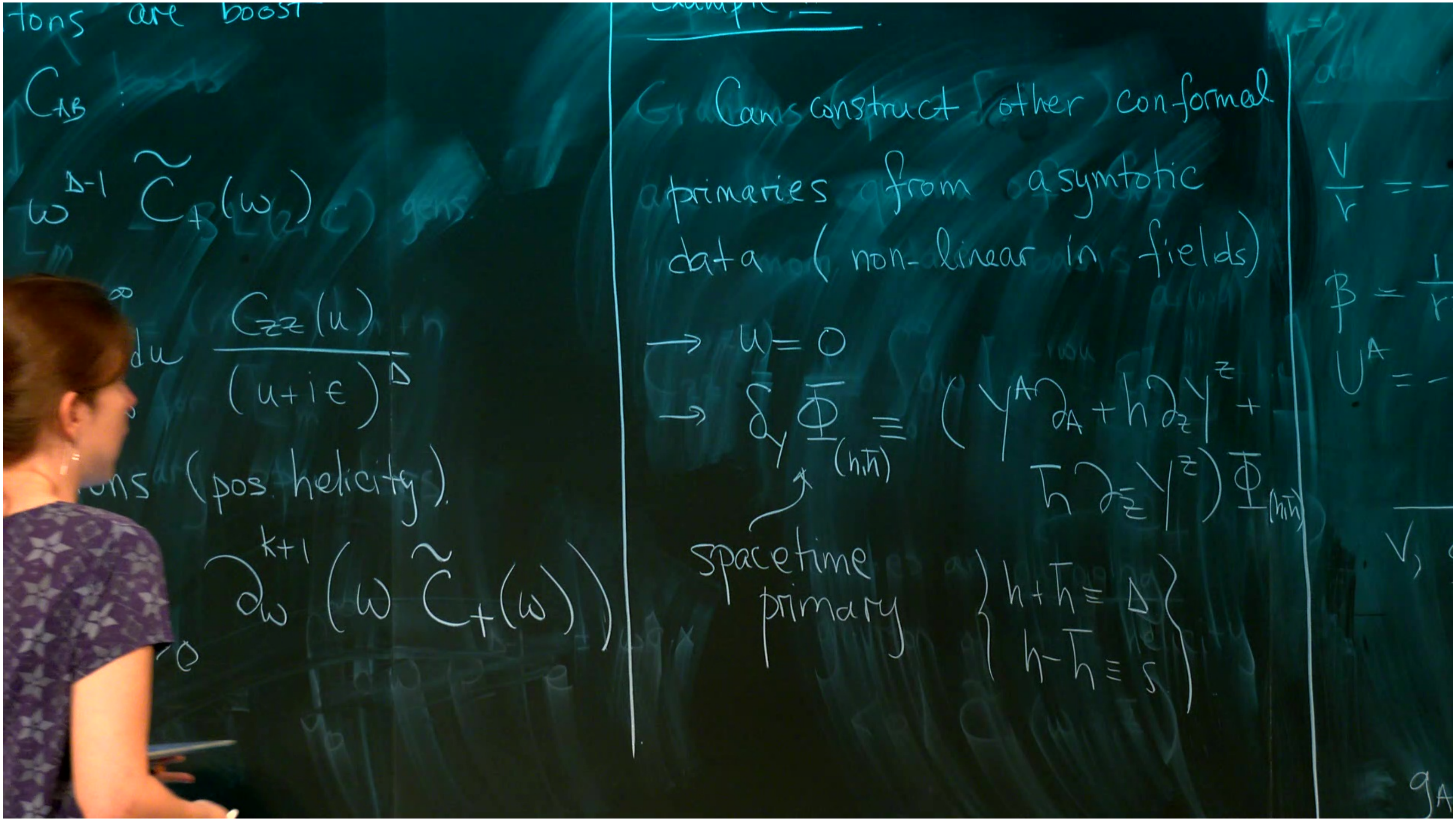
$$= i^\Delta \Gamma(\Delta) \int_{-\infty}^{\infty} du \frac{C_{zz}(u)}{(u+i\epsilon)^\Delta}$$

- Conformally soft gravitons (pos. helicity)

$$\underset{\Delta=-k}{\text{Res}}\, \hat{C}_+(\Delta) = \lim_{\omega\to 0} \partial_\omega^{k+1}\left(\omega \tilde{C}_+(\omega)\right)$$

$k = -1, 0, 1, \ldots$

Ex


tons are boost
C_AB :
ω^(Δ-1) C̃_+(ω)
du C_zz(u) / (u+iε)^Δ
ons (pos helicity)
∂_ω^(k+1) (ω C̃_+(ω))
Can construct other conformal
primaries from asymtotic
data (non-linear in fields)
→ u = 0
→ δ_Y Φ_(h,h̄) ≡ (Y^A ∂_A + h ∂_z Y^z + h̄ ∂_z̄ Y^z̄) Φ_(h,h̄)
spacetime
primary
h + h̄ ≡ Δ
h − h̄ ≡ s
V/r = −
β = 1/r
U^A = −

tons are boost

C_{AB} :

$\omega^{\Delta-1}\,\tilde{C}_+(\omega)$

$\Delta)\int_{-\infty}^{\infty} du\, \frac{C_{zz}(u)}{(u+i\epsilon)^{\Delta}}$

avitons (pos. helicity).

$= \lim_{\omega\to 0} \partial_\omega^{k+1}\left(\omega\, \tilde{C}_+(\omega)\right)$

Example:

Can construct other conformal primaries from asymptotic data (non-linear in fields)

$\longrightarrow u = 0$

$\longrightarrow \delta_Y \Phi_{(h,\bar h)} \equiv \left(Y^A\partial_A + h\,\partial_z Y^z + \bar h\,\partial_{\bar z} Y^{\bar z}\right)\Phi_{(h,\bar h)}$

spacetime primary

$\left\{\begin{array}{l} h+\bar h \equiv \Delta \\ h-\bar h \equiv s \end{array}\right\}$

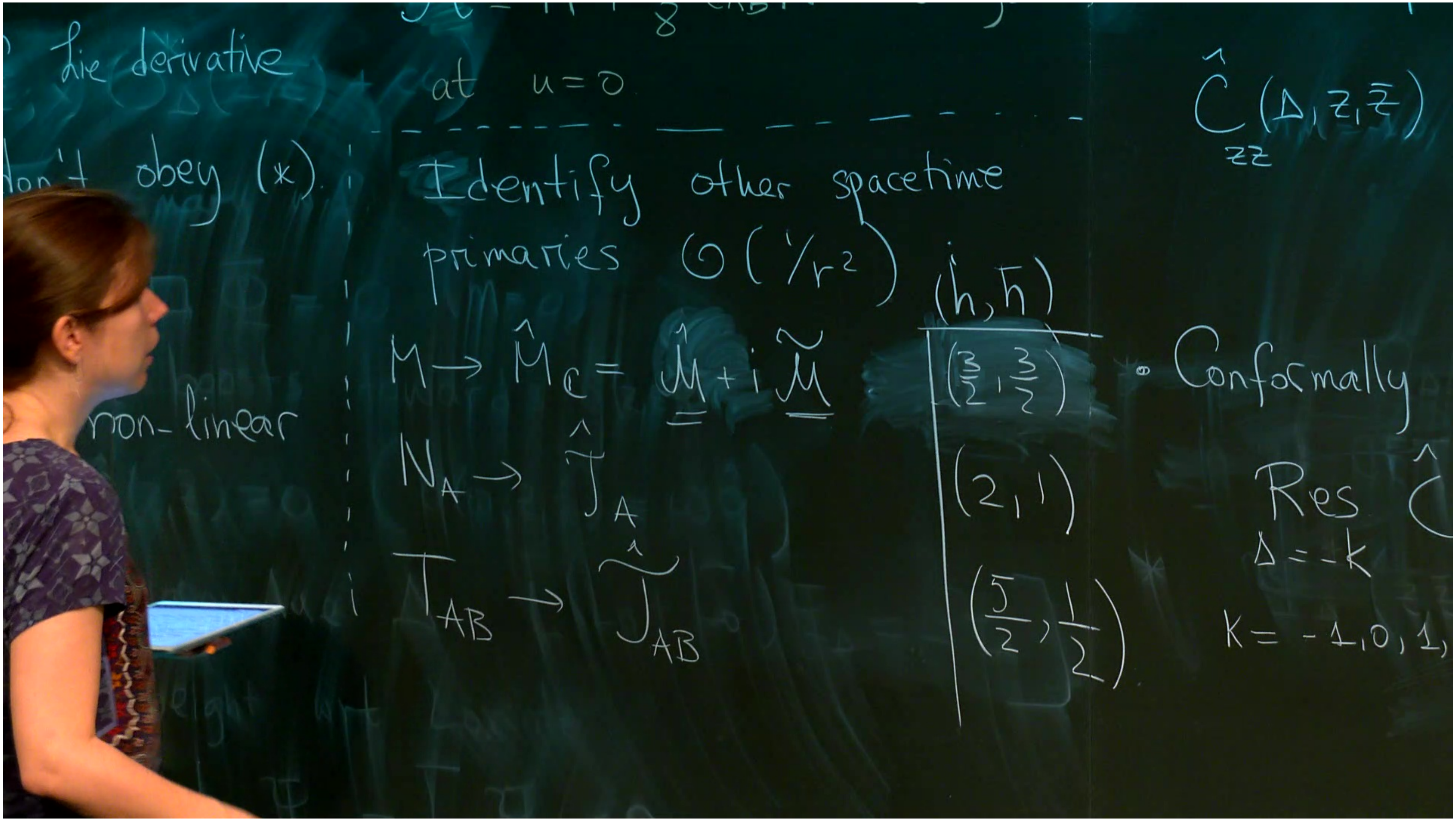
$\frac{V}{r} = -$

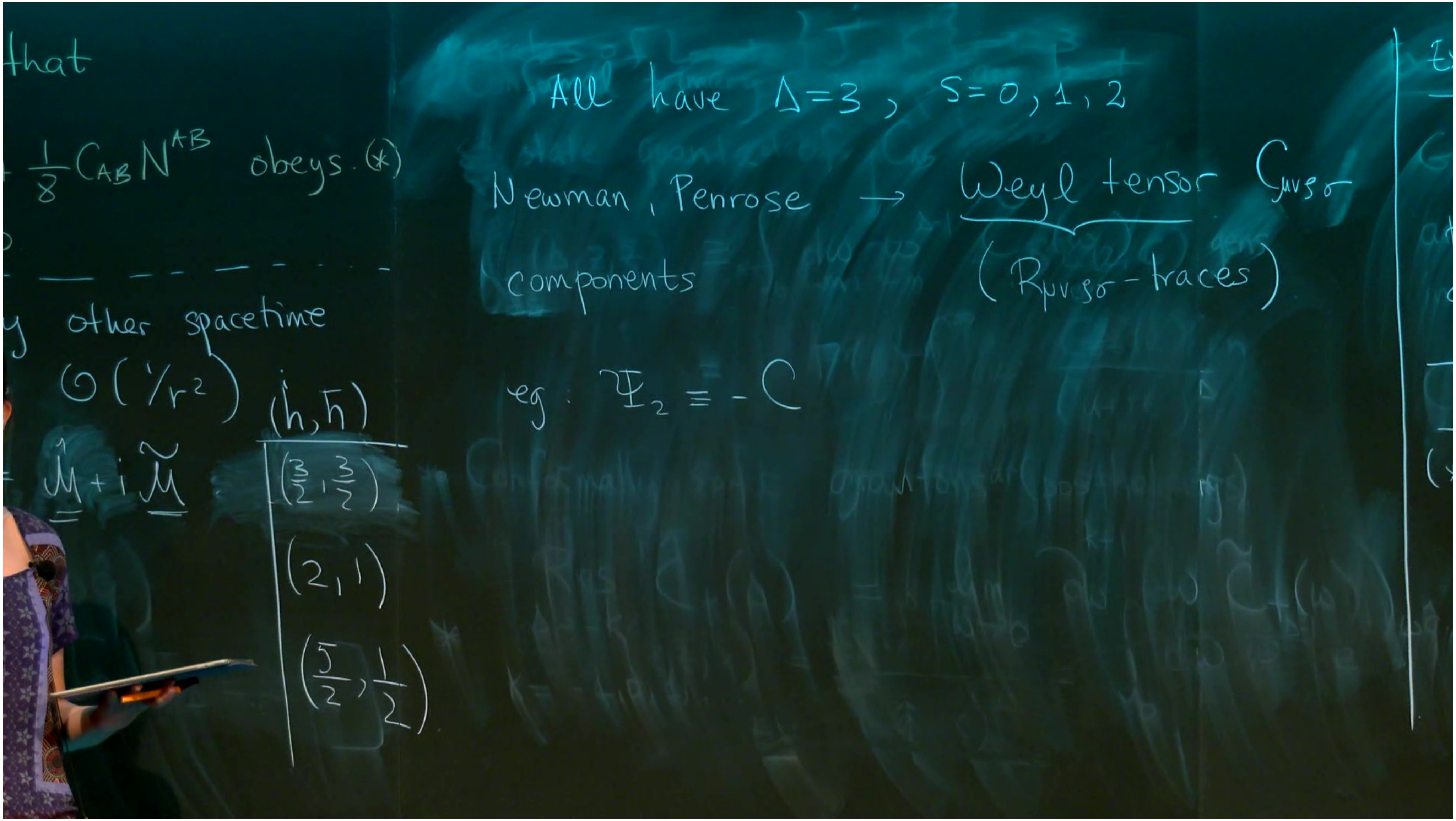
$\beta = \frac{1}{r}$

$U^A = -$

$V,$

g_A


Lie derivative
at u=0
obey (*)
Identify other spacetime
primaries O(1/r²)
(h, h̄)
M → M̂_C = M + i M̃
(3/2, 3/2)
• Conformally
non-linear
N_A → Ĵ_A
(2, 1)
Res
Δ = -k
T_AB → J̃_AB
(5/2, 1/2)
K = -1, 0, 1,
Ĉ_zz(Δ, z, z̄)


that
$\frac{1}{8} C_{AB} N^{AB}$ obeys. (*)
other spacetime
$\mathcal{O}(1/r^2)$
$(h, \bar{h})$
$(\frac{3}{2}, \frac{3}{2})$
$(2, 1)$
$(\frac{5}{2}, \frac{1}{2})$
All have $\Delta = 3$, $s = 0, 1, 2$
Newman, Penrose $\rightarrow$ Weyl tensor $C_{\mu\nu\rho\sigma}$
components
$(R_{\mu\nu\rho\sigma} - \text{traces})$
eg: $\Psi_2 \equiv - C$

$C_{AB} N^{AB}$ obeys $(*)$

other spacetime

$O(1/r^2)$

$(h, \bar{h})$
$\left(\frac{3}{2}, \frac{3}{2}\right)$
$(2, 1)$
$\left(\frac{5}{2}, \frac{1}{2}\right)$

All have $\Delta = 3$, $s = 0, 1, 2$

Newman, Penrose $\longrightarrow$ $\underbrace{\underline{\text{Weyl tensor}}}_{(R_{\mu\nu\rho\sigma} - \text{traces})}$ $C_{\mu\nu\rho\sigma}$

components

eg: $\Psi_2 \equiv - C_{lm\bar{m}n}$

$(l, n, m, \bar{m})$ – null frame

$g_{ab} = -l_a n_b - l_b n_a + m_a \bar{m}_b + m_b \bar{m}_a$

Exam

prin

dat

$(*)$

Ex: show that

$\hat{\mathcal{M}} \equiv M + \frac{1}{8} C_{AB} N^{AB}$ obeys (*)

at $u=0$

Identify other spacetime primaries $\mathcal{O}(1/r^2)$

	$(h, \bar{h})$
$M \to \hat{M}_{\mathbb{C}} = \hat{\mathcal{M}} + i\tilde{\mathcal{M}} = \Psi_2^{(0)}$	$\left(\frac{3}{2}, \frac{3}{2}\right)$
$N_A \to \hat{\mathcal{J}}_A \supset \Psi_1^{(0)}$	$(2, 1)$
$T_{AB} \to \hat{\tilde{\mathcal{J}}}_{AB} \supset \Psi_0^{(0)}$	$\left(\frac{5}{2}, \frac{1}{2}\right)$

$$\Psi_i \equiv \sum_{n \geq 0} \Psi_i^{(n)} r^{-n-5+i}$$

All have $\Delta = 3$, $s = 0, 1, 2$

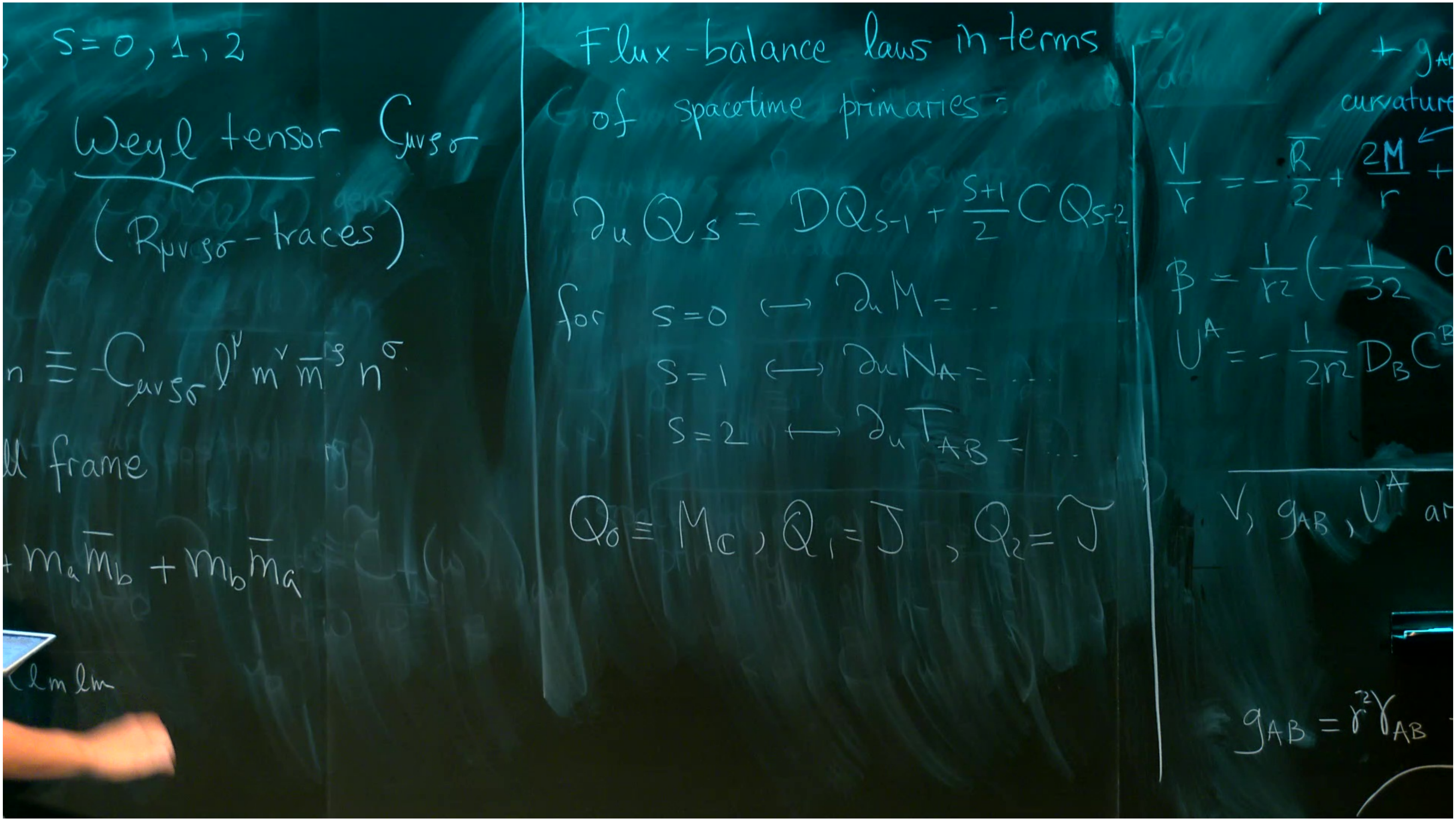
Newman, Penrose $\to$ $\underbrace{\text{Weyl tensor}}_{(R_{\mu\nu\rho\sigma} - \text{trace})}$ components

eg: $\Psi_2 \equiv -C_{\ell m \bar{m} n} \equiv -C_{\mu\nu\rho\sigma} \ell^\mu m^\nu \bar{m}^\rho$

$(\ell, n, m, \bar{m})$ - null frame

$g_{ab} = -l_a n_b - l_b n_a + m_a \bar{m}_b + m_b \bar{m}_a$

$\Psi_1 \equiv C_{\ell n \ell m}$, $\Psi_0 = C_{\ell m \ell m}$


s = 0, 1, 2
Weyl tensor $C_{\mu\nu\rho\sigma}$
$(R_{\mu\nu\rho\sigma}$ - traces$)$
$\equiv C_{\mu\nu\rho\sigma} l^\mu m^\nu \bar{m}^\rho n^\sigma$
frame
$+ m_a \bar{m}_b + m_b \bar{m}_a$
Flux-balance laws in terms
of spacetime primaries :
$\partial_u Q_s = D Q_{s-1} + \frac{s+1}{2} C Q_{s-2}$
for $s=0 \longleftrightarrow \partial_u M = \dots$
$s=1 \longleftrightarrow \partial_u N_A = \dots$
$s=2 \longleftrightarrow \partial_u T_{AB} = \dots$
$Q_0 \equiv M_C, Q_1 = J, Q_2 = \tilde{J}$
curvature
$\frac{V}{r} = -\frac{\bar{R}}{2} + \frac{2M}{r} +$
$\beta = \frac{1}{r^2}\left(-\frac{1}{32}\right.$
$U^A = -\frac{1}{2r^2} D_B C$
V, g_{AB}, U^A
$g_{AB} = r^2 \gamma_{AB}$

$(R_{\mu\nu\rho\sigma} - \text{traces})$

$\ldots m \bar{m} n \equiv C_{\mu\nu\rho\sigma} l^\mu m^\nu \bar{m}^\rho n^\sigma$

null frame

$\ldots_a + m_a \bar{m}_b + m_b \bar{m}_a$

$= C_{lmlm}$

$$\partial_u Q_s = D Q_{s-1} + \frac{s+1}{2} C Q_{s-2}$$

for $s=0 \longleftrightarrow \partial_u M = \ldots$

$s=1 \longleftrightarrow \partial_u N_A = \ldots$

$s=2 \longleftrightarrow \partial_u T_{AB} = \ldots$

$$Q_0 \equiv M_{\mathbb{C}}, \quad Q_1 = \mathcal{J} \equiv m^A \hat{\mathcal{J}}_A, \quad Q_2 = \tilde{\mathcal{J}} \equiv m^A m^B \hat{\mathcal{J}}'_{AB}$$

$$\left[\begin{array}{l} C \equiv m^A m^B C_{AB} \\ N \equiv \bar{m}^A \bar{m}^B N_{AB} \end{array}\right]$$

$\frac{V}{r} = -\frac{R}{2} + \frac{2M}{r} + O$

$\beta = \frac{1}{r^2}\left(-\frac{1}{32} C_{AB} \ldots\right.$

$U^A = -\frac{1}{2r^2} D_B C^{BA} \ldots$

V, g_{AB}, U^A are f…

$g_{AB} = r^2 \gamma_{AB} + r \ldots$

shear : $\partial_u C_{AB} = \ldots$

$N_{AB} \neq 0$

$$\left\{\begin{aligned} Q_{-1} &= \tfrac{1}{2} DN \\ Q_{-2} &= \tfrac{1}{2} \partial_u N \end{aligned}\right\} \leftarrow \text{boundary cond. for recursive diff eq.}$$

<u>Assume $s \in \mathbb{N}$</u> ; $s > 2$ can be shown

truncations of evolution eq. $\Psi_0^{(n)}$, $n \geq 1$

$$Q_s = \underbrace{Q_s^{(1)}}_{\text{lin.}} + \underbrace{Q_s^{(2)}}_{\text{quadratic}} + \underbrace{\dots}_{\text{higher poly. in fields}}$$

<u>truncate</u>

— boundary cond.
for recursive diff eq.

$S > 2$ can be shown

olution eq. $\Psi_0^{(n)}, n \geq 1$

$Q_s^{(1)} + Q_s^{(2)} + \ldots$

truncate

quadratic

higher poly. in fields

$Q_s^{(i)} \equiv Q_s^{(i)}[C, N]$.

$* \quad Q_s \Big|_{u \to \infty} \equiv 0.$

$* \quad Q_s \Big|_{u \to -\infty} \to$ charges

(eg $s = 0 \to$ supertransl charge aspect)

$\hookrightarrow$ for $S \geq 1 \to$ diverge as $u \to -\infty$

All hav

Newman, Pe

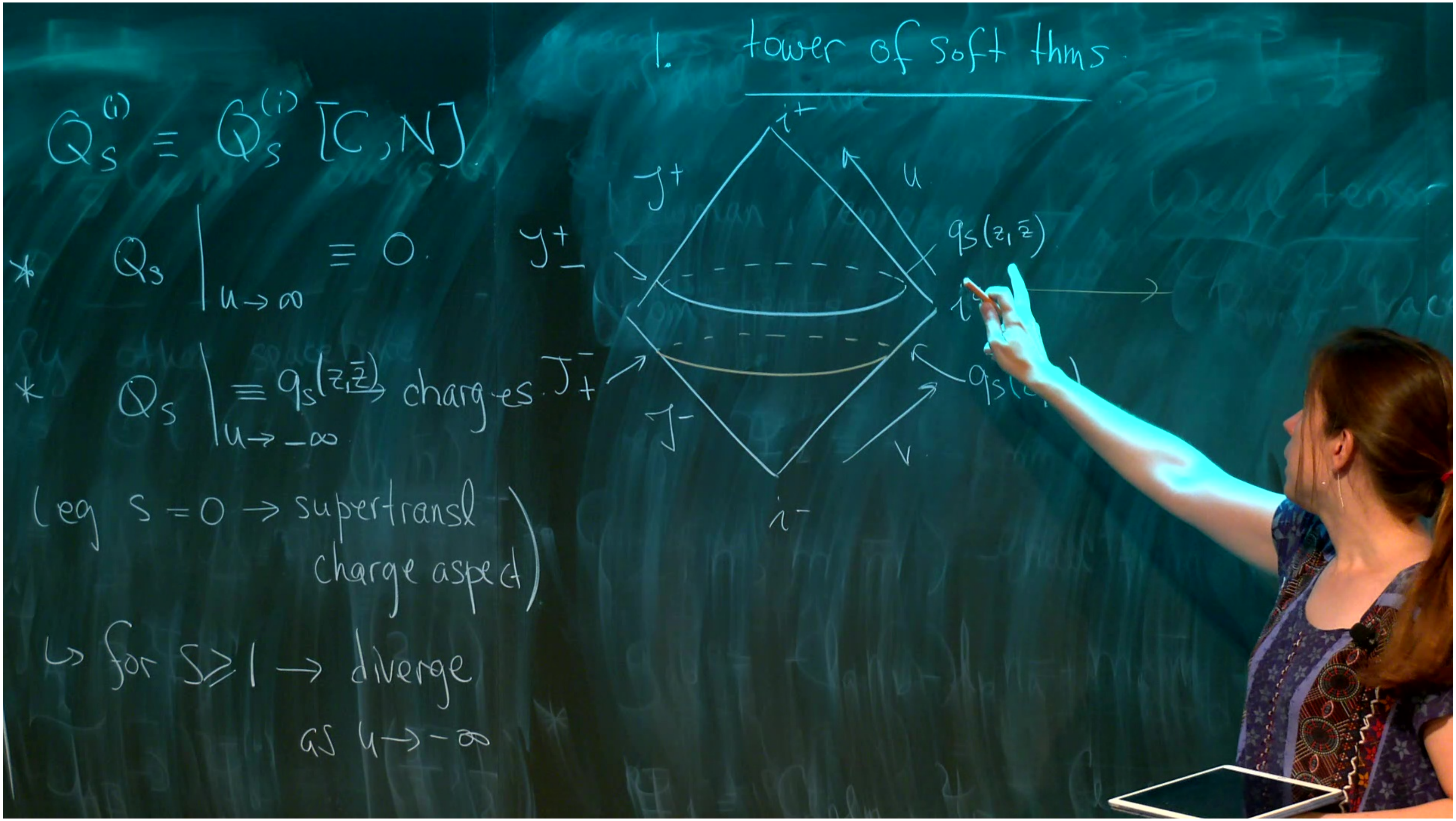
components

eg: $\Psi_2 \equiv$

$(l, n, m,$

$g_{ab} = -l_a$

$\Psi_1 \equiv C_{enl}$


1. tower of soft thms
$Q_s^{(i)} \equiv Q_s^{(i)}[C,N]$
$* \; Q_s \big|_{u\to\infty} \equiv 0$
$* \; Q_s \big|_{u\to-\infty} \equiv q_s(z,\bar z)$ charges
(eg $s=0 \to$ supertransl charge aspect)
$\hookrightarrow$ for $s \geq 1 \to$ diverge as $u \to -\infty$
i^+
i^-
$\mathcal{J}^+$
$\mathcal{J}^-$
$\mathcal{J}^+_-$
$\mathcal{J}^-_+$
u
v
$q_s(z,\bar z)$

1. tower of soft thms

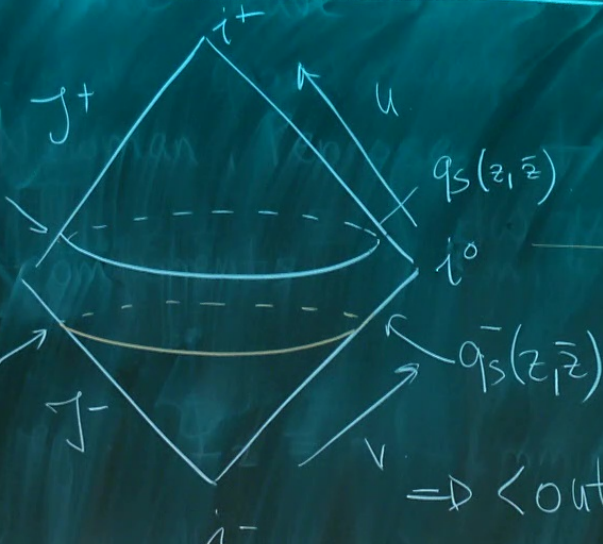
$Q_s^{(i)} \equiv Q_s^{(i)}[C,N]$

$Q_s \Big|_{u\to\infty} \equiv 0$ $\qquad \mathcal{J}^+_-$

$Q_s \Big|_{u\to-\infty} \equiv q_s(z,\bar z)$ charges $\qquad \mathcal{J}^-_+$

$s=0 \to$ supertransl charge aspect

for $s \geq 1 \to$ diverge as $u \to -\infty$



$\xrightarrow[\text{through } i^0]{\text{no flux}}$

antipodal identif. $q_s = q_s^-$

$$\Rightarrow \langle out | Q_s S - S Q_s^- | in \rangle = 0$$

$$\Updownarrow$$

$$\lim_{\omega\to 0} \partial_\omega^s \left(\omega \langle out | a(\omega) S | in \rangle \right) = S^{(s)} \langle out | S | in \rangle$$

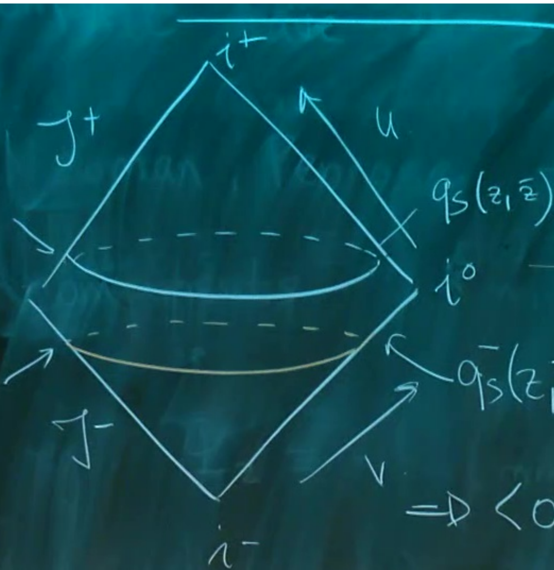
$\hat{Q}_s^{(i)} \equiv \hat{Q}_s^{(i)}[C,N]$

$Q_s\Big|_{u\to\infty} \equiv 0.$

$Q_s\Big|_{u\to-\infty} \equiv q_s(z,\bar{z})$ charges.

eg $s=0 \to$ supertransl charge aspect)

$\Rightarrow$ for $s \geq 1 \to$ diverge as $u \to -\infty$



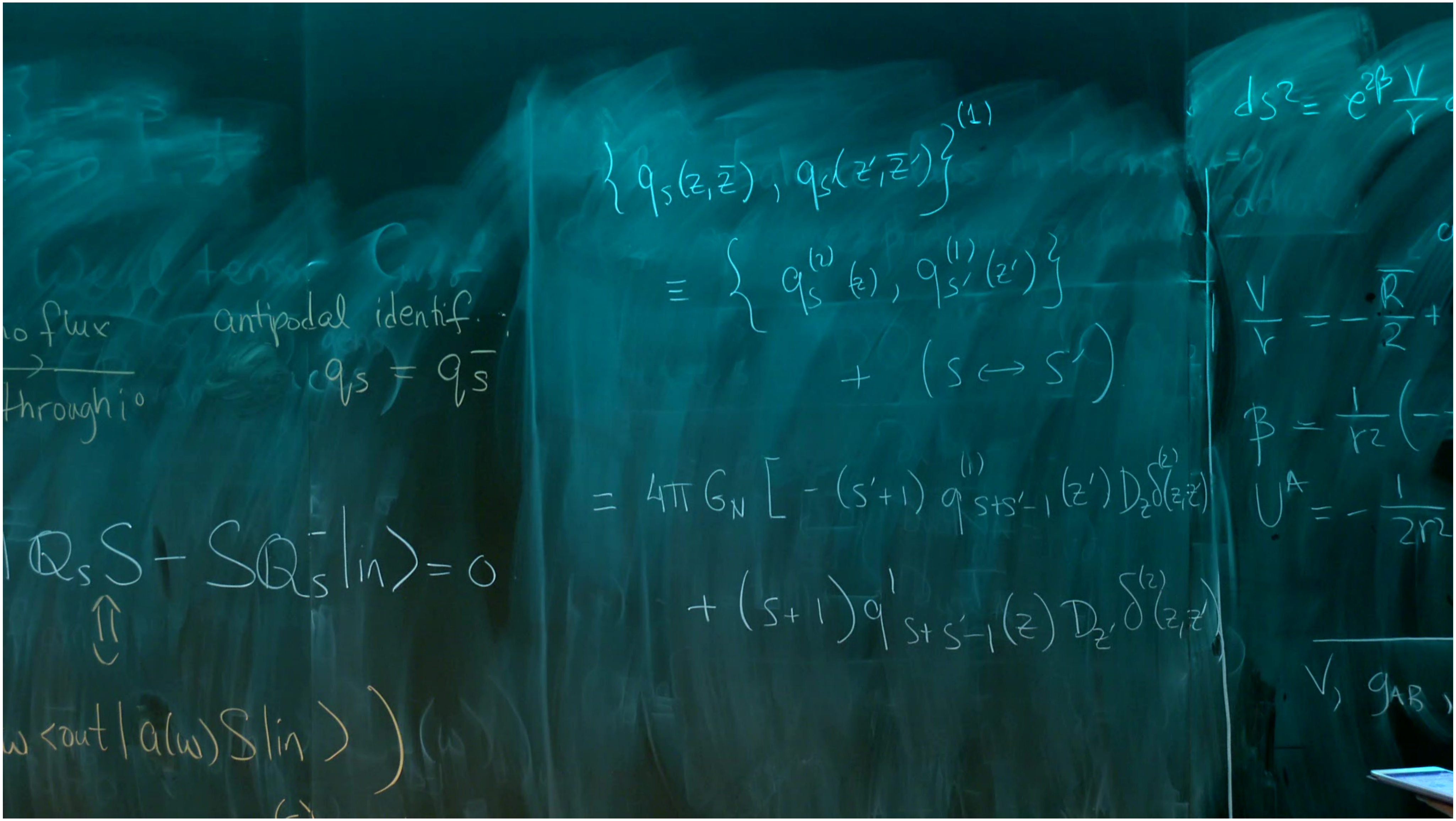
no flux through i^0

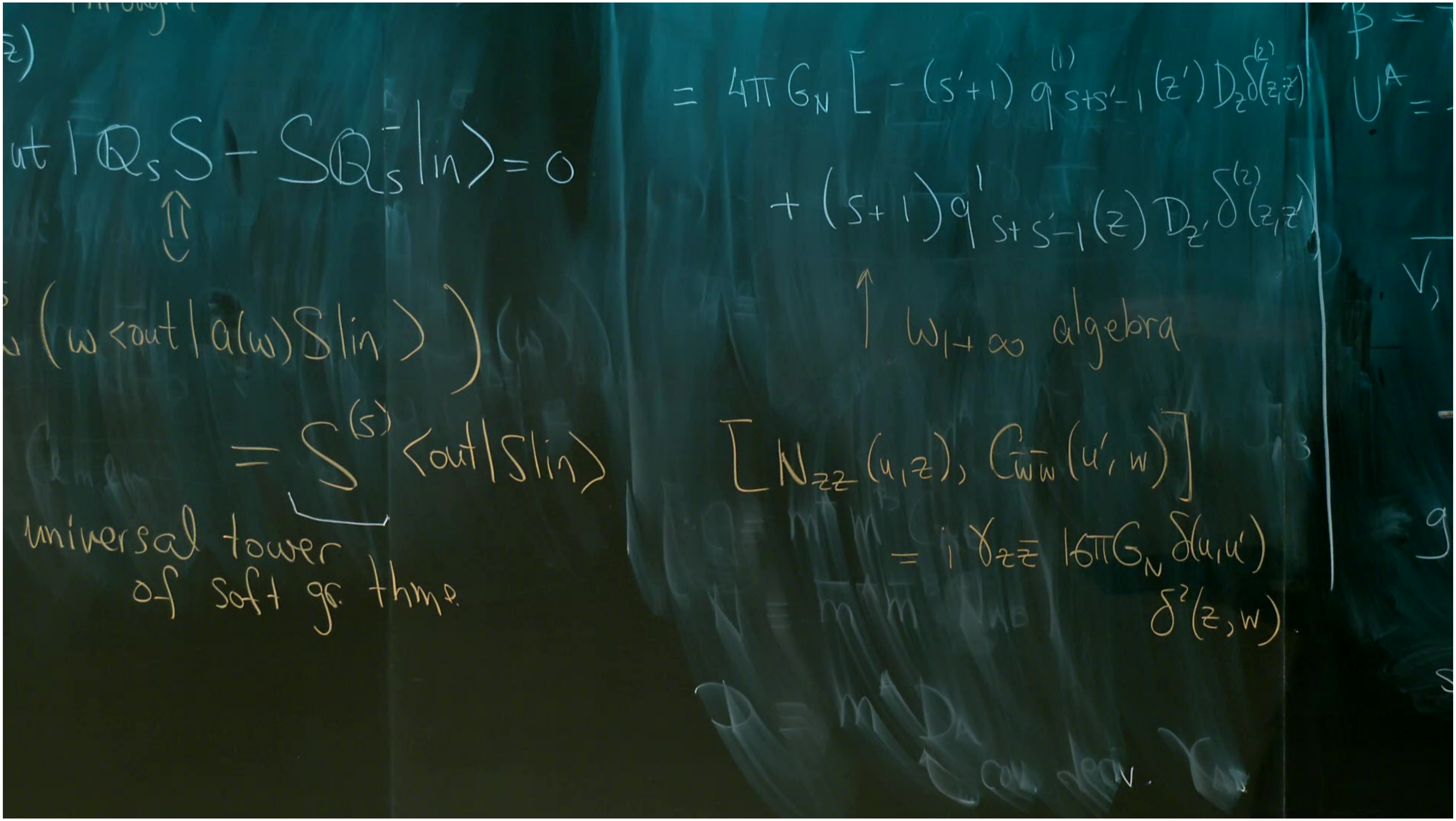
antipodal identif.: $q_s = \bar{q}_{\bar{s}}$

$\Rightarrow \langle out | Q_s S - S \bar{Q}_s | in \rangle = 0$

$\Updownarrow$

$$\lim_{\omega\to 0} \partial_\omega^s \left(\omega \langle out | a(\omega) S | in \rangle \right) = \underbrace{S^{(s)}}_{\text{universal tower of soft gr. thms}} \langle out | S | in \rangle$$


antipodal identif.
$q_s = q_{\bar{s}}$
through i^0
$Q_s S - S Q_s^- |in\rangle = 0$
$\{q_s(z,\bar{z}), q_{s'}(z',\bar{z}')\}^{(1)}$
$\equiv \{q_s^{(2)}(z), q_{s'}^{(1)}(z')\}$
$+ (s \leftrightarrow s')$
$= 4\pi G_N [-(s'+1) q^{(1)}_{s+s'-1}(z') D_z \delta^{(2)}(z,z')$
$+ (s+1) q^1_{s+s'-1}(z) D_{z'} \delta^{(2)}(z,z')$
$ds^2 = e^{2\beta} \frac{V}{r}$
$\frac{V}{r} = -\frac{R}{2} +$
$\beta = \frac{1}{r^2}($
$U^A = -\frac{1}{2r^2}$
$V, g_{AB},$


$\langle \text{out} | Q_s S - S Q_s^- | \text{in} \rangle = 0$
$\Updownarrow$
$(\omega \langle \text{out} | a(\omega) S | \text{in} \rangle)$
$= \underbrace{S^{(s)}} \langle \text{out} | S | \text{in} \rangle$
universal tower
of soft gr. thms.
$= 4\pi G_N [-(s'+1) q^{(1)}_{s+s'-1}(z') D_z \delta^{(2)}(z,z')$
$+ (s+1) q^1_{s+s'-1}(z) D_{z'} \delta^{(2)}(z,z')$
$w_{1+\infty}$ algebra
$[N_{\bar{z}\bar{z}}(u,z), C_{\bar{w}\bar{w}}(u',w)]$
$= i \gamma_{z\bar{z}} 16\pi G_N \delta(u,u')$
$\delta^2(z,w)$.