Title: Lecture - Celestial Holography Ia

Speakers:

Collection: Celestial Holography Summer School 2024

Date: July 22, 2024 - 9:00 AM

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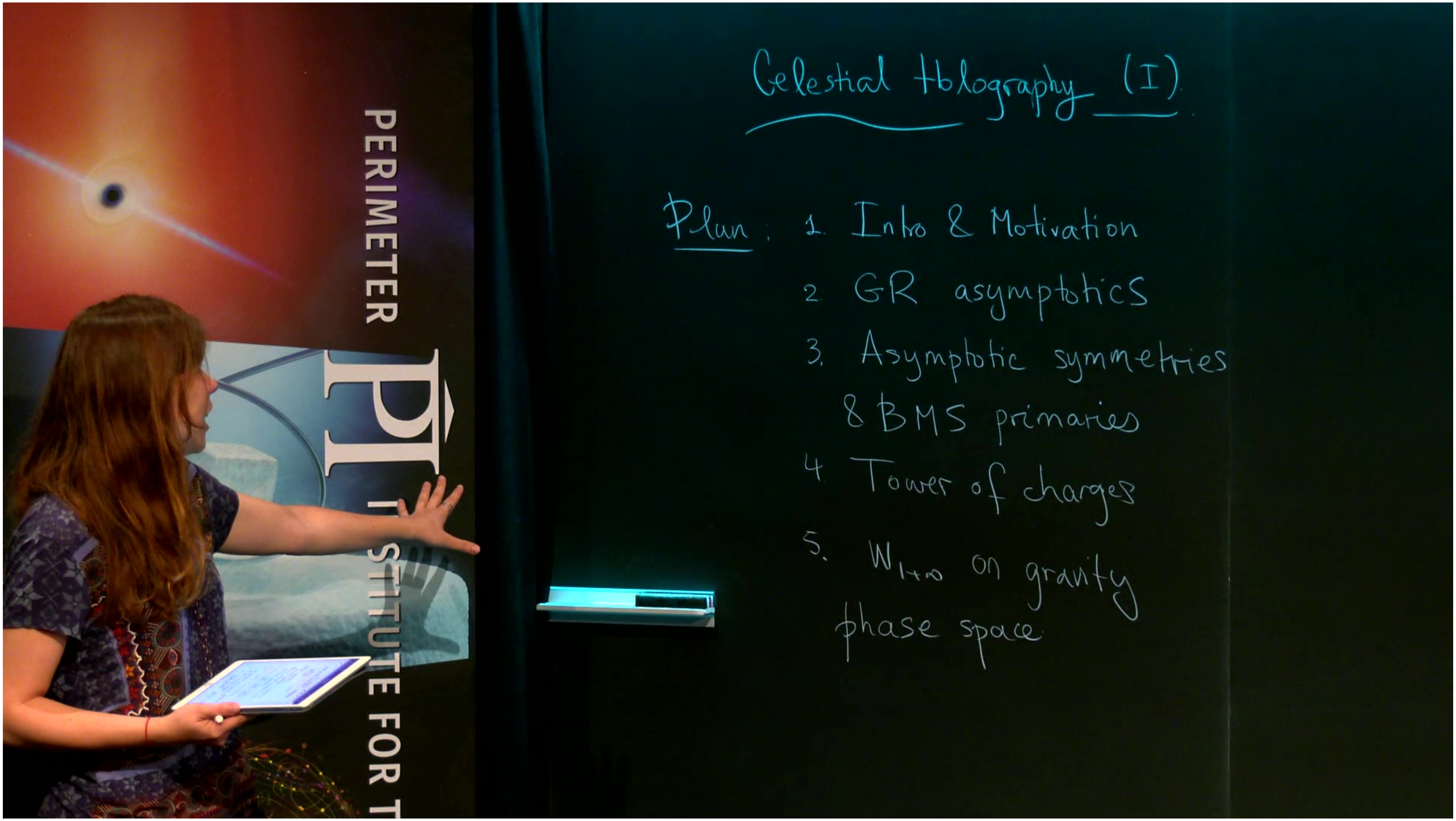

PERIMETER
Celestial Holography (I)
Plan: 1. Intro & Motivation
2. GR asymptotics
3. Asymptotic symmetries
& BMS primaries
4. Tower of charges
5. $W_{1+\infty}$ on gravity
phase space

Celestial Holography (I)

Plan:

1. Intro & Motivation
2. GR asymptotics
3. Asymptotic symmetries & BMS primaries
4. Tower of charges
5. $W_{1+\infty}$ on gravity phase space

Holography

gravity

QFT / CFT

fixed background

dictionary

fluctuating metric

AdS/CFT: a) match symmetries

(isometries of $AdS_{d+1} \simeq$ conformal symm. in d-dim.)

b) match observables

$$\langle O_1(x_1) \ldots O_4(x_4) \rangle$$

2. GR asymptotics

3. Asymptotic symmetries & BMS primaries

4. Tower of charges

5. $W_{1+\infty}$ on gravity phase space

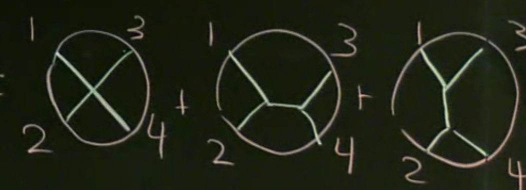
dictionary

fluctuating metric

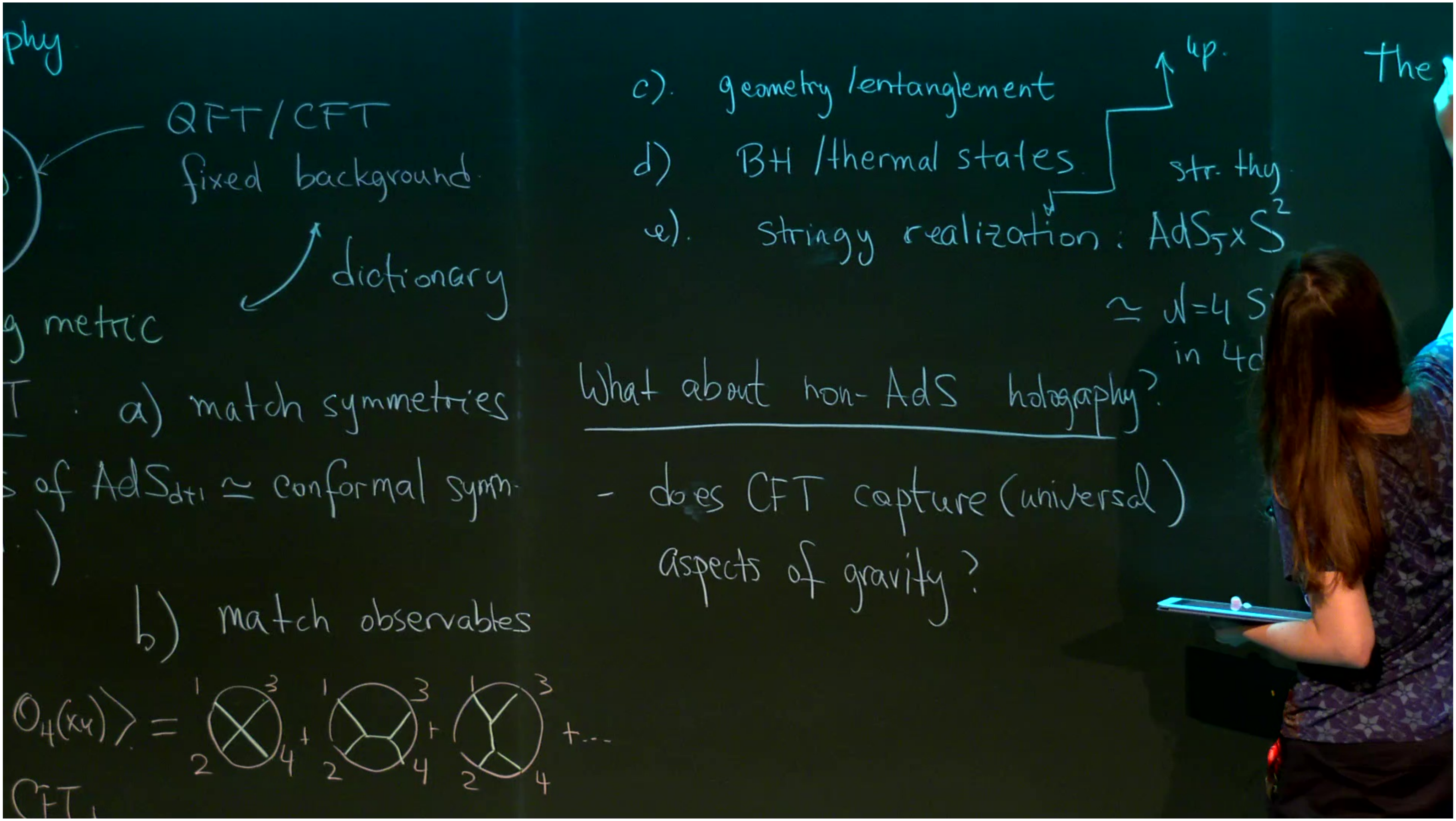
AdS/CFT : a) match symmetries

(isometries of $AdS_{d+1} \simeq$ conformal symm. in d-dim.)

b) match observables

$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_4(x_4) \rangle =$  $+ \dots$

primary ops. in CFT_d


phy
QFT/CFT
fixed background
dictionary
metric
a) match symmetries
of AdS_{d+1} ≃ conformal symm.
b) match observables
CFT
c). geometry/entanglement
d) BH/thermal states
e). stringy realization: AdS5 x S
str. thy
up.
≃ N=4 S
in 4d
What about non-AdS holography?
- does CFT capture (universal) aspects of gravity?
The

tanglement

mal states

str. thy

up.

realization : $AdS_5 \times S^2$

$\simeq \mathcal{N}=4$ SYM in 4d

AdS holography?

pture (universal)

vity?

These lectures: asymptotically flat spacetimes (AFS) - $\Lambda = 0$

* gravitational waves
* astrophysical BH

Goal: explain how a) & b) are realized in (3+1)-dim. AFS

a) suggests dual CFT in 2d

$+ g_{AB}($

$\frac{V}{r} = -\frac{\bar{R}}{2} + \frac{2M}{r} + \ldots$

$\beta = \frac{1}{r^2}\left(-\frac{1}{32} C_{AB} \ldots\right.$

$U^A = -\frac{1}{2r^2} D_B C^{BA} \ldots$

<u>These lectures</u> : asymptotically flat spacetimes (AFS) - $\Lambda = 0$

* gravitational waves
* astrophysical BH

<u>Goal</u> : explain how a) & b) are realized in (3+1)-dim. AFS

a) suggests dual CFT in 2d

b) suggests that gravitational obs. computed from corr. fct's of primaries in 2d CFT.

$$+ g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

$$\frac{V}{r} = -\frac{\bar{R}}{2} + \frac{2M}{r} + \mathcal{O}(r^{-2})$$

$$\beta = \frac{1}{r^2}\left(-\frac{1}{32} C_{AB} C^{AB}\right) + \mathcal{O}(r^{-3})$$

$$U^A = -\frac{1}{2r^2} D_B C^{BA} - \frac{2}{3}\frac{1}{r^3}\left[N^A - \frac{1}{2} C^{AB} D^C C_{BC}\right] + \mathcal{O}(r^{-4})$$

These lectures: asymptotically

flat spacetimes (AFS) $- \Lambda = 0$

* gravitational waves

* astrophysical BH

Goal: explain how a) & b) are realized in (3+1)-dim. AFS

a) suggests dual CFT in 2d

b) suggests that gravitational obs computed from corr. fct's of primaries in 2d CFT

$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du\,dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

$$\frac{V}{r} = -\frac{\bar{R}}{2} + \frac{2M}{r} + \mathcal{O}(r^{-2})$$

$$\beta = \frac{1}{r^2}\left(-\frac{1}{32}C_{AB}C^{AB}\right) + \mathcal{O}(r^{-3})$$

$$U^A = -\frac{1}{2r^2}D_B C^{BA} - \frac{2}{3}\frac{1}{r^3}\left[N^A - \frac{1}{2}C^{AB}D^C C_{BC}\right] + \mathcal{O}(r^{-4})$$

2. Asymptotics & Einstein's equations (E.E.)

* asymptotic structure of background

Look for solutions to E.E.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T^{M}_{\mu\nu} \xrightarrow{\text{today}} 0$$

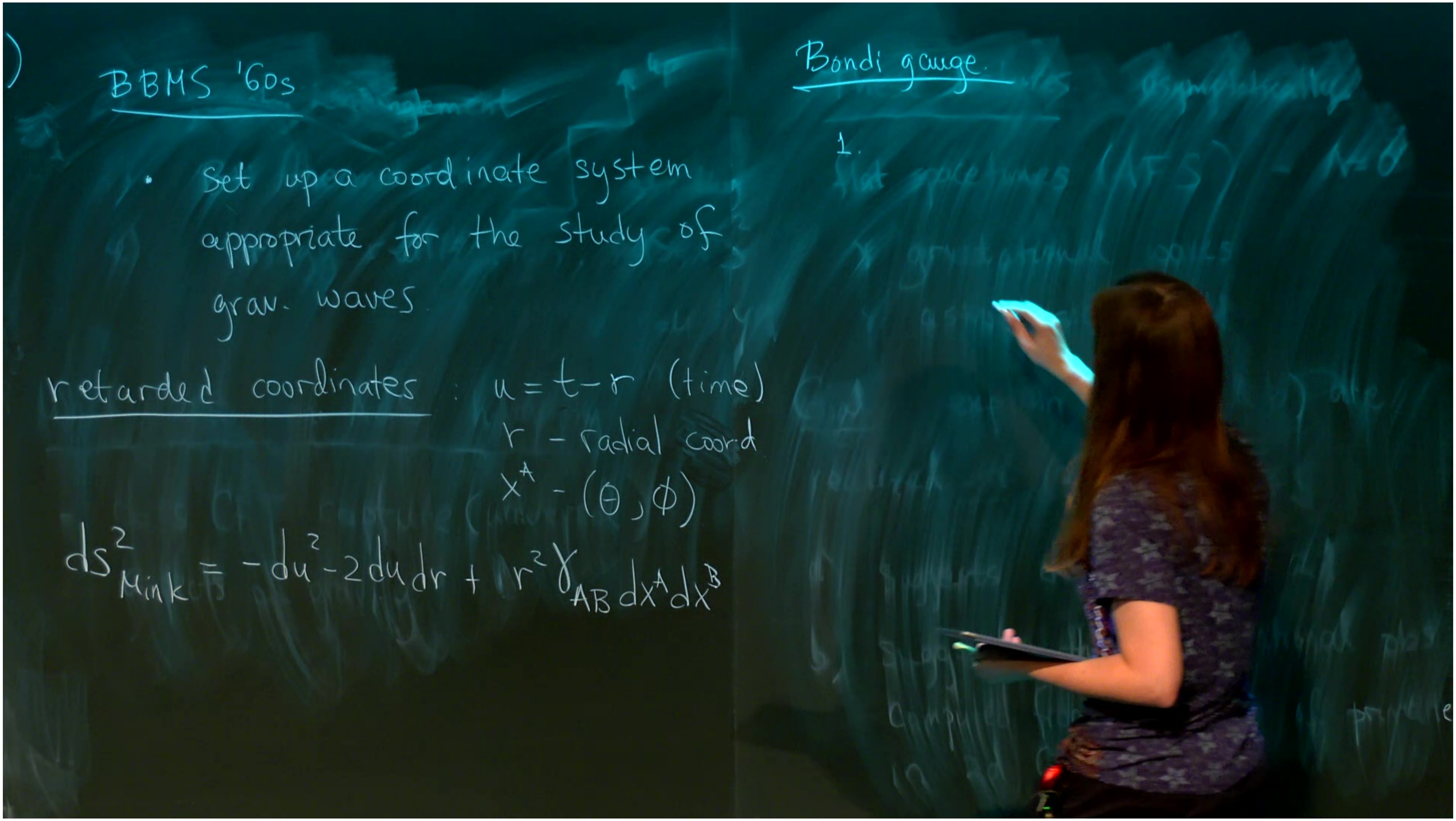
(vacuum E.E.)

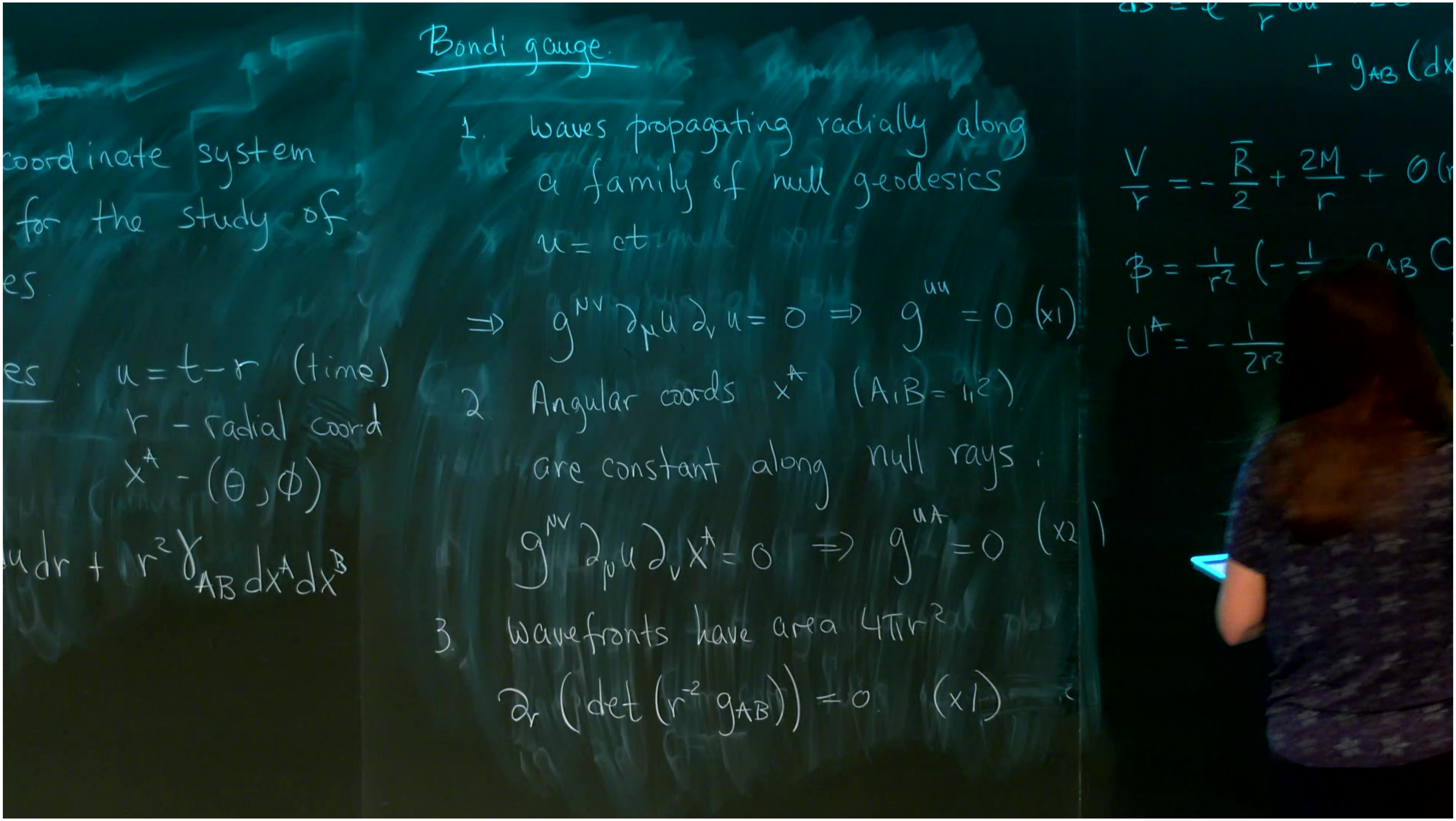
that asymptote to Minkowski at large distances (r)

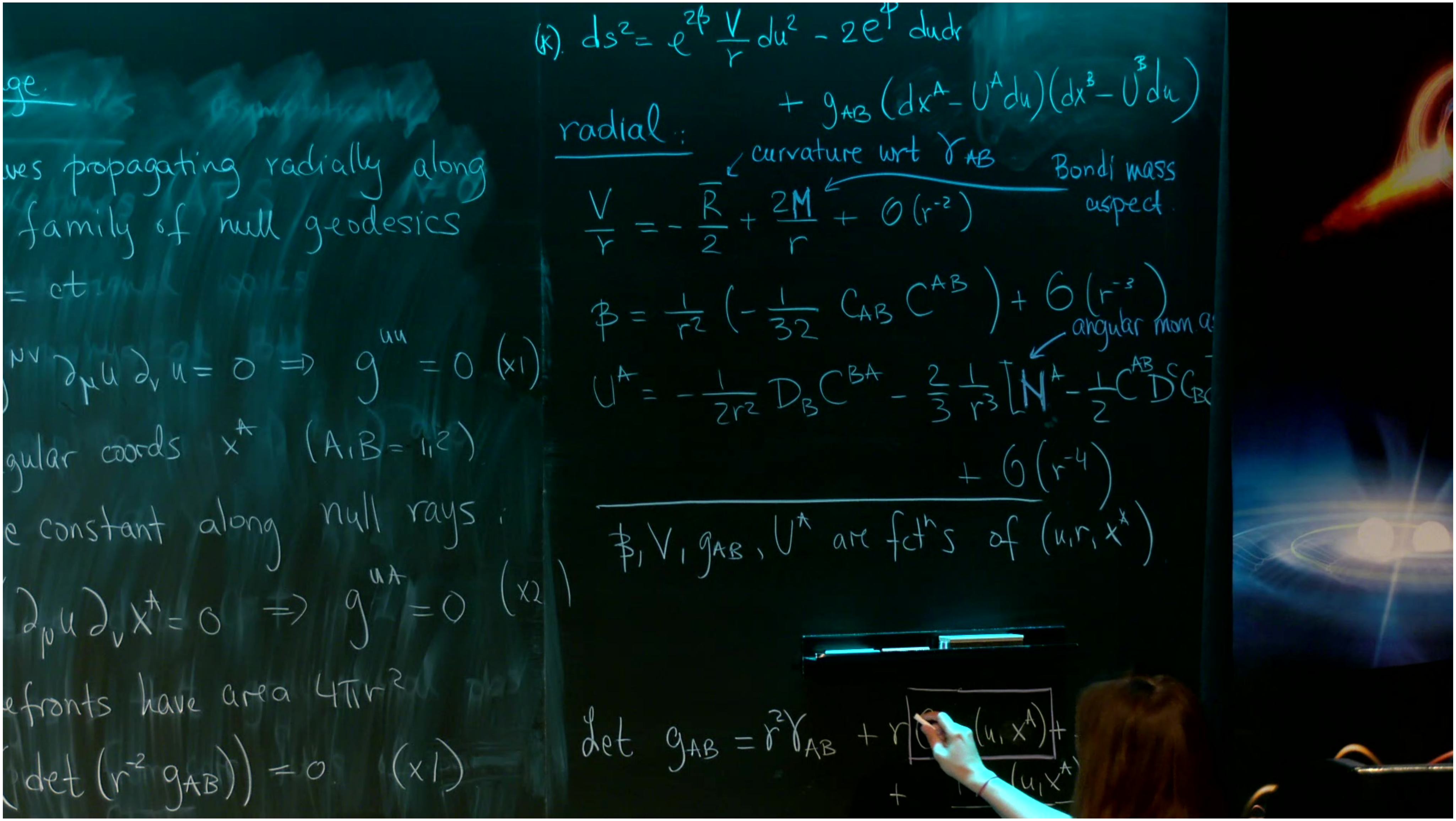
$$ds^2 \xrightarrow[r \to \infty]{} ds^2_{Mink.}$$

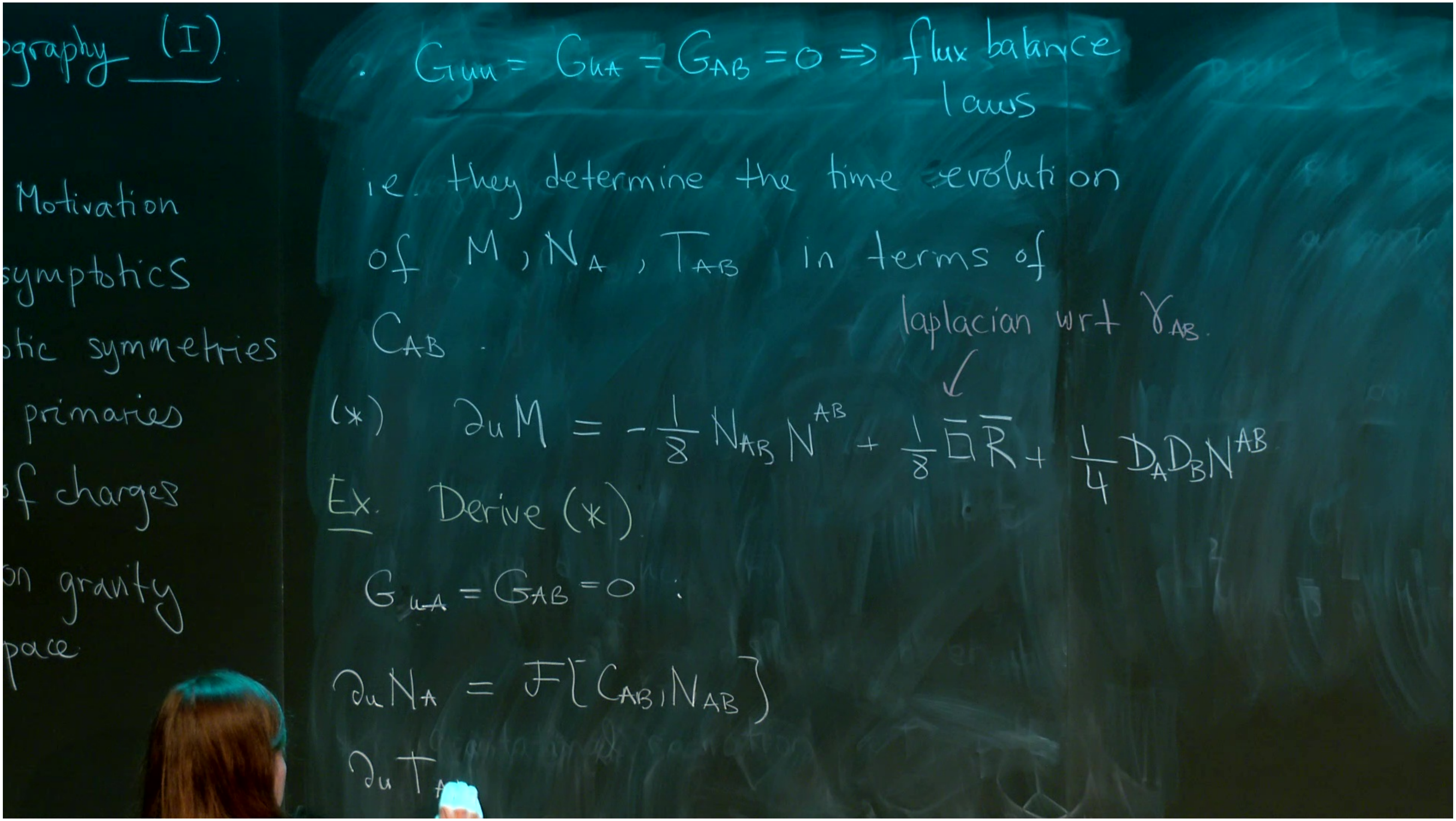
* gravitational radiation ~ TT pert

$$\sim \frac{1}{r}$$


BBMS '60s
• Set up a coordinate system appropriate for the study of grav. waves
retarded coordinates : $u = t - r$ (time)
r - radial coord
x^A - (θ, ϕ)
$ds^2_{Mink} = -du^2 - 2du\,dr + r^2 \gamma_{AB} dx^A dx^B$
Bondi gauge.
1.


coordinate system
for the study of
es
es : u = t − r (time)
r − radial coord
x^A − (θ, φ)
udr + r^2 γ_AB dx^A dx^B
Bondi gauge.
1. waves propagating radially along
a family of null geodesics
u = ct
⇒ g^{μν} ∂_μ u ∂_ν u = 0 ⇒ g^{uu} = 0 (x1)
2 Angular coords x^A (A, B = 1, 2)
are constant along null rays :
g^{μν} ∂_μ u ∂_ν x^A = 0 ⇒ g^{uA} = 0 (x2)
3 wavefronts have area 4πr^2
∂_r (det (r^{-2} g_AB)) = 0 (x1)
+ g_AB (dx
V/r = − R̄/2 + 2M/r + O(
β = 1/r^2 (− 1/… C_AB C
U^A = − 1/2r^2


ge.
ves propagating radially along
family of null geodesics
= ct
$g^{\mu\nu}\partial_\mu u \partial_\nu u = 0 \Rightarrow g^{uu} = 0$ (x1)
gular coords x^A (A, B = 1, 2)
e constant along null rays :
$\partial_\mu u \partial_\nu x^A = 0 \Rightarrow g^{uA} = 0$ (x2)
efronts have area $4\pi r^2$
(det $(r^{-2} g_{AB})$) = 0 (x1)
(x). $ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du\,dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$
radial :
curvature wrt γ_{AB}
Bondi mass aspect
$\frac{V}{r} = -\frac{\bar{R}}{2} + \frac{2M}{r} + O(r^{-2})$
$\beta = \frac{1}{r^2}\left(-\frac{1}{32} C_{AB} C^{AB}\right) + O(r^{-3})$
angular mom as
$U^A = -\frac{1}{2r^2} D_B C^{BA} - \frac{2}{3}\frac{1}{r^3}\left[N^A - \frac{1}{2} C^{AB} D^C C_{BC}\right] + O(r^{-4})$
β, V, g_{AB}, U^A are fct^n s of (u, r, x^A)
det $g_{AB} = r^2\gamma_{AB} + r\,(u, x^A) +$
$+ \quad (u, x^A)$


...ography (I)
Motivation
...symptotics
...tic symmetries
primaries
...f charges
...on gravity
...pace
• $G_{uu} = G_{uA} = G_{AB} = 0 \Rightarrow$ flux balance laws
ie they determine the time evolution
of M, N_A, T_{AB} in terms of
C_{AB}.
laplacian wrt γ_{AB}.
(*) $\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{8} \bar{\Box} \bar{R} + \frac{1}{4} D_A D_B N^{AB}$
Ex. Derive (*)
$G_{uA} = G_{AB} = 0$:
$\partial_u N_A = \mathcal{F}[C_{AB}, N_{AB}]$
$\partial_u T_A$

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ie. they determine the time evolution of M, N_A, T_{AB} in terms of C_{AB}.

laplacian wrt γ_{AB}.

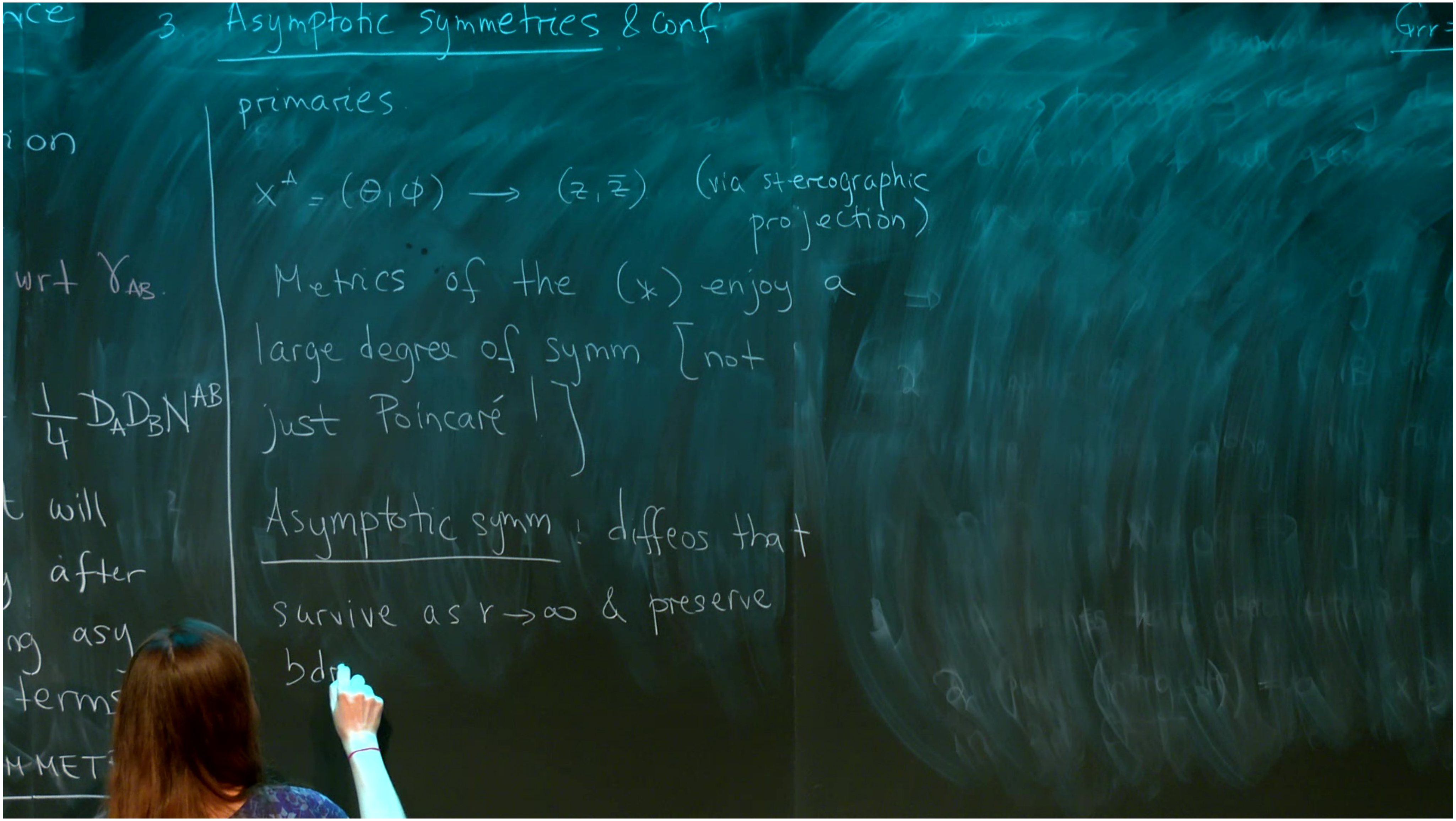
(*) $$\partial_u M = -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{8} \bar{\Box} \bar{R} + \frac{1}{4} D_A D_B N^{AB}$$

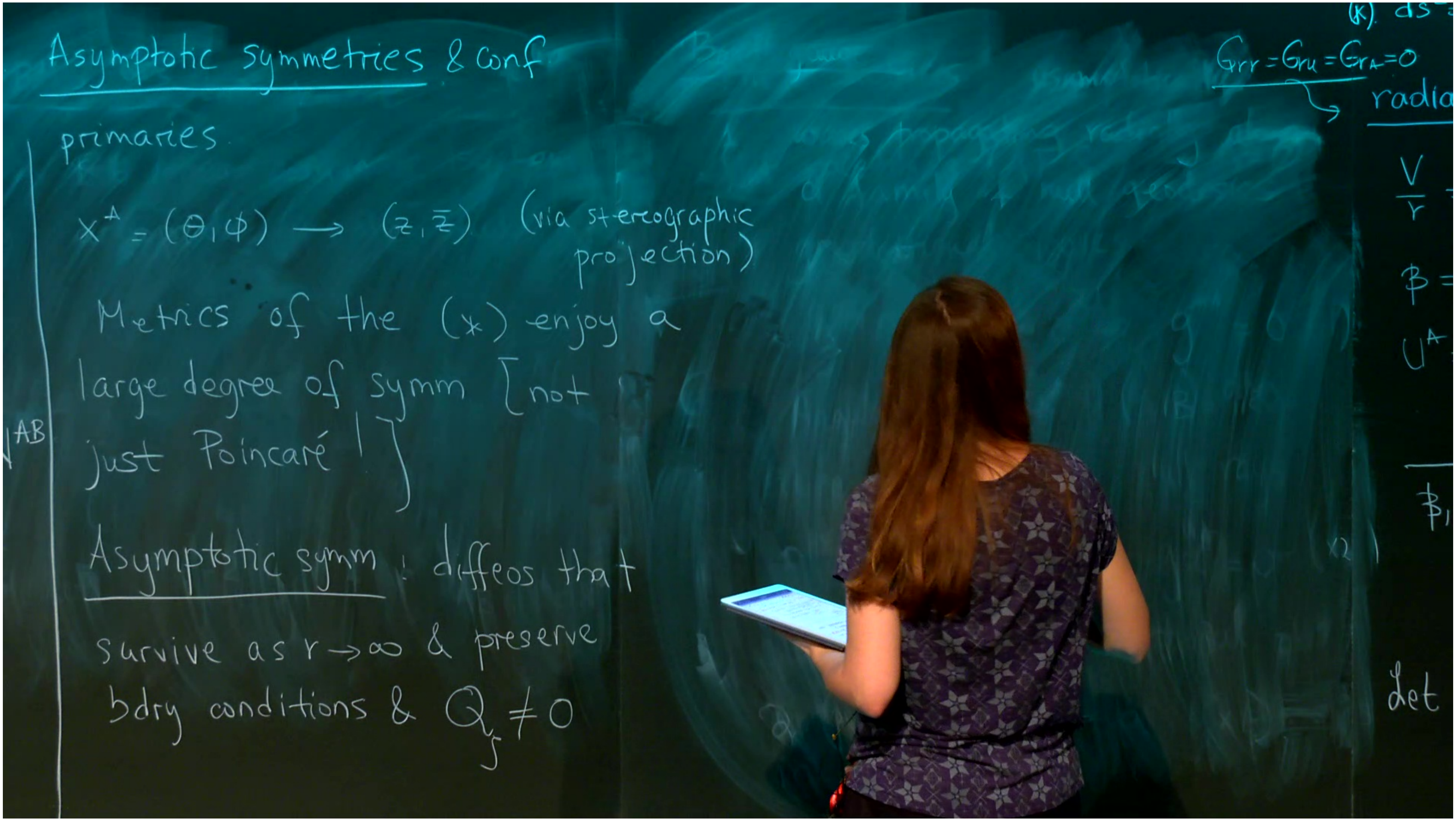
<u>Ex.</u> Derive (*)

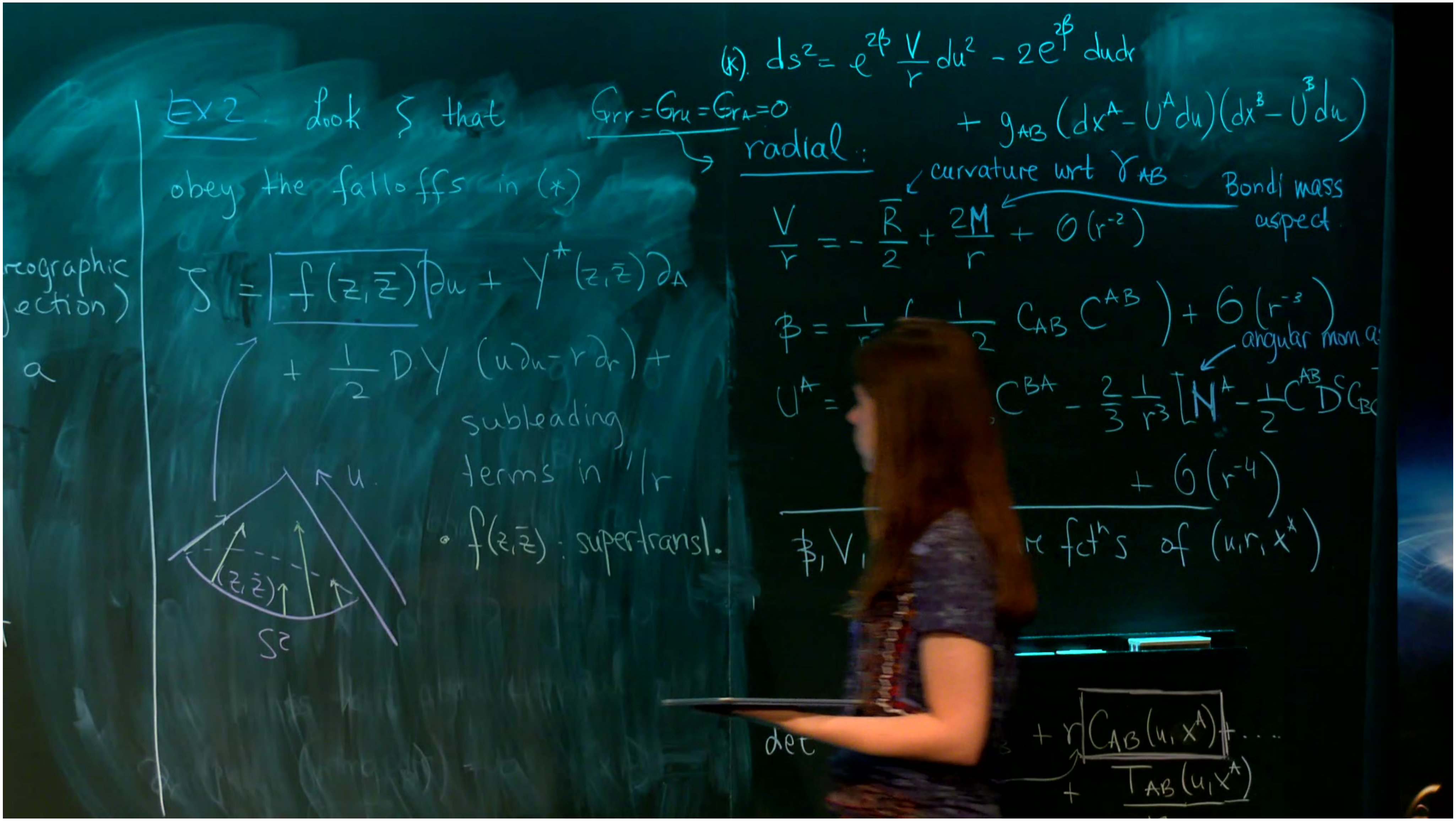
$G_{uA} = G_{AB} = 0$:

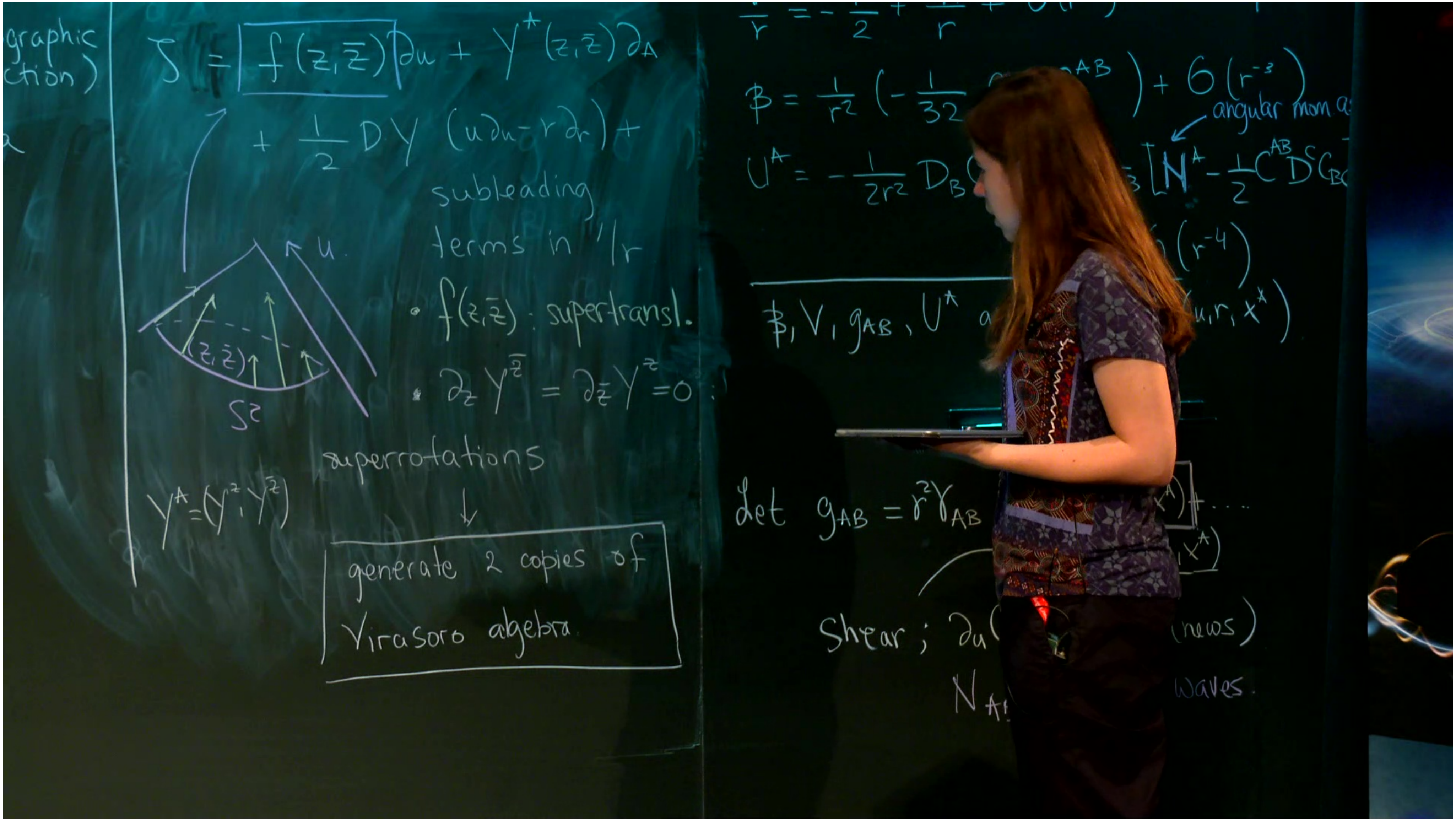
$$\left.\begin{aligned} \partial_u N_A &= \mathcal{F}[C_{AB}, N_{AB}] \\ \partial_u T_{AB} &= \mathcal{G}[C_{AB}, N_{AB}] \end{aligned}\right\}$$

messy, but will simplify after reorganizing asy. data in terms of <u>SYMMETRY</u>


3. Asymptotic symmetries & conf
primaries
$x^A = (\theta, \phi) \longrightarrow (z, \bar{z})$ (via stereographic projection)
Metrics of the (*) enjoy a large degree of symm [not just Poincaré!]
Asymptotic symm : diffeos that survive as $r \to \infty$ & preserve bdr
wrt γ_{AB}.
$\frac{1}{4} D_A D_B N^{AB}$
will
after
asy
terms


Asymptotic symmetries & conf
primaries
x^A = (θ, φ) → (z, z̄) (via stereographic projection)
Metrics of the (*) enjoy a large degree of symm [not just Poincaré!]
Asymptotic symm: diffeos that survive as r→∞ & preserve bdry conditions & Q_ξ ≠ 0
G_rr = G_ru = G_rA = 0
radia
det


Ex 2 look ξ that Grr = Gru = GrA = 0
obey the falloffs in (*)
ξ = f(z, z̄) ∂u + Y^A(z, z̄) ∂A
+ 1/2 D·Y (u∂u − r∂r) +
subleading terms in 1/r
• f(z, z̄) : supertransl.
u
(z, z̄)
S²
reographic
ection)
(*) ds² = e^{2β} V/r du² − 2e^{2β} du dr
+ g_AB (dx^A − U^A du)(dx^B − U^B du)
radial :
curvature wrt γ_AB
Bondi mass aspect
V/r = − R/2 + 2M/r + O(r^−2)
β = O(r^−3)
angular mom as
U^A = − 2/3 1/r³ [N^A − 1/2 C^AB D^C C_BC
+ O(r^−4)
β, V, fct^n s of (u, r, x^A)
det
+ r C_AB(u, x^A) + …
+ T_AB(u, x^A)


graphic
ction)
subleading
terms in 1/r
f(z, z̄) : supertransl.
superrotations
generate 2 copies of
Virasoro algebra
angular mom as
det
Shear
news)
waves

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Asy. symmetry algebra $\supset$ Virasoro^2

↳ signature of 2d CFT!

Good idea (?) to organize asy. data in <u>representations</u> of Vir^2

↳ this is not what one usually does when constructing scattering observables!

soro ^{2}

ure of

CFT.

r^2

one

en

ering

What is usually done

as $t \to \pm\infty$: fields are $\underline{\text{free}}$

(scalars)

$$\Phi = \int \tilde{d^3p} \left[e^{-ip\cdot x} c^*_{\vec{p}} + e^{-ip\cdot x} c_{\vec{p}} \right]$$

Casimir of Poincaré

$$\hookrightarrow (\Box + m^2)\Phi = 0 \Leftrightarrow P_\mu P^\mu \Phi = -m^2 \Phi \quad (a)$$

$\Rightarrow$ quantize $c^*_{\vec{p}}$, $c_{\vec{p}}$ $\to$ promote to $a_{\vec{p}}, a^+_{\vec{p}}$

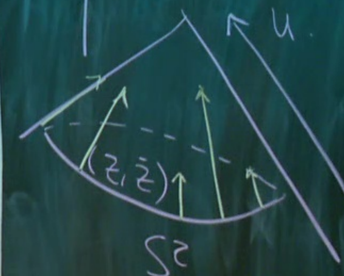
$$(b)\ P_\mu \Phi = p_\mu \Phi$$

$\underline{\text{Ex 2}}$: look ξ that obey the falloffs in (*

$$\xi = \boxed{f(z,\bar{z})}\partial_u + Y^A$$

$$+ \frac{1}{2} D \cdot Y (u\partial_u - $$

subleading terms



- $f(z,\bar{z})$:
- $\partial_z Y^{\bar{z}}$

$Y^A = (Y^z, Y^{\bar{z}})$

superrotations

$\downarrow$

generate 2 co

organize asy.
tions of Vir^2

is not what one
lly does when
tructing scattering
ervables !

(scalars)

$$\Phi = \int \widetilde{d^3 p}\,\big[\, e^{-ip\cdot x}\, c^*_{\vec p} + e^{-ip\cdot x}\, c_{\vec p} \,\big]$$

p^{μ}

Casimir of Poincaré

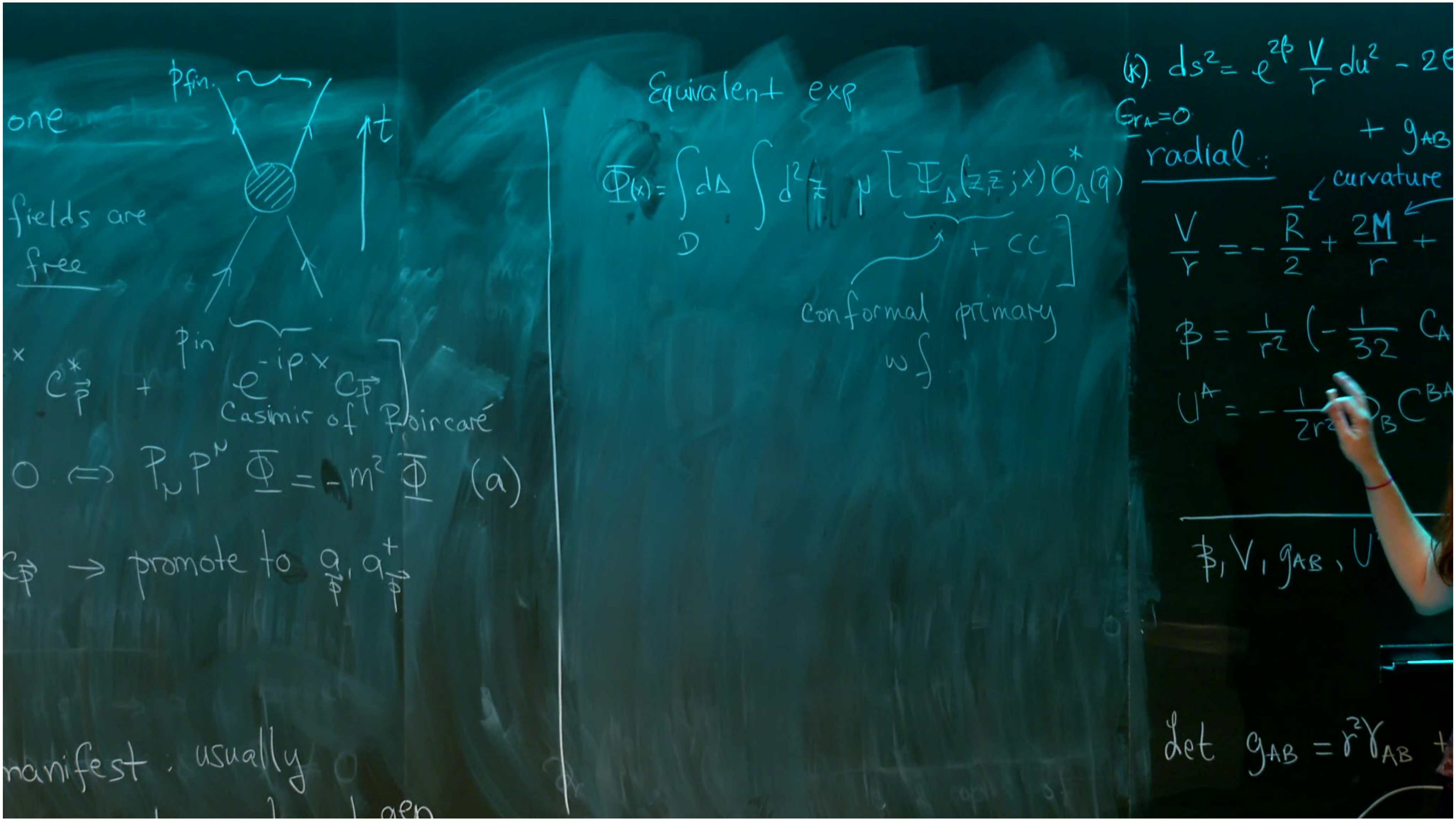
$$\hookrightarrow (\Box + m^2)\,\Phi = 0 \iff P_\mu P^\mu\, \Phi = -m^2\, \Phi \quad \text{(a)}$$

$\Rightarrow$ quantize $c^*_{\vec p}$, $c_{\vec p}$ $\Rightarrow$ promote to $a_{\vec p}$, $a^\dagger_{\vec p}$

(b) $P_\mu \Phi = p_\mu \Phi$

to make Virasoro manifest : usually
diagonalize scaling generator $\iff$ boost gen.
K^3
along $\hat{q}(z,\bar z)$

$y^A = (y^z, y^{\bar z})$


Equivalent exp
one
fields are free
conformal primary
Casimir of Poincaré
promote to
manifest. usually
radial:
curvature
det