Title: Upper Bound on Thermal Gravitational Wave Backgrounds from Hidden Sectors

Speakers: Juraj Klaric

Series: Particle Physics

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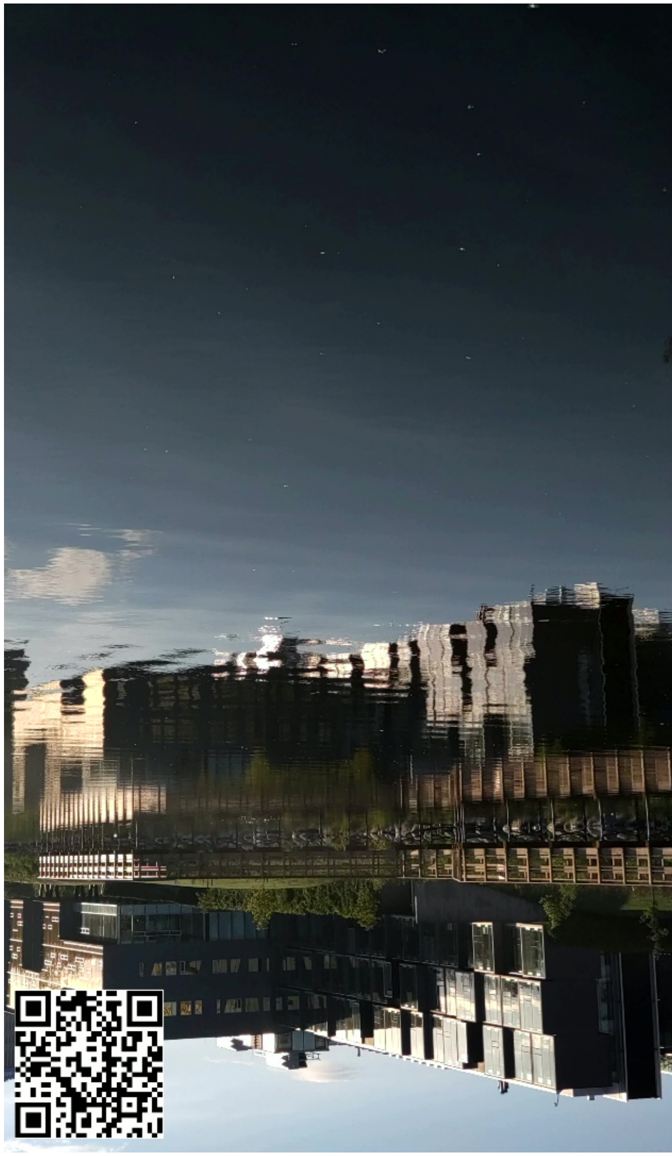
Abstract: Hot viscous plasmas unavoidably emit a gravitational wave background, similar to electromagnetic black body radiation. In this talk we will discuss the contribution from hidden particles to the diffuse background emitted by the primordial plasma in the early universe. While this contribution can easily dominate over that from Standard Model particles, both are capped by a generic upper bound that makes them difficult to detect with interferometers in the foreseeable future. We will illustrate our results on the examples of axion-like particles and heavy neutral leptons. We will also discuss how this bound affects the previous estimates of gravitational wave backgrounds from particle decays out of thermal equilibrium.

Zoom link



Upper bound on Thermal Gravitational Waves from Hidden Sectors

based on [arXiv:2312.13855]
with *M. Drewes, Y. Georis and P. Klose*

PI particle physics seminar, May 31st 2024

Juraj Klarić

University of Zagreb
Universiteit van Amsterdam
Nikhef

Gravitational Waves

- Excitations of the metric
- Sourced by anisotropies in the stress-energy tensor

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2 h_{ij}}{a^2} = \frac{16\pi T_{ij}}{a^2 m_{\rm Pl}^2}$$

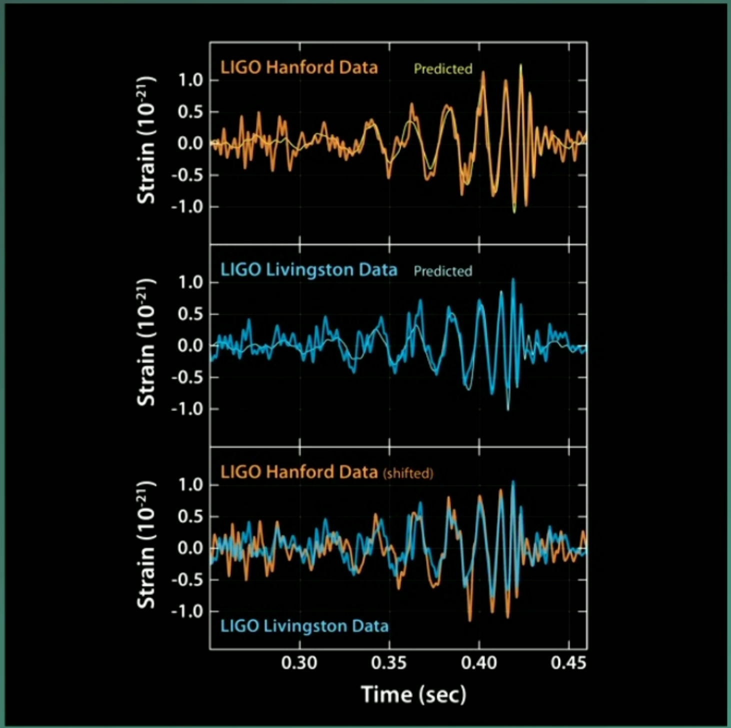
- In the local Minkowski metric:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(\delta_{ij} + h_{ij})\mathrm{d}x^i \mathrm{d}x^j$$

$$h_i^i = \partial_i h_{ij} = 0$$

Sources of Gravitational Waves

- Various sources observed so far:
  - Black Hole mergers
  - Neutron stars
- Well studied potential sources predicted in the **Early Universe**:
  - Inflation
  - Preheating
  - Phase transitions



[Image credit: LIGO]

Sources of Gravitational Waves

- Various sources observed so far:
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- Well studied potential sources predicted in the **Early Universe**:
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- What about the **SM thermal plasma?**

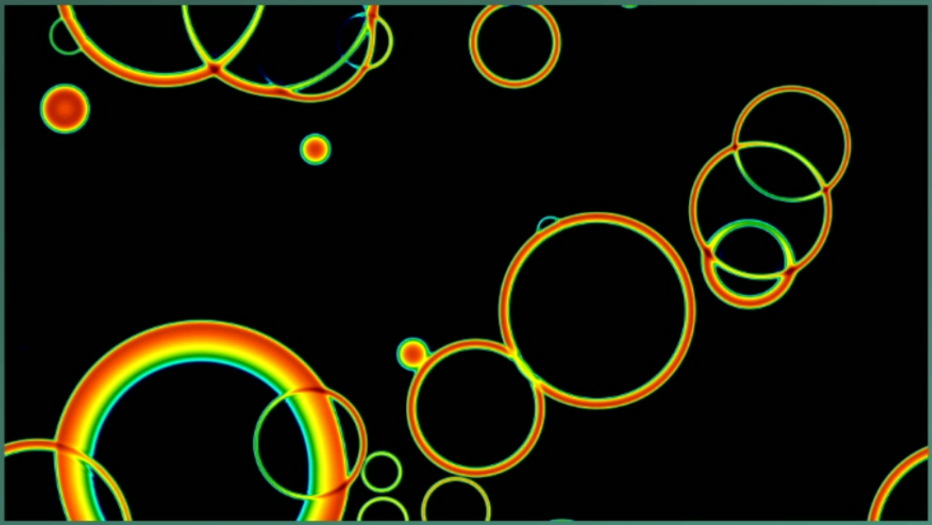

Fig. From [D. Weir '19]

GW spectral density in the Early Universe

- The spectral density $e_{\mathrm{gw}} \equiv \frac{m_{\mathrm{Pl}}^2}{32\pi} \left\langle \dot{h}^{ij}(t,0)\dot{h}_{ij}(t,0) \right\rangle$ grows as:

$$\frac{\mathrm{d}\dot{e}_{\mathrm{gw}}}{\mathrm{d}\ln f} + 4H \frac{\mathrm{d}e_{\mathrm{gw}}}{\mathrm{d}\ln f} = 16\pi^2 \left(\frac{f a_0}{a}\right)^3 \frac{\Pi(2\pi f a_0 a)}{m_{\mathrm{Pl}}^2}$$

[Laine, Ghiglieri]

- The production rate Π is sourced by the fluctuations in the stress-energy tensor T_{ij} :

$$\Pi(k) = \frac{1}{2} \int \mathrm{d}t\, \mathrm{d}^3x\; e^{\imath(kt-kx)}\; \mathbb{L}^{ij;kl} \left\langle \{ T_{ij}(t,x), T_{kl}(0,0) \} \right\rangle$$

- And leads to the overall GW background:

$$h^2\Omega_{\mathrm{gw}}(f) \approx 2.02 \cdot 10^{-38} \times \left(\frac{f}{Hz}\right)^3 \times \int_{T_{\mathrm{min}}}^{T_{\mathrm{max}}} \frac{\mathrm{d}T'}{m_{\mathrm{Pl}}} \frac{\Pi\left(2\pi f a_0 a'\right)}{8T'^4}$$

Thermal Production of Gravitational Waves in the Standard Model

- Just like any other particle, GWs are produced thermally through scatterings
- This process is extremely slow

$$\Pi \sim \frac{T^3}{m_{\rm Pl}^2} \ll H \sim \frac{T^2}{m_{\rm Pl}}$$

- GWs never reach thermal equilibrium

$$\mathcal{L} \supset \frac{1}{2}\frac{\sqrt{8\pi}}{m_{\rm Pl}} h_{ij} T^{ij}$$

[J. Ghiglieri, G. Jackson, M. Laine, Y. Zhu]

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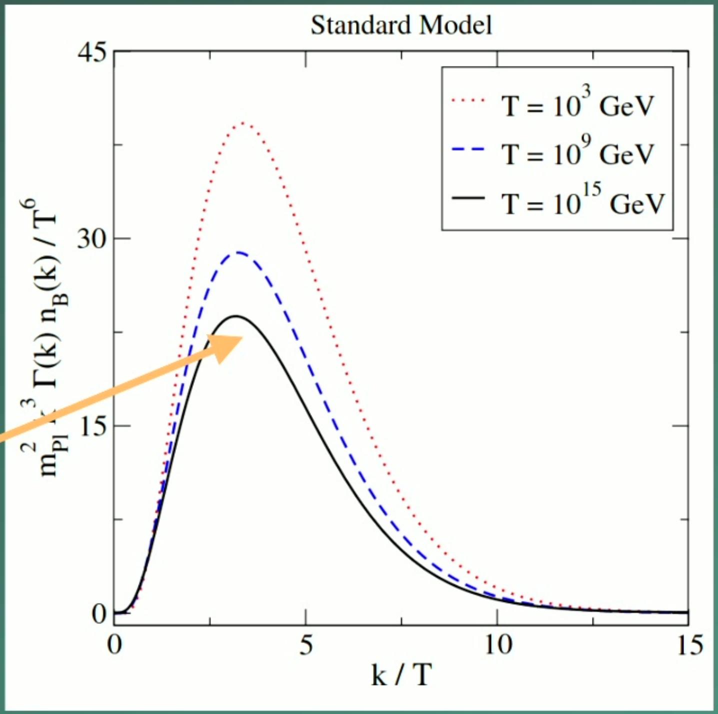

[J. Ghiglieri, G. Jackson, M. Laine, Y. Zhu]

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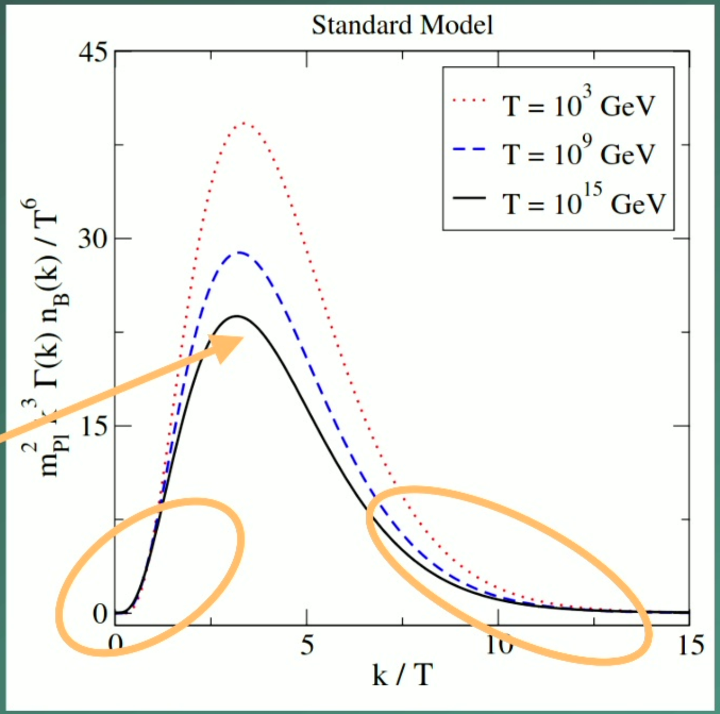
- GWs **never reach thermal equilibrium**
- Production optimal for $k \simeq T$
- Modes with $k \gg T$ **Boltzmann suppressed**
- **IR modes are enhanced** $\Pi \propto \frac{T^5 \Upsilon_{av}}{k^2}$



[J. Ghiglieri, G. Jackson, M. Laine, Y. Zhu]

The IR regime: GWs from hydrodynamic fluctuations

- GWs with $k \ll T$ can be sourced by macroscopic hydrodynamic fluctuations
- Similar phenomenon exists in QED:

$$\frac{\mathrm{d}\Gamma_\gamma(\vec{k})}{\mathrm{d}^3\vec{k}} = \frac{1}{(2\pi)^3 2k} \sum_\lambda \epsilon^{(\lambda)}_{\mu,\vec{k}} \epsilon^{(\lambda)*}_{\nu,\vec{k}} \int_{\mathcal{X}} e^{i\mathcal{K}\cdot\mathcal{X}} \langle J^\mu_{em}(0) J^\nu_{em}(\mathcal{X})\rangle$$

~ $\chi_{em}\, T$

- While the mean charge density vanishes $\langle n(x)\rangle = 0$
  thermal fluctuations give non-zero root-mean-square value

$$\langle n(x) n(y)\rangle = T\chi_{\mathrm{em}}$$

EM susceptibility

- In GR the stress energy tensor has the role of the current

$$\Pi(k) \sim \langle T_{ij} T_{kl}\rangle \to 8T\eta$$

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Shear viscosity

The Shear Viscosity

- The shear viscosity η scales with the **mean free path l_{av}** of the most weakly interacting plasma constituent:

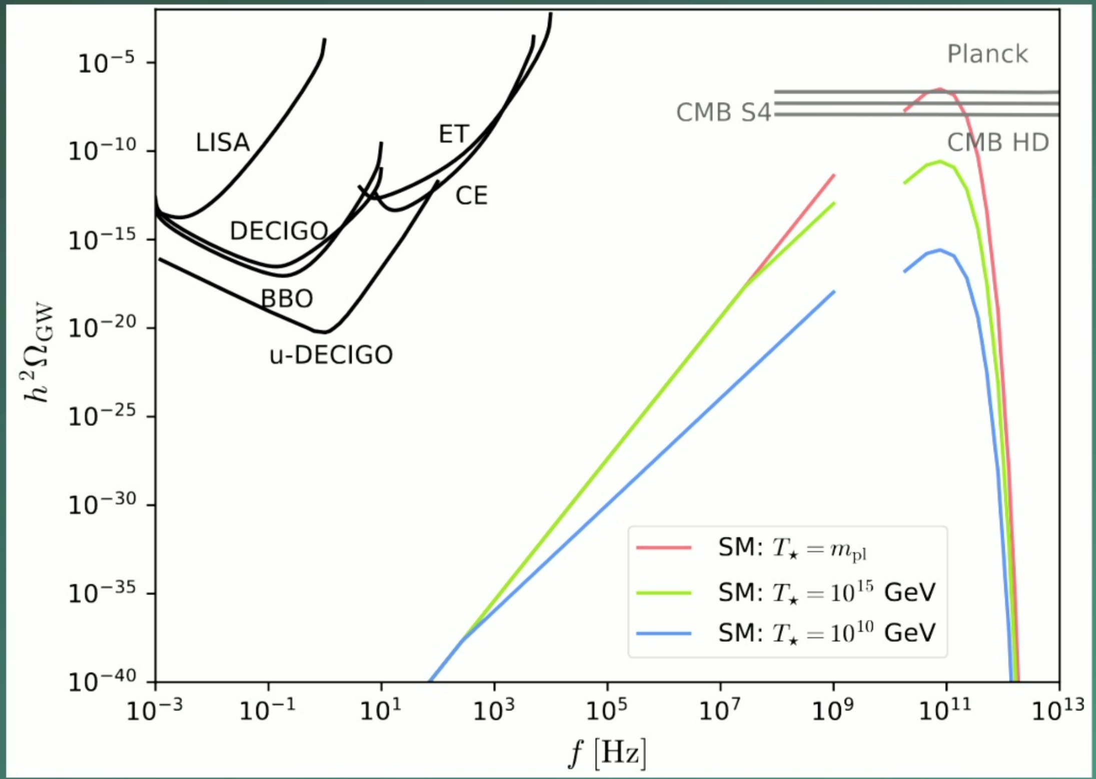
$$\eta \sim l_{\mathrm{av}} v_{\mathrm{av}} T^4 \sim \frac{T^4}{\Upsilon}$$

mean velocity → v_{av}; particle width → Υ; Finite for small k !

- Particles with **feeble interactions** (long free paths) can **dominate GW production**
- GWs can be a promising **probe for hidden sectors**
- The scaling $\Pi \sim 8\eta T \sim 8T^5/\Upsilon$ only holds in the **hydrodynamic limit** ($k \ll \Upsilon$).
- For smaller width $k \gg \Upsilon$, the scaling changes to $\Pi \propto \Upsilon$ - **without the enhancement** for feebly interacting particles.
- Feebly interacting particles can dominate the primordial GW production in the **hydrodynamic regime** but not necessarily in other regimes

The SM GW background

- Guaranteed GW background
- Amplitude highly sensitive to the **maximal temperature $T_\star$**
- Possible to probe with **CMB_S4** by measuring N_{eff}
- Signal peaked at $f \approx 10^{11}$ **Hz**
- Well understood **hydrodynamic** and **Boltzmann** limits
- moderate uncertainty in the intermediate regime



[Laine, Ghiglieri][J. Ghiglieri, G. Jackson, M. Laine, Y. Zhu] [Ringwald Schutte-Engel Tamarit]

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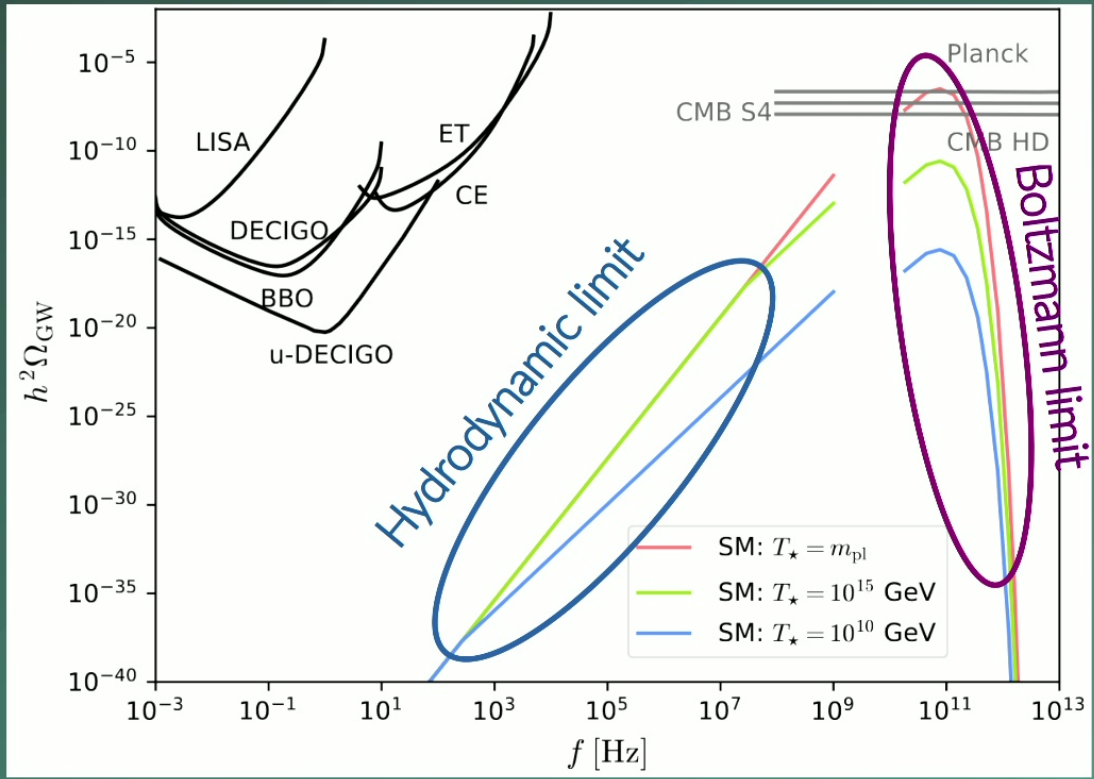
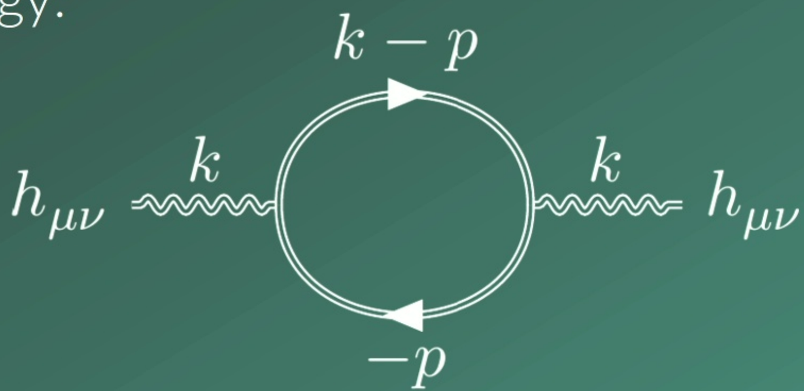


[Laine, Ghiglieri][J. Ghiglieri, G. Jackson, M. Laine, Y. Zhu] [Ringwald Schutte-Engel Tamarit]

GWs from Hidden Sectors

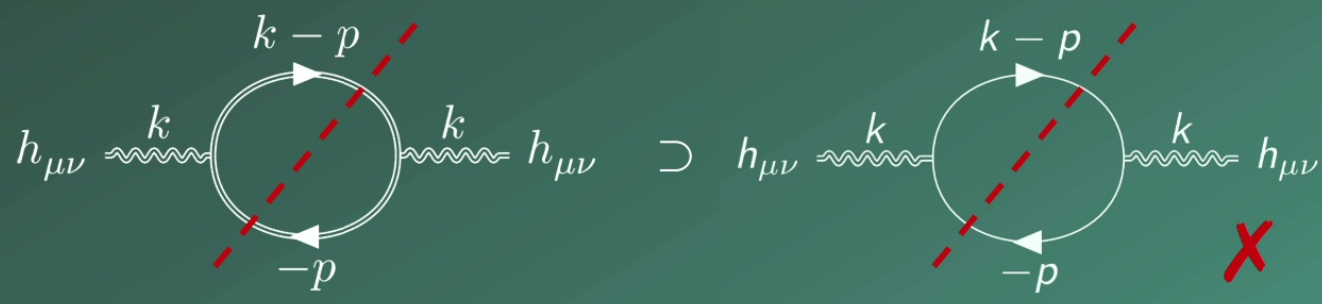
Towards a unified description of GW production from Hidden Sectors

- To capture both elements of the **hydrodynamic** and **Boltzmann regimes**, we compute the GW production rate in the real-time formalism
- Optical theorem: the production rate is just the imaginary part of the graviton self-energy:



The need for a finite width

- At tree-level, the process is **kinematically forbidden**
- By using resummed propagators, we effectively include an entire class of diagrams



$$\Pi(k) = -\frac{c_D^2}{8}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int \frac{\mathrm{d}p_0}{2\pi}\, \mathbb{L}^{ij;kl} p_i p_k [\gamma_j 1 S_p^{>} \gamma_l 1 S_{p-k}^{<} + \gamma_j 1 S_p^{<} \gamma_l 1 S_{p-k}^{>}]$$

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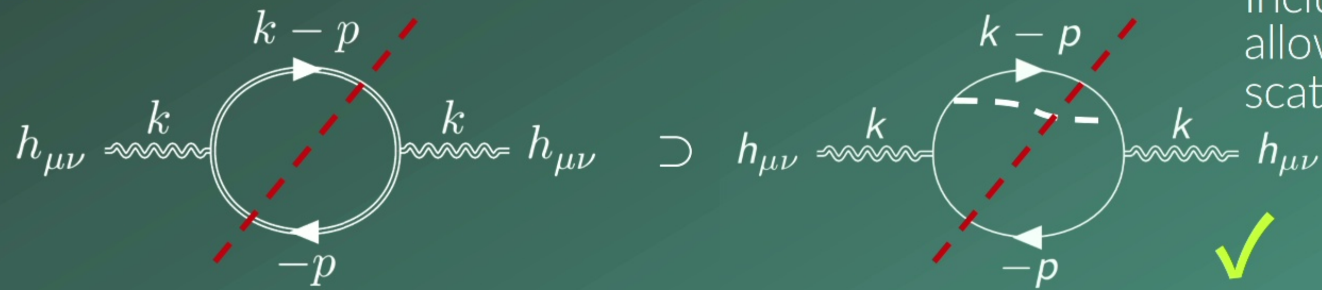
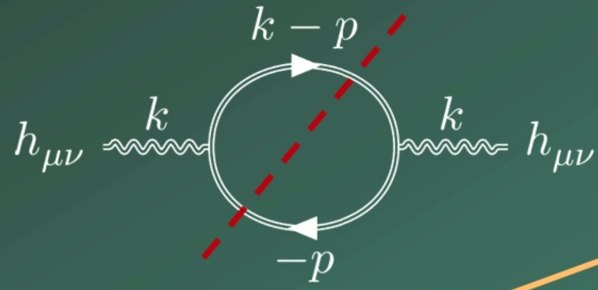


Including the allowed $2 \to 2$ scatterings

$$\Pi(k) = -\frac{c_D^2}{8} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}p_0}{2\pi} \, \mathbb{L}^{ij;kl} p_i p_k [\gamma_j 1 S^>_p \gamma_l 1 S^<_{p-k} + \gamma_j 1 S^<_p \gamma_l 1 S^>_{p-k}]$$

The GW production rate in the real-time formalism



# of internal DOFs

Derivative couplings

$$\Pi(k) = -\frac{c_D^2}{8}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int \frac{\mathrm{d}p_0}{2\pi}\, \mathbb{L}^{ij;kl} p_i p_k [\gamma_j \mathbb{1} S_p^{>} \gamma_l \mathbb{1} S_{p-k}^{<} + \gamma_j \mathbb{1} S_p^{<} \gamma_l \mathbb{1} S_{p-k}^{>}]$$

Traceless transverse projector

Finite width propagator

Finite width
$\Gamma_p \approx 2 E_p \Upsilon$

$$\mathbb{1} S^{>} \propto \frac{\Gamma_p}{\Omega_p^2 + \Gamma_p^2}$$

Mass pole
$p^2 - m^2$

The GW production rate

- Putting everything together, we find:

$$\Pi(k) \simeq c_X \int \frac{d^3p}{(2\pi)^3} p_\perp^4 \left[n(\epsilon)(1-n(\epsilon)) + \bar{n}(\epsilon)(1-\bar{n}(\epsilon))\right] \frac{2\Upsilon_p}{k^2(\epsilon - p_{||})^2 + 4\epsilon^2\Upsilon_p^2}$$

(anti)-particle distribution functions

GW wave number

In the relativistic limit:

$$\Pi(k) \overset{m \ll T}{\simeq} g_X \frac{16\pi^2}{225} T^5 \frac{5\Upsilon_{\rm av}}{k^2 + 10\Upsilon_{\rm av}^2}$$

mean width $\Upsilon_{av} \approx <\sigma v> N$

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Boltzmann regime $k \gg \Upsilon_{av}$

$$\Pi \propto \frac{T^5 \Upsilon_{av}}{k^2}$$

Hydrodynamic regime $k \ll \Upsilon_{av}$

$$\Pi \propto \frac{T^5}{\Upsilon_{av}}$$

Horizon suppression

- The GW production rate is sensitive to the Horizon scale:
  - Super-horizon (k < H) modes are static
  - Sub-horizon modes (k > H) are produced efficiently
- Effectively a given frequency mode $k = 2\pi a_0/a\, f$ is only produced when

$$k < H \sim T^2/m_{Pl}$$

Importance of this effect for GW production pointed out in [Tokareva]

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{k^2}{a^2}h_{ij} = \frac{16\pi\, T_{ij}}{a^2 m_{\rm Pl}^2}$$

$$T < T_{\rm entry}(f) = 4 \cdot 10^7 \mathrm{GeV}\, \frac{f}{Hz}$$

$$h^2\Omega_{\rm gw}(f) \approx 2.02 \cdot 10^{-38} \times \left(\frac{f}{Hz}\right)^3 \times \int_{T_{\rm min}}^{T_{\rm max}} \frac{\mathrm{d}T'}{m_{\rm Pl}} \frac{\Pi\,(2\pi f a_0 a')}{8T'^4}$$

Modifies the scaling of the IR modes!

Horizon effect on the SM GW background

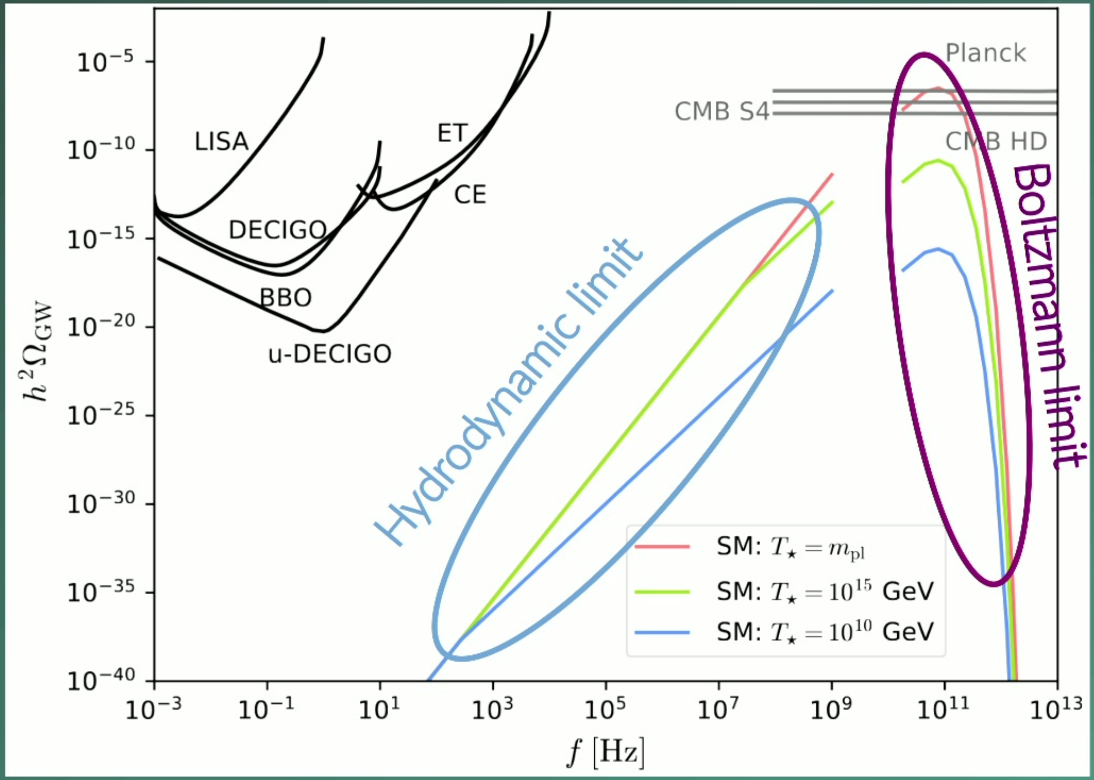
- The scaling in the hydrodynamic limit is usually

$$h^2\Omega_{\rm gw}(f) \propto \left(\frac{f}{Hz}\right)^3 \frac{T_\star}{\Upsilon_{SM}}$$

- But changes slope for

$$T_\star < T_{\rm entry}(f) = 4 \cdot 10^7 \mathrm{GeV} \frac{f}{Hz}$$

$$h^2\Omega_{\rm gw}(f) \propto \left(\frac{f}{Hz}\right)^4$$



Model-independent upper bound

- The optimal production rate is realized when $\Upsilon_{av} \to \frac{k}{\sqrt{10}}$

$$\Pi(k) \overset{m \ll T}{\simeq} g_X \frac{16\pi^2}{225} T^5 \frac{5\Upsilon_{\rm av}}{k^2 + 10\Upsilon_{\rm av}^2} \leq g_X \frac{4\sqrt{2/5}\pi^2}{45} \frac{T^5}{k}$$

- This can be used to compute a generic upper bound

$$h^2\Omega_{\rm gw}(f) < 4.9 \cdot 10^{-40} \times g_X \left(\frac{f}{\rm Hz}\right)^2 \min\left(\frac{f}{\rm Hz}, \frac{f_\star}{\rm Hz}\right)$$
$$\leq 4.9 \cdot 10^{-40} \times g_X \left(\frac{f}{\rm Hz}\right)^3$$

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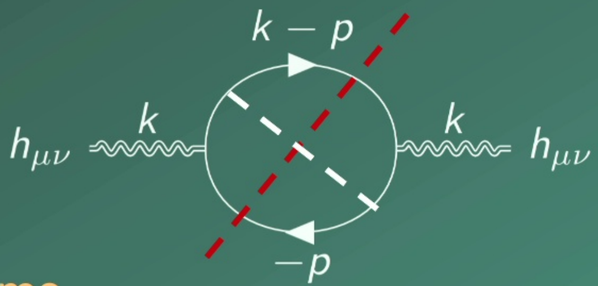
Caveats!

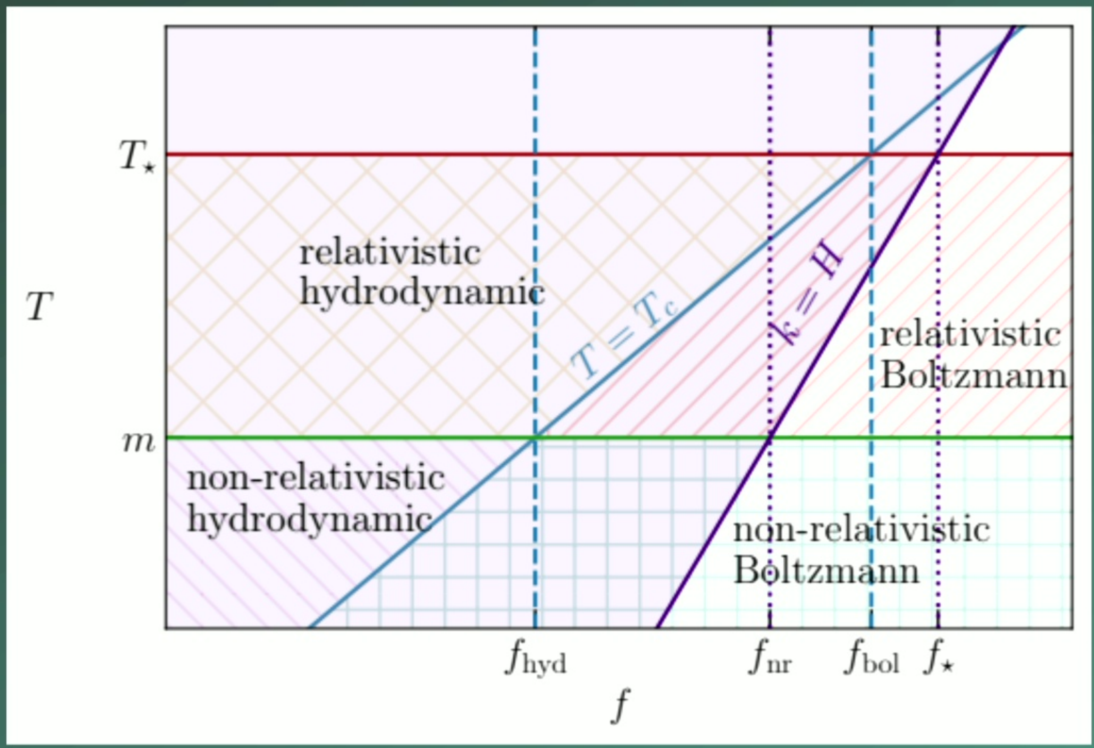


- We assume a **standard cosmic history**
  - SM radiation domination
  - Relatively straightforward to generalize
- We neglect the contribution from **vertex-type diagrams**
- We assume thermal equilibrium
  - **Quasi-particle ansatz** possible
  - Distribution functions are the equilibrium ones
    - Generic formula valid also for arbitrary distributions

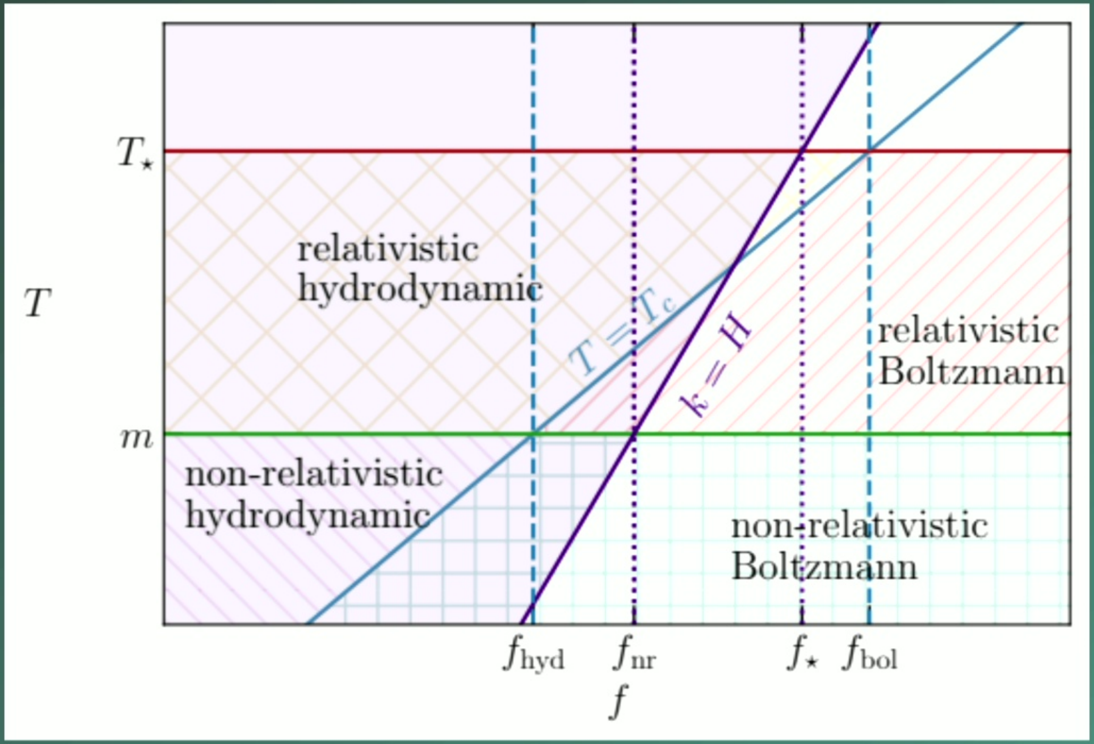
Different GW production regimes

- For renormalizable interactions we assume that the width scales with the temperature:
$$\Upsilon_{av} \propto T \to \frac{\Upsilon_{av}}{k} = \text{const.}$$
Hence a given frequency mode is always either in the hydrodynamic or Boltzmann regime
- In the case of non-renormalizable interactions we expect the rate to monotonically decrease with the temperature in the relativistic regime
$$\Upsilon_{av} \propto T \left(\frac{T}{\Lambda}\right)^{2(d-4)} \to \frac{\Upsilon_{av}}{k} \neq \text{const.}$$

Different GW production regimes

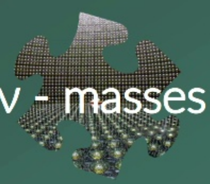


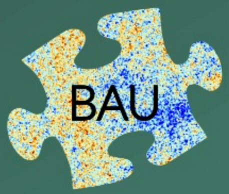
Different GW production regimes

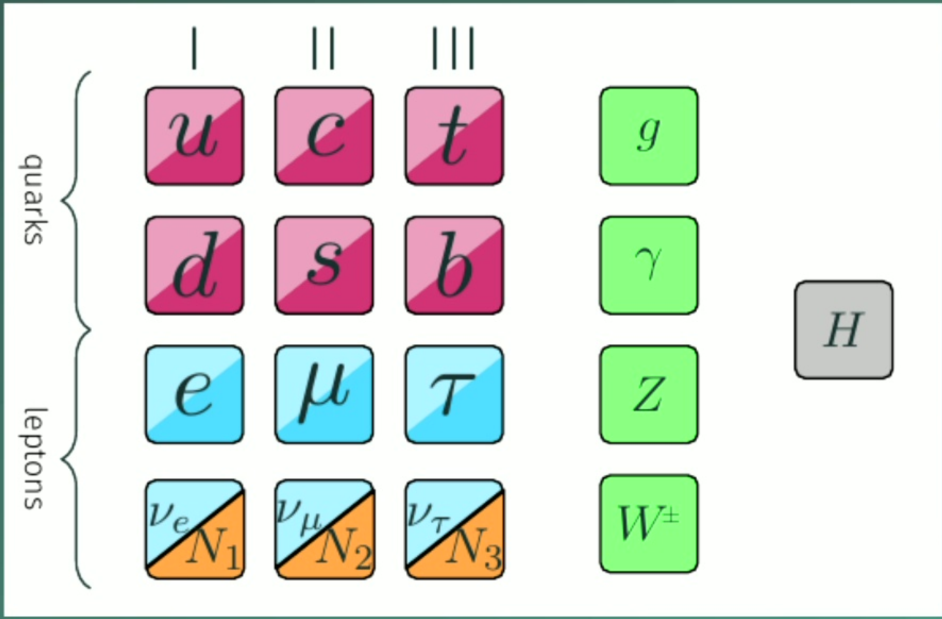


Example 1: Heavy Neutral Leptons

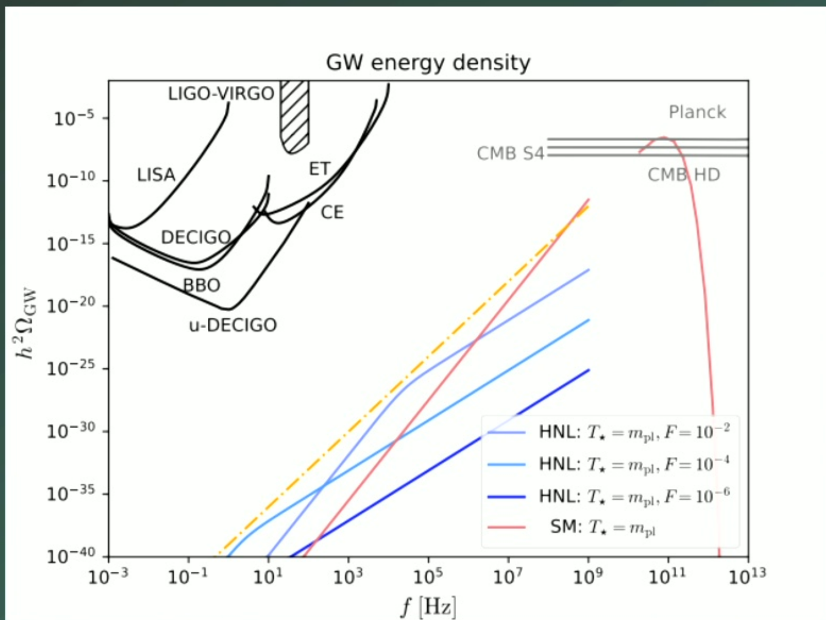
- Minimal extension of the SM
- One of the four *feebly interacting particle portal scenarios*
- Can simultaneously solve several problems of BSM physics
- If light, expected to have feeble interactions to the SM

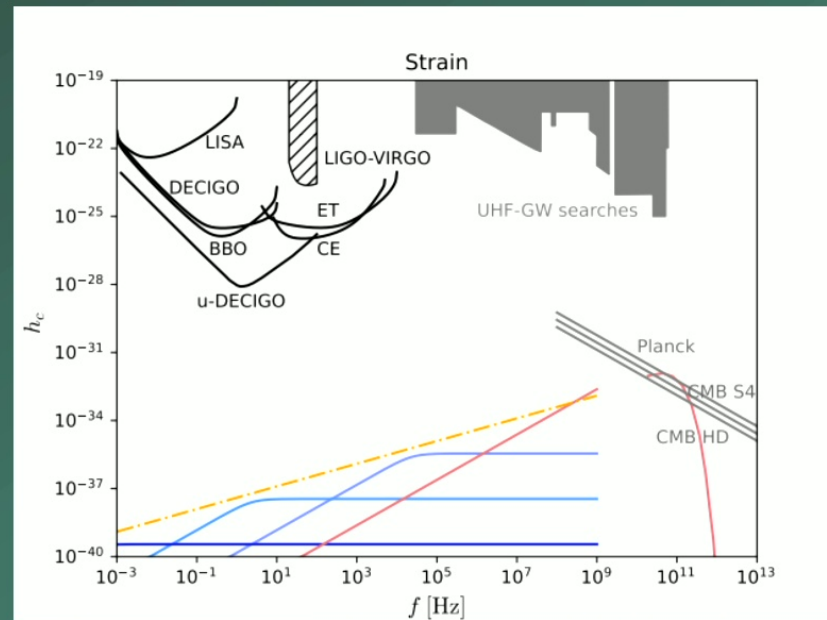






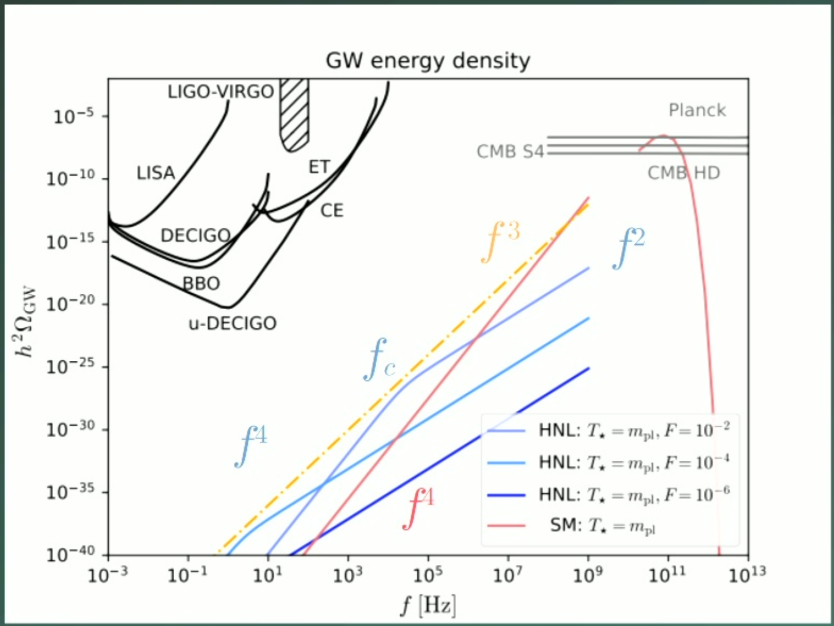
Heavy Neutral Leptons

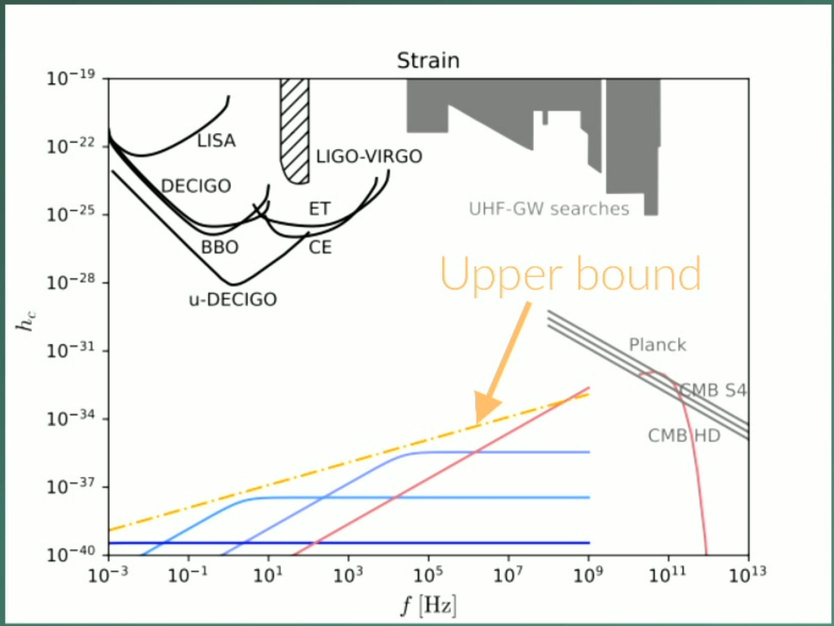




- HNLs have a renormalizable Yukawa coupling F to the SM $\Upsilon_{av} \approx 0.2T\frac{F^2}{8\pi}$
- They saturate the bound for a specific frequency f_c

Heavy Neutral Leptons





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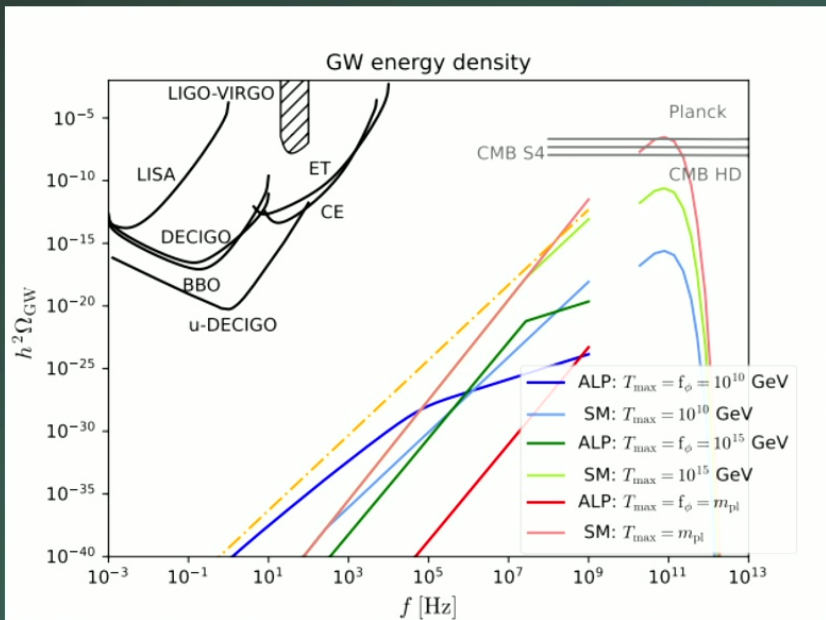
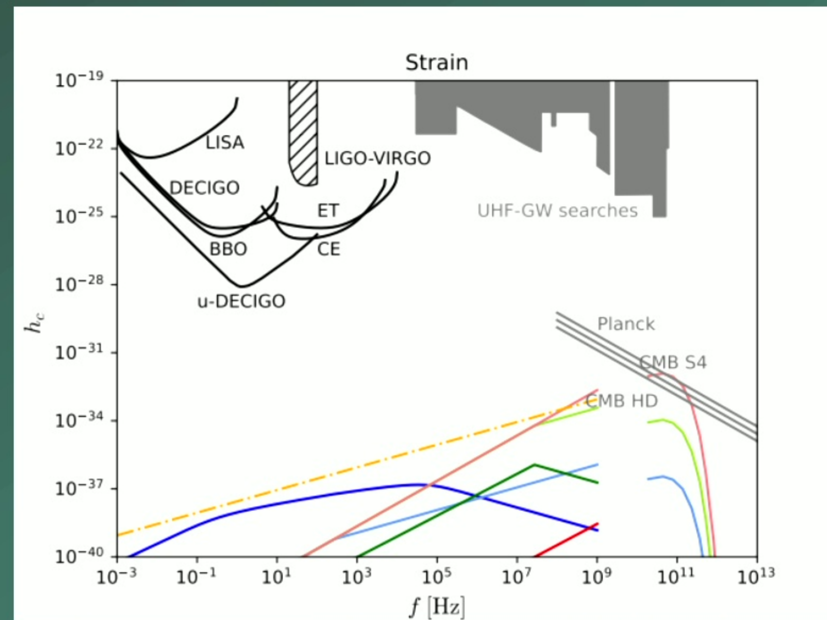
Example 2: ALPs

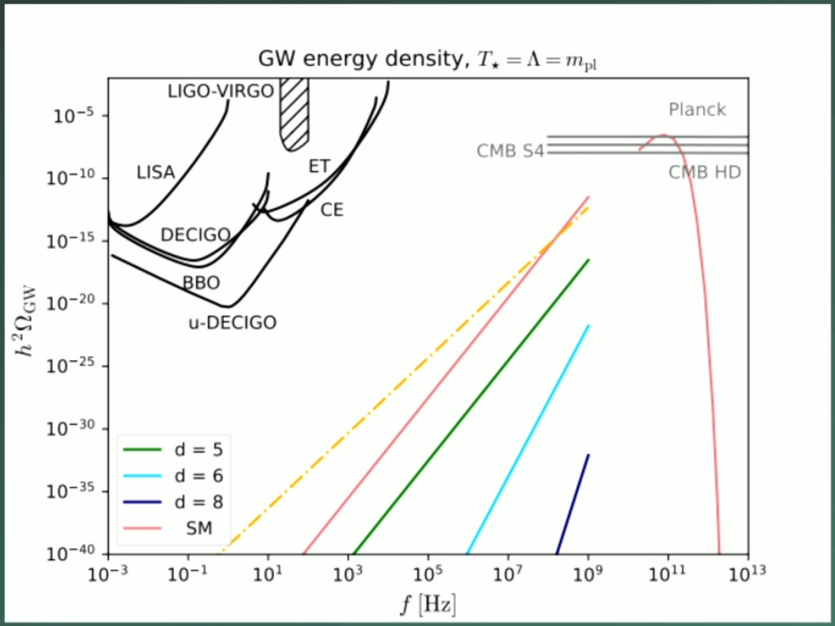


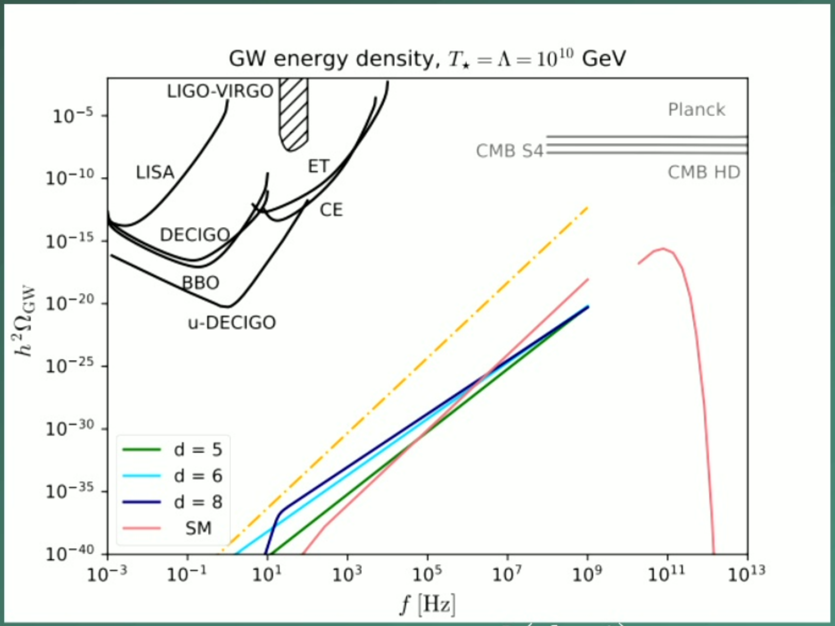


- Non-renormalizable interactions:

$$\Upsilon_{\rm av} \overset{m \ll T}{=} \kappa\, n_c^3 (n_c^2 - 1) \frac{\alpha^5 T^3}{f_\phi^2}\ , \kappa \approx 1.5\ , \frac{1}{\alpha} \approx \frac{22 n_c}{12\pi} \ln\left(\frac{2\pi T}{\Lambda_{\rm IR}}\right)$$

Example 3: Other non-renormalizable interactions





- For a generic NR interaction we expect: $\Upsilon_{av} = T\left(\frac{T}{\Lambda}\right)^{2(d-4)}$
- For d=8 there are unavoidable interactions coming from graviton exchanges

Outlook and conclusions

- Thermal primordial gravitational waves can lead to high-frequency gravitational waves with **no known astrophysical backgrounds**
- Thermal fluctuations lead to an **unavoidable SM background**
- Feebly coupled particles can dominate of the SM background by orders of magnitude
- Detection still very challenging!
- **Hydrodynamic & horizon supersessions** are extremely important for accurate predictions!
- Same formalism could be applied to other scenarios **out thermal of equilibrium** (e.g. reheating)